LI 4102

ALGORITHMS

AUG 27 2013

ALGOR
ITHMS

i love algorithms
so much

you curse my name

i love it so much

i’m in this maze
with you

you will crack the code

one day you are
 hard
one day you are
 there

Holy
grail

anthem

let me intro myself

first goal: create
an amazing
learning
experience

second goal: instill my enthusiasm for this area

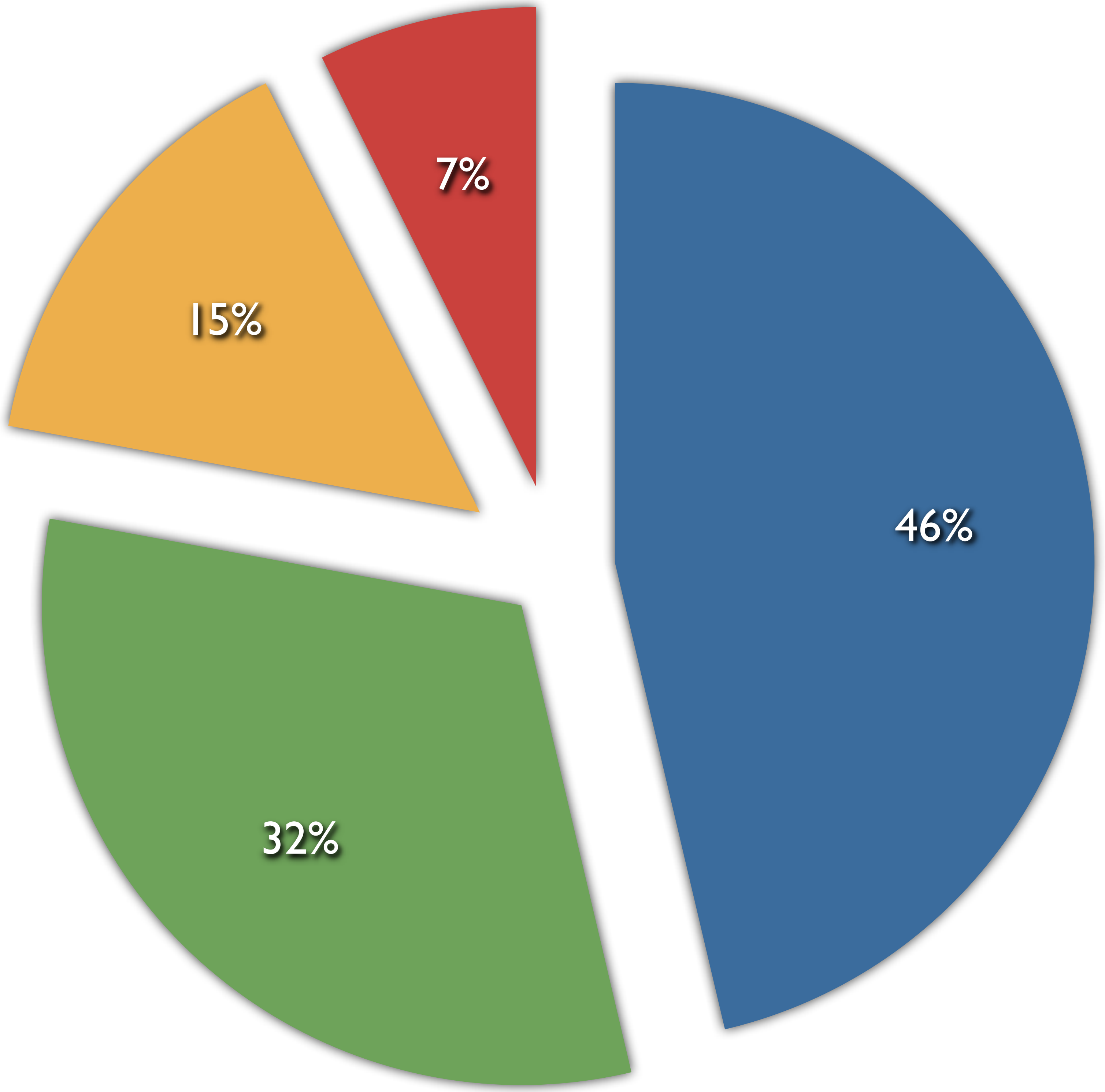
third goal: enjoy
every second of
this semester

caveat
emptor

This was one of the most brutally difficult courses I have taken. Almost every homework ended with me staying up all night before it was due in order to get it finished. However, all told, this has also been one of the most worthwhile classes I have taken. The work is very difficult, but because of that it was eve more rewarding every time I solved a problem. Abhi is incredibly enthusiastic about the topi and really does his best to get the class to actually learn something. He also really knows the subject, and is almost always able to quickly and accurately respond to any student questions.

Professor Shelat put an outstanding amount of effort into this class; he is one of the few professors I've had that have made their own slides, which were all very helpful. The homeworks were all very challenging, but really pushed me understand the material. The very theoretical perspective Prof. Shelat brought to the class was great- it was nice to have this not just be another programming class. This class has definitely been my favorite CS class at UVA as a 4th yr major.

Shelat turned this formerly-easy class into pure hell. All the assignments have been stupid hard, throw-up-your-hands-in-frustration level difficulty. And they rarely have anything to do with the lectures. And the problems are poorly written. And the assignment grading is excessively harsh, frequently arbitrary, and often inconsistent. And Shelat has been completely unresponsive to the many student complaints about all this. This has been the worst kind of hard class; the kind where you work insanely hard only to accomplish nothing meaningfull....Bottom line: Shelat should never be allowed to teach an undergraduate course ever again, at any school!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F'11

4.6

49ERS

http://www.sportslogos.net/logos/view/1008/San_Francisco_49ers/1972/Wordmark_Logo

WHAT IS THIS COURSE ABOUT?

CHISTMAS MORNING ✓

Stockings✓

Step 1.) Cammy gets everybodys out
Step 2.) Cammy looks at hers taking One thing out at a time & Showing it to everyone.
step 3.) Then she puts them neatly back in the stocking.
Step. 4.) Connie does this also. Then Bill.

Presents

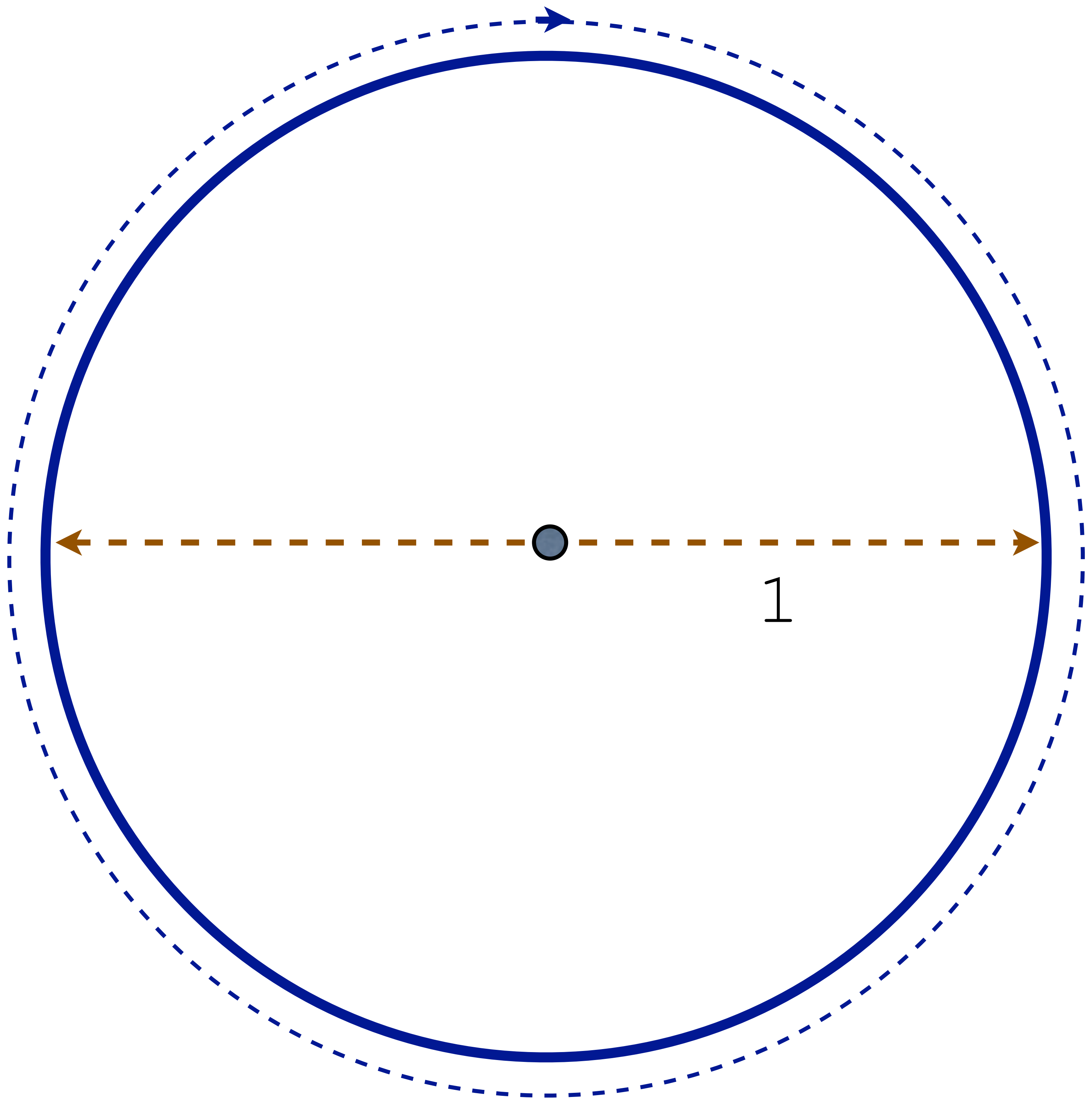
1.) Cammy is appointed present finder.
2.) Cammy finds herself a present & after looking it over & saying the nessary thankyous she passes it around for everyone to see.
3.) then she puts the wraping paper in a pile & puts the present in a place where all her present will go. (everyone has a spot like this). She does this for everyone.
4.) Cammy finds a present for Connie.
5.) Connie does the second part of ~~two~~ #2 then gives the wraping paper & present to Cammy.
6.) Cammy finds a present For Bill.
7.) Bill does the second part of #2 then gives the wrapping paper & present to Cammy.
8.) This is repeated till there are no more presents.

GREAT PYRAMID AT GIZA 2500BC

IMAGE FROM WIKIMEDIA

π

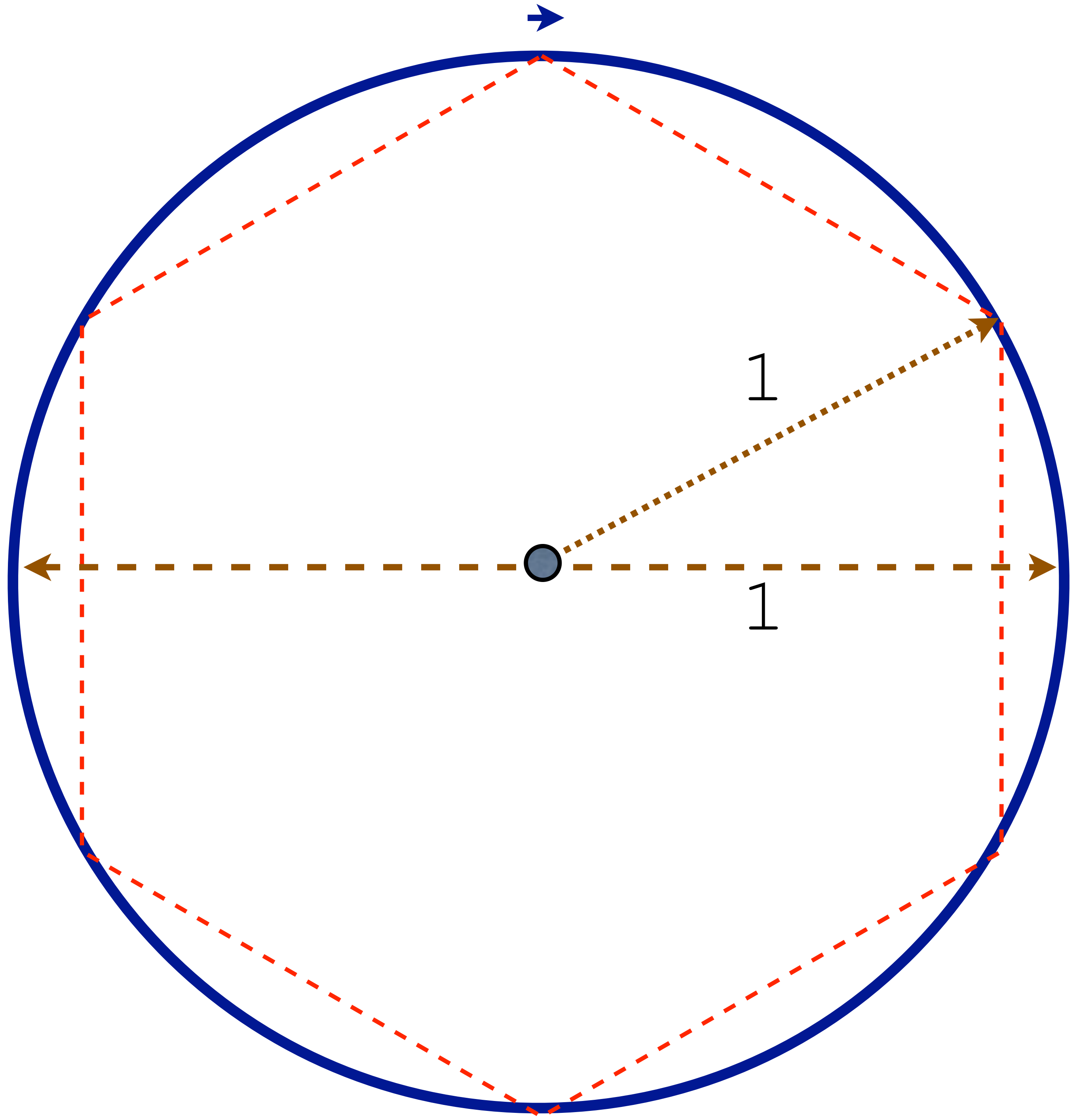
http://www.cupertino.org/inc/pdf/apple/Renderings.pdf



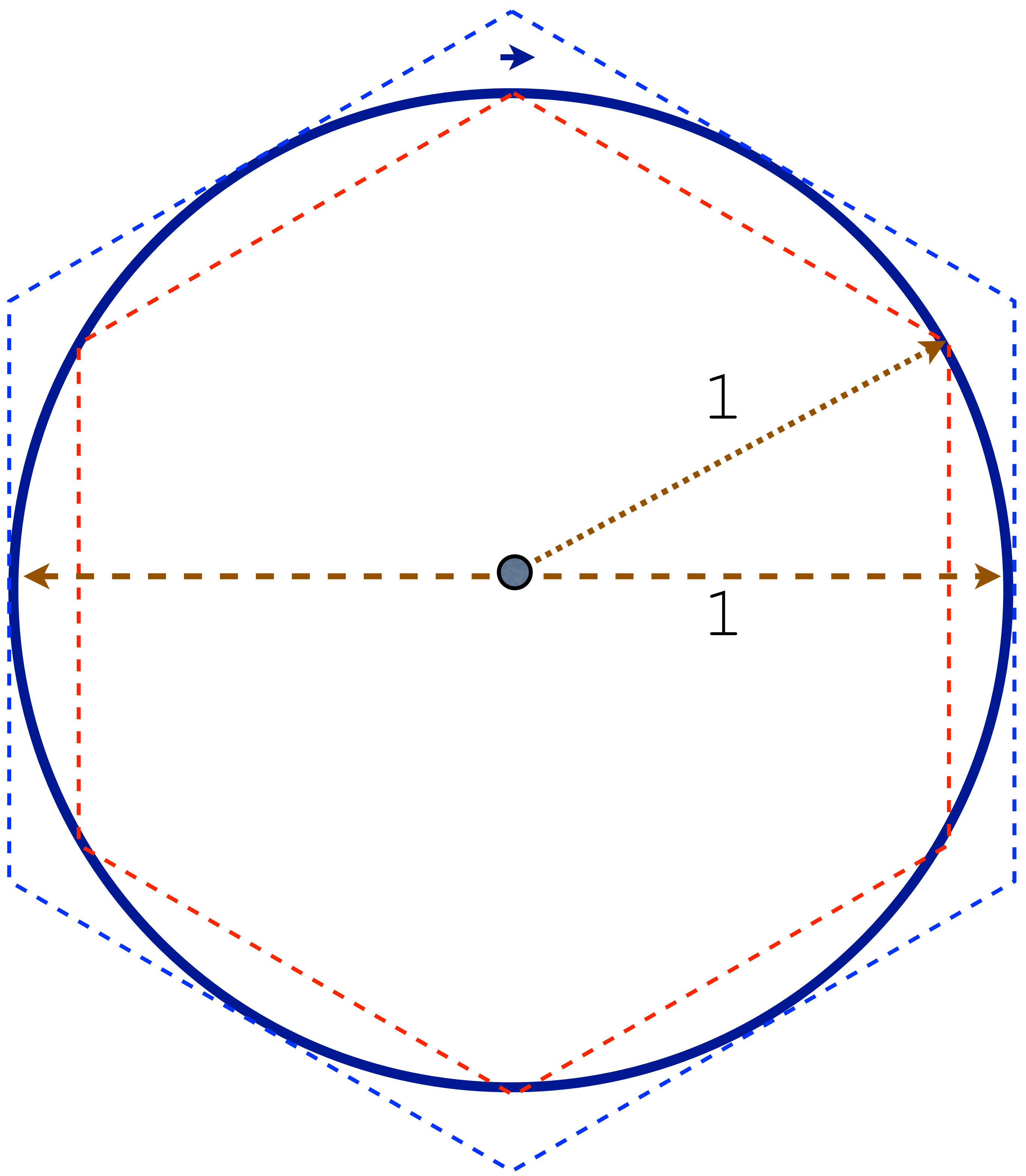
“HOW MUCH GRANITE/GLASS DO I NEED?”

ALGORITHM TO COMPUTE

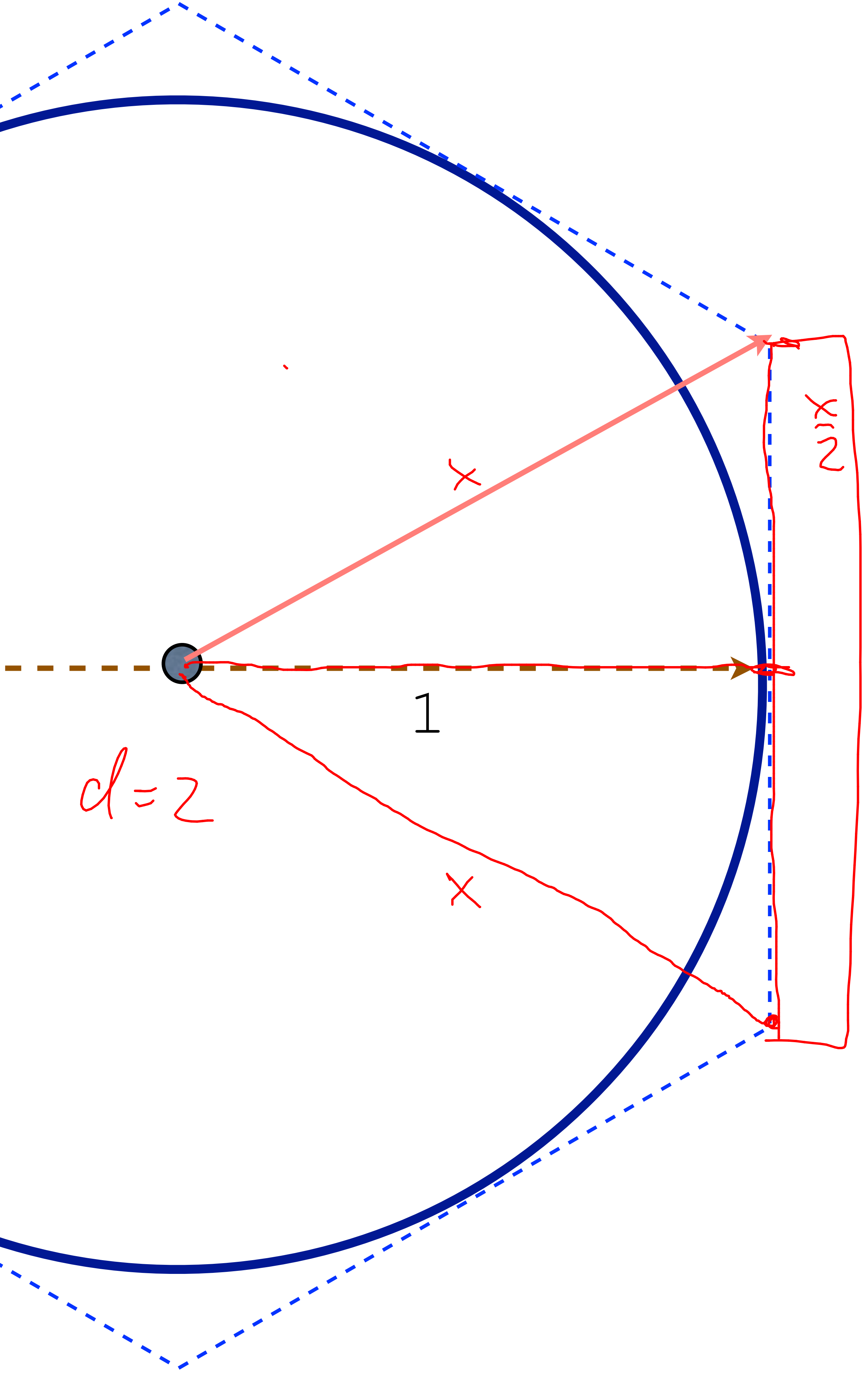
$$\pi$$



RED PERIMETER $< \pi d <$ BLUE PERIMETER



RED PERIMETER < πd < BLUE PERIMETER



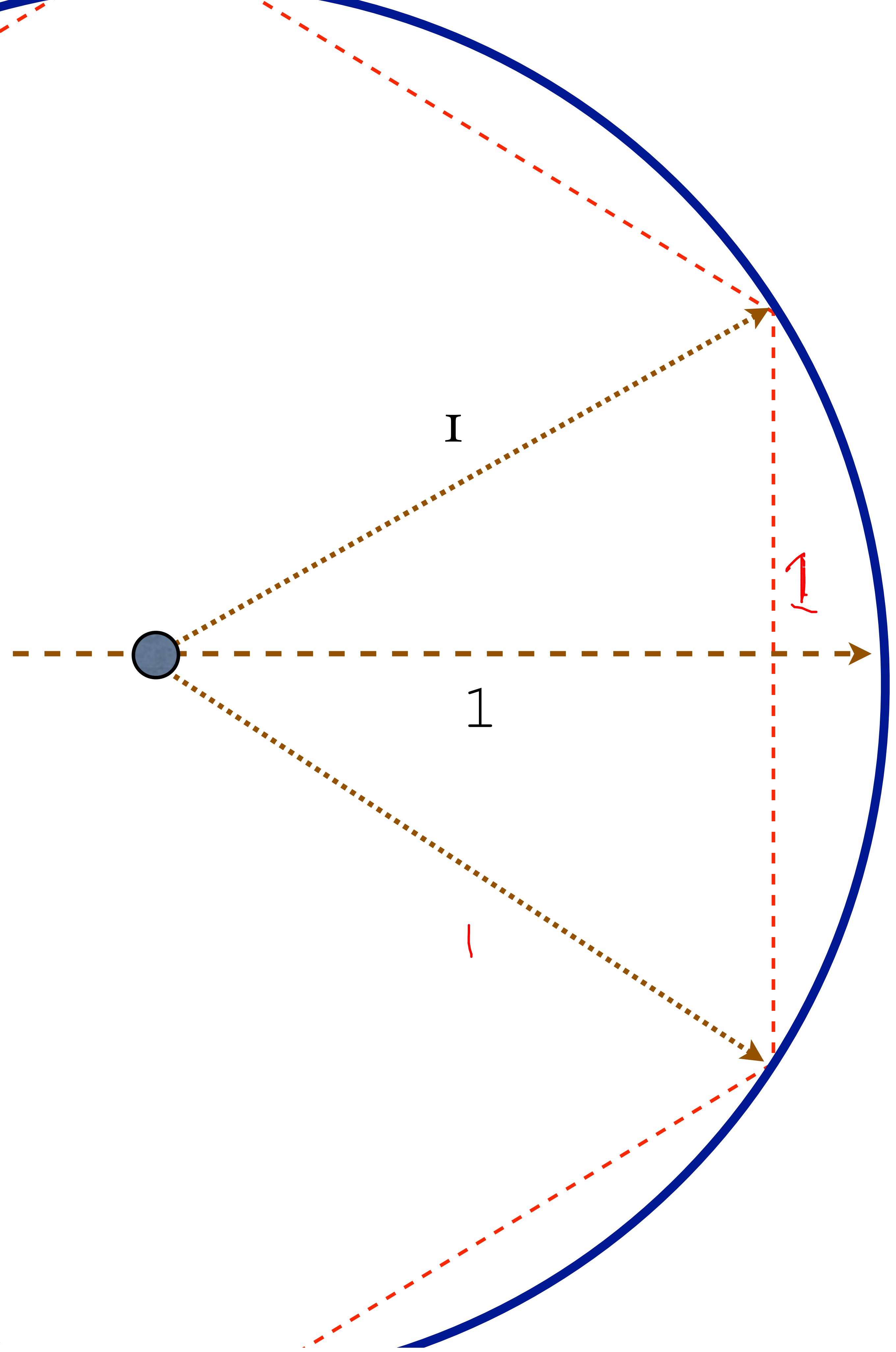
$$1^2 + \left(\frac{x}{2}\right)^2 = x^2$$

$$4 + x^2 = 4x^2 \Rightarrow 3x^2 = 4 \Rightarrow x = \frac{2}{\sqrt{3}}$$

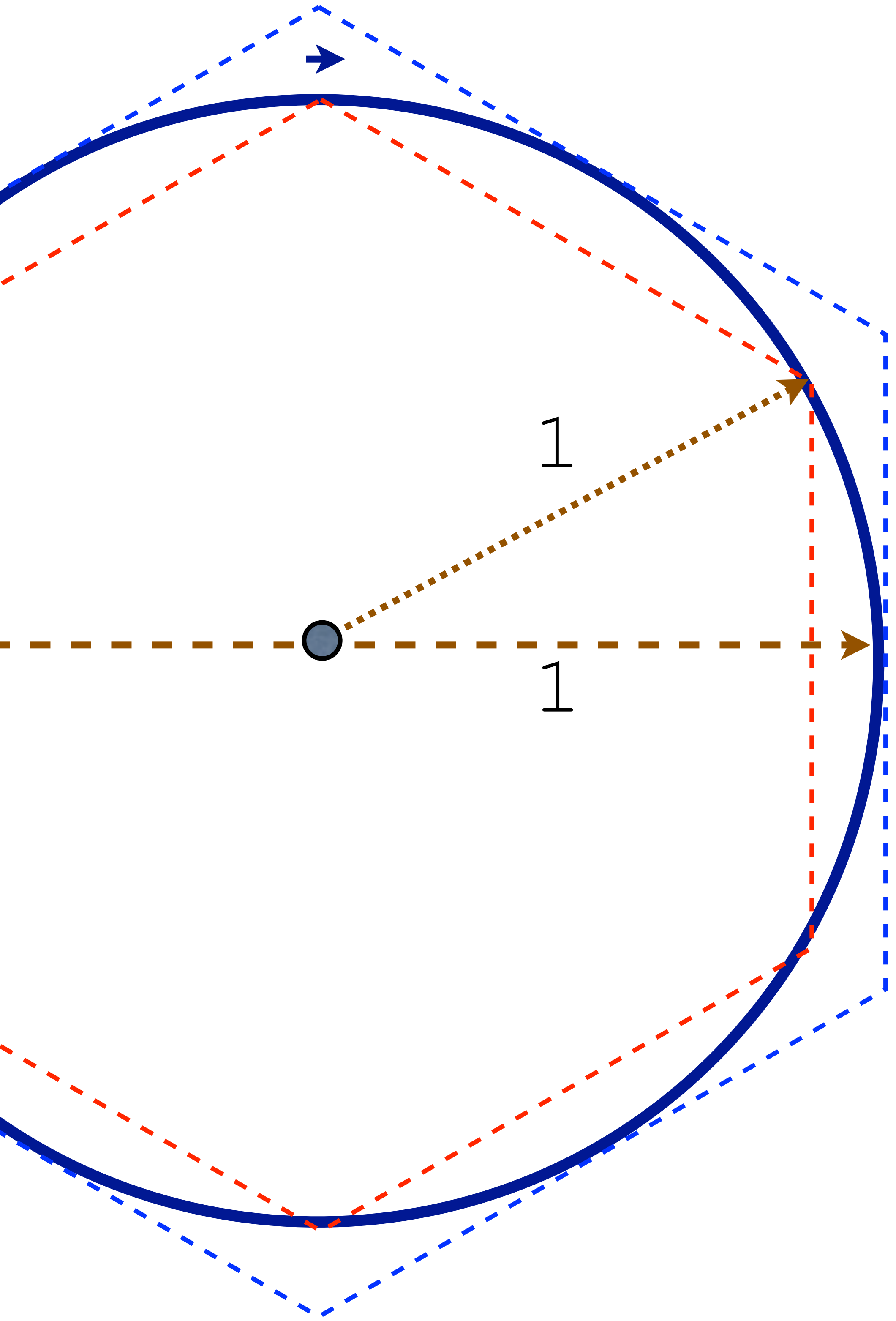
perimeter of the hex: $\frac{12}{\sqrt{3}} > 2\pi$

$$\Rightarrow \frac{6}{\sqrt{3}} > \pi$$

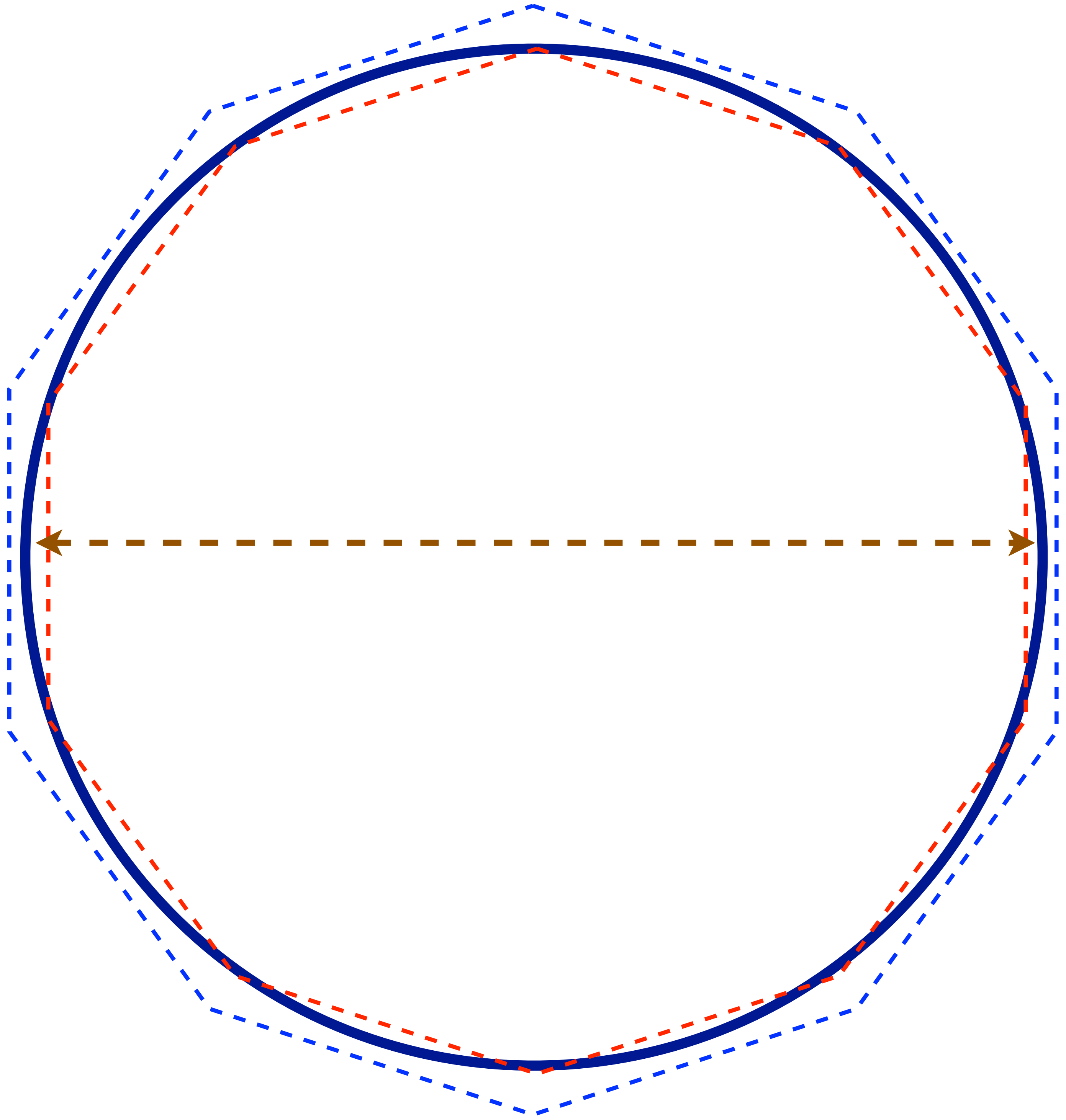
$$\frac{265}{153} \approx \sqrt{3}$$



red perimeter: 6



TER < πd < BLUE PERIMETER



$$3\frac{10}{70} > \pi > 3\frac{10}{71}$$

$\underbrace{3.14}2$

3 digits correct

HOW TO ANALYZE THIS APPROACH?

- How close are we to the answer?

- How much work is needed to do better??

$$\frac{1}{\pi} = \boxed{\frac{2\sqrt{2}}{9801}} \underline{\sum_{k=0}^{\infty} \frac{(4k)!(1103 + 26390k)}{(k!)^4 396^{4k}}}$$

1910.

$\underline{\underline{k=0}}$

$$\frac{2\sqrt{2}}{9801}\left(\frac{0!\,(1103)}{(0!)(396^0)}\right) = \frac{2\sqrt{2}}{9801}\cdot 1103 = \frac{2206\sqrt{2}}{9801}$$

$$\underline{3.141592}\underset{\uparrow}{\boxed{7}}3001$$

K=O

3.14159273001330576017

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801}\sum_{k=0}^{\infty}\frac{(4k)!(1103+26390k)}{(k!)^4 396^{4k}}$$

K=I

$$\frac{2\sqrt{2}}{9801}\left(1103 + \frac{4!(1103+26390)}{1\cdot 396^{4}}\right)$$

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801}\sum_{k=0}^{\infty}\frac{(4k)!(1103+26390k)}{(k!)^4 396^{4k}}$$

K=I

$$\frac{2\sqrt{2}}{9801}\left[1103+\frac{24\cdot 27493}{396^4}\right]$$

$$3.141592\underbrace{653589793}97787$$

↑

every step of this series produces

8 digits of pi

BENEFITS?

GOOD ALGORITHMS TOUCH EVERY ASPECT OF OUR LIVES

Google

FedEx

TESLA

TESLA MOTORS

amazon

NETFLIX

at&t

GOOD ALGORITHMS DEFEND FREEDOM

IMAGE:HISTORY OF AIR CARGO

WHAT SKILLS DO YOU NEED FOR THIS COURSE?

PRECISION

CREATIVITY

IN·GE·NU·I·TY

THEME

“SMALL PROBLEMS ARE EASY TO SOLVE.”

THEME

"SMALL PROBLEMS ARE EASY TO SOLVE."

"SOLVE BIG PROBLEMS BY MAKING THEM INTO SMALLER ONES."

THEME 2

“TO CONVINCE WITH PURE REASON IS THE BEST MARK OF UNDERSTANDING” *honor policy. ① write up on your own.

HOW TO LEARN IN THIS CLASS

① groupwork

② discussion →

③ Ask for help

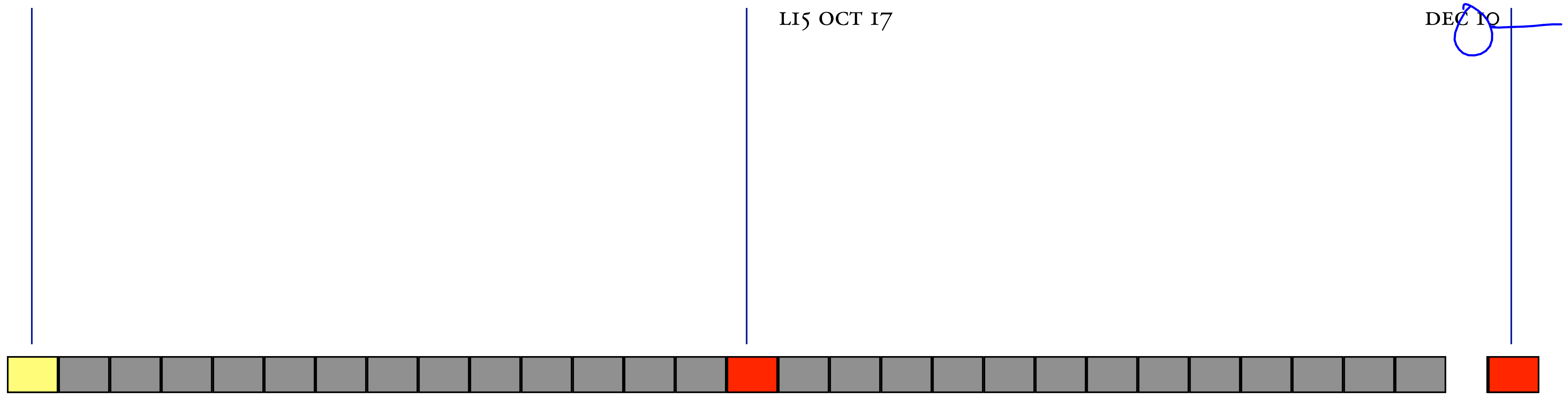
NO COOKBOOK

DEVELOP GENERAL PROBLEM SOLVING SKILLS

UNDERSTAND KNOWN TECHNIQUES

WORK WITH YOUR PEERS

https://crypto.cs.virginia.edu/13f4102/



TODAY

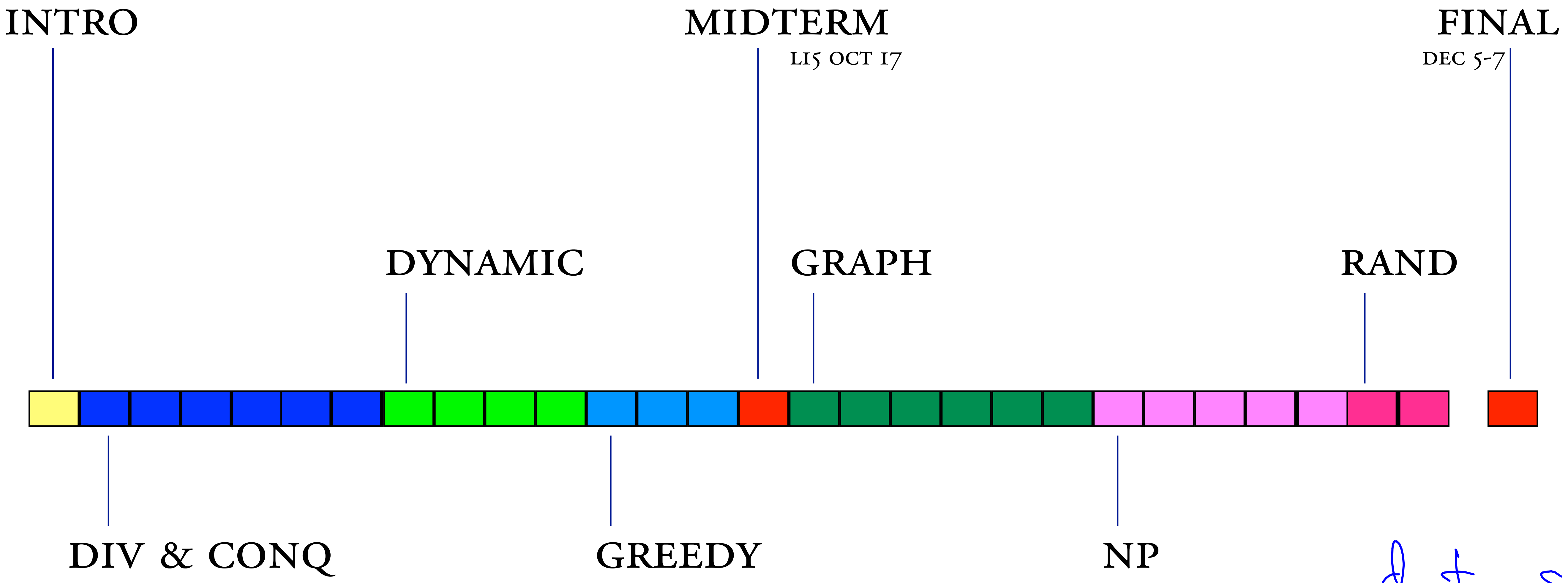
MIDTERM

L15 OCT 17

FINAL

DEC 10

8 — 5-7



INTRO

MIDTERM
L15 OCT 17

FINAL
DEC 5-7

DYNAMIC

GRAPH

RAND

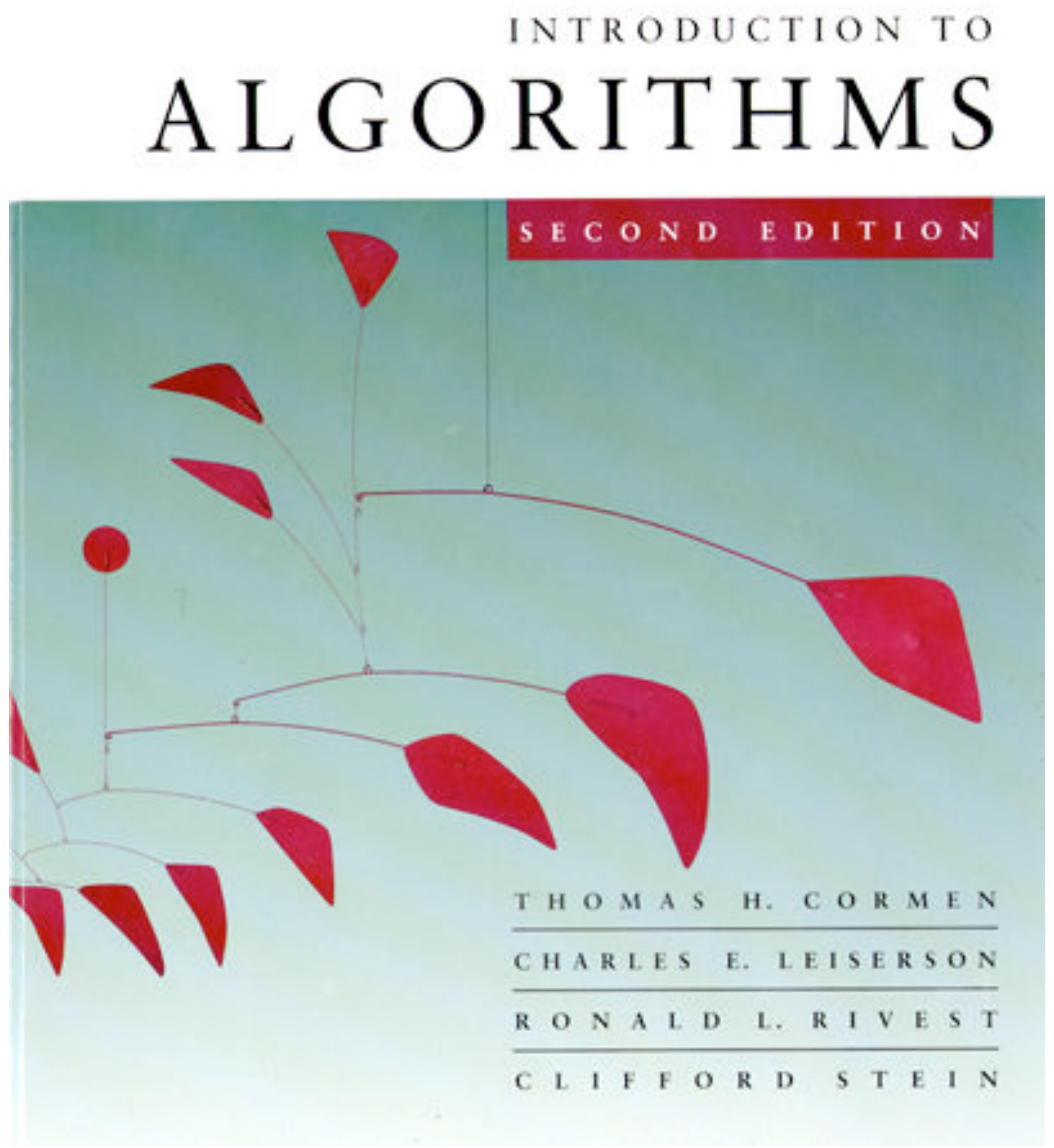
DIV & CONQ

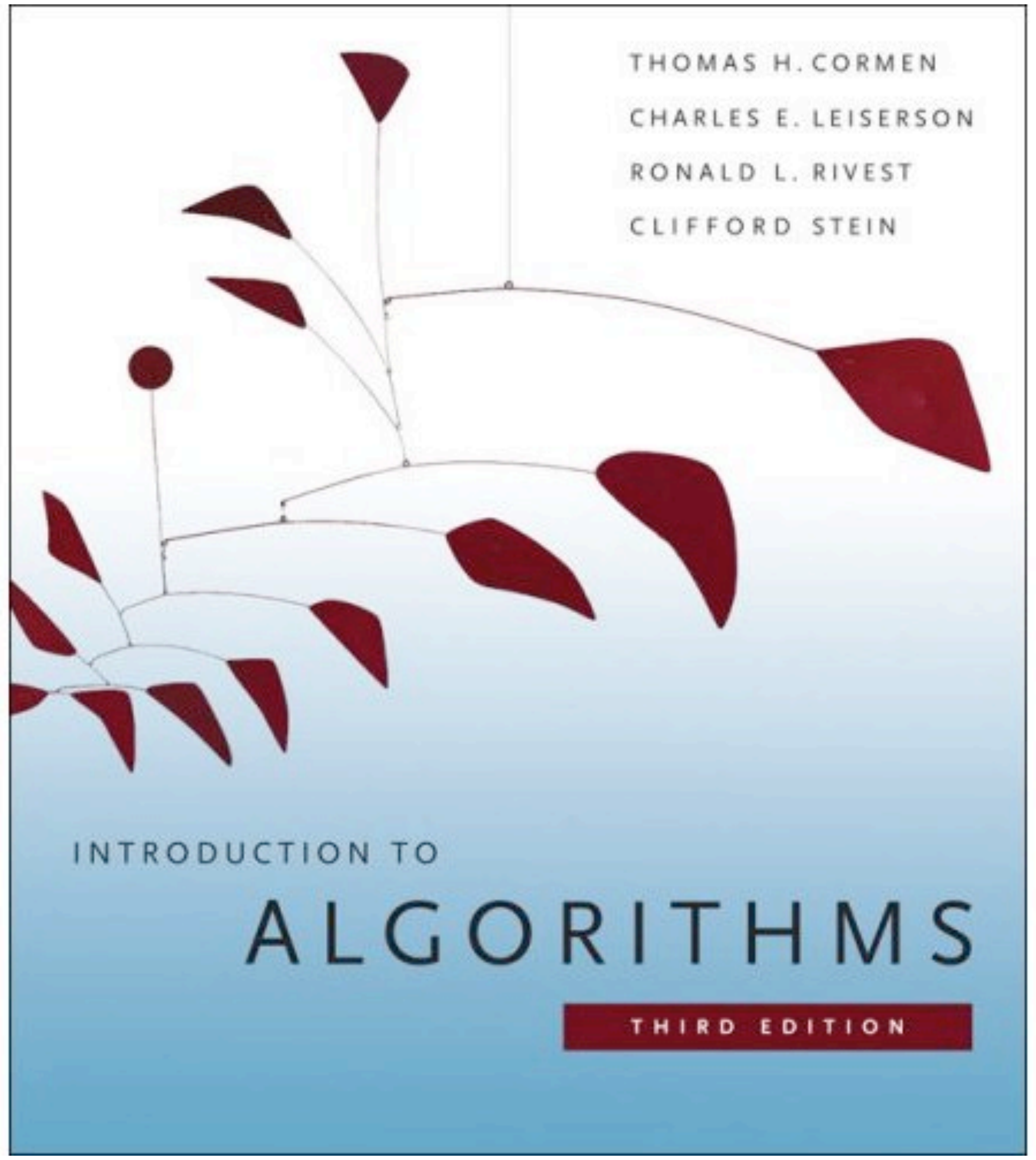
GREEDY

NP

- data structures
- hashing
- numerical algorithm

cryptographic algorithms

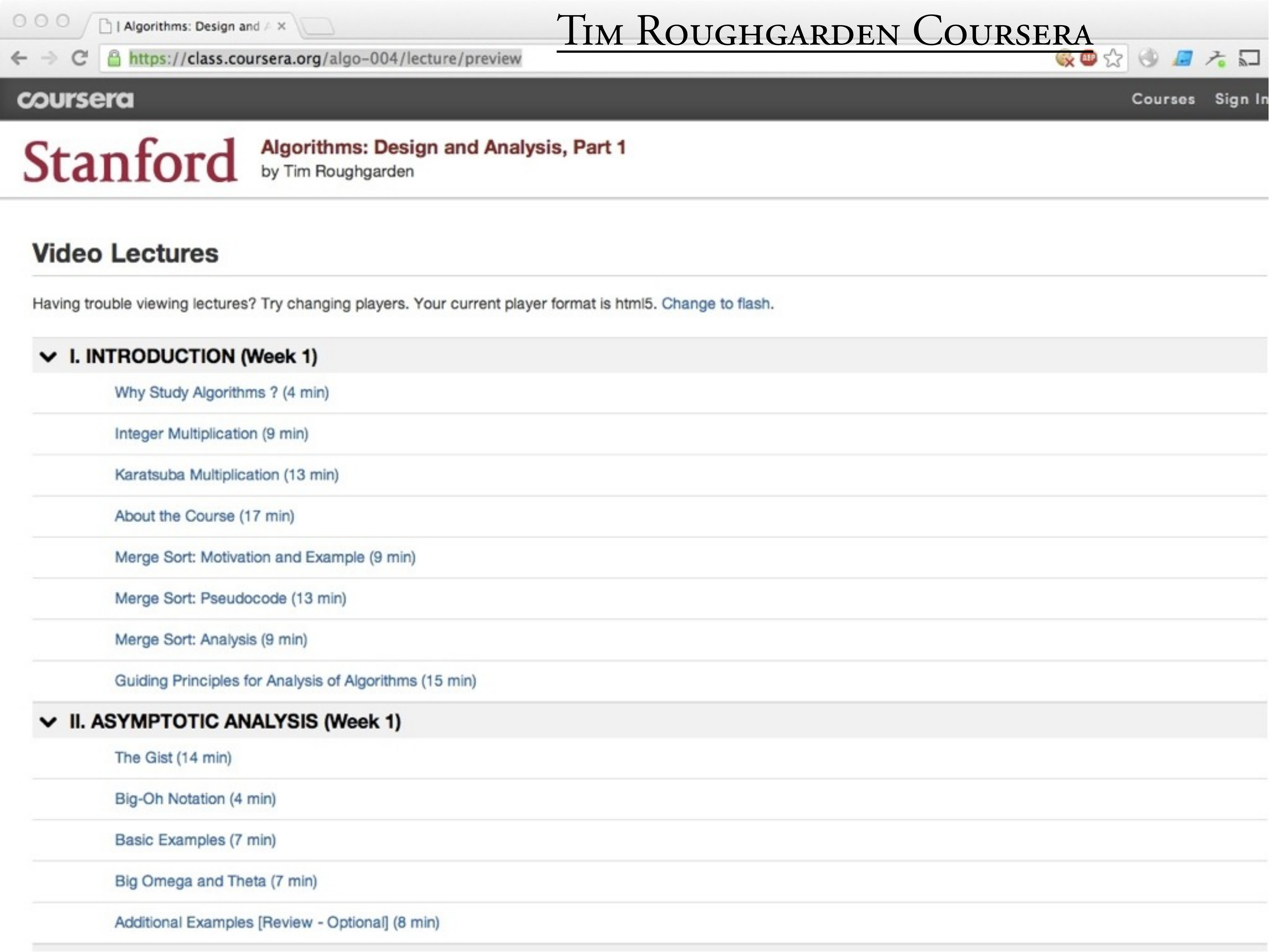




CLRS

Kleinberg Tardos

TIM ROUGHGARDEN COURSERA



Algorithms: Design and Analysis, Part 1
by Tim Roughgarden

Video Lectures

Having trouble viewing lectures? Try changing players. Your current player format is html5. Change to flash.

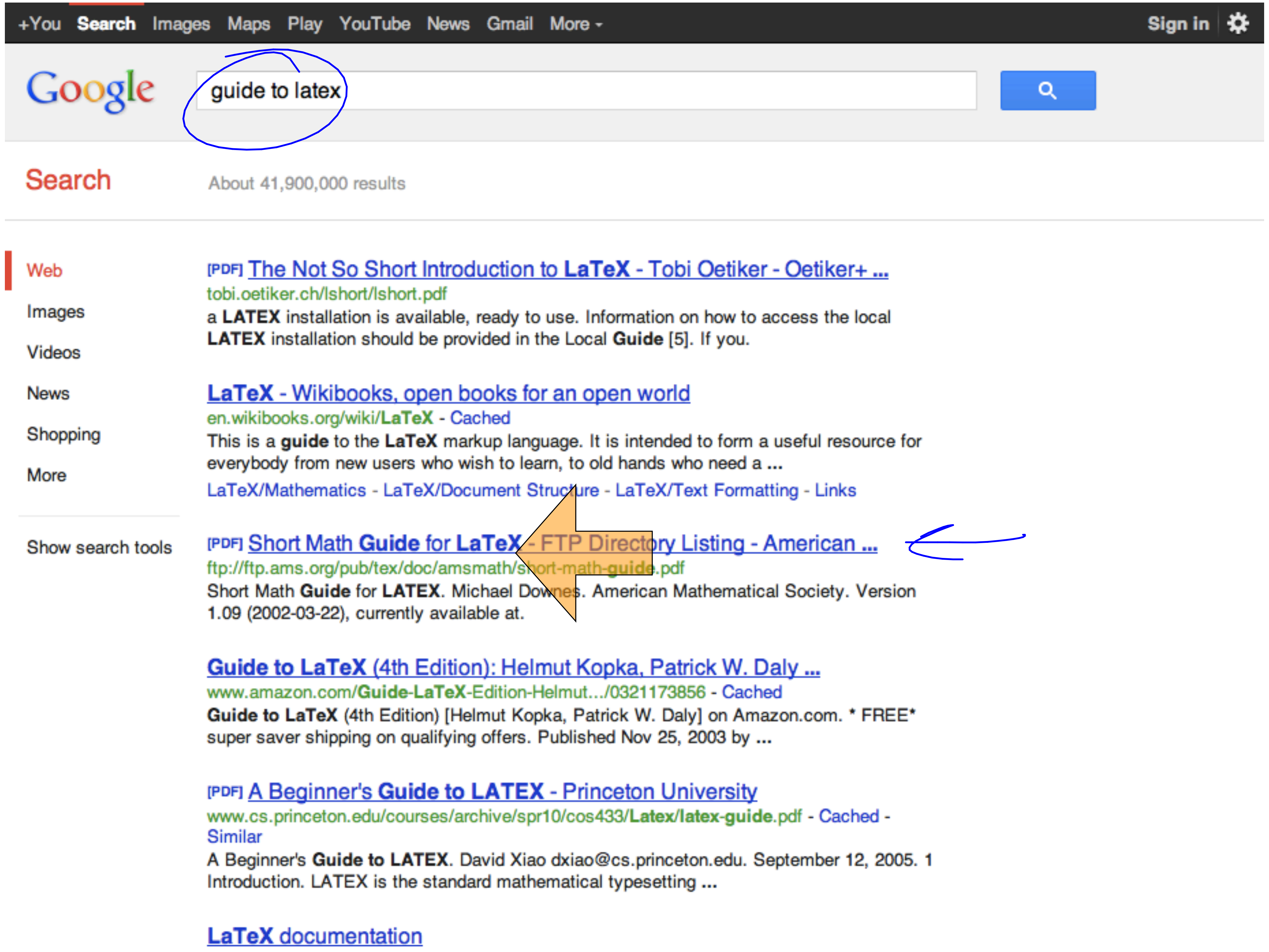
I. INTRODUCTION (Week 1)

- Why Study Algorithms ? (4 min)
- Integer Multiplication (9 min)
- Karatsuba Multiplication (13 min)
- About the Course (17 min)
- Merge Sort: Motivation and Example (9 min)
- Merge Sort: Pseudocode (13 min)
- Merge Sort: Analysis (9 min)
- Guiding Principles for Analysis of Algorithms (15 min)

II. ASYMPTOTIC ANALYSIS (Week 1)

- The Gist (14 min)
- Big-Oh Notation (4 min)
- Basic Examples (7 min)
- Big Omega and Theta (7 min)
- Additional Examples [Review - Optional] (8 min)

LaTeX



guide to latex

About 41,900,000 results

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tobi.oetiker.ch/lshort/lshort.pdf
a **LATEX** installation is available, ready to use. Information on how to access the local **LATEX** installation should be provided in the Local **Guide** [5]. If you.

LaTeX - Wikibooks, open books for an open world
en.wikibooks.org/wiki/**LaTeX** - Cached
This is a **guide** to the **LaTeX** markup language. It is intended to form a useful resource for everybody from new users who wish to learn, to old hands who need a **...**
LaTeX/Mathematics - LaTeX/Document Structure - LaTeX/Text Formatting - Links

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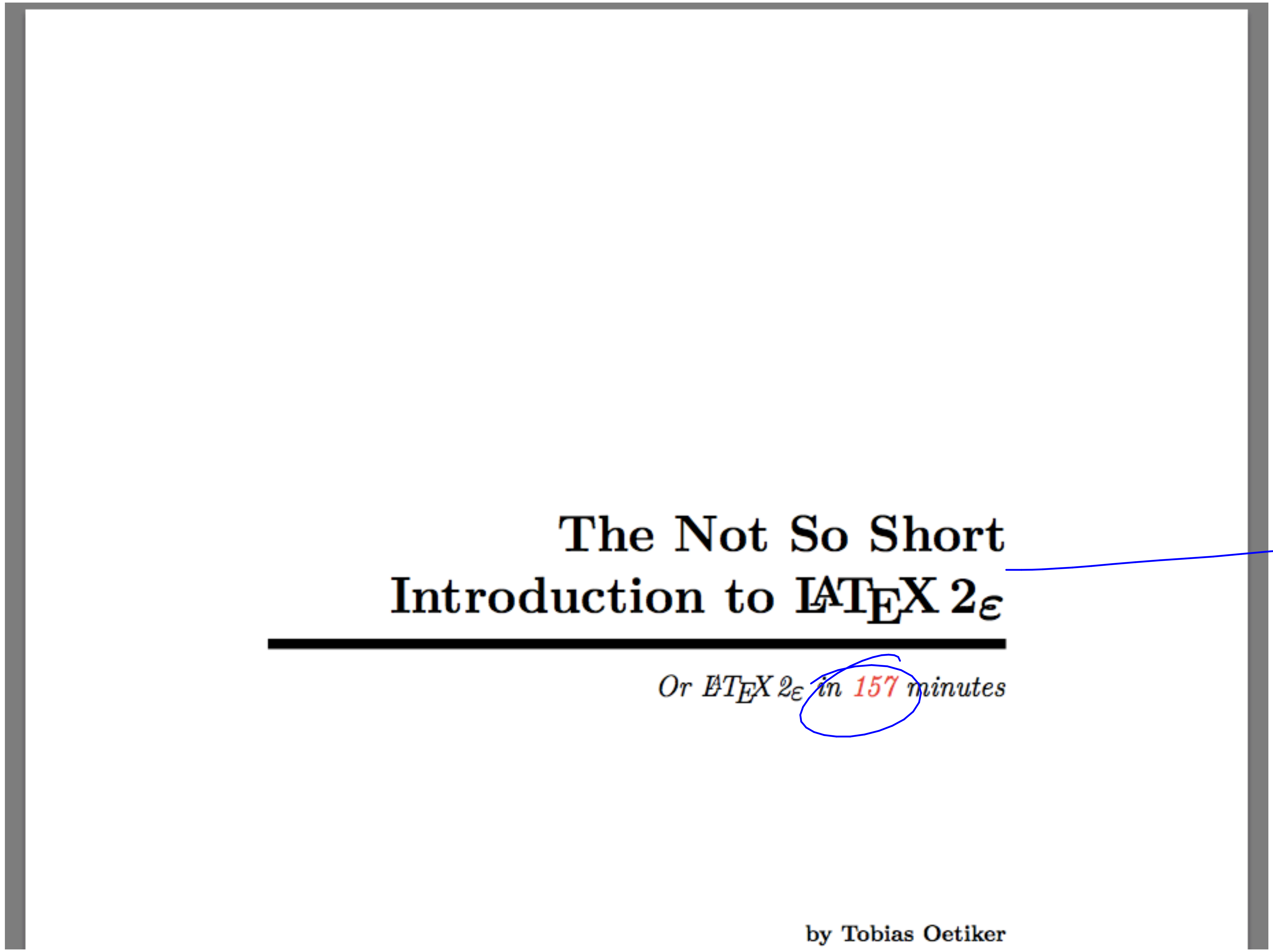
www.cs.princeton.edu/courses/archive/spr10/cos433/**Latex**/**latex**-**guide**.pdf - Cached - Similar
A Beginner's **Guide to LATEX**. David Xiao dxiao@cs.princeton.edu. September 12, 2005. 1 Introduction. LATEX is the standard mathematical typesetting **...**

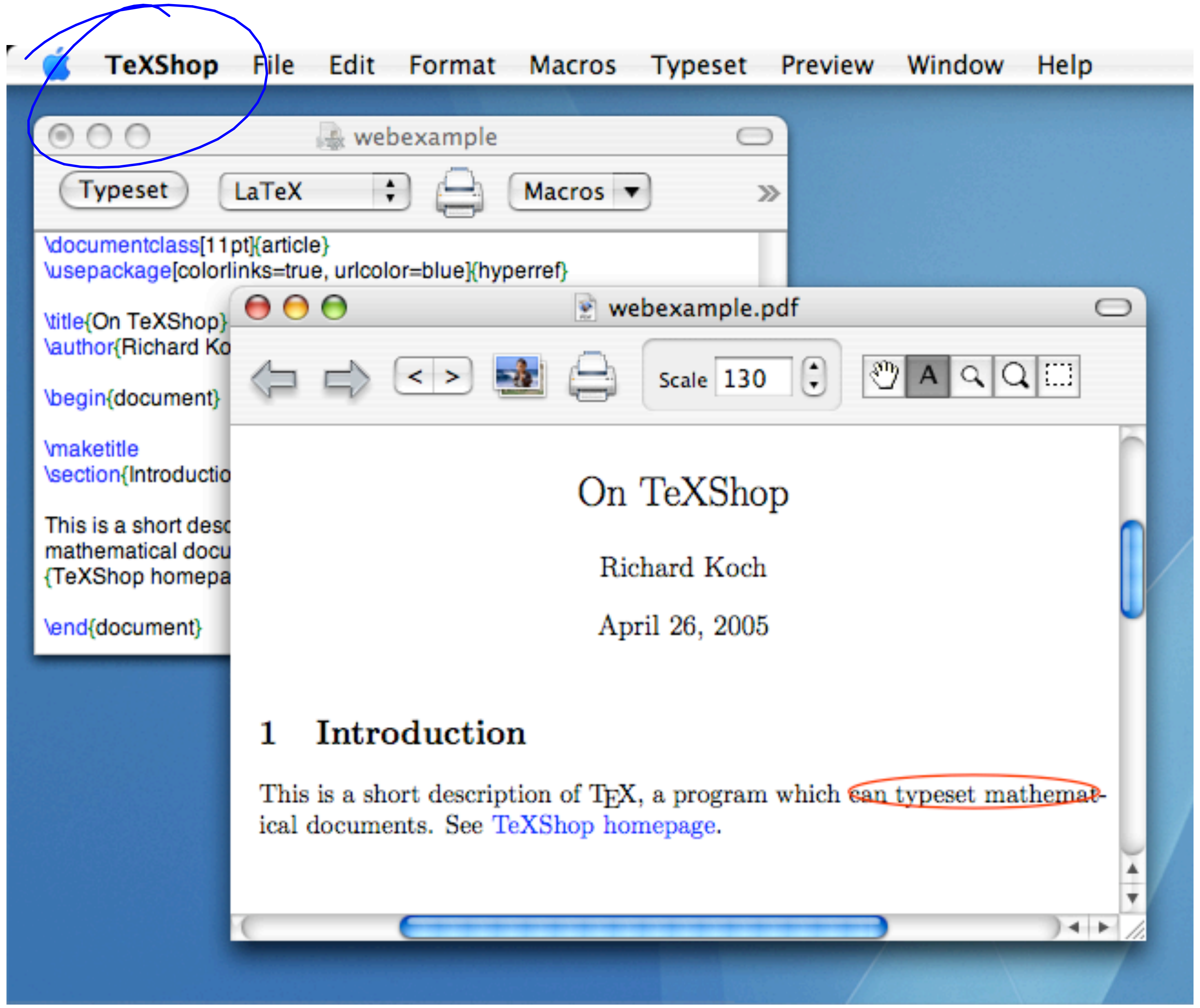
LaTeX documentation



The Not So Short Introduction to $\LaTeX\,2_\varepsilon$

Or $\LaTeX\,2_\varepsilon$ in 157 minutes

by Tobias Oetiker



```
\documentclass[11pt]{article}
\usepackage[colorlinks=true, urlcolor=blue]{hyperref}

\title{On TeXShop}
\author{Richard Ko

\begin{document}

\maketitle
\section{Introductio

This is a short desc
mathematical docu
{TeXShop homepa

\end{document}
```

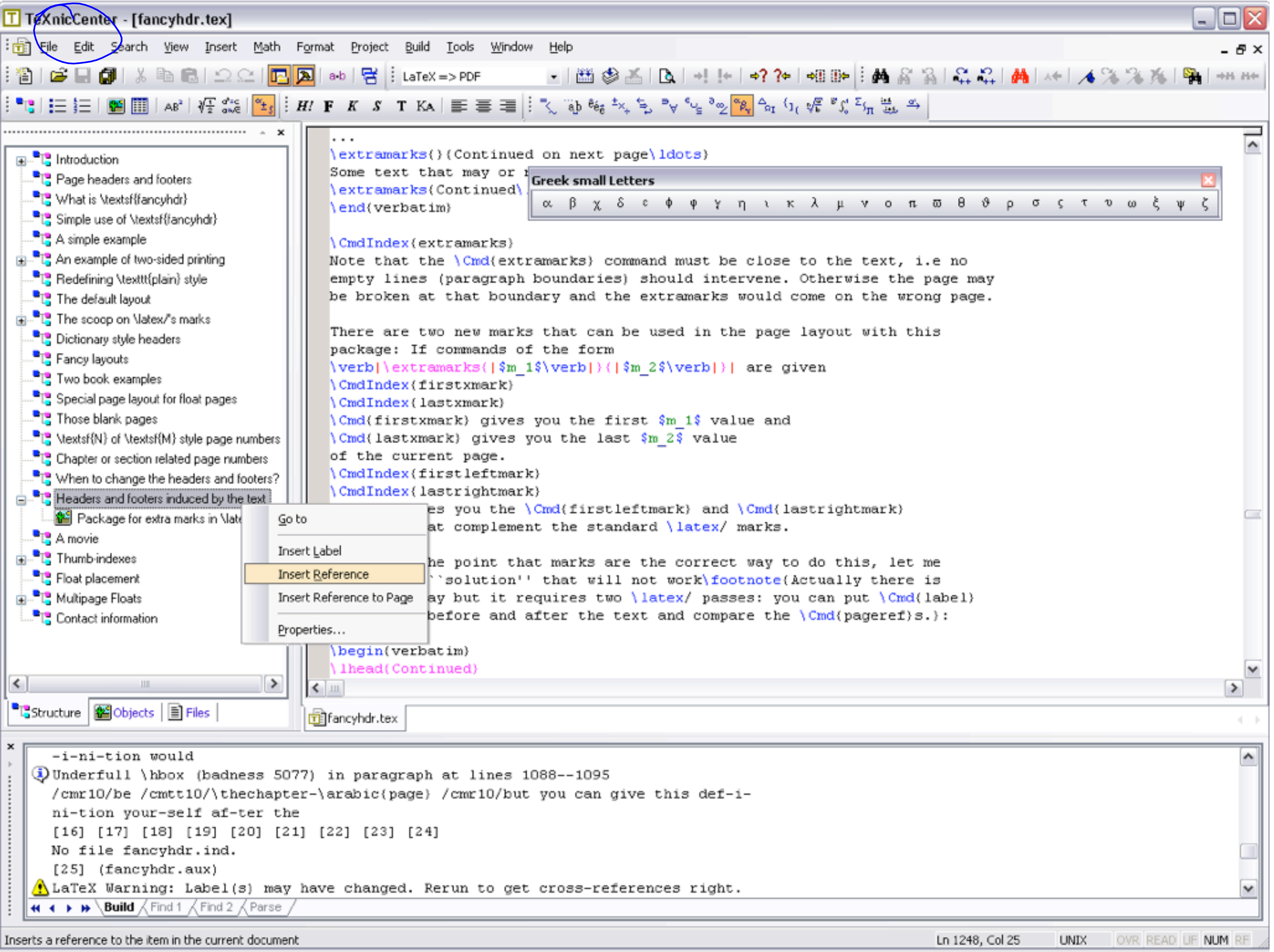
On TeXShop

Richard Koch

April 26, 2005

1 Introduction

This is a short description of TEX, a program which can typeset mathematical documents. See TeXShop homepage.



```
...
\extramarks{}{Continued on next page\ldots}
Some text that may or 
\extramarks{Continued\ 
\end{verbatim}

\CmdIndex{extramarks}
Note that the \Cmd{extramarks} command must be close to the text, i.e no
empty lines (paragraph boundaries) should intervene. Otherwise the page may
be broken at that boundary and the extramarks would come on the wrong page.

There are two new marks that can be used in the page layout with this
package: If commands of the form
\verb|\extramarks{|$m_1$\verb|}{|$m_2$\verb|}| are given
\CmdIndex{firstxmark}
\CmdIndex{lastxmark}
\Cmd{firstxmark} gives you the first $m_1$ value and
\Cmd{lastxmark} gives you the last $m_2$ value
of the current page.
\CmdIndex{firstleftmark}
\CmdIndex{lastrightmark}
                es you the \Cmd{firstleftmark} and \Cmd{lastrightmark}
                at complement the standard \latex/ marks.

                he point that marks are the correct way to do this, let me
                ``solution'' that will not work\footnote{Actually there is
                ay but it requires two \latex/ passes: you can put \Cmd{label}
                before and after the text and compare the \Cmd{pageref}s.}:

\begin{verbatim}
\lhead{Continued}
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ni-tion your-self af-ter the
[16] [17] [18] [19] [20] [21] [22] [23] [24]
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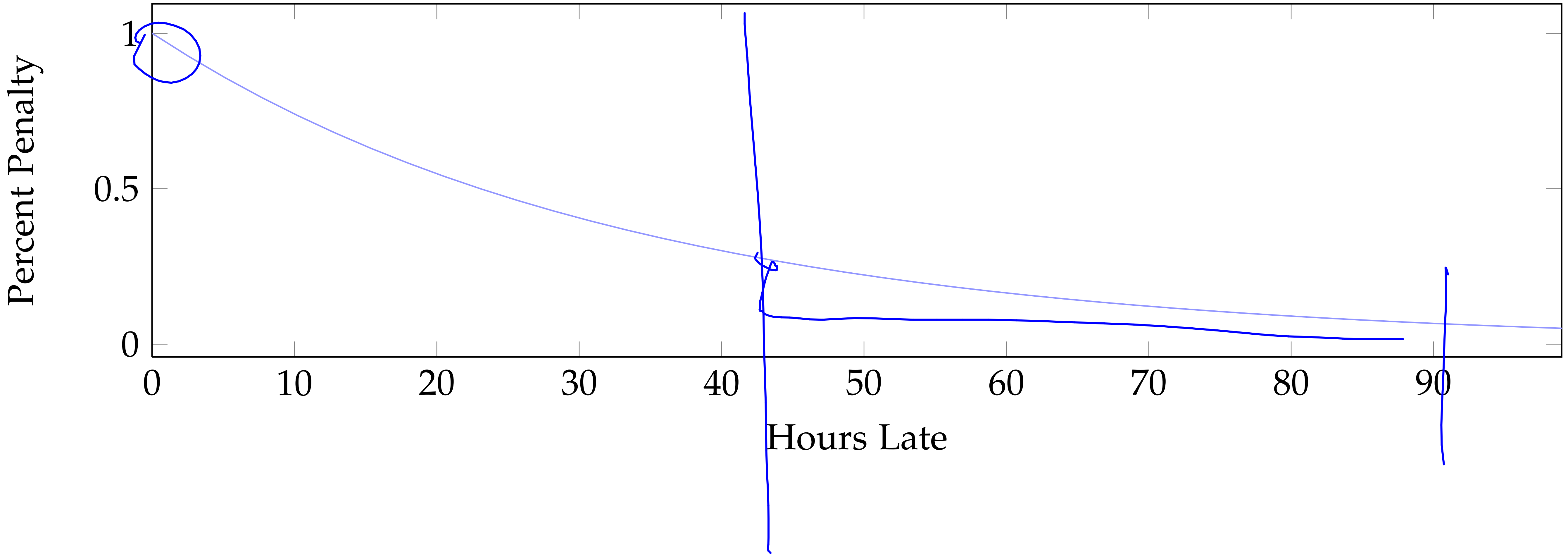
LATE POLICY



COUNTING

1 STAND

❶ STAND

❷ SET YOUR “NUMBER” TO ONE

1. STAND
2. SET YOUR “NUMBER” TO ONE
3. GREET A NEIGHBOR (PAUSE IF ODD PERSON OUT)

❶ STAND

❷ SET YOUR “NUMBER” TO ONE

❸ GREET A NEIGHBOR (PAUSE IF ODD PERSON OUT)

❹ IF YOU ARE OLDER, GIVE YOUR “NUMBER” TO YOUNG AND SIT
IF YOU ARE YOUNGER, ADD “NUMBERS”

123 people

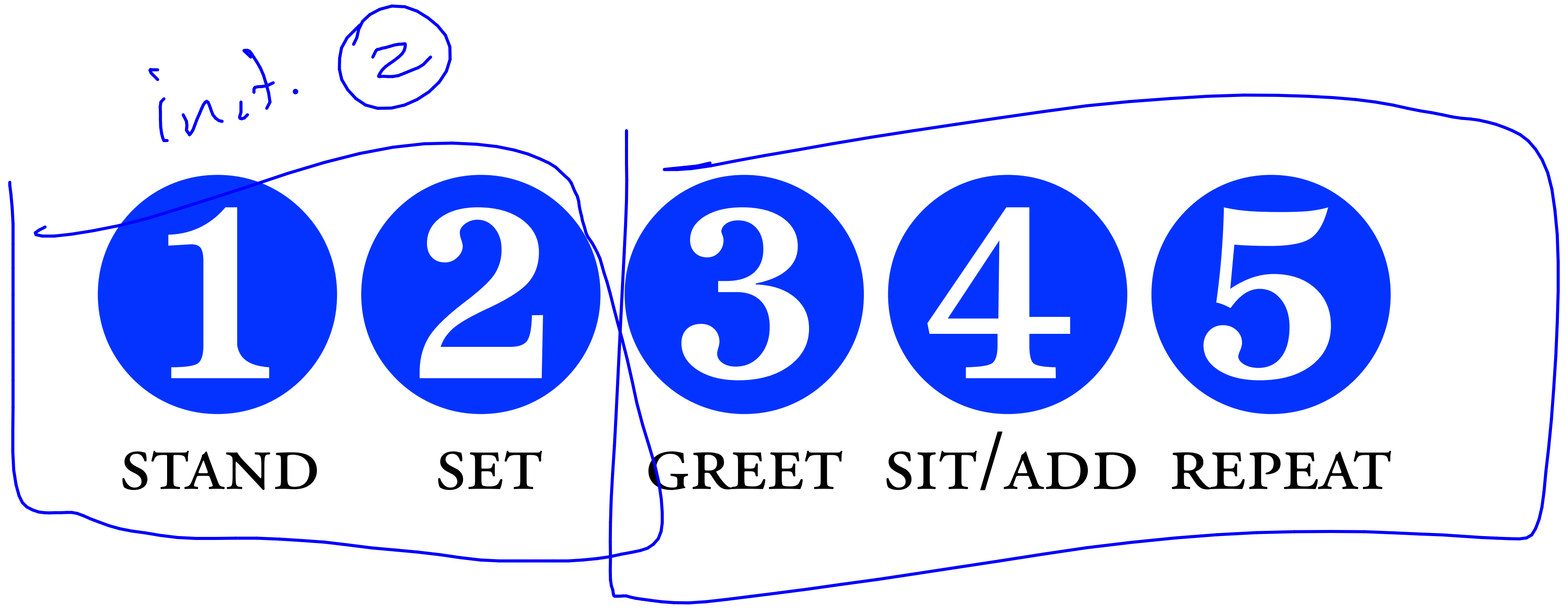
1. STAND
2. SET YOUR "NUMBER" TO ONE
3. GREET A NEIGHBOR (PAUSE IF ODD PERSON OUT)
4. IF YOU ARE OLDER, GIVE YOUR "NUMBER" TO YOUNG AND SIT
   IF YOU ARE YOUNGER, ADD "NUMBERS"
5. IF YOU ARE STANDING & YOU HAVE A NEIGHBOR,
   GOTO 3

① STAND
② SET
③ GREET
④ SIT/ADD
⑤ REPEAT

LETS ANALYZE THIS ALG

①	②	③	④	⑤
STAND	SET	GREET	SIT/ADD	REPEAT

HOW FAST DOES IT WORK:



init. ②

1. STAND
2. SET
3. GREET
4. SIT/ADD
5. REPEAT

HOW FAST DOES IT WORK:

$$\underbrace{T(n)}$$

TIME TO FINISH FOR A ROOM OF SIZE N

1	2	3	4	5
STAND	SET	GREET	SIT/ADD	REPEAT
		1	1	↺

HOW FAST DOES IT WORK:

$$\underline{\underline{T(n)}} = 1 + 1 + T(\lceil \tfrac{n}{2} \rceil)$$

$$2 + T(\lceil \tfrac{n}{2} \rceil)$$

① STAND ② SET

③ GREET ④ SIT/ADD ⑤ REPEAT

HOW FAST DOES IT WORK:

$$T(n)$$

TIME TO FINISH FOR A ROOM OF SIZE N

1 STAND 2 SET 3 GREET 4 SIT/ADD 5 REPEAT

HOW FAST DOES IT WORK:

$$\underbrace{T(n) = 1 + 1 + T(\lceil n/2 \rceil)}_{}$$

Recurrences.

Define.

application of the function on a smaller argument.

$$T(1) = \underline{\underline{2}}$$

$$T(2^k) = 2 + T(2^{k-1})$$

RECURRENCE?

$$T(2^k) = 2 + T(2^{k-1})$$

“INTUITION HERE”

$$\begin{aligned} T(2^k) &= 2 + T(2^{k-1}) \\ &= 2 + 2 + T(2^{k-2}) \end{aligned}$$

“INTUITION HERE”

$$
\begin{aligned}
T(2^k) &= 2 + T(2^{k-1}) \\
&= 2 + 2 + T(2^{k-2}) \\
&= \overbrace{2 + 2 + \cdots + 2}^{k} + T(2)
\end{aligned}
$$

"INTUITION HERE"

$$\begin{aligned} T(2^k) &= 2 + T(2^{k-1}) \\ &= 2 + 2 + T(2^{k-2}) \\ &= \overbrace{2 + 2 + \cdots + 2}^{k} + T(2) \\ &= 2k + 2 \ = O(\log(2^k)) \end{aligned}$$

"INTUITION HERE"

$$
\begin{aligned}
T(2^k) &= 2 + T(2^{k-1}) \\
&= 2 + 2 + T(2^{k-2}) \\
&= \overbrace{2 + 2 + \cdots + 2}^{k} + T(2) \\
&= 2k + 2 \ = O(\log(2^k))
\end{aligned}
$$

"INTUITION HERE"

$$\forall 0 < n < m, T(n) \leq T(m)$$

$$
\begin{aligned}
T(2^k) &= 2 + T(2^{k-1}) \\
&= 2 + 2 + T(2^{k-2}) \\
&= \overbrace{2 + 2 + \cdots + 2}^{k} + T(2) \\
&= 2k + 2 \;= O(\log(2^k))
\end{aligned}
$$

"INTUITION HERE"

$$\forall 0 < n < m, T(n) \leq T(m)$$

$$T(m) \leq T(2^{\lceil \log(m) \rceil})$$

$$
\begin{aligned}
T(2^k) &= 2 + T(2^{k-1}) \\
&= 2 + 2 + T(2^{k-2}) \\
&= \overbrace{2 + 2 + \cdots + 2}^{k} + T(2) \\
&= 2k + 2 \ = O(\log(2^k))
\end{aligned}
$$

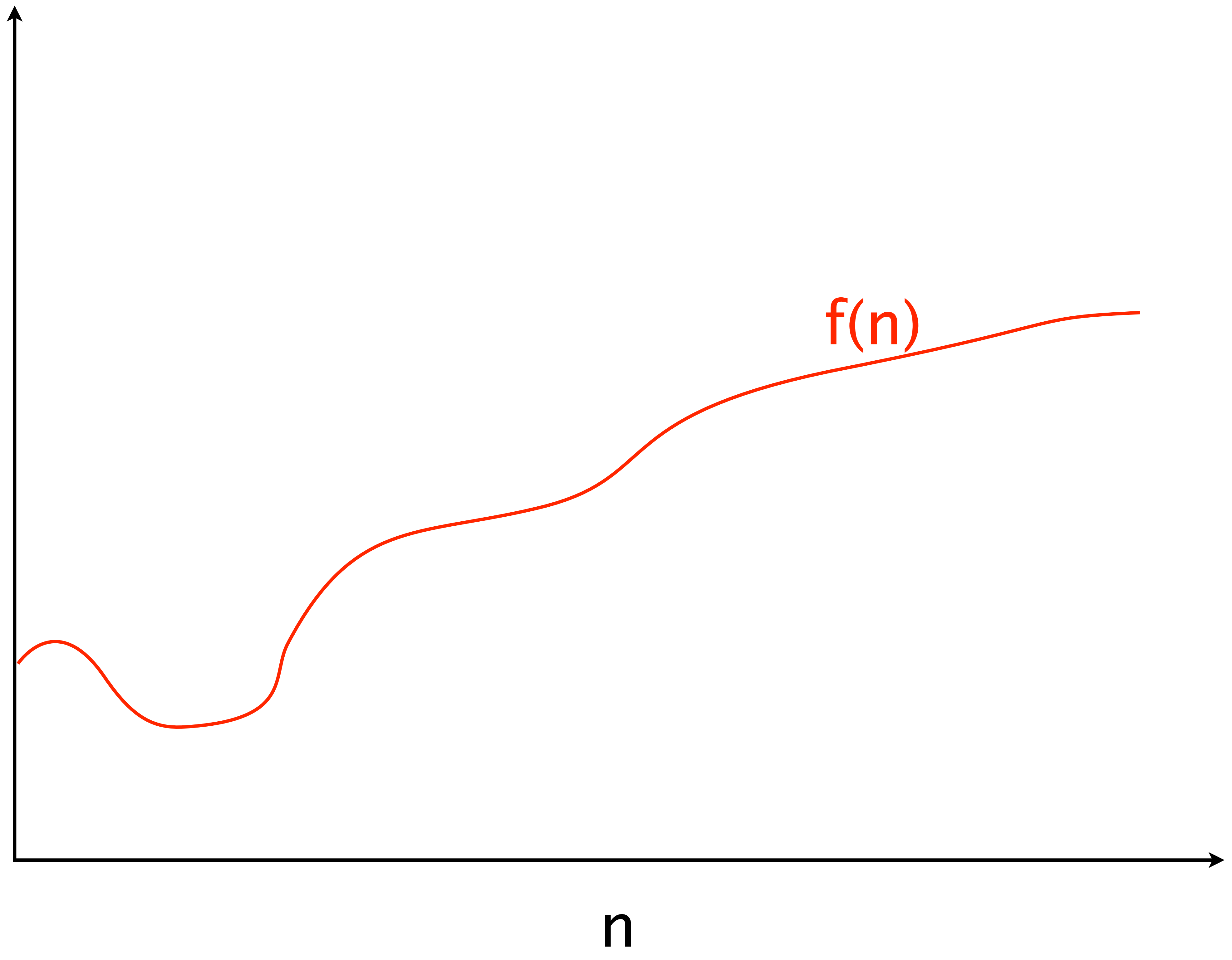
“INTUITION HERE”

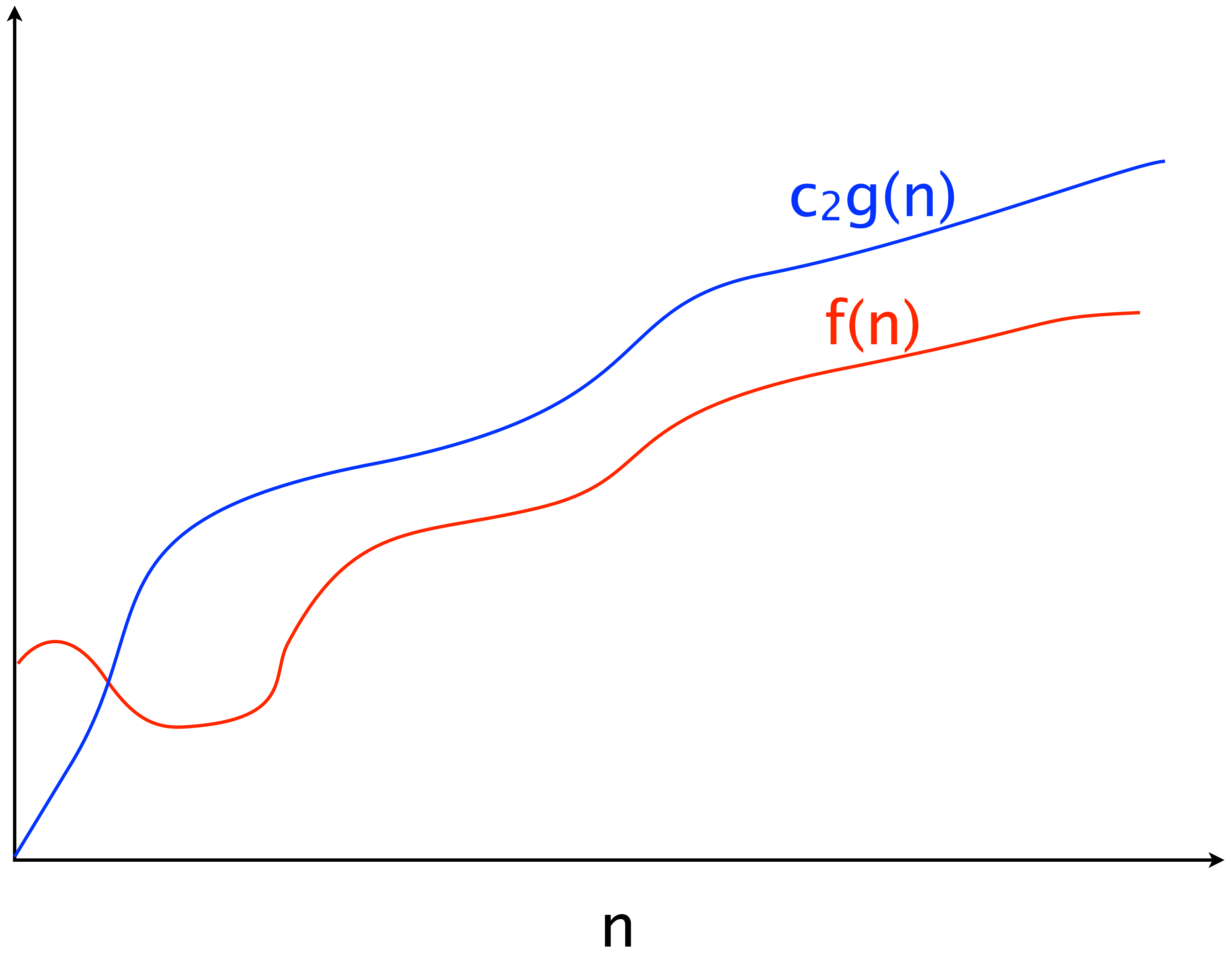
$$\forall 0 < n < m, T(n) \leq T(m)$$

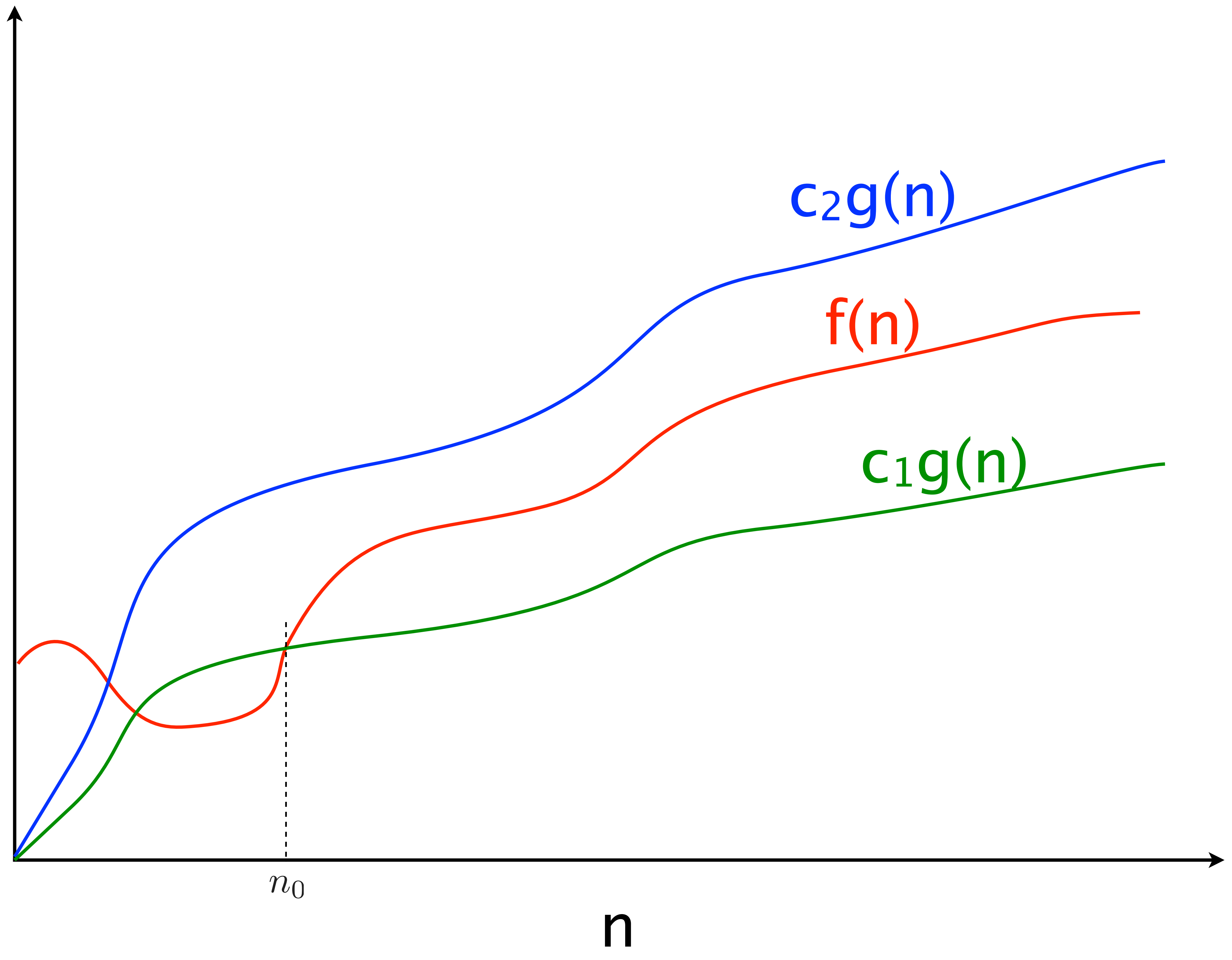
$$T(m) \leq T(2^{\lceil \log(m) \rceil}) = 2\lceil \log(m) \rceil + 2$$

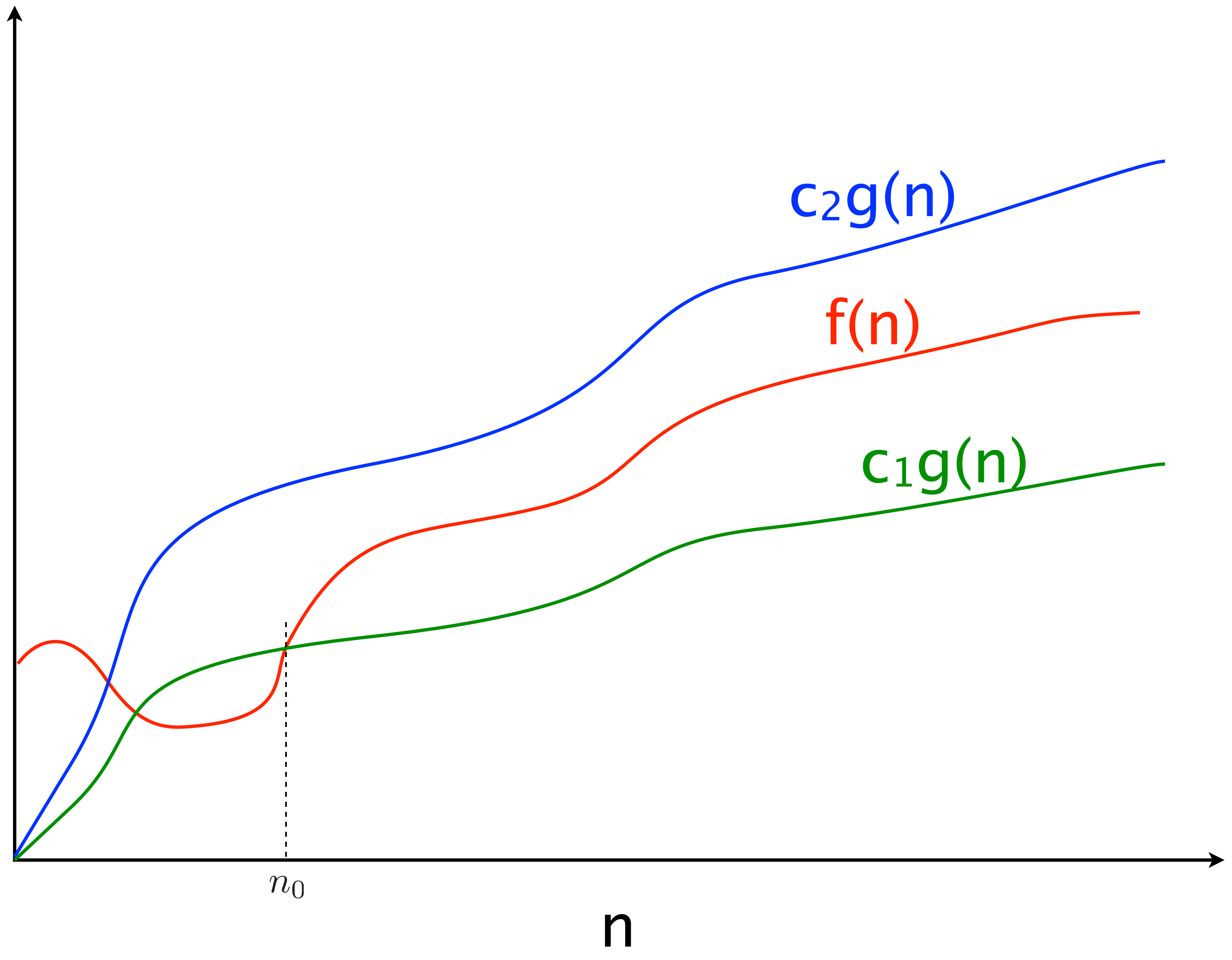
ASYMPTOTIC NOTATION

$O(f)$	at most within const of f	for large n
$\Omega(f)$	at least within const of f	for large n
$\Theta(f)$	within a const of f	for large n

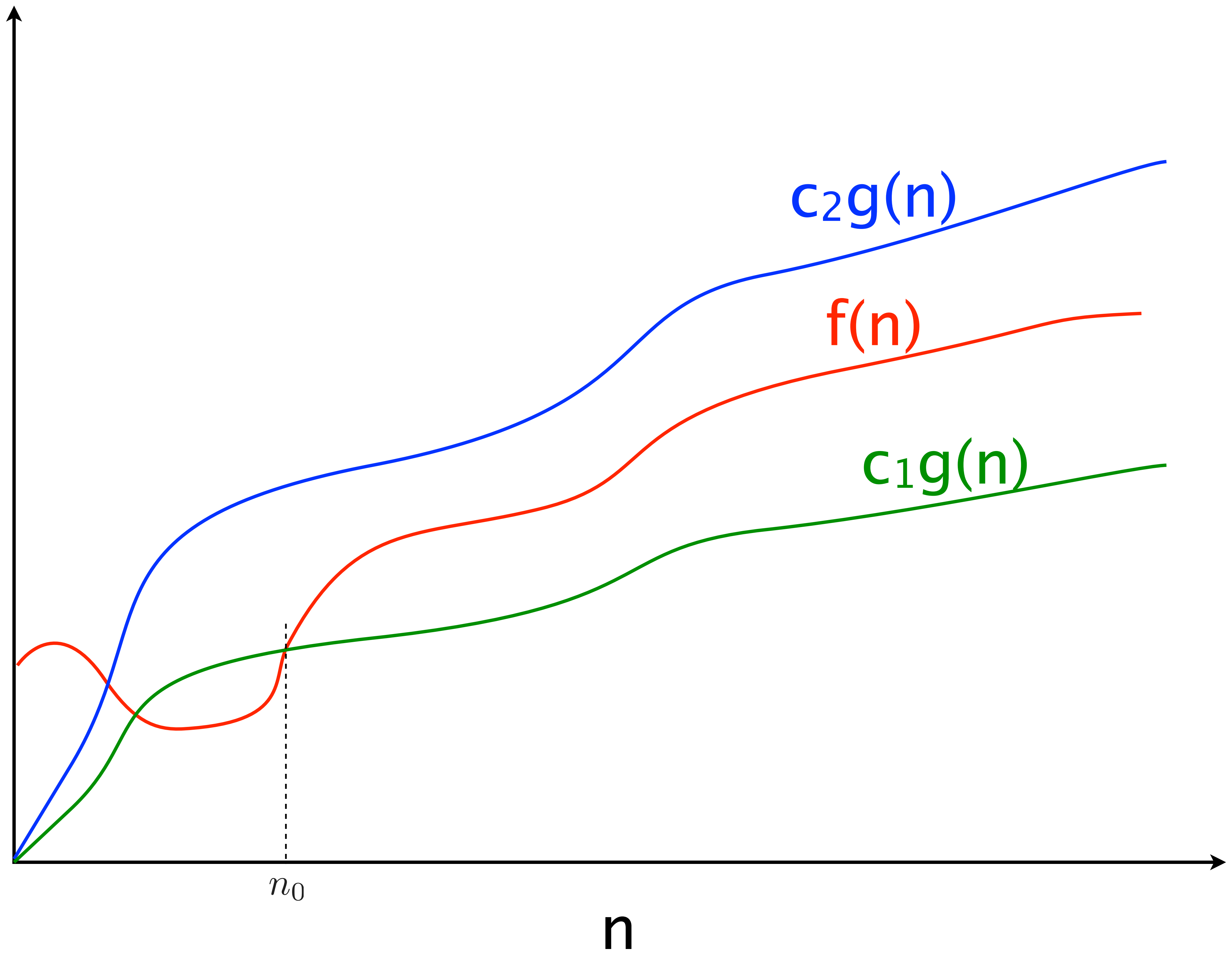

f(n)
n


$c_2g(n)$
$f(n)$
n


$c_2g(n)$
$f(n)$
$c_1g(n)$
n_0
n

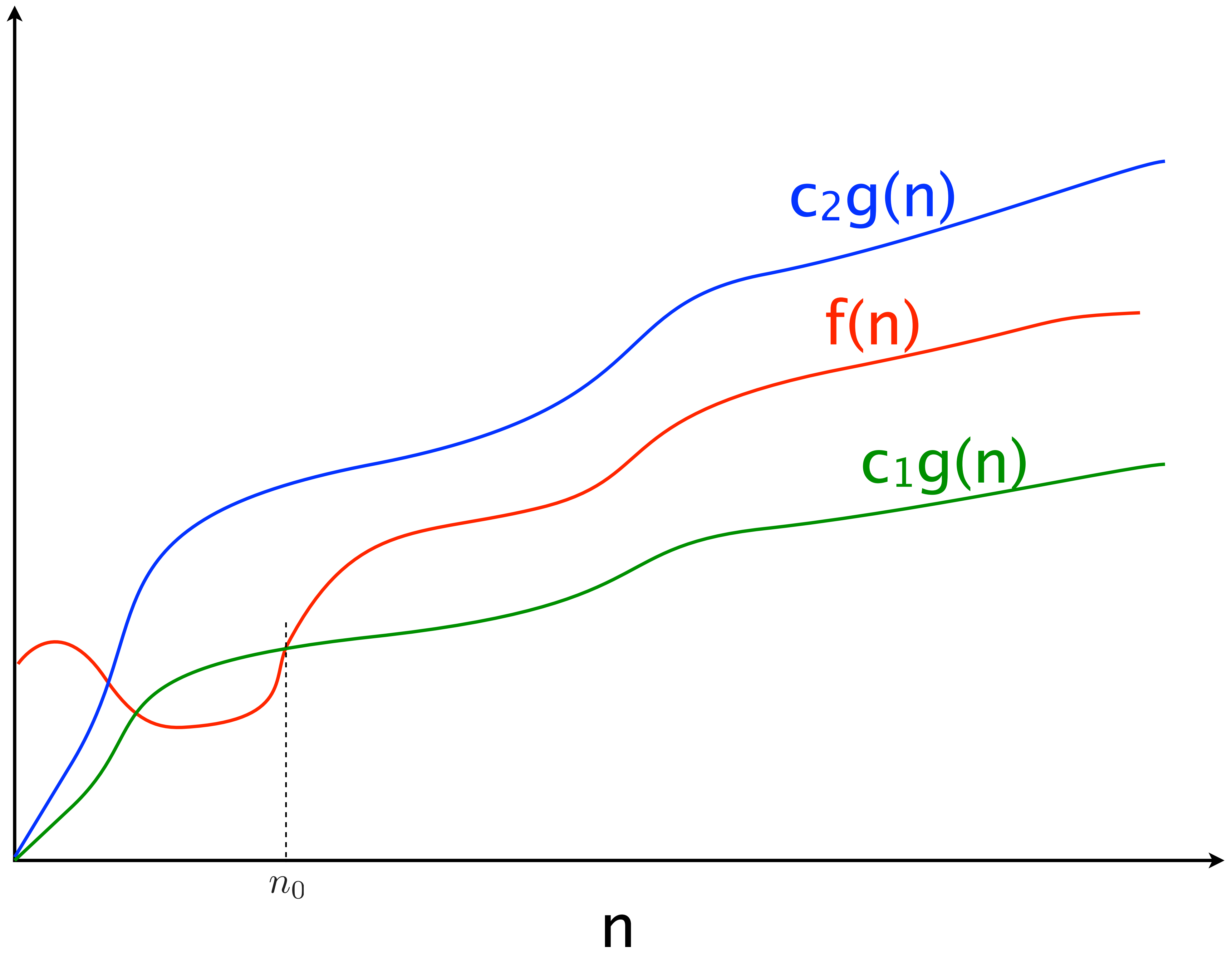


$$f(n) = O(g(n))$$



$$f(n) = O(g(n))$$

$$f(n) = \Omega(g(n))$$



$$f(n) = O(g(n))$$

$$f(n) = \Theta(g(n)$$

$$f(n) = \Omega(g(n))$$

$$
\begin{aligned}
T(2^k) &= 2 + T(2^{k-1}) \\
&= 2 + 2 + T(2^{k-2}) \\
&= \overbrace{2 + 2 + \cdots + 2}^{k} + T(2) \\
&= 2k + 2 \ = O(\log(2^k))
\end{aligned}
$$

“INTUITION HERE”

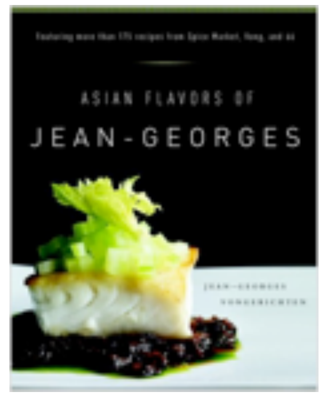
$$\forall 0 < n < m, T(n) \leq T(m)$$

$$T(m) \leq T(2^{\lceil \log(m) \rceil}) = 2\lceil \log(m) \rceil + 2 \ = O(\log(m))$$

$$
\begin{aligned}
T(m) &= \Omega(\log(m)) \\
&= \Theta(\log(m))
\end{aligned}
$$

How to
solve
recurrence

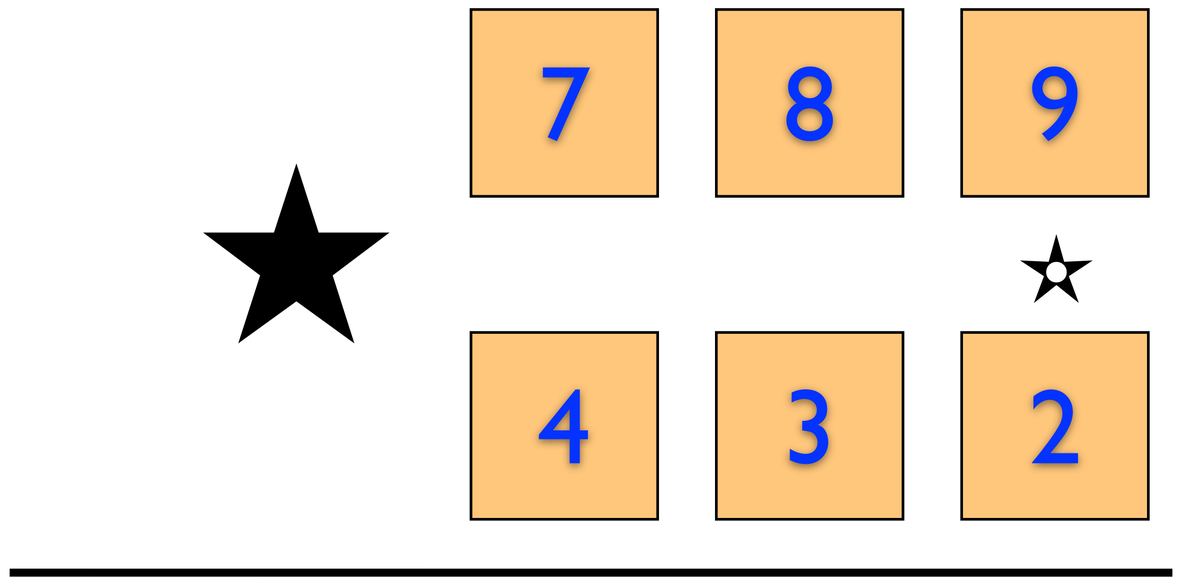
?-✓

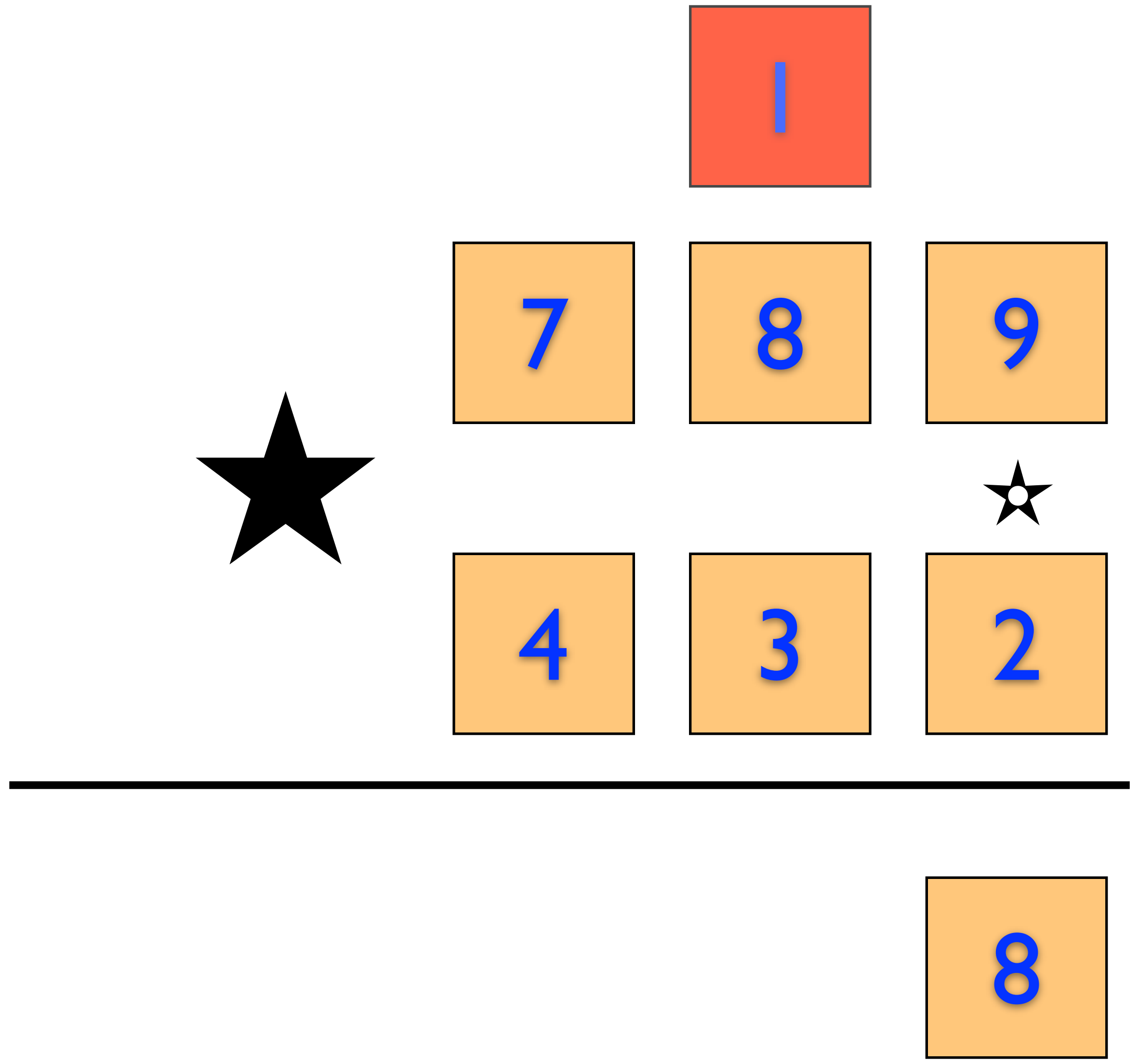


ASIAN FLAVORS OF
JEAN-GEORGES

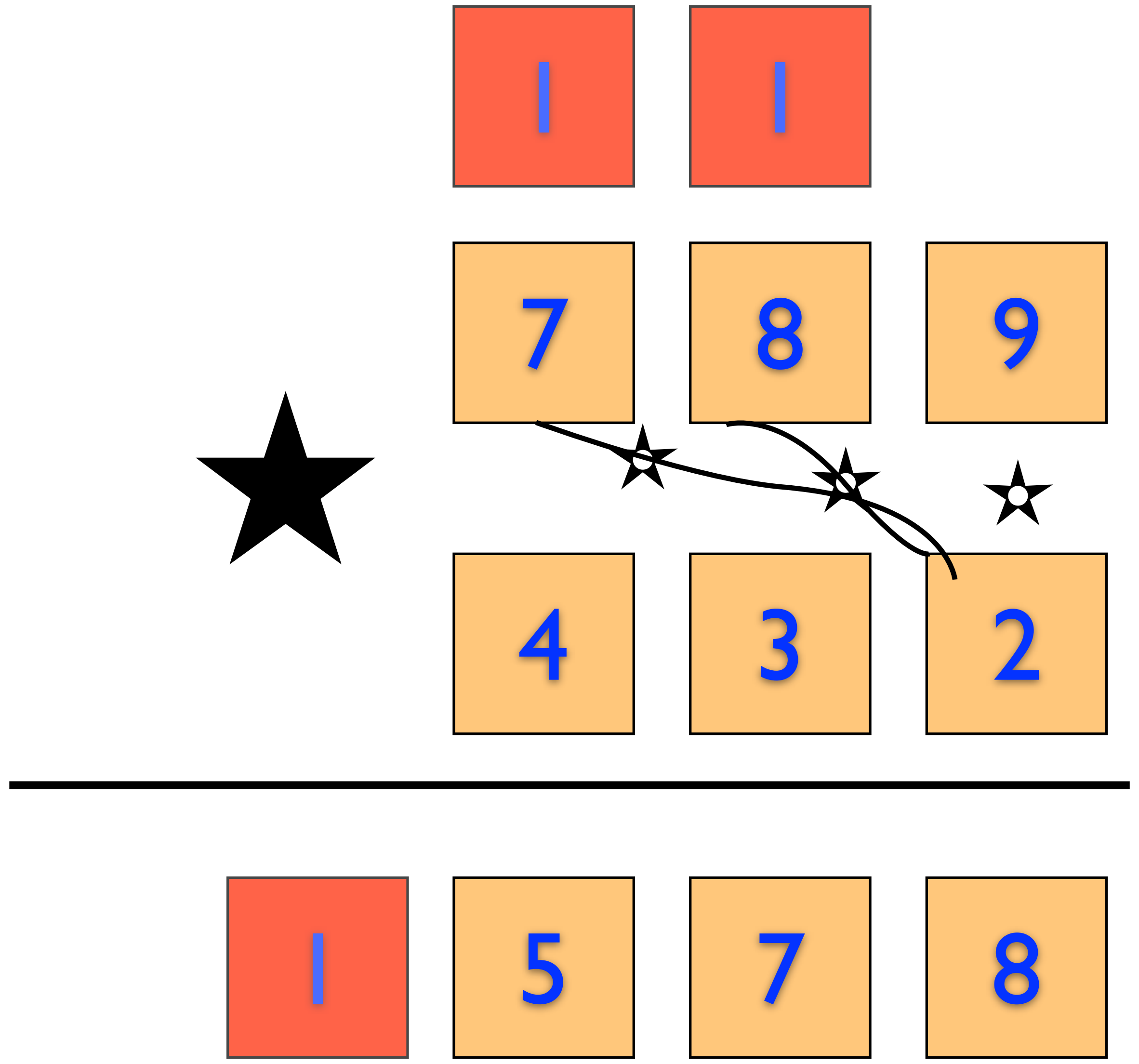
http://www.drblank.com/law301.jpg

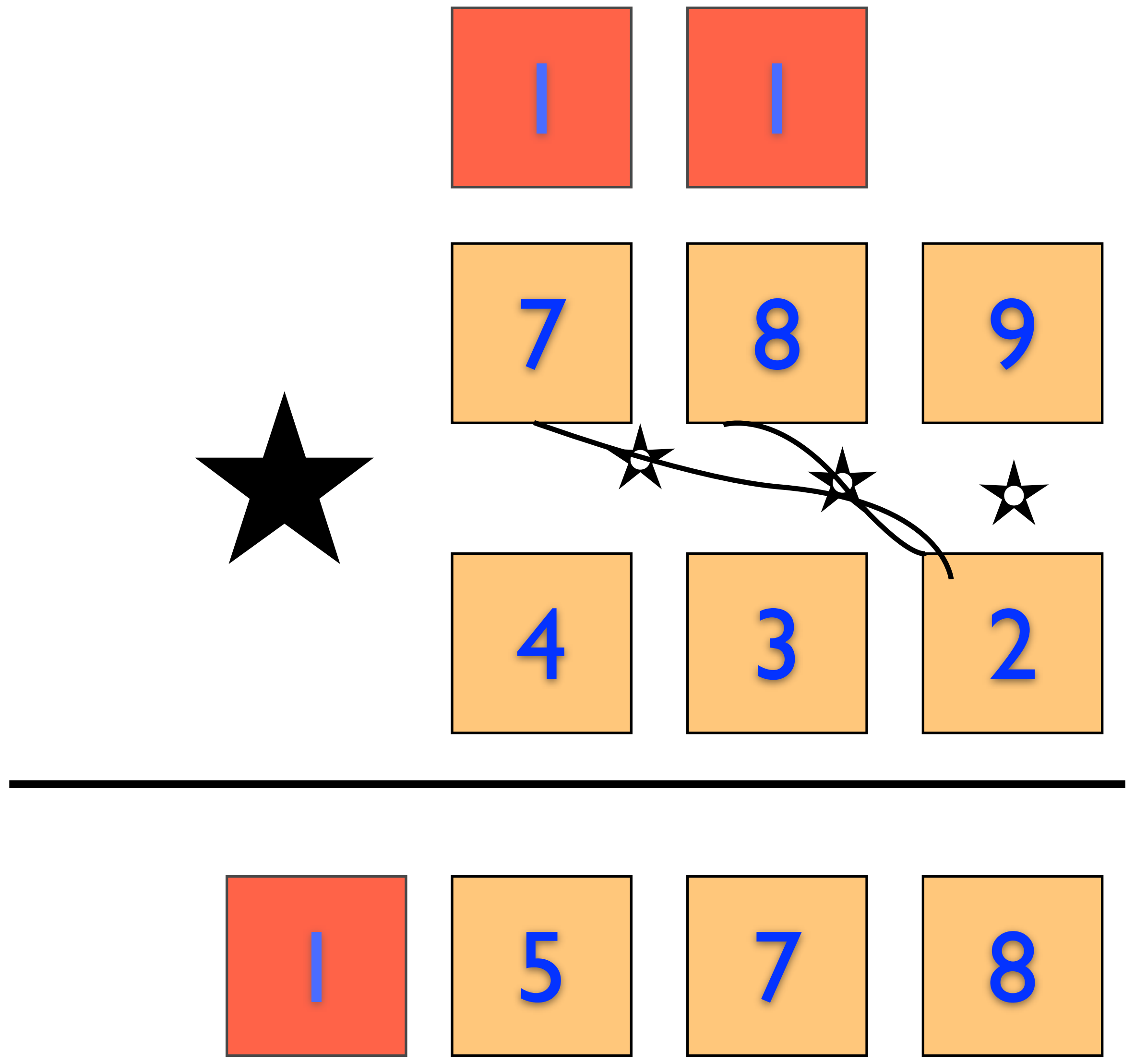
$$\begin{array}{cccc} & 7 & 8 & 9 \\ \bigstar & & & \\ & 4 & 3 & 2 \\ \hline \end{array}$$


7
8
9
4
3
2

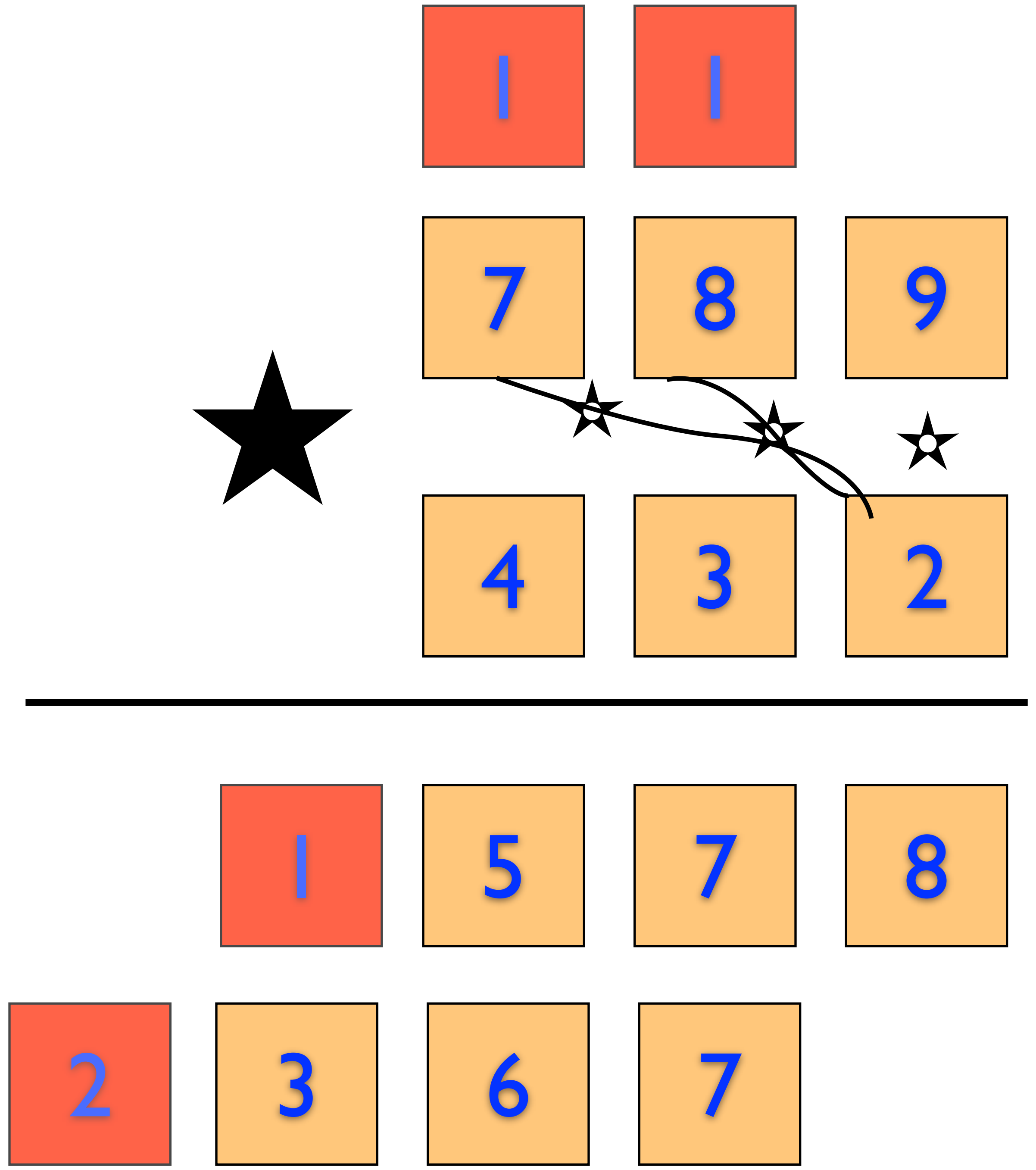


$$\begin{array}{rccc} & & 1 & \\ & 7 & 8 & 9 \\ \star & 4 & 3 & 2 \\ \hline & & & 8 \end{array}$$


1 1
7 8 9
4 3 2
1 5 7 8

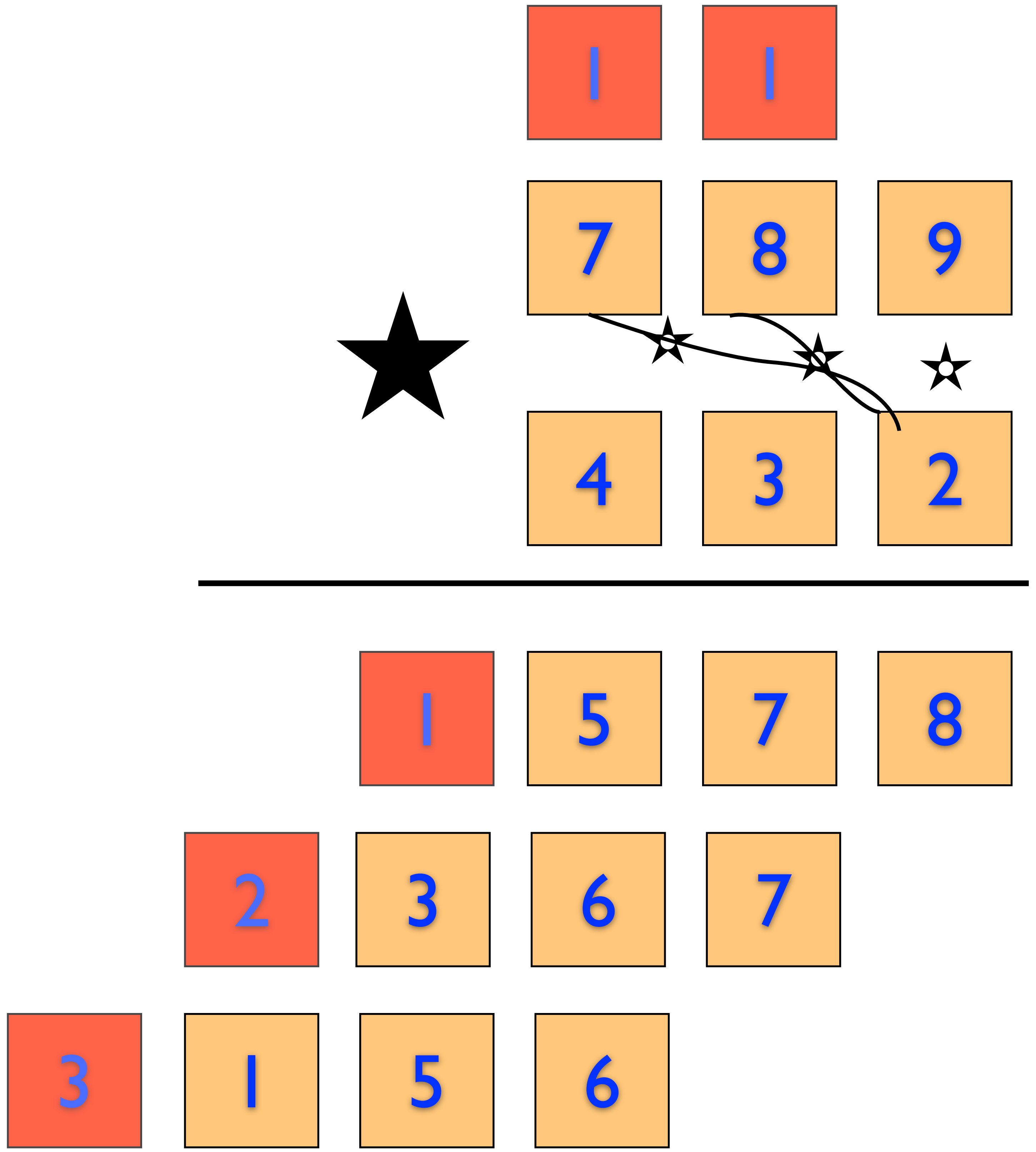


n✫ n−1✢



n✫ n–1✢

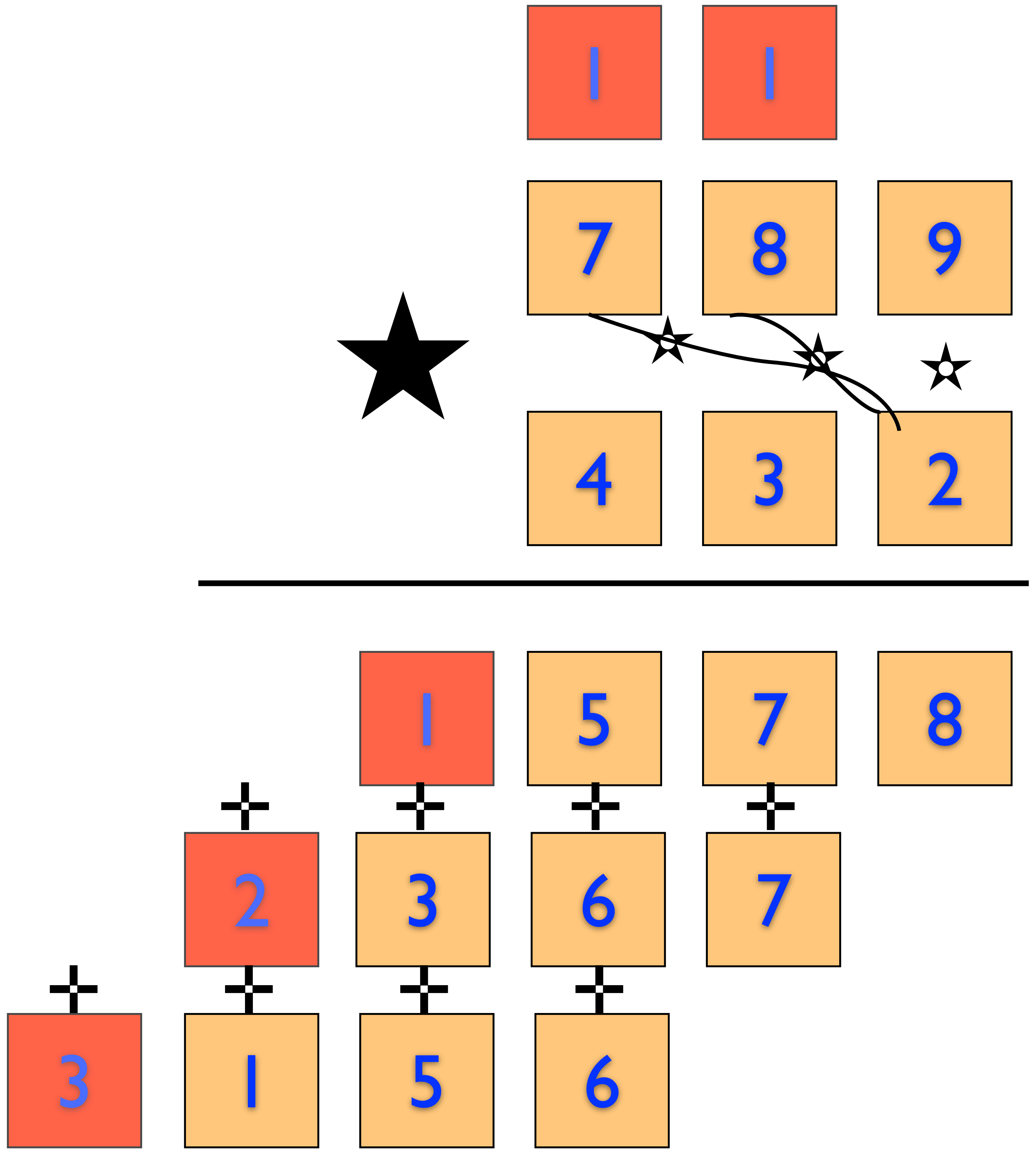
n✫ n–1✢



n✫ n–1✢

n✫ n–1✢

n✫ n–1✢

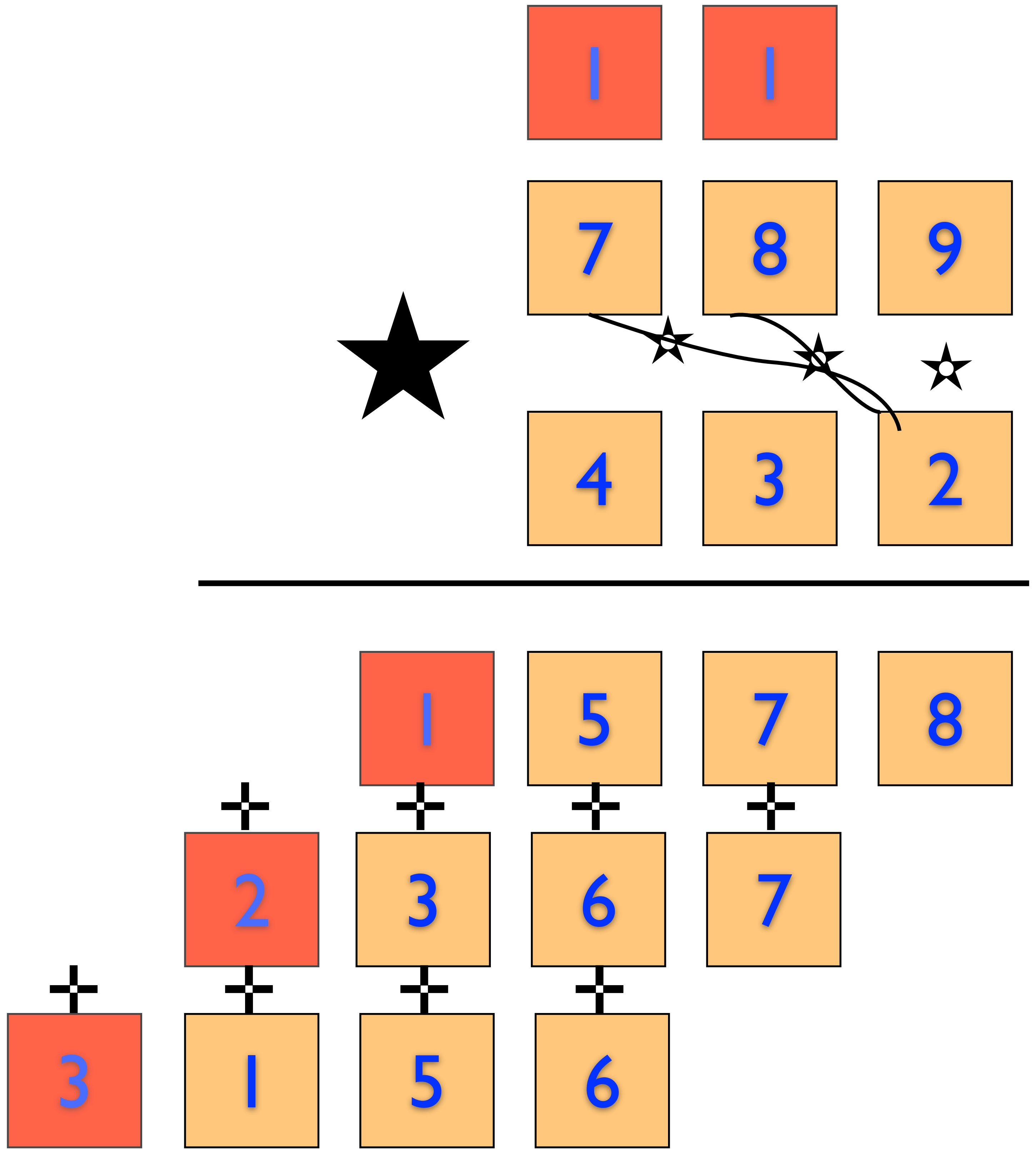


n☆ n−1✛

n☆ n−1✛

n☆ n−1✛

(n−1)(n+1)✛



(n−1)(n+1)✛

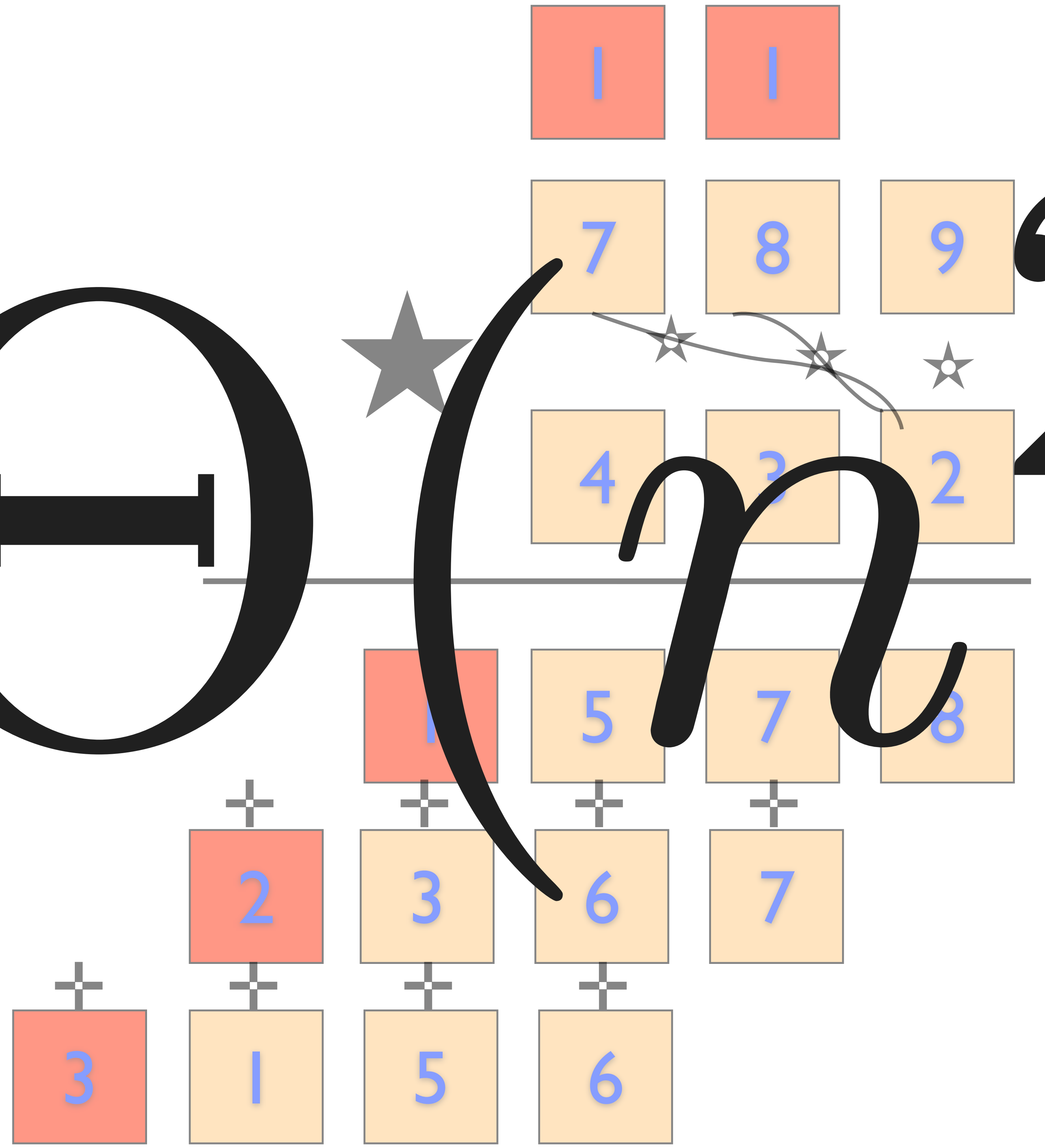
n☆ n−1✛

n☆ n−1✛

n☆ n−1✛

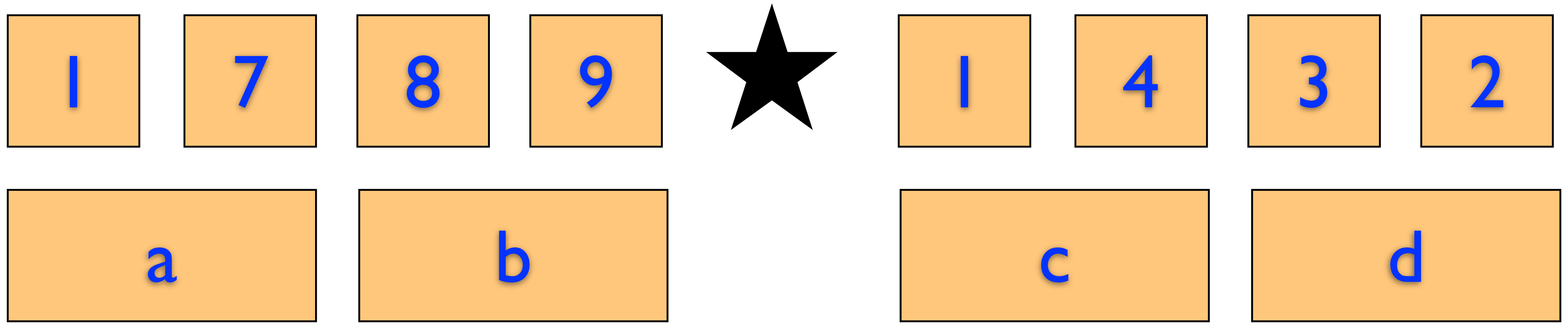
n^2 $2n^2 - n - 1$

$$\Theta^{\star}(n^2)$$



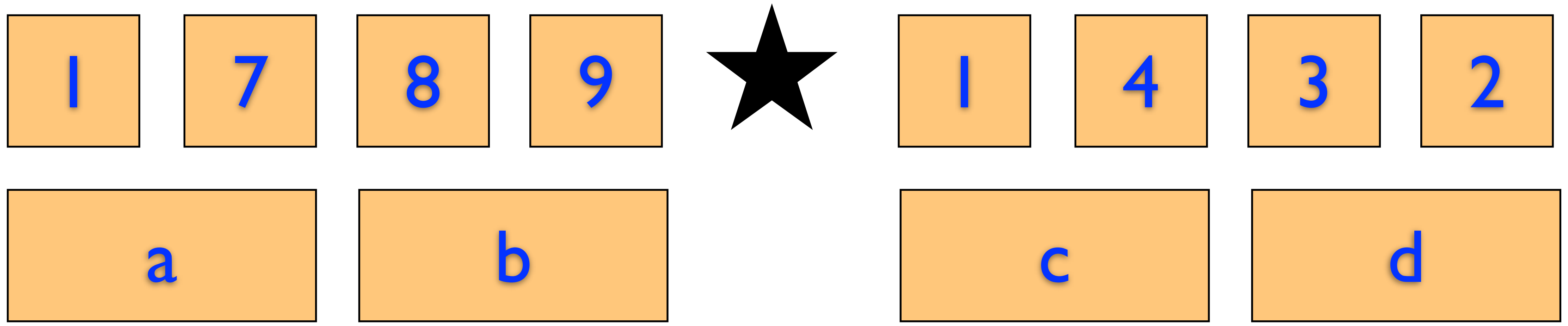
(n−1)(n+1)✢

n☆	n-1✢
n☆	n-1✢
n☆	n-1✢
n^2	$2n^2 - n - 1$



1 7 8 9 ★ 1 4 3 2

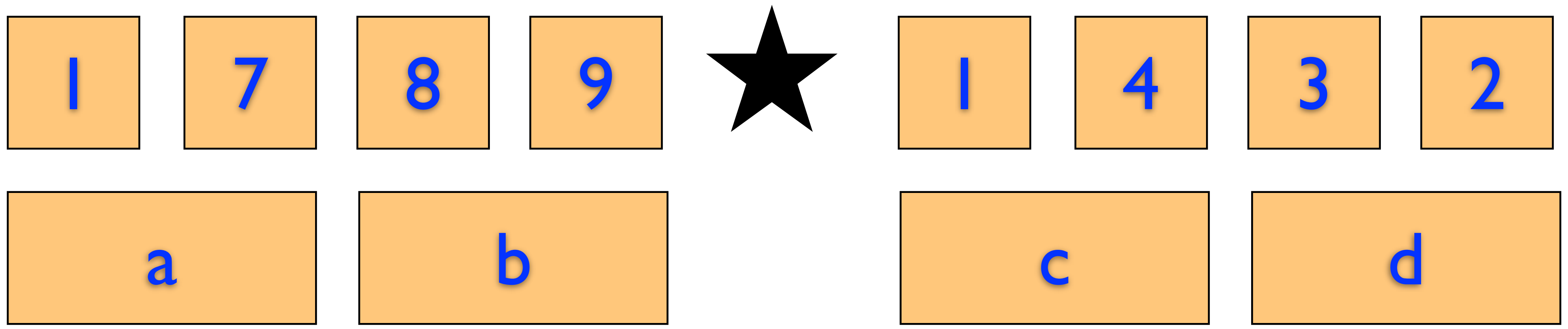
a b c d



1 7 8 9 ★ 1 4 3 2

a b c d

$$ac100^2 + (ad + bc)100 + bd$$



$$ac100^2 + (ad + bc)100 + bd$$

4✫ 3✜

1 7 8 9 ★ 1 4 3 2

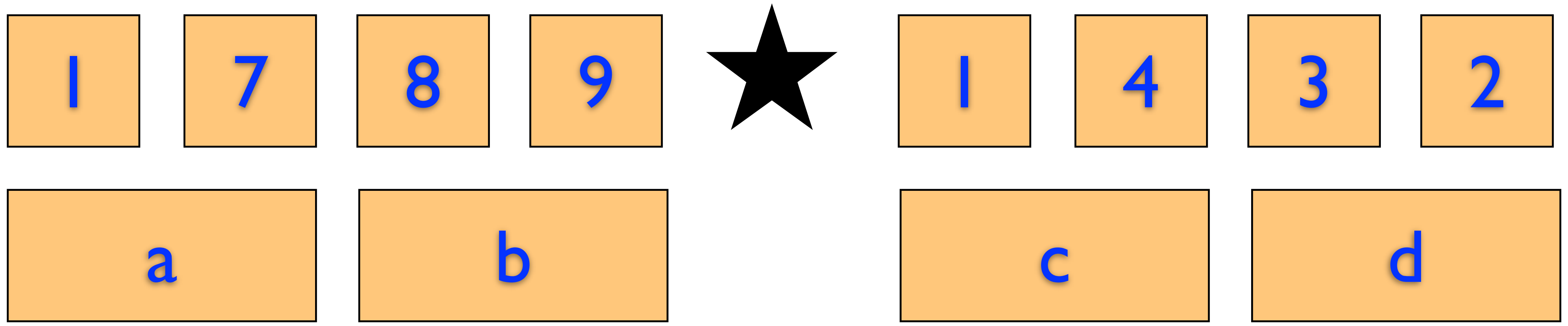
a b ★ c d

$$ac100^2 + (ad + bc)100 + bd$$

4☆ 3✛

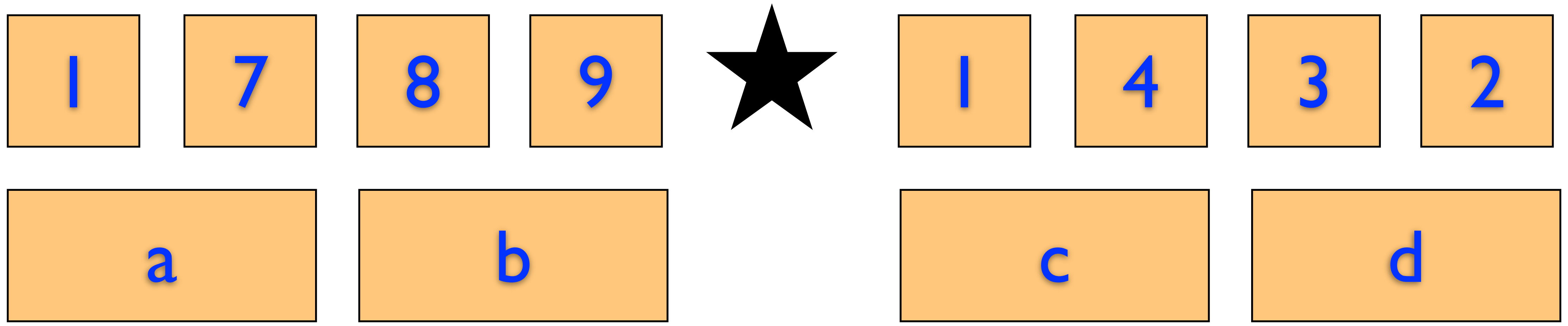
$$T(n) = 4T(n/2) + 3O(n)$$

KARATSUBA



$$ac100^2 + (ad + bc)100 + bd$$

KARATSUBA



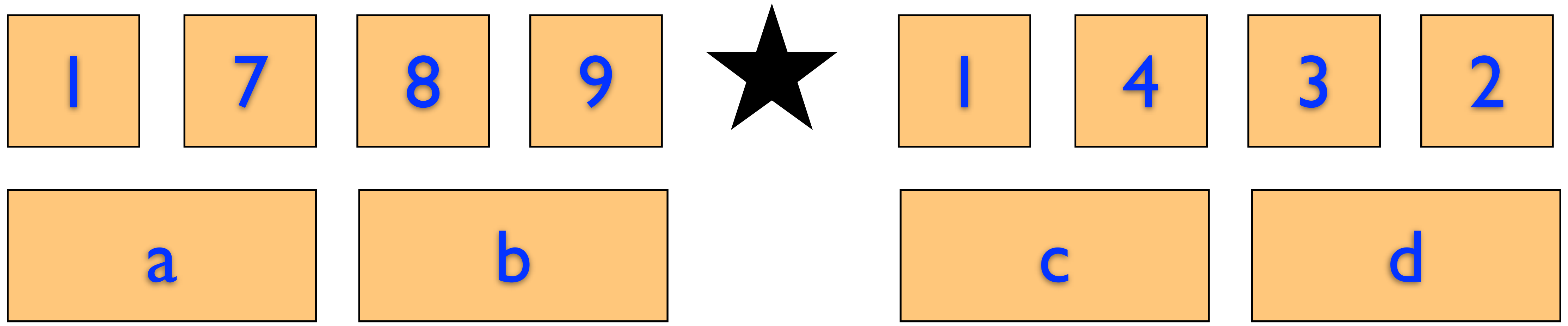
1 7 8 9 ★ 1 4 3 2

a b c d

$$ac100^2 + (ad + bc)100 + bd$$

$$(a + b)(c + d) = ac + ad + bc + bd$$

KARATSUBA



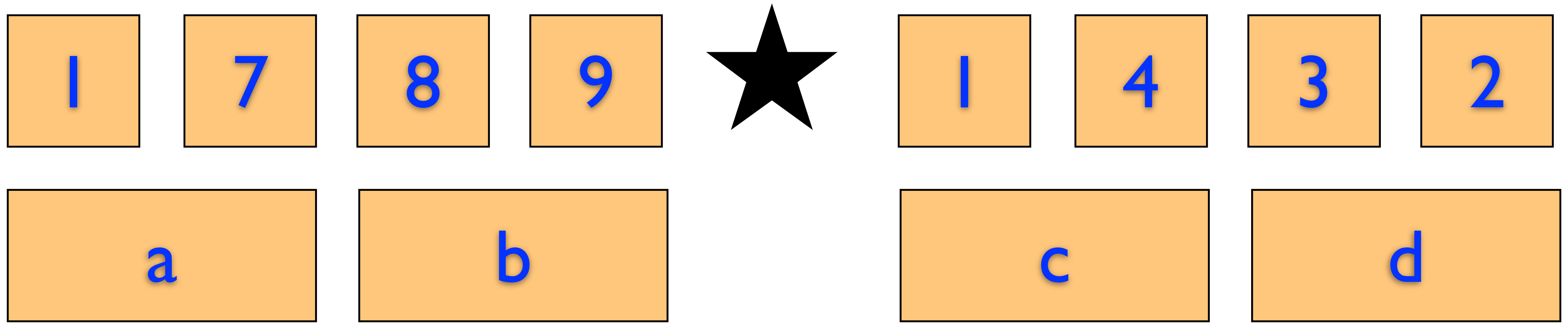
1 7 8 9 ★ 1 4 3 2

a b c d

$$ac100^2 + (ad + bc)100 + bd$$

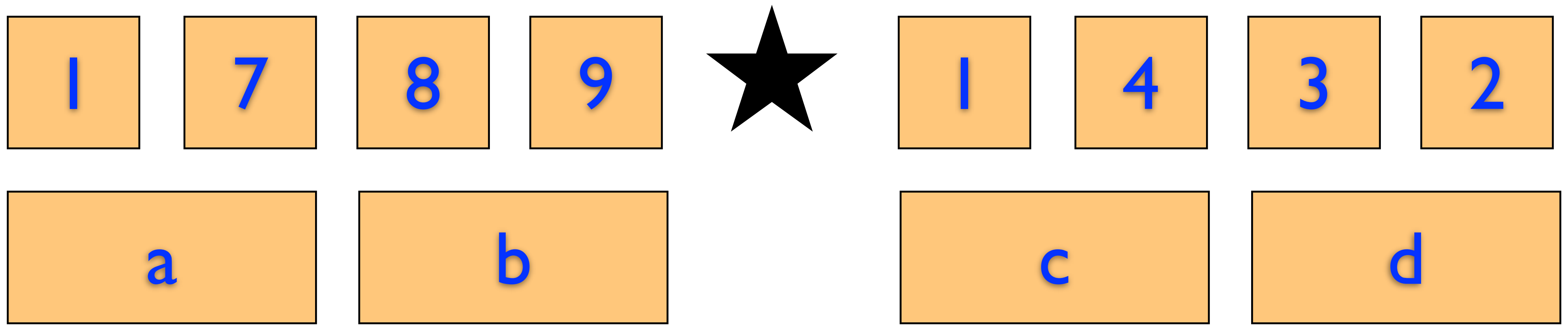
$$(a + b)(c + d) = ac + ad + bc + bd$$

$$ad + bc = (a + b)(c + d) - ac - bd$$



1 7 8 9 ★ 1 4 3 2

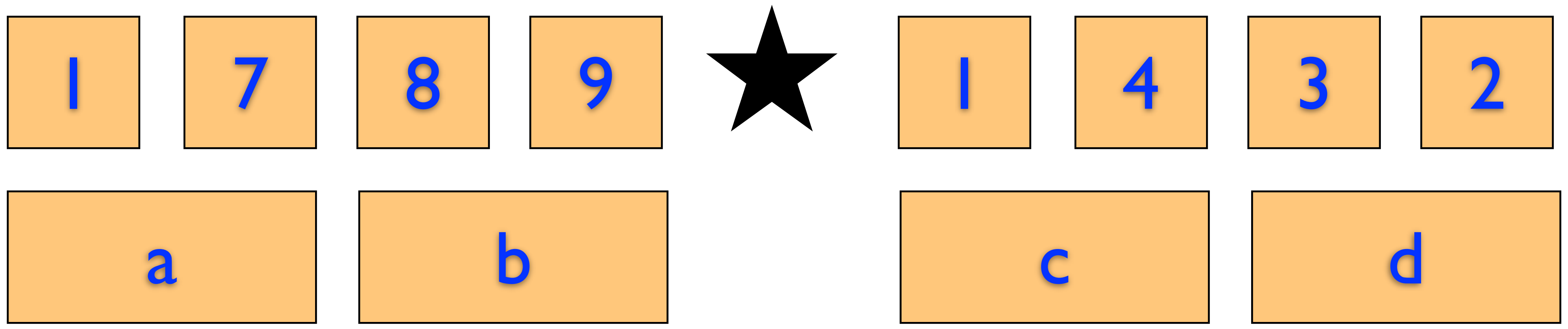
a b c d



1 7 8 9 ★ 1 4 3 2

a b c d

① $ac, bd, (a+b)(c+d)$

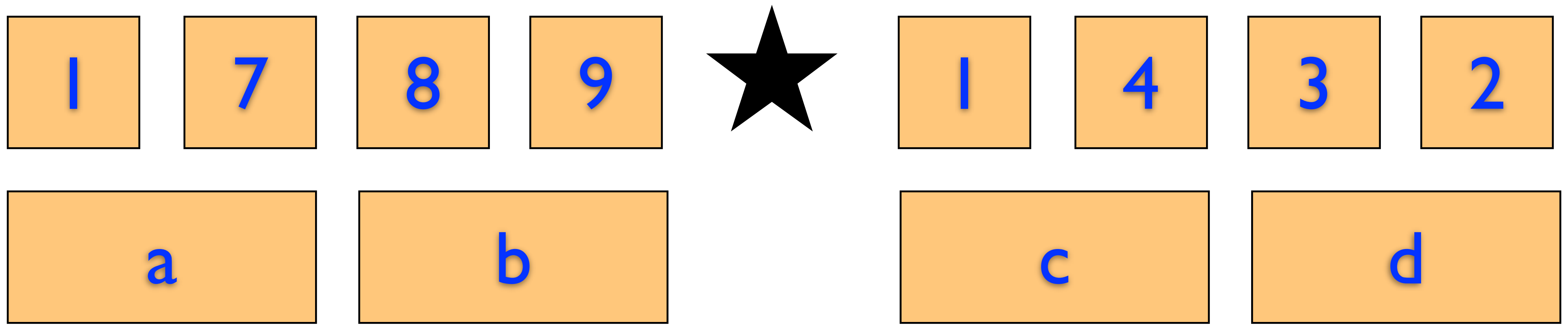


1 7 8 9 ★ 1 4 3 2

a b c d

❶ $ac, bd, (a+b)(c+d)$

❷ $ad+bc = (a+b)(c+d) - ac - bd$



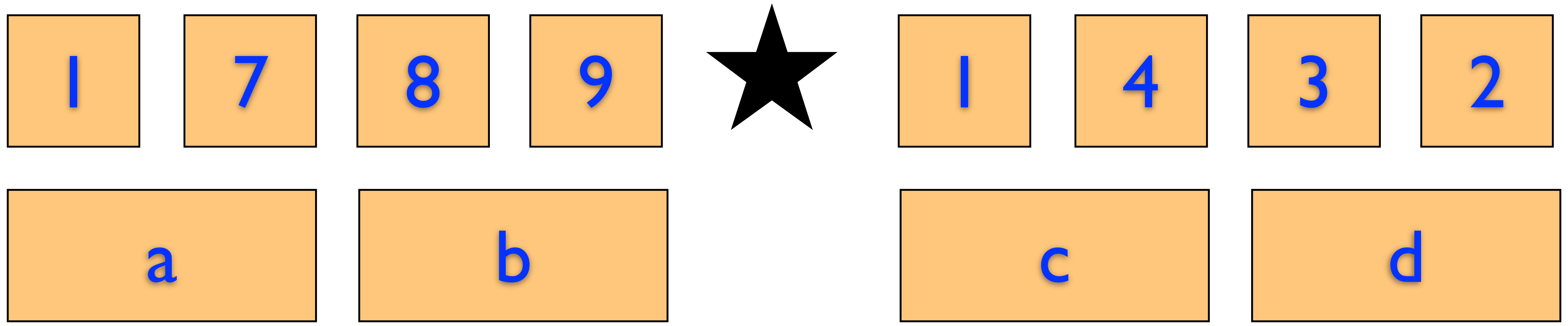
1 7 8 9 ★ 1 4 3 2

a b c d

❶ $ac, bd, (a+b)(c+d)$

❷ $ad + bc = (a+b)(c+d) - ac - bd$

❸ $ac100^2 + (ad+bc)100 + bd$



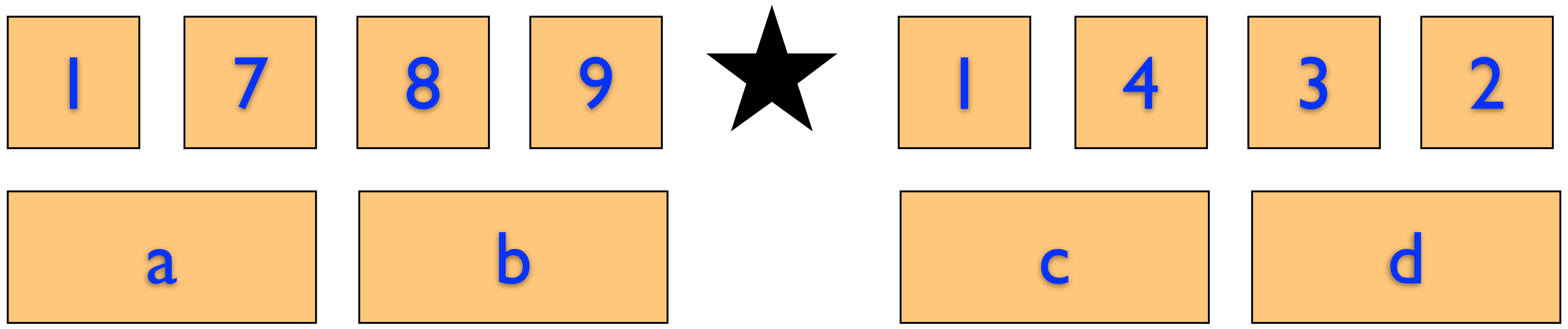
1 7 8 9 ★ 1 4 3 2

a b c d

❶ $ac, bd, (a+b)(c+d)$

❷ $ad + bc = (a+b)(c+d) - ac - bd$

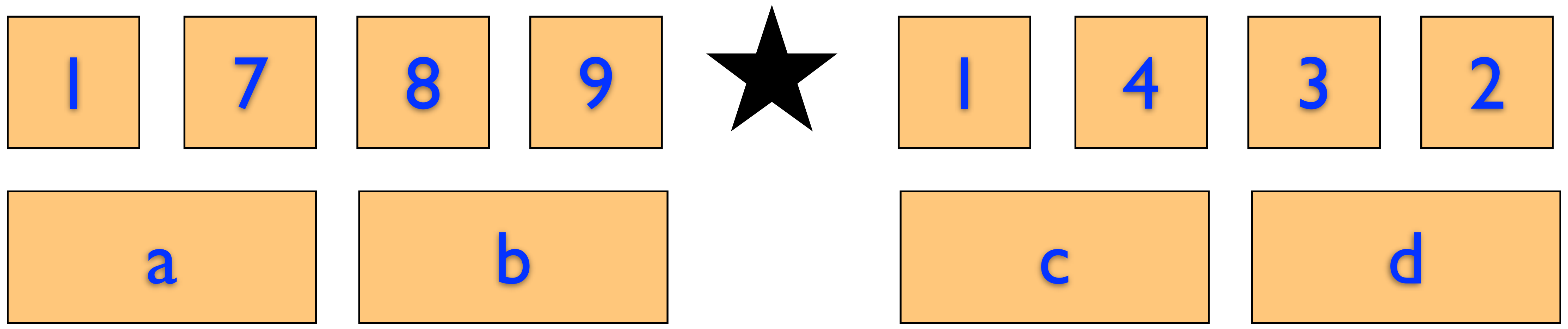
❸ $ac100^2 + (ad+bc)100 + bd$



❶ $ac, bd, (a+b)(c+d)$ $3T(n/2) + 2O(n)$

❷ $ad + bc = (a+b)(c+d) - ac - bd$

❸ $ac100^2 + (ad + bc)100 + bd$



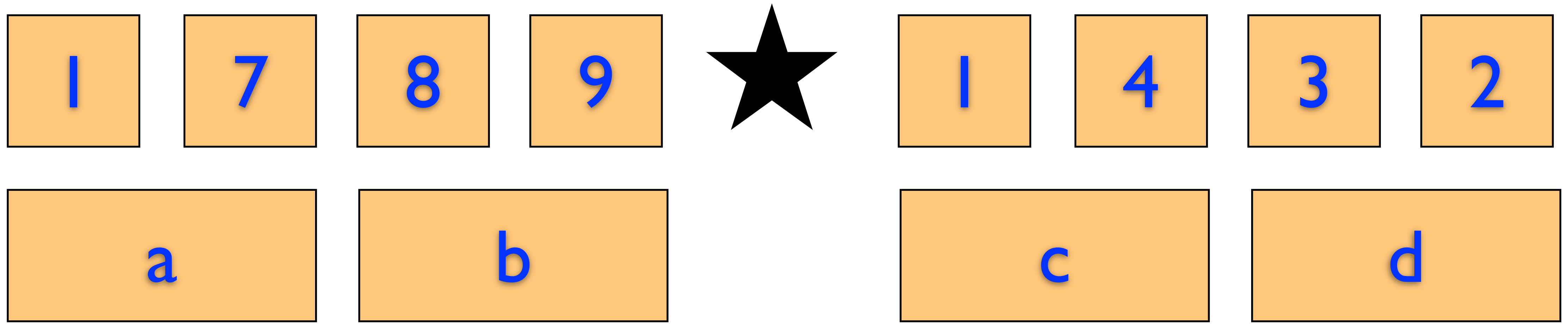
1 7 8 9 ★ 1 4 3 2

a b c d

❶ $ac, bd, (a+b)(c+d)$ $3T(n/2) + 2O(n)$

❷ $ad + bc = (a+b)(c+d) - ac - bd$ $2O(n)$

❸ $ac100^2 + (ad+bc)100 + bd$



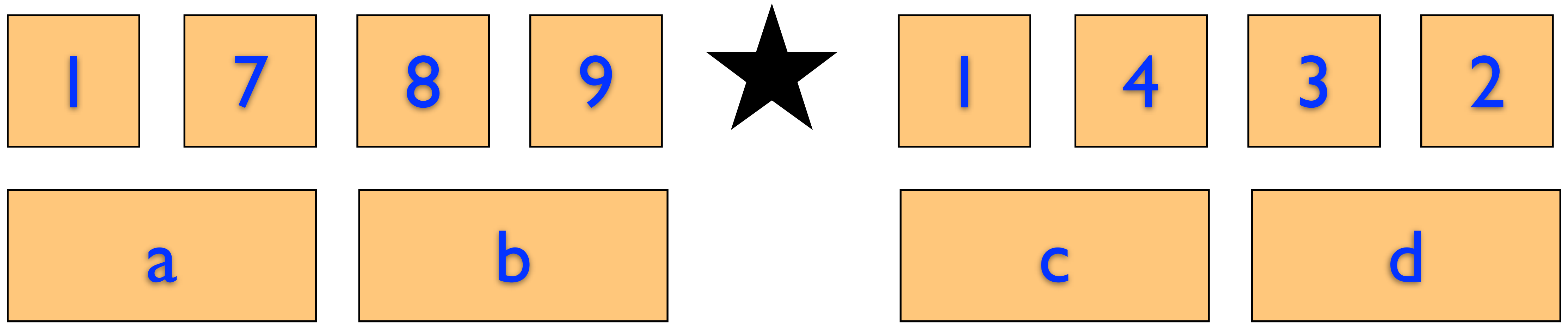
1 7 8 9 ★ 1 4 3 2

a b c d

❶ $ac, bd, (a+b)(c+d)$ $3T(n/2) + 2O(n)$

❷ $ad + bc = (a+b)(c+d) - ac - bd$ $2O(n)$

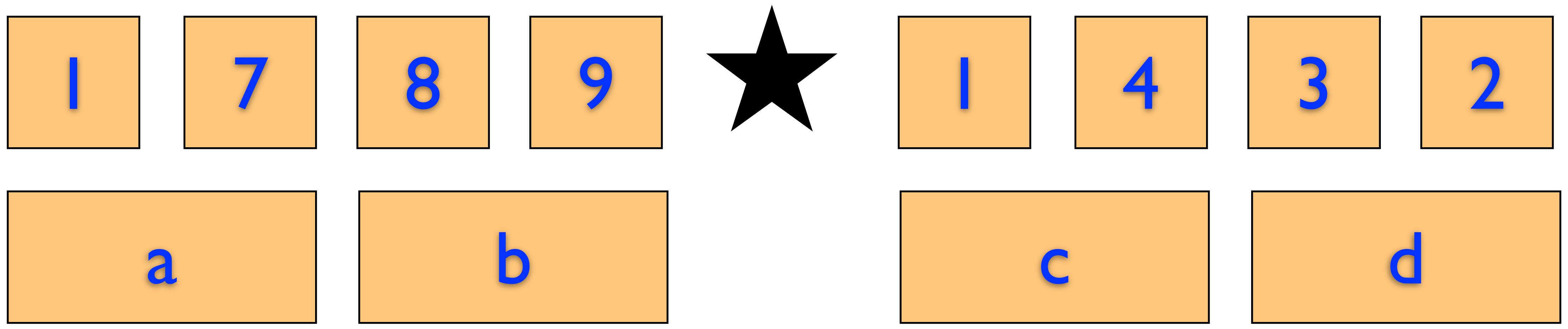
❸ $ac100^2 + (ad+bc)100 + bd$ $2O(n)$



1 7 8 9 ★ 1 4 3 2

a b c d

$$T(n) = 3T(n/2) + 6O(n)$$



$$T(n) = 3T(n/2) + 6O(n)$$

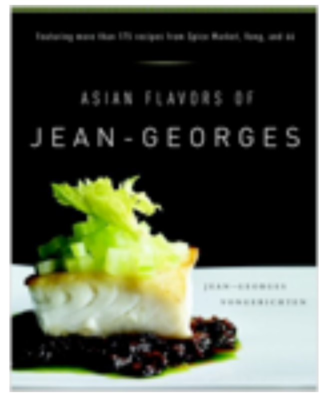
$$\Theta(n^{1.585})$$

$$T(n) = 3T(n/2) + 6O(n)$$

$$T(n) = 3T(n/2) + 6O(n)$$

$$T(n) = f(n) + af\left(\frac{n}{b}\right) + a^2 f\left(\frac{n}{b^2}\right) + a^3 f\left(\frac{n}{b^3}\right) + \cdots + a^L f\left(\frac{n}{b^L}\right)$$

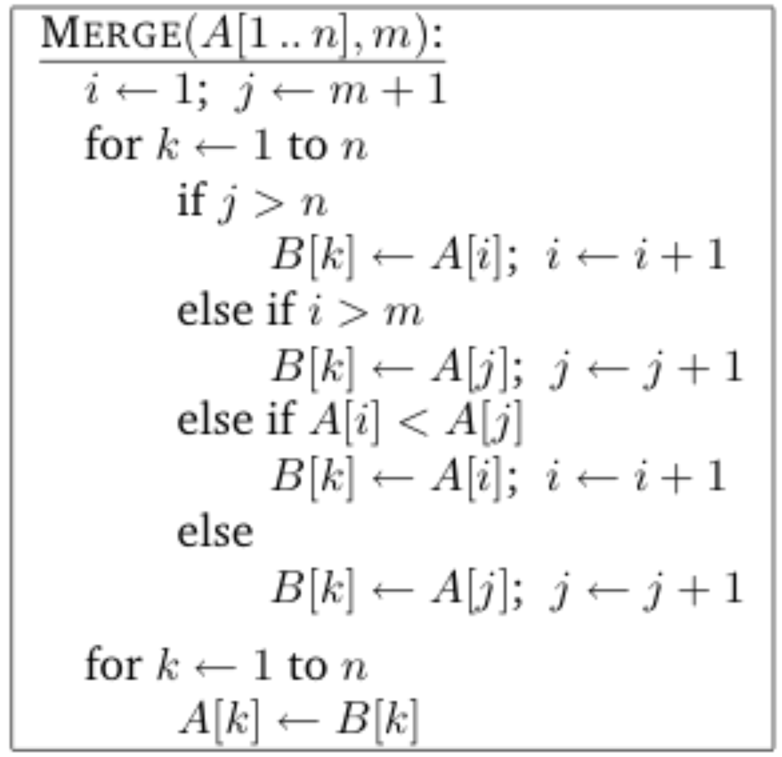
?-✓



ASIAN FLAVORS OF
JEAN-GEORGES

http://www.drblank.com/law301.jpg

```
MERGE-SORT $(A, p, r)$
    IF $p < r$
        $q \leftarrow \lfloor (p + r)/2 \rfloor$
        MERGE-SORT $(A, p, q)$
        MERGE-SORT $(A, q + 1, r)$
        MERGE $(A, p, q, r)$
   ...
```



```
MERGE($A[1..n], m$):
    $i \leftarrow 1$;  $j \leftarrow m + 1$
    for $k \leftarrow 1$ to $n$
        if $j > n$
            $B[k] \leftarrow A[i]$;  $i \leftarrow i + 1$
        else if $i > m$
            $B[k] \leftarrow A[j]$;  $j \leftarrow j + 1$
        else if $A[i] < A[j]$
            $B[k] \leftarrow A[i]$;  $i \leftarrow i + 1$
        else
            $B[k] \leftarrow A[j]$;  $j \leftarrow j + 1$

    for $k \leftarrow 1$ to $n$
        $A[k] \leftarrow B[k]$
```

JEFF ERICKSON