L19

4800

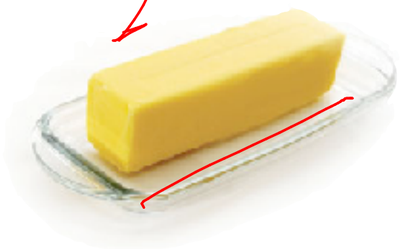
11.14.2016

abhi shelat

Guns and butter

x

y



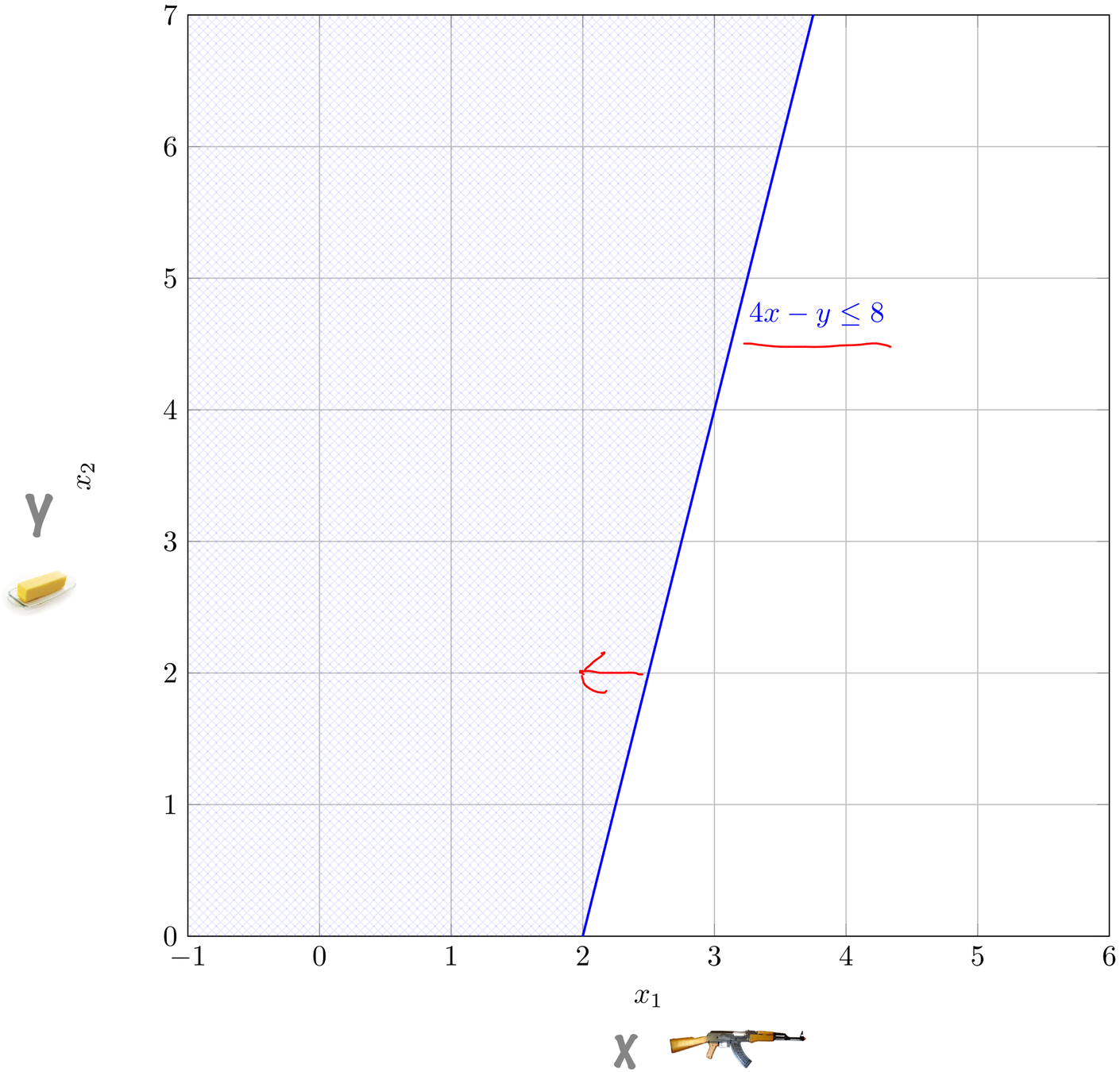
$$\max x + y$$

$$\begin{aligned} 4x - y &\leq 8 \\ 2x + y &\leq 10 \\ 5x - 2y &\geq -2 \\ x, y &\geq 0 \end{aligned}$$

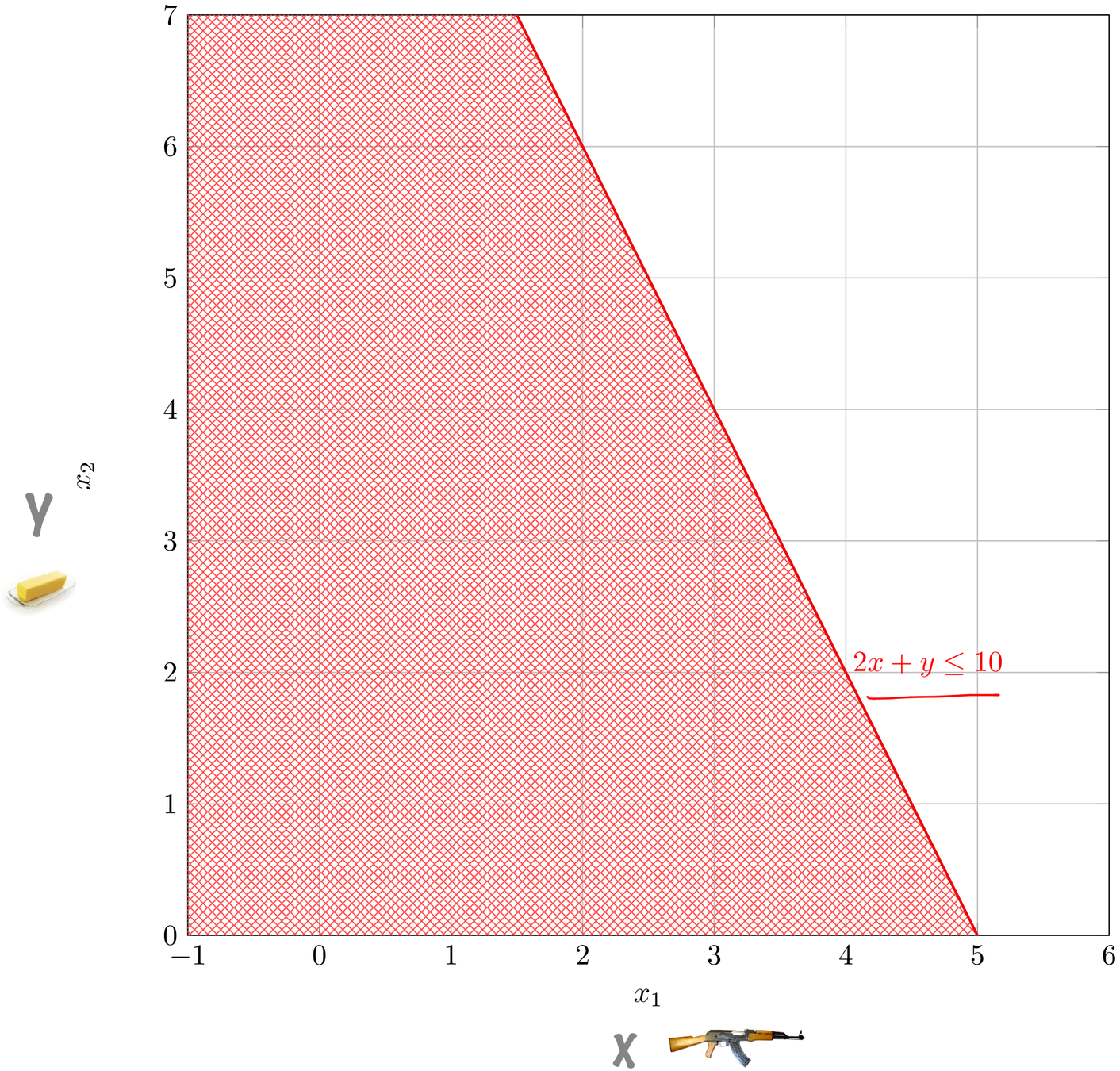
stamina

http://i16.photobucket.com/albums/b20/safebuy/ak47/ak47-electric_lg.jpg

http://2.bp.blogspot.com/_NX4zcMNX4VE/Sb8MQffflI/AAAAAAAAAL0/eu4J0dfFhJE/s400/gourmet-butter.jpg



$$
\begin{aligned}
4x - y &\leq 8 \\
2x + y &\leq 10 \\
5x - 2y &\geq -2 \\
x, y &\geq 0
\end{aligned}
$$



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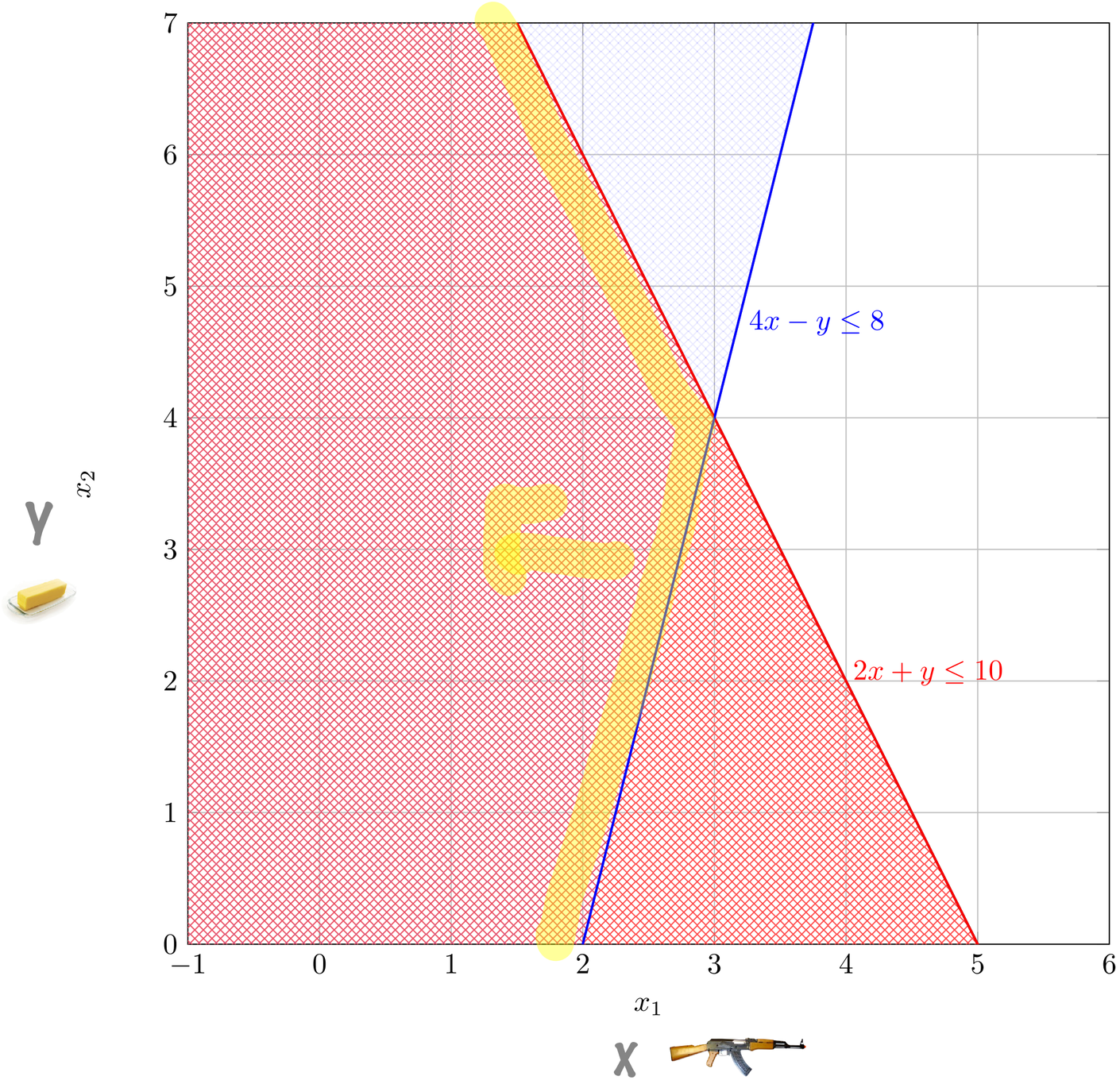
$$



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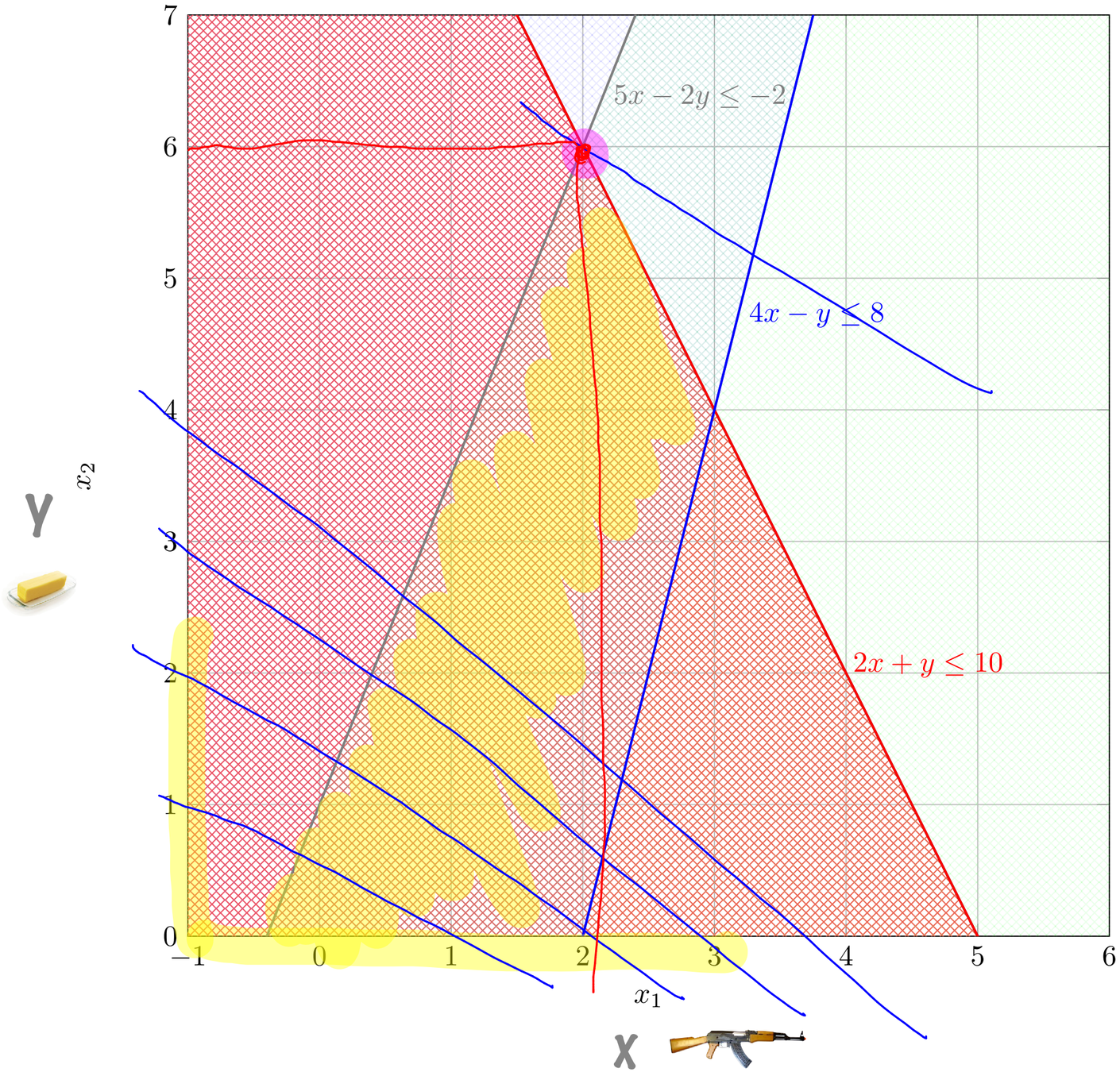
$$



$$
\begin{aligned}
4x - y &\leq 8 \\
2x + y &\leq 10 \\
5x - 2y &\geq -2 \\
x, y &\geq 0
\end{aligned}
$$

max $x+y$

$2+6 = 8$

Certificate of optimality

$$\max x + y$$

$$\begin{aligned} 4x - y &\leq 8 \\ 2x + y &\leq 10 \\ 5x - 2y &\geq -2 \\ x, y &\geq 0 \end{aligned}$$

$$7 \Rightarrow -14x + 7y \leq 70$$

$$-1 \Rightarrow -5x + 2y \leq 2$$

$$9x + 9y \leq 72$$

$$\Rightarrow x + y \leq 8$$

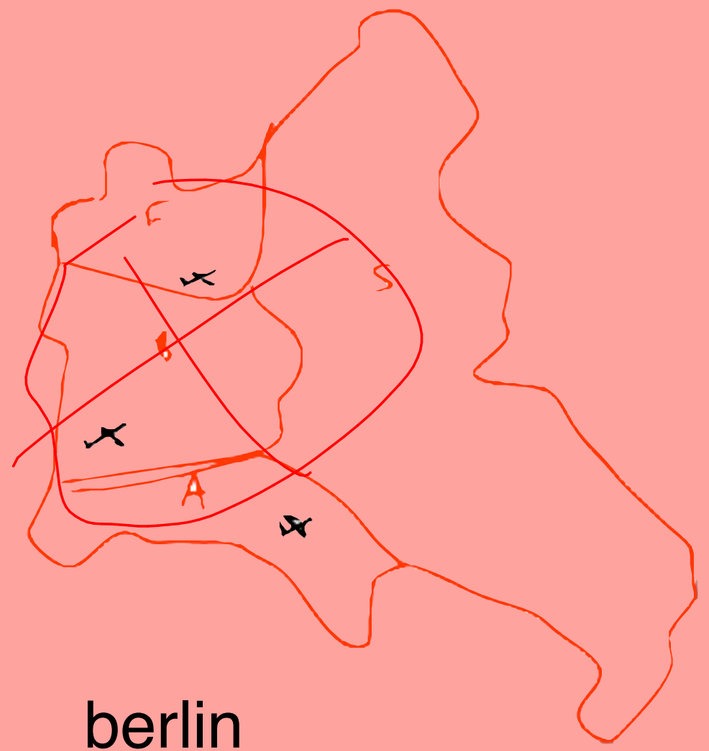
Certificate of optimality

$$\max x + y$$

$$\begin{aligned} 4x - y &\leq 8 \\ 2x + y &\leq 10 \\ 5x - 2y &\geq -2 \\ x, y &\geq 0 \end{aligned}$$

7	$14x + 7y \leq 70$
-1	$-5x + 2y \leq 2$
	$9x + 9y \leq 72$

44 plans



berlin

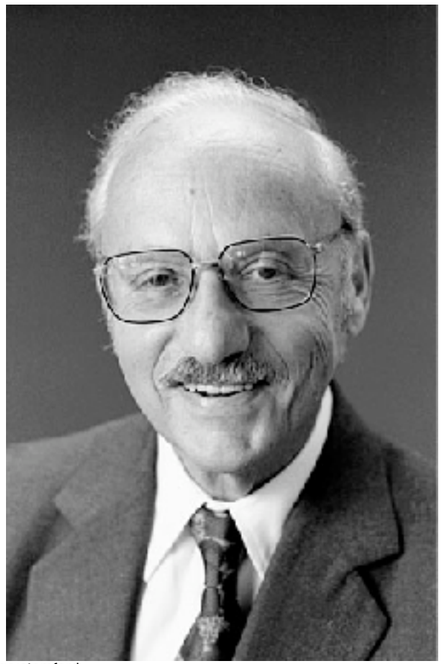


image:stamford

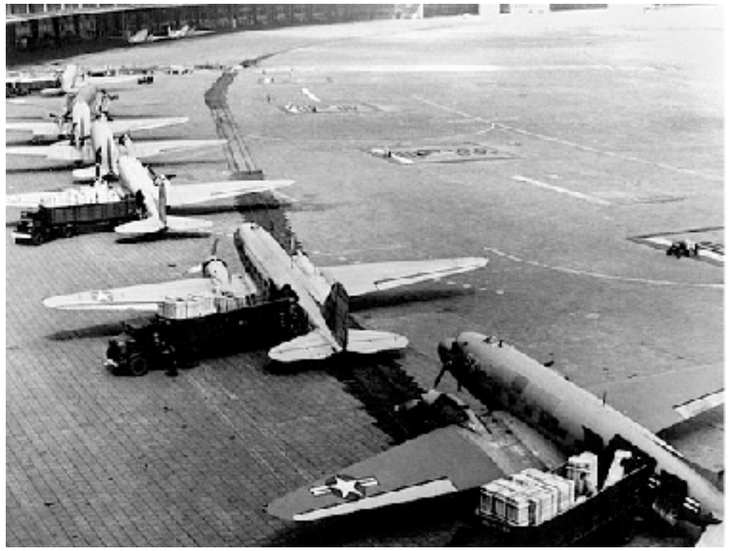


image:history of air cargo

linear programming saved Berlin

Stigler diet

calories	3000
protein	70g
calcium	.8g
iron	19mg
vitamin A	5000iu
thiamine	1.8mg
riboflavin	2.7mg
niacin	18mg
ascorbic acid	75mg

TABLE A. NUTRITIVE VALUES OF COMMON FOODS PER DOLLAR OF EXPENDITURE, AUGUST 15, 1939

Commodity	Unit	Price Aug. 15, 1939 (cents)	Edible Weight per $1.00 (grams)	Calories (1,000)	Protein (grams)	Calcium (grams)	Iron (mg.)	Vitamin A (1,000 I.U.)	Thiamine (mg.)	Riboflavin (mg.)	Niacin (mg.)	Ascorbic Acid (mg.)
**1. Wheat Flour (Enriched)	10 lb.	36.0	12,600	44.7	1,411	2.0	365		55.4	33.3	441	
2. Macaroni	1 lb.	14.1	3,217	11.6	418	.7	54		3.2	1.9	68	
3. Wheat Cereal (Enriched)	28 oz.	24.2	5,280	11.8	377	14.4	175		14.4	8.8	114	
4. Corn Flakes	8 oz.	7.1	3,194	11.4	232	.1	56		13.5	2.3	68	
5. Corn Meal	1 lb.	4.6	9,861	36.0	897	1.7	99	30.0	17.4	7.9	106	
6. Hominy Grits	24 oz.	8.5	8,065	28.6	680	.8	80		10.6	1.6	110	
7. Rice	1 lb.	7.5	6,048	21.2	400	.6	41		2.0	4.8	60	
8. Rolled Oats	1 lb.	7.1	6,389	25.3	907	5.1	541		37.1	8.9	64	
9. White Bread (Enriched)	1 lb.	7.9	5,742	15.6	488	2.5	115		13.8	8.5	126	
10. Whole Wheat Bread	1 lb.	9.1	4,985	12.2	484	2.7	125		13.9	6.4	160	
11. Rye Bread	1 lb.	9.2	4,930	12.4	439	1.1	82		9.9	3.0	66	
12. Pound Cake	1 lb.	24.8	1,829	8.0	130	.4	31	18.9	2.8	3.0	17	
13. Soda Crackers	1 lb.	15.1	3,004	12.5	288	.5	50					
14. Milk	1 qt.	11.0	8,867	6.1	310	10.5	18	16.8	4.0	16.0	7	177
**15. Evaporated Milk (can)	14½ oz.	6.7	6,085	8.4	422	15.1	9	26.0	3.0	23.5	11	60
16. Butter	1 lb.	30.8	1,473	10.8	9	.2	3	44.2		.2	2	
*17. Oleomargarine	1 lb.	16.1	2,817	20.6	17	.6	6	55.6	.2			
18. Eggs	1 doz.	32.6	1,857	2.9	238	1.0	52	18.6	2.8	6.5	1	
**19. Cheese (Cheddar)	1 lb.	24.2	1,874	7.4	448	16.4	19	28.1	.8	10.3	4	
20. Cream	½ pt.	14.1	1,689	3.5	49	1.7	3	16.9	.6	2.5		17
21. Peanut Butter	1 lb.	17.9	2,534	15.7	661	1.0	48		9.6	3.1	471	
22. Mayonnaise	½ pt.	16.7	1,198	8.6	18	.2	9	2.7	.4	.5		
23. Crisco	1 lb.	20.3	2,234	20.1								
24. Lard	1 lb.	9.8	4,628	41.7				.2		.5	5	
25. Sirloin Steak	1 lb.	39.6	1,145*	2.9	166	.1	34	.2	2.1	2.9	69	
26. Round Steak	1 lb.	36.4	1,246*	2.2	214	.1	32	.4	2.5	2.4	87	
27. Rib Roast	1 lb.	29.2	1,553*	3.4	213	.1	33			2.0		
28. Chuck Roast	1 lb.	22.6	2,007*	3.6	309	.2	46	.4	1.0	4.0	120	
29. Plate	1 lb.	14.6	3,107*	8.5	404	.2	62		.9			
**30. Liver (Beef)	1 lb.	26.8	1,692*	2.2	333	.2	139	169.2	6.4	50.8	316	525
31. Leg of Lamb	1 lb.	27.6	1,643*	3.1	245	.1	20		2.8	3.9	36	
32. Lamb Chops (Rib)	1 lb.	36.6	1,239*	3.3	140	.1	15		1.7	2.7	34	
33. Pork Chops	1 lb.	30.7	1,477*	3.5	196	.2	30		17.4	2.7	60	
34. Pork Loin Roast	1 lb.	24.2	1,874*	4.4	240	.3	37		18.2	3.6	79	
35. Bacon	1 lb.	25.6	1,772*	10.4	152	.2	23		1.8	1.8	71	
36. Ham—smoked	1 lb.	27.4	1,655*	6.7	212	.2	31		9.9	3.3	50	
37. Salt Pork	1 lb.	16.0	2,835*	18.8	104	.1	26		1.4	1.8		
38. Roasting Chicken	1 lb.	30.3	1,497*	1.8	184	.1	30	.1	.9	1.3	68	46
39. Veal Cutlets	1 lb.	42.3	1,072*	1.7	156	.1	24		1.4	2.4	57	
40. Salmon, Pink (can)	16 oz.	15.0	3,489	5.8	705	6.6	45	3.5	1.0	4.9	209	
41. Apples	1 lb.	4.4	9,072	5.8	27	.5	36	7.3	3.6	2.7	5	544
42. Bananas	1 lb.	6.1	4,982	4.9	60	.4	30	17.4	2.5	3.5	28	498
43. Lemons	1 doz.	26.0	2,380	1.0	21	.6	14		.5		4	952
44. Oranges	1 doz.	30.9	4,439	2.2	40	1.1	18	11.1	3.6	1.3	10	1,998
*45. Green Beans	1 lb.	7.1	5,750	2.4	138	3.7	80	69.0	4.3	5.8	37	862
**46. Cabbage	1 lb.	3.7	8,949	2.6	125	4.0	36	7.2	9.0	4.5	26	5,369
47. Carrots	1 bunch	4.7	6,080	2.7	73	2.8	43	188.5	6.1	4.3	89	608
48. Celery	1 stalk	7.3	3,915	.9	51	3.0	23	.9	1.4	1.4	9	313
49. Lettuce	1 head	8.2	2,247	.4	27	1.1	22	112.4	1.8	3.4	11	449
*50. Onions	1 lb.	5.6	11,844	5.8	166	3.8	59	16.6	4.7	5.9	21	1,184

*51. Potatoes	15 lb.	34.0	16,810	14.3	336	1.8	118	6.7	29.4	7.1	198	2,522
**52. Spinach	1 lb.	8.1	4,592	1.1	106	—	138	918.4	5.7	13.8	33	2,755
**53. Sweet Potatoes	1 lb.	5.1	7,649	9.6	138	2.7	54	290.7	8.4	5.4	83	1,912
54. Peaches (can)	No. 2½	16.8	4,894	3.7	20	.4	10	21.5	.5	1.0	31	196
55. Pears (can)	No. 2½	20.4	4,050	3.0	8	.3	8	.8	.8	.8	5	81
56. Pineapple (can)	No. 2½	21.3	3,993	2.4	16	.4	8	2.0	2.8	.8	7	399
57. Asparagus (can)	No. 2	27.7	1,045	.4	33	.3	12	16.3	1.4	2.1	17	272
58. Green Beans (can)	No. 2	10.0	5,386	1.0	54	2.0	65	53.9	1.6	4.3	32	481
59. Pork and Beans (can)	16 oz.	7.1	6,389	7.5	364	4.0	134	3.5	8.3	7.7	56	
60. Corn (can)	No. 2	10.4	5,452	5.2	136	.2	16	12.0	1.6	2.7	42	218
61. Peas (can)	No. 2	13.8	4,109	2.3	136	.6	45	34.9	4.9	2.5	37	370
62. Tomatoes (can)	No. 2	8.6	6,263	1.3	63	.7	38	53.2	3.4	2.5	36	1,253
63. Tomato Soup (can)	10½ oz.	7.6	3,017	1.6	71	.6	43	57.9	3.5	2.4	67	802
*64. Peaches, Dried	1 lb.	15.7	2,889	8.5	87	1.7	173	86.9	1.2	4.3	55	57
*65. Prunes, Dried	1 lb.	9.0	4,284	12.8	99	2.5	154	85.7	3.9	4.3	65	257
66. Raisins, Dried	15 oz.	9.4	4,524	13.5	104	2.5	136	4.5	6.3	1.4	24	136
67. Peas, Dried	1 lb.	7.9	5,742	20.0	1,367	4.2	345	2.0	28.7	18.4	162	
**68. Lima Beans, Dried	1 lb.	8.9	5,007	17.4	1,055	3.7	459	5.1	26.9	38.2	93	
**69. Navy Beans, Dried	1 lb.	5.9	7,888	26.9	1,691	11.4	792		38.4	24.6	217	
70. Coffee	1 lb.	22.4	2,025	—	—	—	—		4.0	5.1	50	
71. Tea	¼ lb.	17.4	652	—	—	—	—			2.3	42	
72. Cocoa	8 oz.	8.6	2,657	8.7	237	3.0	72		2.0	11.9	40	
73. Chocolate	8 oz.	16.2	1,409	8.0	77	1.3	39		.9	3.4	14	
74. Sugar	10 lb.	51.7	8,773	34.9	—	—	—					
75. Corn Sirup	24 oz.	13.7	4,968	14.7	—	.5	74				5	
76. Molasses	18 oz.	13.6	3,752	9.0	—	10.3	244		1.9	7.5	146	
77. Strawberry Preserves	1 lb.	20.5	2,213	6.4	11	.4	7	.2	.2	.4	3	

* Quantities including inedible portions.

TABLE B. NUTRITIVE VALUES OF COMMON FOODS PER DOLLAR OF EXPENDITURE, AUGUST 15, 1944

Commodity	Price Aug. 15, 1944 (cents)	Calories (1,000)	Protein (grams)	Calcium (grams)	Iron (mg.)	Vitamin A (1,000 I.U.)	Thiamine (mg.)	Riboflavin (mg.)	Niacin (mg.)	Ascorbic Acid (mg.)
1. Wheat Flour	64.6	24.9	786	1.1	203		30.9	18.6	246	
3. Wheat Cereal	23.2	12.3	398	15.0	183		15.0	9.2	119	
5. Corn Meal	6.3	26.3	655	1.2	72	22.6	12.7	5.8	77	
8. Rolled Oats	9.9	18.1	651	3.7	245		26.6	6.4	46	
15. Evaporated Milk	10.0	5.6	283	10.1	6	17.4	2.0	15.7	7	40
46. Cabbage	4.9	2.0	94	3.0	27	5.4	6.8	3.4	20	4,054
51. Potatoes	80.1	6.1	143	.8	50	2.8	12.5	3.0	84	1,071
52. Spinach	11.6	.8	74	—	96	641.3	4.0	9.6	23	1,924
53. Sweet Potatoes	12.3	4.0	57	1.1	22	120.5	3.5	2.2	34	793
69. Navy Beans	10.8	14.7	924	6.2	433		21.0	13.4	119	
74. Sugar	67.0	26.9	—	—	—					
78. Pancake Flour[1]	12.2	16.0	479	18.1	46		3.7	1.9	41	
79. Beets[2]	7.3	2.2	85	1.1	70	132.3	2.9	6.3	29	895
80. Liver (Pork)[3]	21.9	2.7	408	.2	518	145.0	10.4	51.8	472	580

[1] Unit: 20 oz.; edible weight: 4,647 g. [2] Unit: 1 bunch; edible weight: 4,971 g. [3] Unit: 1 lb.; edible weight: 2,071 g.

	x_1 Brownie	x_2 Truck	x_3 Pavement	x_4 Amelia
cost	5	2	3	8
cals	400	200	150	500
choc	3	2	0	0
sugar	2	2	4	4
fat	2	4	0	5

requirements: * 500 calories, 6 oz choc, 10 oz sugar, 8 oz fat

LUNCH

$$\min \quad 5x_1 + 2x_2 + 3x_3 + 8x_4$$

$$\begin{pmatrix} 400x_1 + 200x_2 + 150x_3 + 500x_4 \geq 500 \\ 3x_1 + 2x_2 + 0x_3 + 0x_4 \geq 6 \\ 2x_1 + 2x_2 + 4x_3 + 4x_4 \geq 10 \end{pmatrix}$$

$$Ax \geq b$$

	Brownie	Truck	Pavement	Amelia
cost	5	2	3	8
cals	400	200	150	500
choc	3	2	0	0
sugar	2	2	4	4
fat	2	4	0	5

requirements: 500 calories, 6 oz choc, 10 oz sugar, 8 oz fat

	Brownie	Truck	Pavement	Amelia
cost	5	2	3	8
cals	400	200	150	500
choc	3	2	0	0
sugar	2	2	4	4
fat	2	4	0	5

requirements: 500 calories, 6 oz choc, 10 oz sugar, 8 oz fat

$$\min 5x_1 + 2x_2 + 3x_3 + 8x_4$$

$$A \begin{bmatrix} 400 & 200 & 150 & 500 \\ 3 & 2 & 0 & 0 \\ 2 & 2 & 4 & 4 \\ 2 & 4 & 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \geq \begin{bmatrix} 500 \\ 6 \\ 10 \\ 8 \end{bmatrix}$$

LP

Linear Program

$$\min 5x_1 + 2x_2 + 3x_3 + 8x_4$$

$$\begin{bmatrix} 400 & 200 & 150 & 500 \\ 3 & 2 & 0 & 0 \\ 2 & 2 & 4 & 4 \\ 2 & 4 & 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \geq \begin{bmatrix} 500 \\ 6 \\ 10 \\ 8 \end{bmatrix}$$

$$x_1, x_2, x_3, x_4 \geq 0$$

$$\min 5x_1 + 2x_2 + 3x_3 + 8x_4$$

$$\begin{bmatrix} 400 & 200 & 150 & 500 \\ 3 & 2 & 0 & 0 \\ 2 & 2 & 4 & 4 \\ 2 & 4 & 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \geq \begin{bmatrix} 500 \\ 6 \\ 10 \\ 8 \end{bmatrix}$$

```
H-representation
begin
8 4 rational
-500 400 200 150 500
-6   3   2   0    0
-10  2   2   4    4
-8   2   4   0    5
0    1   0   0    0
0    0   1   0    0
0    0   0   1    0
0    0   0   0    1
end
minimize
0 5 2 3 8
```

$$\min 5x_1 + 2x_2 + 3x_3 + 8x_4$$

$$\begin{bmatrix} 400 & 200 & 150 & 500 \\ 3 & 2 & 0 & 0 \\ 2 & 2 & 4 & 4 \\ 2 & 4 & 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \geq \begin{bmatrix} 500 \\ 6 \\ 10 \\ 8 \end{bmatrix}$$

$$\min 5x_1 + 2x_2 + 3x_3 + 8x_4$$

$$\begin{bmatrix} 400 & 200 & 150 & 500 \\ 3 & 2 & 0 & 0 \\ 2 & 2 & 4 & 4 \\ 2 & 4 & 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \geq \begin{bmatrix} 500 \\ 6 \\ 10 \\ 8 \end{bmatrix}$$

```
H-representation
begin
8 4 rational
-500 400 200 150 500
-6   3   2   0    0
-10  2   2   4    4
-6   2   4   0    5
0    1   0   0    0
0    0   1   0    0
0    0   0   1    0
0    0   0   0    1
end
minimize
0 5 2 3 8
```

$$\min 5x_1 + \overset{3}{2}x_2 + \overset{1}{3}x_3 + 8x_4$$

$$\begin{array}{c} \begin{array}{cccc} & 3 & 1 & \end{array} \\ \begin{bmatrix} \cancel{400} & 200 & 150 & 500 \\ \cancel{3} & 2 & 0 & 0 \\ \cancel{2} & 2 & 4 & 4 \\ \cancel{2} & 4 & 0 & 5 \end{bmatrix} \end{array} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \geq \begin{bmatrix} 500 \\ 6 \\ 10 \\ 8 \end{bmatrix}$$

```
H-representation
begin
8 4 rational
-500 400 200 150 500
-6   3   2   0   0
-10  2   2   4   4
-6   2   4   0   5
0    1   0   0   0
0    0   1   0   0
0    0   0   1   0
0    0   0   0   1
end
minimize
0 5 2 3 8
```

```
*Objective function is
 0 + 5 X[1] + 2 X[2] + 3 X[3] + 8 X[4]
*LP status: a dual pair (x, y) of optimal solutions fo
begin
  primal_solution
  1 :  0
  2 :  3
  3 :  1
  4 :  0
  dual_solution
  2 :  -1/4
  5 :  -11/4
  3 :  -3/4
  8 :  -5
  optimal_value : 9
end
*number of pivot operations = 4
```

max flow as lp

input: (G, c, s, t) $G = (V, E)$ $c : E \to \mathbb{Z}_+$

find a flow f_e → flow on edge e

objective $\max \sum_{v \in V} f(s,v) - \sum_{u \in V} f(u,s)$

$f(u,s) \to$ variable for edge (u,s)

$\forall e \quad f_e \leq c_e$

$\forall v \in V - \{s,t\} \quad \sum_{u \in V} f(u,v) = \sum_{u \in V} f(v,u)$

$f_e \geq 0$

max flow as lp

$$\max \sum_{v} f(s,v) - \sum_{v} f(v,s)$$

$$f(u,v) \leq c(u,v)$$ for (u,v) in E

$$\sum_{u} f(u,v) = \sum_{w} f(v,w) \quad \forall v$$

$$f(u,v) \geq 0$$ for (u,v) in E

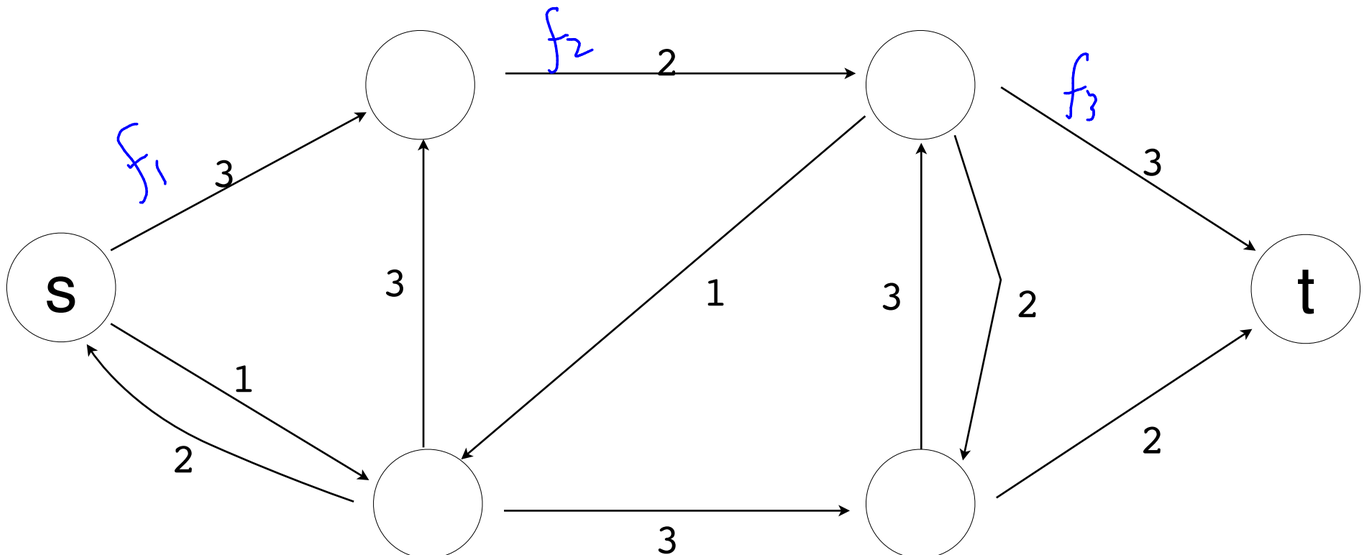
max flow as lp

$$\max \sum_v f(s,v) - \sum_v f(v,s)$$

$$f(u,v) \leq c(u,v)$$ for (u,v) in E

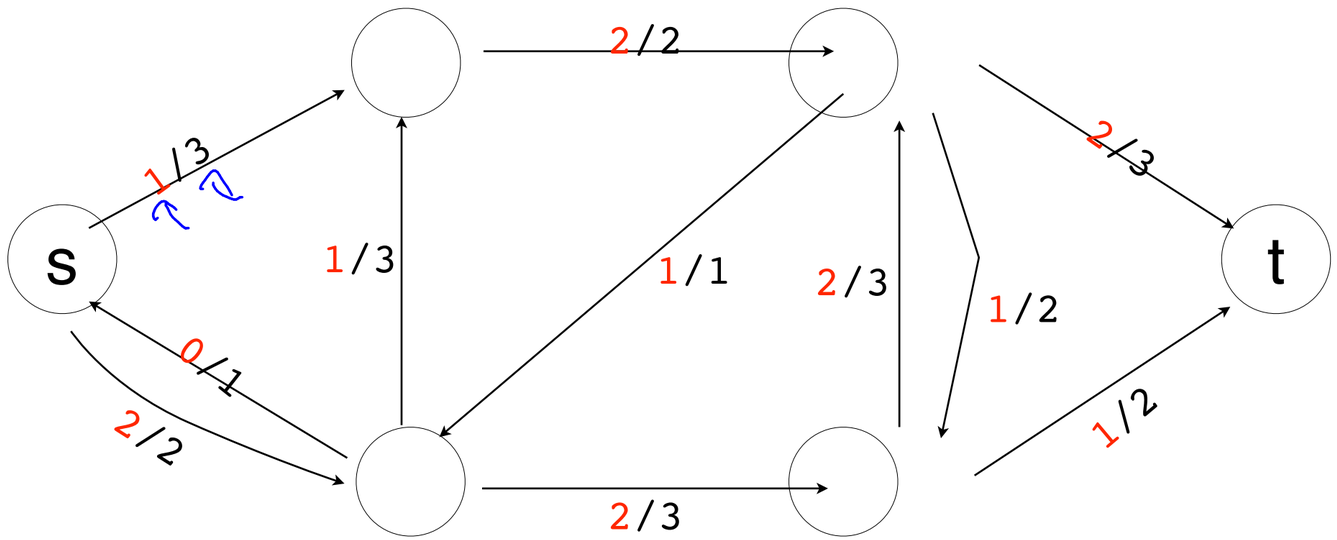
$$\sum_u f(u,v) = \sum_w f(v,w) \quad \forall v$$

$$f(u,v) \geq 0$$ for (u,v) in E



min-cost flow as lp

input: (G, c, s, t) $G = (V, E)$ $c : E \to \mathbb{Z}_+$ $\overset{\text{cost}}{x : E \to \mathbb{Z}_+}$ d



min-cost flow as lp

variables: f_{e_1} f_{e_2} ... f_{e_k} flow on each edge

objective → minimize $\sum f_e \cdot x_e$

x_e → cost of the edge

f_e → flow on that edge

$f_e \geq 0$

$f_e \leq c_e$

$\text{INFLOW}(u) = \text{OUTflow}(u)$ for all $u \in V - \{s, t\}$

$$\sum_{u \in V} f(s,u) - \sum_{u \in V} f(u,s) = 135$$

min-cost flow as lp

$$\min_{e} x_e \cdot f(e)$$

$$f(e) \leq c(e)$$

$$f(e) \geq 0$$

$$\sum_{u} f(u,v) = \sum_{w} f(v,w)$$

$$\sum_{v} f(s,v) - \sum_{v} f(v,s) = d$$ 135

		Seats	Price ($)
CHO	ORD	14	114
CHO	LGA	13	89
CHO	CTL	44	90
IAD	BRU	16	269
IAD	CDG	15	555
ORD	LHR	61	309
ORD	FRA	61	412
LGA	JFK	230	37
JFK	LHR	56	528
CTL	LHR	53	528
CTL	CDG	34	593
ATL	LHR	23	466
BRU	CDG	14	71
LHR	CDG	211	35
MLA	CDG	111	34

		Seats	Price ($)
CHO	IAD	74	64
CHO	ATL	24	72
IAD	ZRH	78	536
IAD	MLA	13	480
ORD	CDG	23	390
ORD	BRU	115	332
JFK	CDG	37	693
JFK	FRA	20	412
CTL	FRA	28	412
ATL	CDG	19	607
ATL	ZRH	18	499
ZRH	CDG	335	53
FRA	CDG	412	57

shortest paths as LP

inputs:

shortest paths as LP

single source (s)

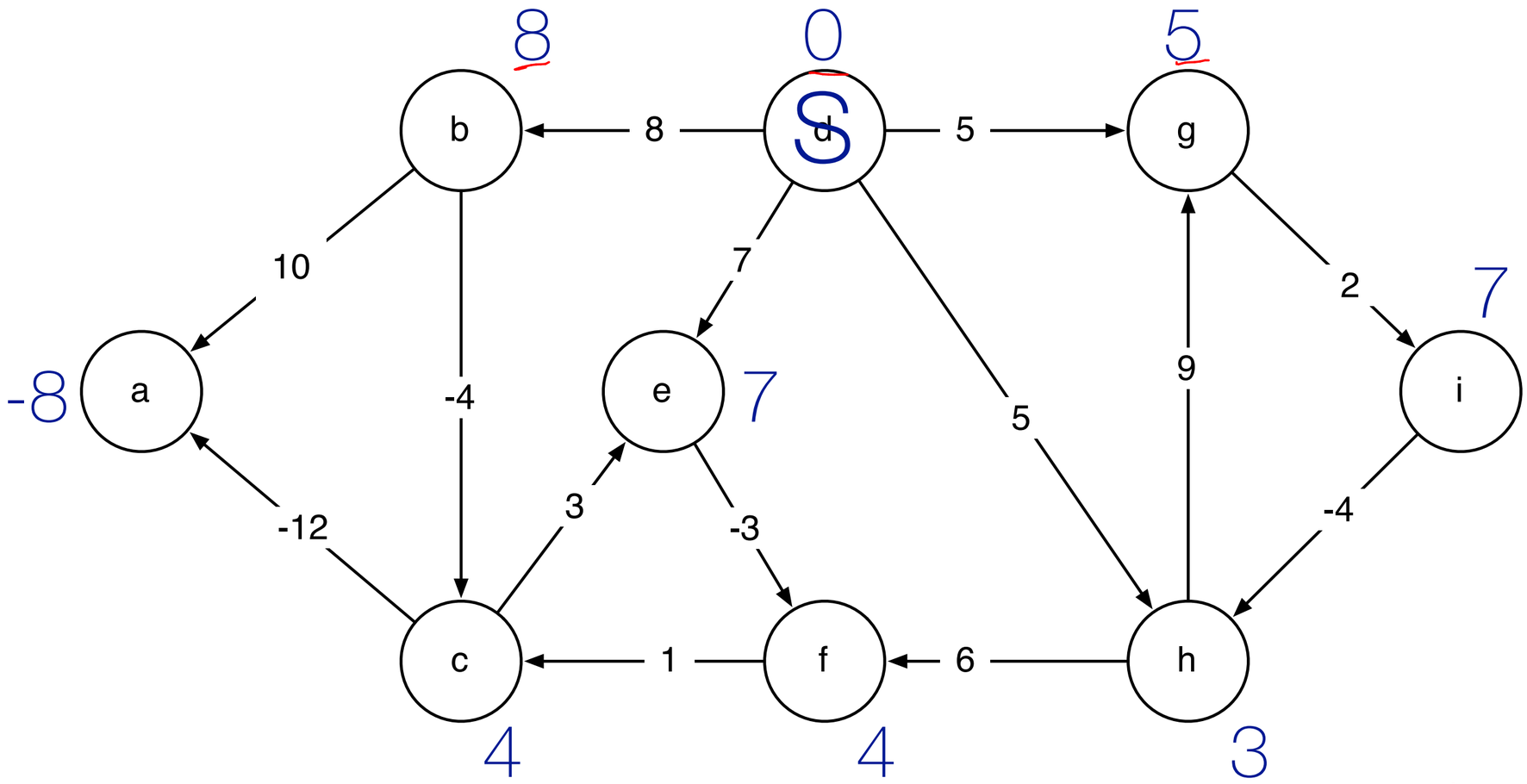
each node gets a label d_u

$$\max d_t \rightarrow \quad d_t = \sum_{u \in V} d_u$$

$$d_y - d_x \leq l(x,y) \qquad \forall e = (x,y) \in E$$

$$d_s = 0$$

8+5+7+3+7+4+4+4+(-4)



$d_y \leq d_x + l(x,y)$

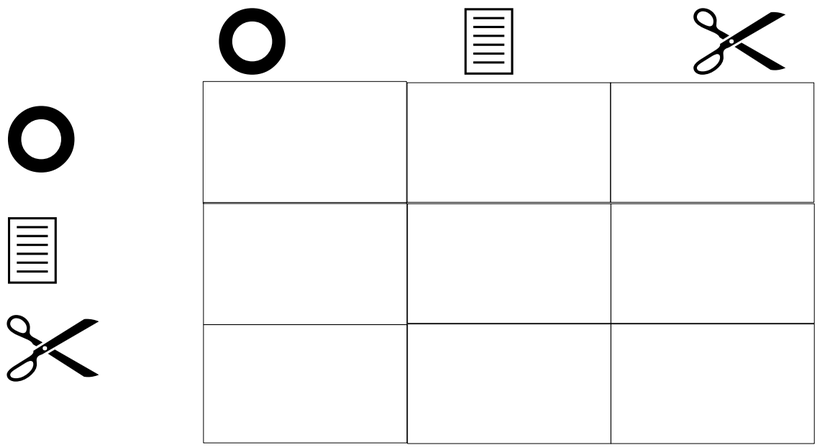
$$\max d_t$$

$$d_y - d_x \leq l(x,y) \qquad \forall e = (x,y) \in E$$

$$d_s = 0$$

dt = 30

zero-sum games



zero-sum games

COLIN

ROWENA

3,-3	-1,1
-2,2	1,-1

zero-sum games

COLIN

-3

ROWENA		
r_1	3	-1
r_2	-2	1

zero-sum games

COLIN

ROWENA

r_1	3	-1
r_2	-2	1

$$\sum_{i,j} G_{ij} r_i c_j$$

zero-sum games

colin

rowena	3	-1
	-2	1

rowena announces
her strategy first:

zero-sum games

colin

rowena

3	-1
-2	1

rowena announces
her strategy first:

(r_1, r_2) $\qquad \min\{\ 3r_1 - 2r_2,\ -r_1 + r_2\ \}$

zero-sum games

colin

rowena

3	-1
-2	1

rowena announces her strategy first:

$$\max z$$

$$z \leq 3r_1 - 2r_2$$

$$z \leq -r_1 + 2r_2$$

$$r_1 + r_2 = 1$$

$$r_1, r_2 \geq 0$$

zero-sum games

colin

rowena	3	-1
	-2	1

colin announces
his strategy first:

(c_1, c_2)

zero-sum games

colin

rowena	3	-1
	-2	1

colin announces
his strategy first:

pick (c_1, c_2) so as to min $\max\{3c_1 - c_2,\ -2c_1 + c_2\}$

zero-sum games

colin

rowena	3	-1
	-2	1

colin announces
his strategy first:

$$\min w$$

$$\begin{aligned} -3c_1 + c_2 + w &\geq 0 \\ 2c_1 - c_2 + w &\geq 0 \\ c_1 + c_2 &= 1 \\ c_1, c_2 &\geq 0 \end{aligned}$$

zero-sum games

colin

rowena

3	-1
-2	1

$$\max z$$

$$\begin{aligned} -3r_1 + 2r_2 + z &\leq 0 \\ r_1 - r_2 + z &\leq 0 \\ r_1 + r_2 &= 1 \\ r_1, r_2 &\geq 0 \end{aligned}$$

zero-sum games

colin

rowena

3	-1
-2	1

$$\max z$$

$$\begin{aligned} -3r_1 + 2r_2 + z &\leq 0 \\ r_1 - r_2 + z &\leq 0 \\ r_1 + r_2 &= 1 \\ r_1, r_2 &\geq 0 \end{aligned}$$

$$\min w$$

$$\begin{aligned} -3c_1 + c_2 + w &\geq 0 \\ 2c_1 - c_2 + w &\geq 0 \\ c_1 + c_2 &= 1 \\ c_1, c_2 &\geq 0 \end{aligned}$$

zero-sum games

colin

	2/7	5/7
3/7 rowena	3	-1
4/7	-2	1

value of the game is : 1/7

FEDERAL RESERVE NOTE
AE 77665544 B
E5
THE UNITED STATES OF AMERICA
AE 77665544 B
100
ONE HUNDRED DOLLARS

FEDERAL RESERVE NOTE
KB 46279860 I
B2
THE UNITED STATES OF AMERICA
KB 46279860 I
100
ONE HUNDRED DOLLARS

$$\max_x \min_y \sum_{i,j} G_{ij} x_i y_j = \min_y \max_x \sum_{i,j} G_{ij} x_i y_j$$

https://www.inf.ethz.ch/personal/fukudak/cdd_home/

Welcome to the cdd and cddplus Homepage

Last update: May 15, 2015

Currently, the C-library version cddlib of cdd packages is the only one being updated, while standalone codes cdd and cddplus are still useful. To know what cdd, cddplus and cddlib are, please read

- cddplus readme
- cddlib readme

Manuals (html version):

- cdd/cdd+ manual
- cddlib manual

Get source codes:

- cdd/cddpuls directory click here
- cdd package cdd-061a.tar.gz
- cddplus package cdd+-077a.tar.gz (to be compled with g++ 4.1. With more recent g++, try patch) NEW . With g++ 3.1, use cdd+-077.tar.gz
- cddlib package cddlib-094h.tar.gz NEW

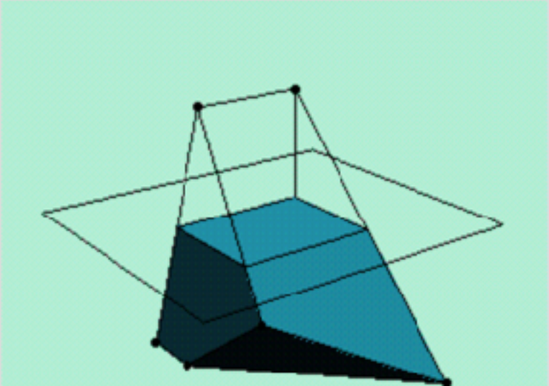
To know the implementation:

- ``The double description revisited'' gzipped ps file

To learn the fundamental concepts of Convex Hull, Vornonoi, Delaunay, etc.:

- ``Polyhedral Computation FAQ'' (still experimental) html version or pdf file

Links to cdd/cdd+/cddlib users and more. NEW



standard form

$$\max_{\boldsymbol{x}} \sum x_i c_i$$

$$\sum a_{ij} x_i \leq b_i$$

$$x_i \geq 0$$

getting to standard form

$$\min_{\boldsymbol{x}} \sum x_i c_i$$

$$\sum a_{ij} x_i \leq b_i$$

$$x_i \geq 0$$

getting to standard form

$$\max_{\boldsymbol{x}} \sum x_i c_i$$

$$\sum a_{ij} x_i \geq b_i$$

$$x_i \geq 0$$

getting to standard form

$$\max_{\boldsymbol{x}} \sum x_i c_i$$

$$\sum a_{ij} x_i = b_i$$

$$x_i \geq 0$$

getting to standard form

$$\max_{\boldsymbol{x}} \sum x_i c_i$$

$$\sum a_{ij} x_i \leq b_i$$

(non-negative)
ie, what if there
is
no constraint
on x?

```
H-representation
begin
24 13 rational
0 0 0 0 0 0 1/2 5/12 1/3 1/4 1/6 1/12 -1
0 0 0 0 0 7/22 5/22 5/33 1/11 1/22 1/66 0 -1
0 0 0 0 0 7/44 1/11 1/22 1/55 1/220 0 0 -1
0 0 0 0 14/99 7/99 1/33 1/99 1/495 0 0 0 -1
0 0 0 0 7/99 7/264 1/132 1/792 0 0 0 0 -1
0 0 0 1/11 1/33 1/132 1/924 0 0 0 0 0 -1
0 0 0 1/22 1/99 1/792 0 0 0 0 0 0 -1
0 0 1/11 1/55 1/495 0 0 0 0 0 0 0 -1
0 0 1/22 1/220 0 0 0 0 0 0 0 0 -1
0 1/6 1/66 0 0 0 0 0 0 0 0 0 -1
0 1/12 0 0 0 0 0 0 0 0 0 0 -1
1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 0
-1 1 1 1 1 1 1 1 1 1 1 1 0
0 1 0 0 0 0 0 0 0 0 0 0 0
0 0 1 0 0 0 0 0 0 0 0 0 0
0 0 0 1 0 0 0 0 0 0 0 0 0
0 0 0 0 1 0 0 0 0 0 0 0 0
0 0 0 0 0 1 0 0 0 0 0 0 0
0 0 0 0 0 0 1 0 0 0 0 0 0
0 0 0 0 0 0 0 1 0 0 0 0 0
0 0 0 0 0 0 0 0 1 0 0 0 0
0 0 0 0 0 0 0 0 0 1 0 0 0
0 0 0 0 0 0 0 0 0 0 1 0 0
0 0 0 0 0 0 0 0 0 0 0 1 0
end
maximize
0 0 0 0 0 0 0 0 0 0 0 0 1
```

```
* Compiled for Rational Exact Arithmetic with GMP
*cdd LP Result
*cdd input file : 12.ine  (24 x 13)
*LP solver: Dual Simplex
*LP status: a dual pair (x, y) of optimal solutions found.
*maximization is chosen.
*Objective function is
 0 + 0 X[1] + 0 X[2] + 0 X[3] + 0 X[4] +
 0 X[5] + 0 X[6] + 0 X[7] + 0 X[8] + 0 X[9] +
 0 X[10] + 0 X[11] + 1 X[12]
*LP status: a dual pair (x, y) of optimal solutions found.
begin
  primal_solution
  1 :  280/1643
  2 :  4217/14787
  3 :  130/477
  4 :  280/1643
  5 :  120/1643
  6 :  140/4929
  7 :  0
  8 :  0
  9 :  0
  10 :  0
  11 :  0
  12 :  70/4929
  dual_solution
  24 :  383/29574
  21 :  599/73935
  20 :  74/14787
  22 :  1003/98580
  23 :  173/14787
  1 :  74/4929
  3 :  99/1643
  5 :  264/1643
  7 :  462/1643
  9 :  1540/4929
  11 :  280/1643
  12 :  70/4929
  optimal_value : 70/4929
end
*number of pivot operations = 8
*Computation starts     at Tue Apr 19 12:54:03 2016
*            terminates at Tue Apr 19 12:54:03 2016
*Total processor time = 0 seconds
*                     = 0h 0m 0s
closing the file 12.lps
closing the file 12.ddl
```