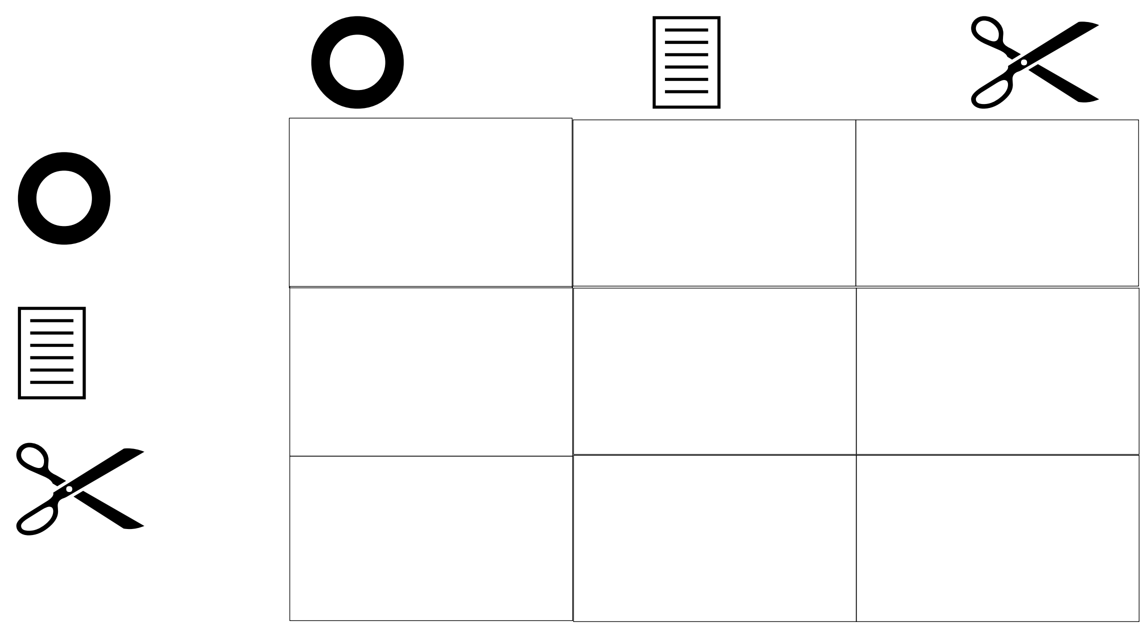
L21

4800

11.22.2016

abhi shelat

zero-sum games



zero-sum games

		COLIN	
ROWENA		3,-3	-1,1
		-2,2	1,-1

zero-sum games

COLIN

ROWENA

r_1	3	-1
r_2	-2	1

zero-sum games

$$\begin{array}{c|c|c|} & \multicolumn{2}{c}{\text{COLIN}} \\ & c_1 & c_2 \\ \hline r_1 & 3 & -1 \\ \hline r_2 & -2 & 1 \\ \hline \end{array}$$

ROWENA

$$\sum_{i,j} G_{ij} r_i c_j$$

zero-sum games

colin

rowena

r_1	3	-1
r_2	-2	1

rowena announces her strategy first:

zero-sum games

colin

r_1	3	-1
r_2	-2	1

rowena

rowena announces her strategy first:

(r_1, r_2)

$$\min\{\ 3r_1 - 2r_2,\ -r_1 + r_2\ \}$$

zero-sum games

colin

rowena r_1	3	-1
r_2	-2	1

rowena announces her strategy first:

$$\max z$$

$$z \leq 3r_1 - 2r_2$$

$$z \leq -r_1 + 2r_2$$

$$r_1 + r_2 = 1$$

$$r_1, r_2 \geq 0$$

zero-sum games

		colin	
rowena		3	-1
		-2	1

colin announces
his strategy first:

(c_1, c_2)

zero-sum games

	colin	
rowena	3	-1
	-2	1

colin announces his strategy first:

pick (c_1, c_2) so as to min $\max\{3c_1 - c_2,\ -2c_1 + c_2\}$

zero-sum games

		colin	
rowena		3	-1
		-2	1

colin announces his strategy first:

$$\min w$$

$$\begin{aligned} -3c_1 + c_2 + w &\geq 0 \\ 2c_1 - c_2 + w &\geq 0 \\ c_1 + c_2 &= 1 \\ c_1, c_2 &\geq 0 \end{aligned}$$

zero-sum games

		colin	
rowena		3	-1
		-2	1

$$\max z$$

$$\begin{aligned} -3r_1 + 2r_2 + z &\leq 0 \\ r_1 - r_2 + z &\leq 0 \\ r_1 + r_2 &= 1 \\ r_1, r_2 &\geq 0 \end{aligned}$$

zero-sum games

	colin	
rowena	3	-1
	-2	1

$$\max z$$

$$\begin{aligned} -3r_1 + 2r_2 + z &\leq 0 \\ r_1 - r_2 + z &\leq 0 \\ r_1 + r_2 &= 1 \\ r_1, r_2 &\geq 0 \end{aligned}$$

$$\min w$$

$$\begin{aligned} -3c_1 + c_2 + w &\geq 0 \\ 2c_1 - c_2 + w &\geq 0 \\ c_1 + c_2 &= 1 \\ c_1, c_2 &\geq 0 \end{aligned}$$

zero-sum games

		colin 2/7	5/7
rowena	3/7	3	-1
	4/7	-2	1

value of the game is : 1/7

$$\max_x \min_y \sum_{i,j} G_{ij} x_i y_j = \min_y \max_x \sum_{i,j} G_{ij} x_i y_j$$


A
A


FEDERAL RESERVE NOTE
AE 77665544 B
E5
THE UNITED STATES OF AMERICA
AE 77665544 B
100
ONE HUNDRED DOLLARS


FEDERAL RESERVE NOTE
KB 46279860 I
B2
THE UNITED STATES OF AMERICA
KB 46279860 I
100
ONE HUNDRED DOLLARS

Colin

		1	2
Rowena	1	-2	3
	2	3	-4

https://www.inf.ethz.ch/personal/fukudak/cdd_home/

Welcome to the cdd and cddplus Homepage

Last update: May 15, 2015

Currently, the C-library version cddlib of cdd packages is the only one being updated, while standalone codes cdd and cddplus are still useful. To know what cdd, cddplus and cddlib are, please read

cddplus readme
cddlib readme

Manuals (html version):

cdd/cdd+ manual
cddlib manual

Get source codes:

cdd/cddpuls directory click here
cdd package cdd-061a.tar.gz
cddplus package cdd+-077a.tar.gz (to be compled with g++ 4.1. With more recent g++, try patch) NEW . With g++ 3.1, use cdd+-077.tar.gz
cddlib package cddlib-094h.tar.gz NEW

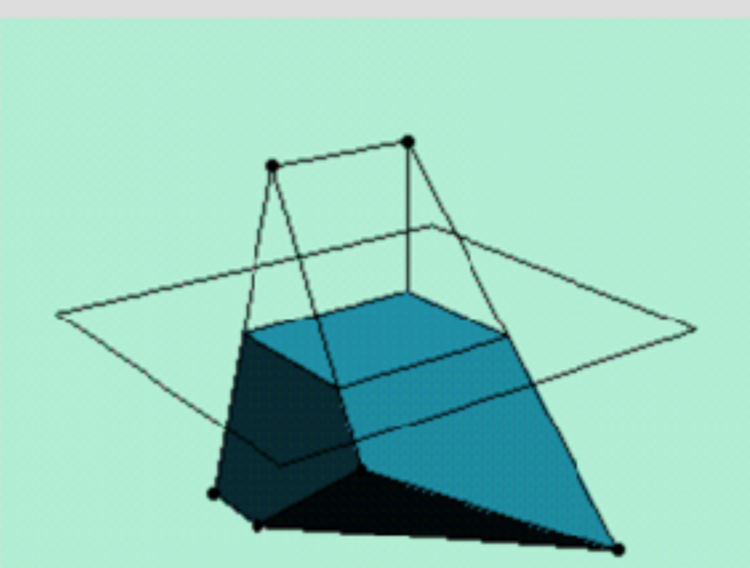
To know the implementation:

``The double description revisited" gzipped ps file

To learn the fundamental concepts of Convex Hull, Vornonoi, Delaunay, etc.:

``Polyhedral Computation FAQ" (still experimental) html version or pdf file

Links to cdd/cdd+/cddlib users and more. NEW



standard form

$$\max_{x} \sum x_i c_i$$

$$\sum a_{ij} x_i \leq b_i$$

$$x_i \geq 0$$

getting to standard form

$$\min_{x} \sum x_i c_i$$

$$\sum a_{ij} x_i \leq b_i$$

$$x_i \geq 0$$

getting to standard form

$$\max_{x} \sum x_i c_i$$

$$\sum a_{ij} x_i \geq b_i$$

$$x_i \geq 0$$

getting to standard form

$$\max_{x} \sum x_i c_i$$

$$\sum a_{ij} x_i = b_i$$

$$x_i \geq 0$$

getting to standard form

$$\max_{x} \sum x_i c_i$$

$$\sum a_{ij} x_i \leq b_i$$

(non-negative)
ie, what if there
is
no constraint
on x?

```
H-representation
begin
24 13 rational
0 0 0 0 0 0 1/2 5/12 1/3 1/4 1/6 1/12 -1
0 0 0 0 0 7/22 5/22 5/33 1/11 1/22 1/66 0 -1
0 0 0 0 0 7/44 1/11 1/22 1/55 1/220 0 0 -1
0 0 0 0 14/99 7/99 1/33 1/99 1/495 0 0 0 -1
0 0 0 0 7/99 7/264 1/132 1/792 0 0 0 0 -1
0 0 0 1/11 1/33 1/132 1/924 0 0 0 0 0 -1
0 0 0 1/22 1/99 1/792 0 0 0 0 0 0 -1
0 0 1/11 1/55 1/495 0 0 0 0 0 0 0 -1
0 0 1/22 1/220 0 0 0 0 0 0 0 0 -1
0 1/6 1/66 0 0 0 0 0 0 0 0 0 -1
0 1/12 0 0 0 0 0 0 0 0 0 0 -1
1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 0
-1 1 1 1 1 1 1 1 1 1 1 1 0
0 1 0 0 0 0 0 0 0 0 0 0 0
0 0 1 0 0 0 0 0 0 0 0 0 0
0 0 0 1 0 0 0 0 0 0 0 0 0
0 0 0 0 1 0 0 0 0 0 0 0 0
0 0 0 0 0 1 0 0 0 0 0 0 0
0 0 0 0 0 0 1 0 0 0 0 0 0
0 0 0 0 0 0 0 1 0 0 0 0 0
0 0 0 0 0 0 0 0 1 0 0 0 0
0 0 0 0 0 0 0 0 0 1 0 0 0
0 0 0 0 0 0 0 0 0 0 1 0 0
0 0 0 0 0 0 0 0 0 0 0 1 0
end
maximize
0 0 0 0 0 0 0 0 0 0 0 0 1
```

```
* Compiled for Rational Exact Arithmetic with GMP
*cdd LP Result
*cdd input file : 12.ine  (24 x 13)
*LP solver: Dual Simplex
*LP status: a dual pair (x, y) of optimal solutions found.
*maximization is chosen.
*Objective function is
 0 + 0 X[1] + 0 X[2] + 0 X[3] + 0 X[4] +
 0 X[5] + 0 X[6] + 0 X[7] + 0 X[8] + 0 X[9] +
 0 X[10] + 0 X[11] + 1 X[12]
*LP status: a dual pair (x, y) of optimal solutions found.
begin
  primal_solution
  1 :  280/1643
  2 :  4217/14787
  3 :  130/477
  4 :  280/1643
  5 :  120/1643
  6 :  140/4929
  7 :  0
  8 :  0
  9 :  0
  10 :  0
  11 :  0
  12 :  70/4929
  dual_solution
  24 :  383/29574
  21 :  599/73935
  20 :  74/14787
  22 :  1003/98580
  23 :  173/14787
  1 :  74/4929
  3 :  99/1643
  5 :  264/1643
  7 :  462/1643
  9 :  1540/4929
  11 :  280/1643
  12 :  70/4929
  optimal_value : 70/4929
end
*number of pivot operations = 8
*Computation starts     at Tue Apr 19 12:54:03 2016
*           terminates at Tue Apr 19 12:54:03 2016
*Total processor time = 0 seconds
*                     = 0h 0m 0s
closing the file 12.lps
closing the file 12.ddl
```

how to “evaluate” an lp

$$\max c^T \vec{x}$$

$$A\vec{x} \leq \vec{b}$$

$$\vec{x} \geq 0$$

definitions

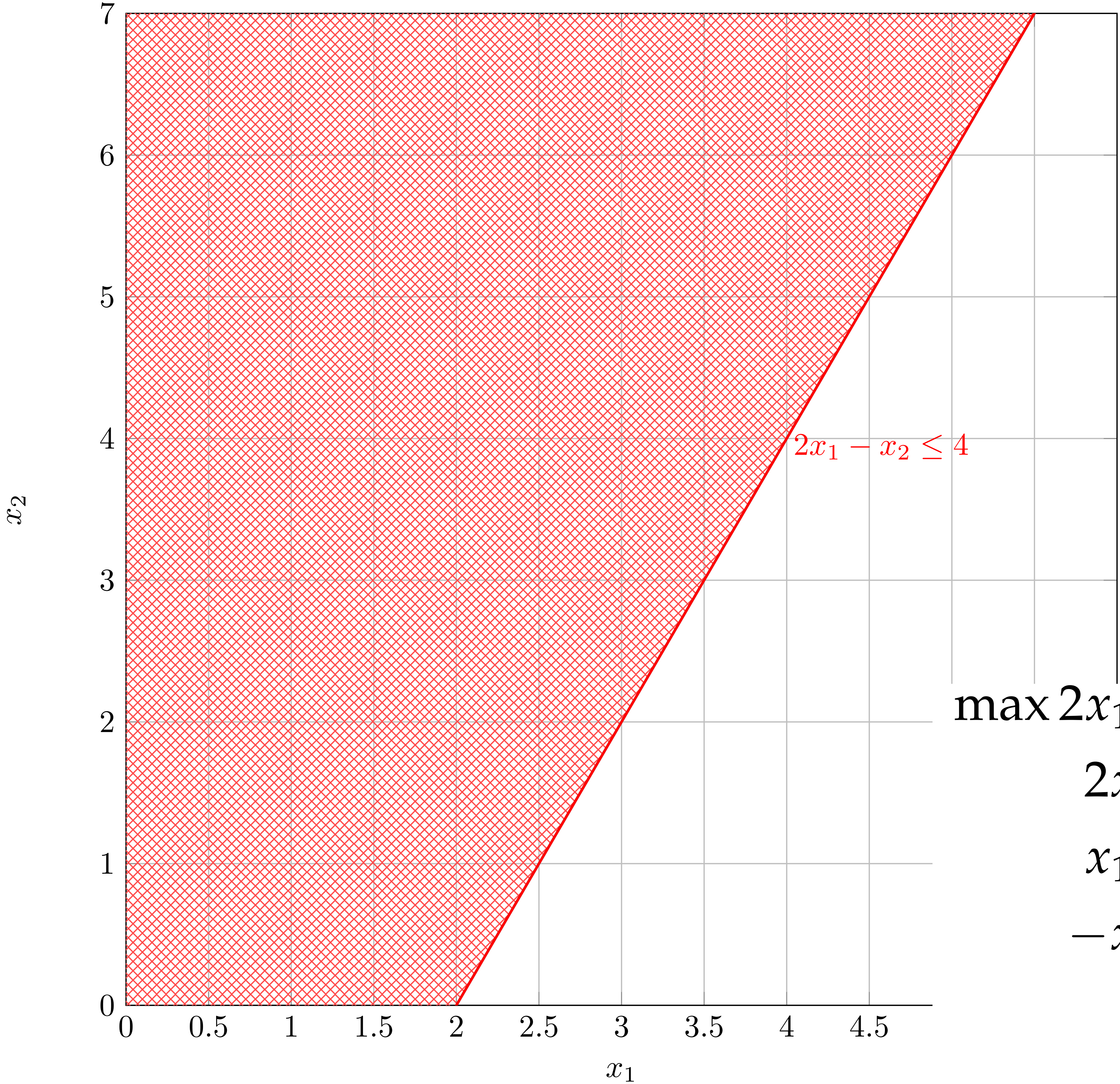
feasible point:

vertex:

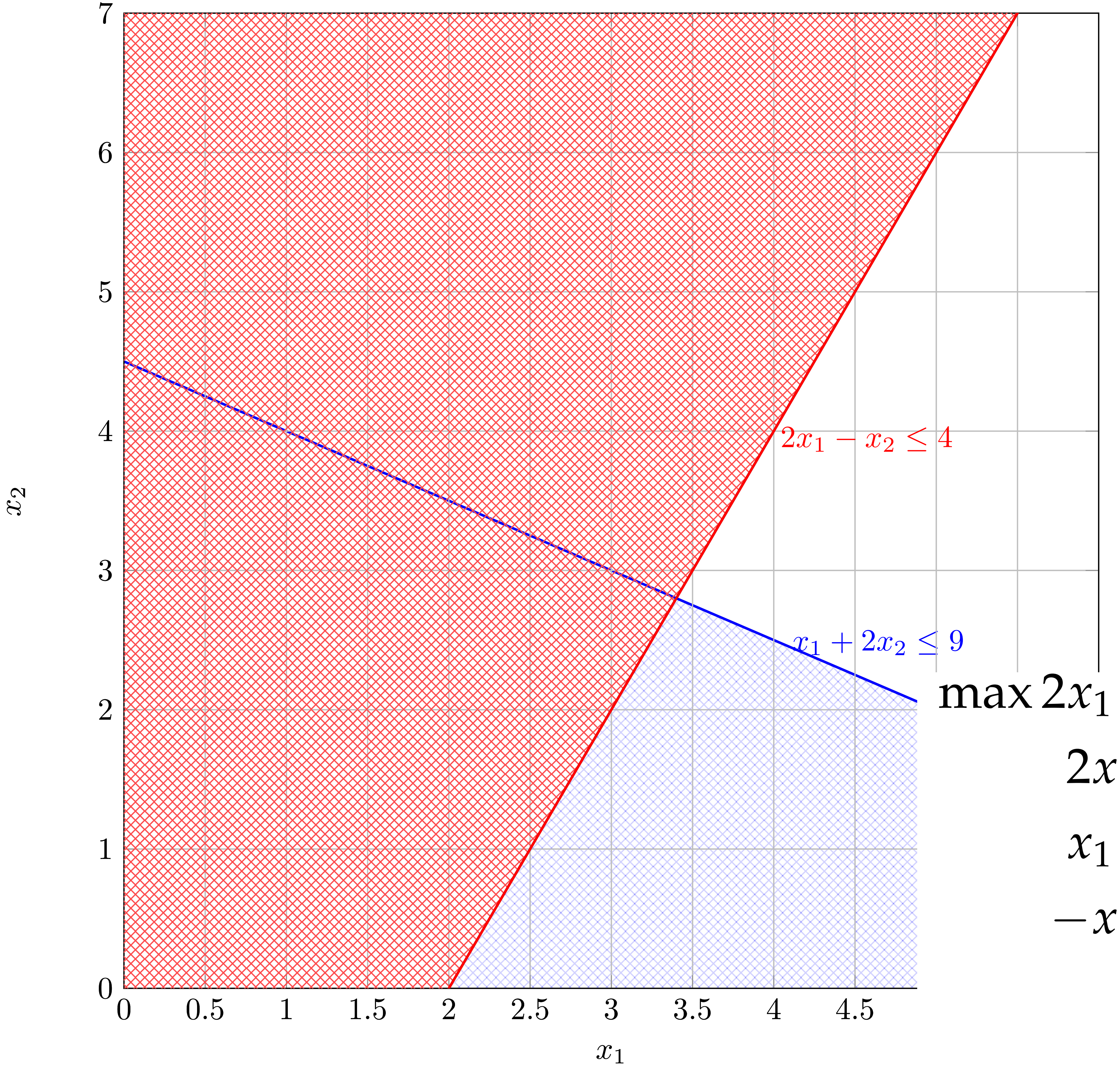
neighbor of vertex v:

simplex

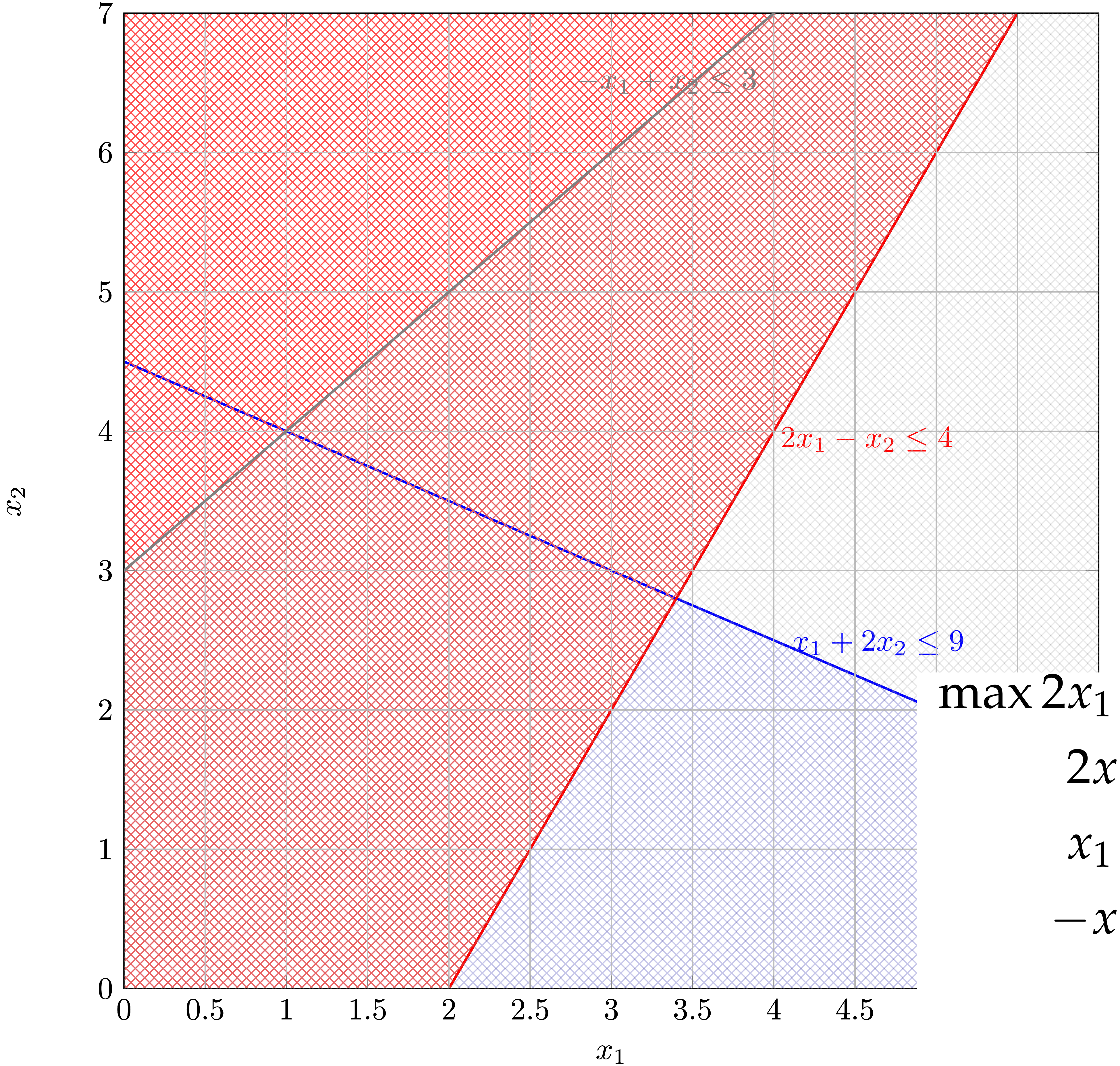
```
init:

while
      do:
```



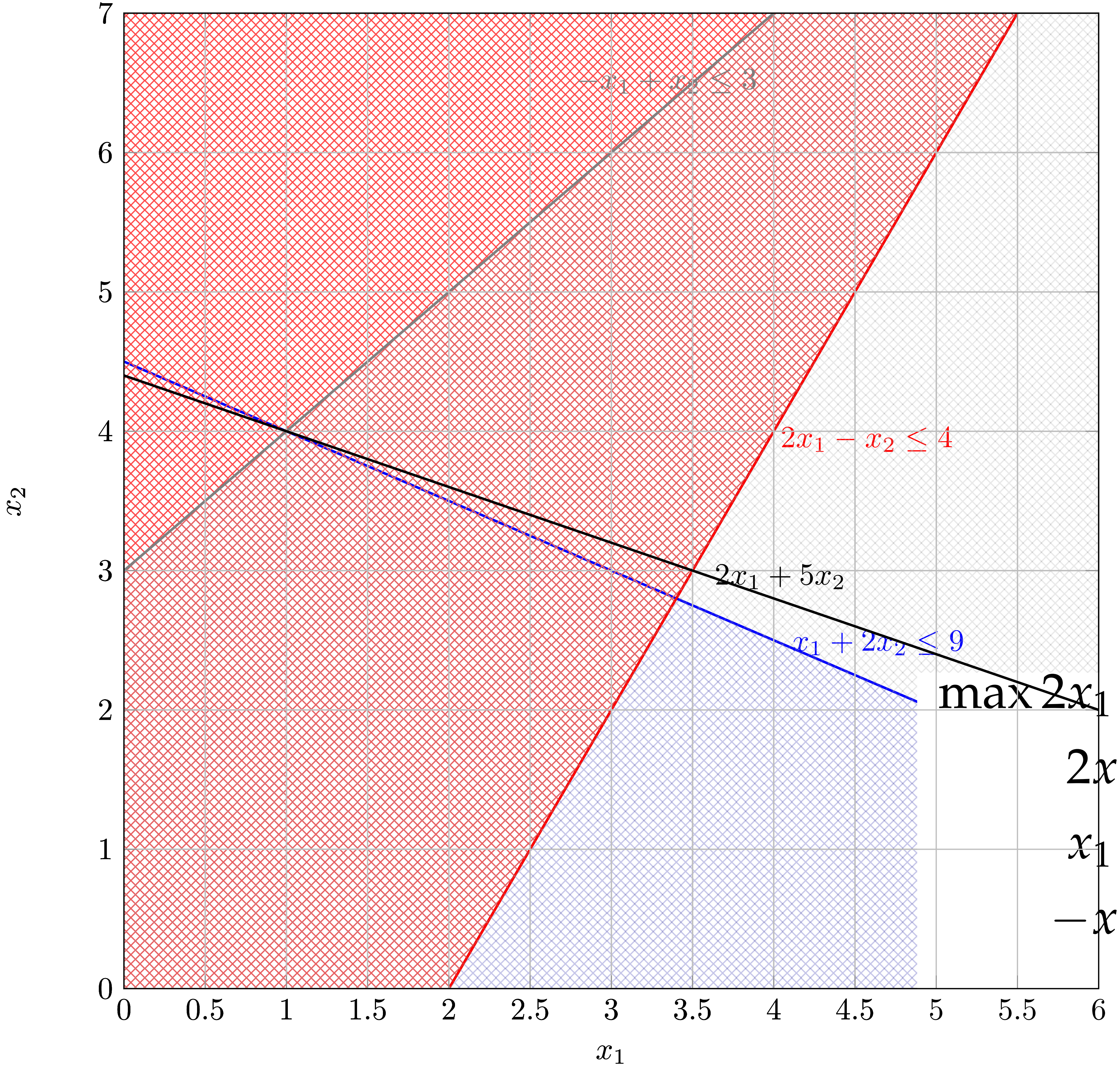
$$\max 2x_1 + 5x_2$$
$$2x_1 - x_2 \leq 4$$
$$x_1 + 2x_2 \leq 9$$
$$-x_1 + x_2 \leq 3$$
$$x_1 \geq 0$$
$$x_2 \geq 0$$



$$\begin{aligned} \max 2x_1 + 5x_2 & \\ 2x_1 - x_2 &\leq 4 \\ x_1 + 2x_2 &\leq 9 \\ -x_1 + x_2 &\leq 3 \\ x_1 &\geq 0 \\ x_2 &\geq 0 \end{aligned}$$



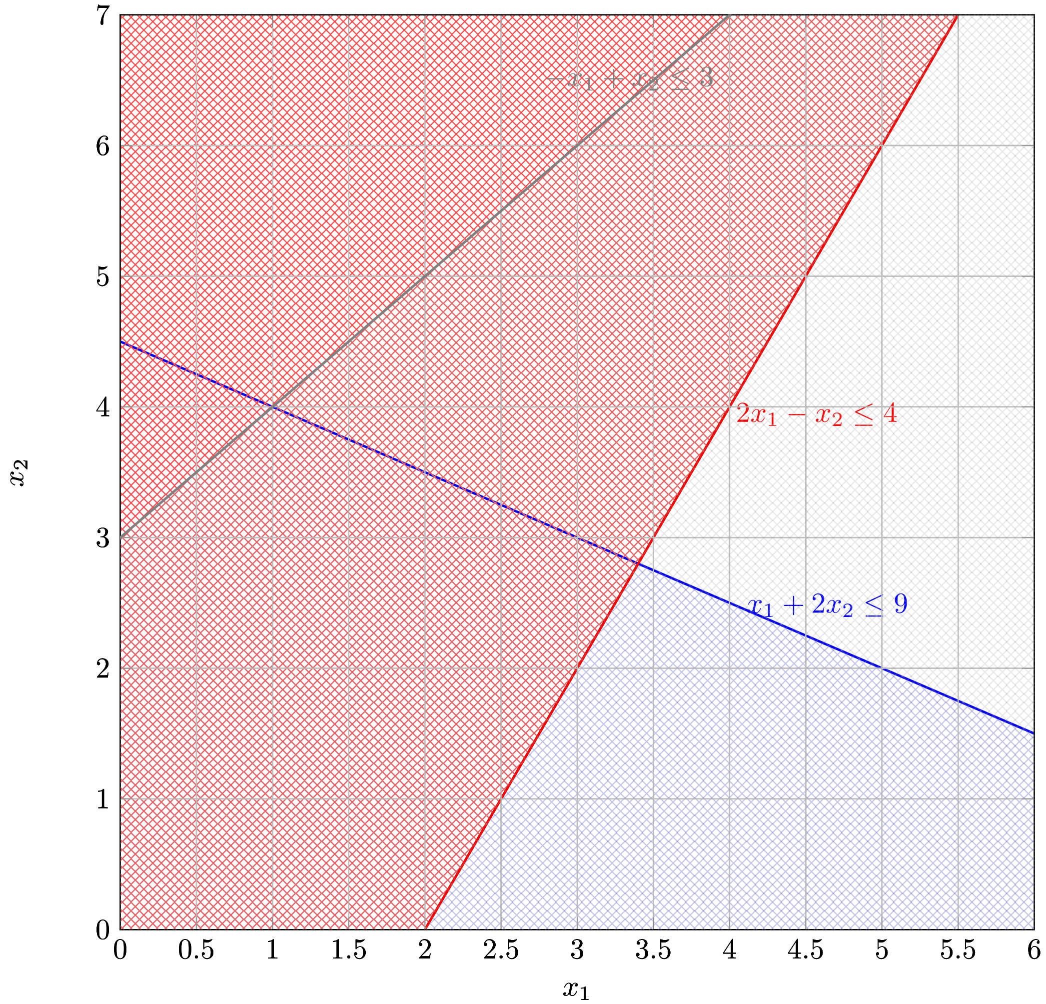
$$
\begin{aligned}
\max 2x_1 + 5x_2 & \\
2x_1 - x_2 &\leq 4 \\
x_1 + 2x_2 &\leq 9 \\
-x_1 + x_2 &\leq 3 \\
x_1 &\geq 0 \\
x_2 &\geq 0
\end{aligned}
$$



$$\max 2x_1 + 5x_2$$
$$2x_1 - x_2 \leq 4$$
$$x_1 + 2x_2 \leq 9$$
$$-x_1 + x_2 \leq 3$$
$$x_1 \geq 0$$
$$x_2 \geq 0$$

Pivot

$$\max 2x_1 + 5x_2$$
$$2x_1 - x_2 \le 4$$
$$x_1 + 2x_2 \le 9$$
$$-x_1 + x_2 \le 3$$
$$x_1 \ge 0$$
$$x_2 \ge 0$$



Pivot

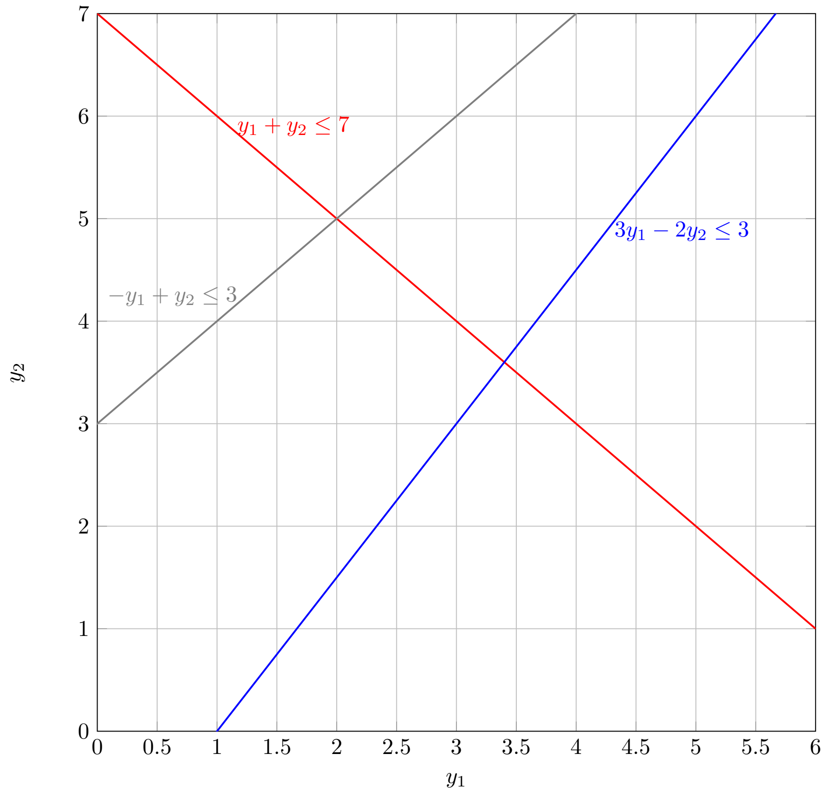
$$
\begin{aligned}
\max 2x_1 + 5x_2 & \\
2x_1 - x_2 &\leq 4 \\
x_1 + 2x_2 &\leq 9 \\
-x_1 + x_2 &\leq 3 \\
x_1 &\geq 0 \\
x_2 &\geq 0
\end{aligned}
$$

$$y_1 = x_1, y_2 = 3 + x_1 - x_2$$

pivot

$$\max 7y_1 - 5y_2 + 15$$
$$y_1 + y_2 \leq 7$$
$$3y_1 - 2y_2 \leq 3$$
$$y_2 \geq 0$$
$$y_1 \geq 0$$
$$-y_1 + y_2 \leq 3$$

$$z_1 =$$
$$z_2 =$$



$$\begin{aligned}\max 7y_1 - 5y_2 + 15 \\ y_1 + y_2 &\leq 7 \\ 3y_1 - 2y_2 &\leq 3 \\ y_2 &\geq 0 \\ y_1 &\geq 0 \\ -y_1 + y_2 &\leq 3\end{aligned}$$

$$z_1 = 3 - 3y_1 + 2y_2, z_2 = y_2$$

$$
\begin{aligned}
\max 22 - \frac{7}{3}z_1 - \frac{1}{3}z_2 & \\
-\frac{1}{3}z_2 + \frac{5}{3}z_2 &\leq 6 \\
z_1 &\geq 0 \\
z_2 &\geq 0 \\
\frac{1}{3}z_1 - \frac{2}{3}z_2 &\leq 1 \\
\frac{1}{3}z_1 + \frac{1}{3}z_2 &\leq 4
\end{aligned}
$$

$$
\begin{aligned}
\max 22 - \frac{7}{3}z_1 - \frac{1}{3}z_2 \\
-\frac{1}{3}z_2 + \frac{5}{3}z_2 \leq 6 \\
z_1 \geq 0 \\
z_2 \geq 0 \\
\frac{1}{3}z_1 - \frac{2}{3}z_2 \leq 1 \\
\frac{1}{3}z_1 + \frac{1}{3}z_2 \leq 4
\end{aligned}
$$

$$z_1 = 3 - 3y_1 + 2y_2, z_2 = y_2$$

$$y_1 = x_1, y_2 = 3 + x_1 - x_2$$

Optimality

$$
\begin{aligned}
\max 2x_1 + 5x_2 & \\
2x_1 - x_2 &\leq 4 \\
x_1 + 2x_2 &\leq 9 \\
-x_1 + x_2 &\leq 3 \\
x_1 &\geq 0 \\
x_2 &\geq 0
\end{aligned}
$$

simplex

simple fact: origin is optimal if and only if

problems

initial vertex

no solution?

run time