L12

4102

3.01.2016

abhi shelat

typesetting question

what is the first rule of typesetting?

what did the variable Sij represent?

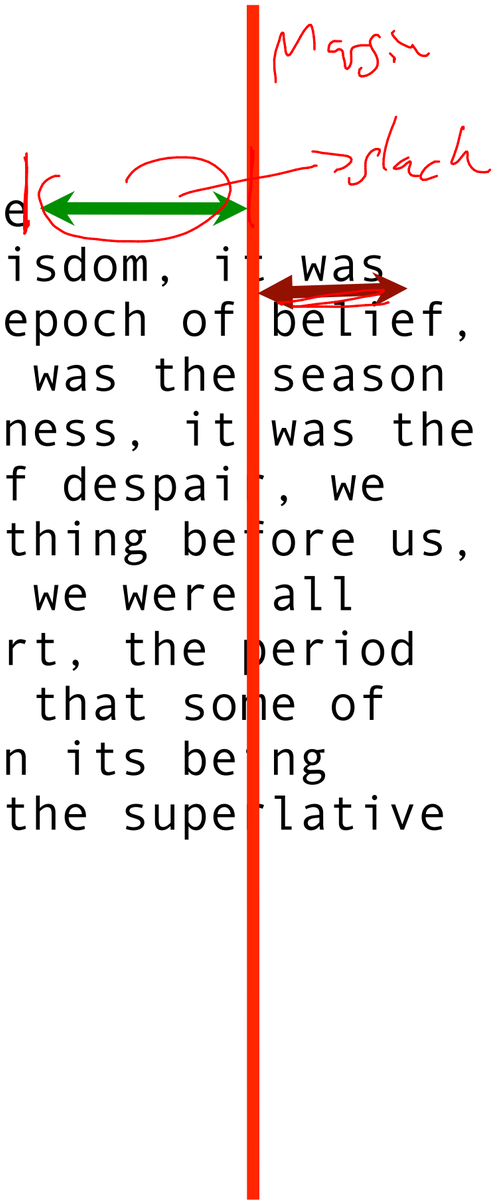
Express a recursive formula for Sij :

userid:

Typesetting

It was the best of times, it was the worst of times, it was the age of wisdom, it was the age of foolishness, it was the epoch of belief, it was the epoch of incredulity, it was the season of Light, it was the season of Darkness, it was the spring of hope, it was the winter of despair, we had everything before us, we had nothing before us, we were all going direct to heaven, we were all going direct the other way - in short, the period was so far like the present period, that some of its noisiest authorities insisted on its being received, for good or for evil, in the superlative degree of comparison only.

It was the best of times, it was the worst of times, it was the age of wisdom, it was the age of foolishness, it was the epoch of belief, it was the epoch of incredulity, it was the season of Light, it was the season of Darkness, it was the spring of hope, it was the winter of despair, we had everything before us, we had nothing before us, we were all going direct to heaven, we were all going direct the other way - in short, the period was so far like the present period, that some of its noisiest authorities insisted on its being received, for good or for evil, in the superlative degree of comparison only.



First rule of typesetting

never print in the margin! ✓

 are simply not allowed

It was the best of times, it was the worst of times, it was the age of wisdom, it was the age of foolishness, it was the epoch of belief, it was the epoch of incredulity, it was the season of Light, it was the season of Darkness, it was the spring of hope, it was the winter of despair, we had everything before us, we had nothing before us, we were all going direct to heaven, we were all going direct the other way - in short, the period was so far like the present period, that some of its noisiest authorities insisted on its being received, for good or for evil, in the superlative degree of comparison only.

_____ is....

Second rule of typesetting

avoid big ugly whitespaces (slack)

Typesetting problem

input: $W = \{w_1, w_2, w_3, \ldots, w_n\}$ M margin

output: $L = (w_1, \ldots, w_{\ell_1}), (w_{\ell_1+1}, \ldots, w_{\ell_2}), \ldots, (w_{\ell_{x+1}, \ldots, w_n})$

list

$$c_i = \left(\sum_{j=\ell_i+1}^{\ell_{i+1}} |w_j| \right) + (\ell_{i+1} - \ell_i - 1)$$

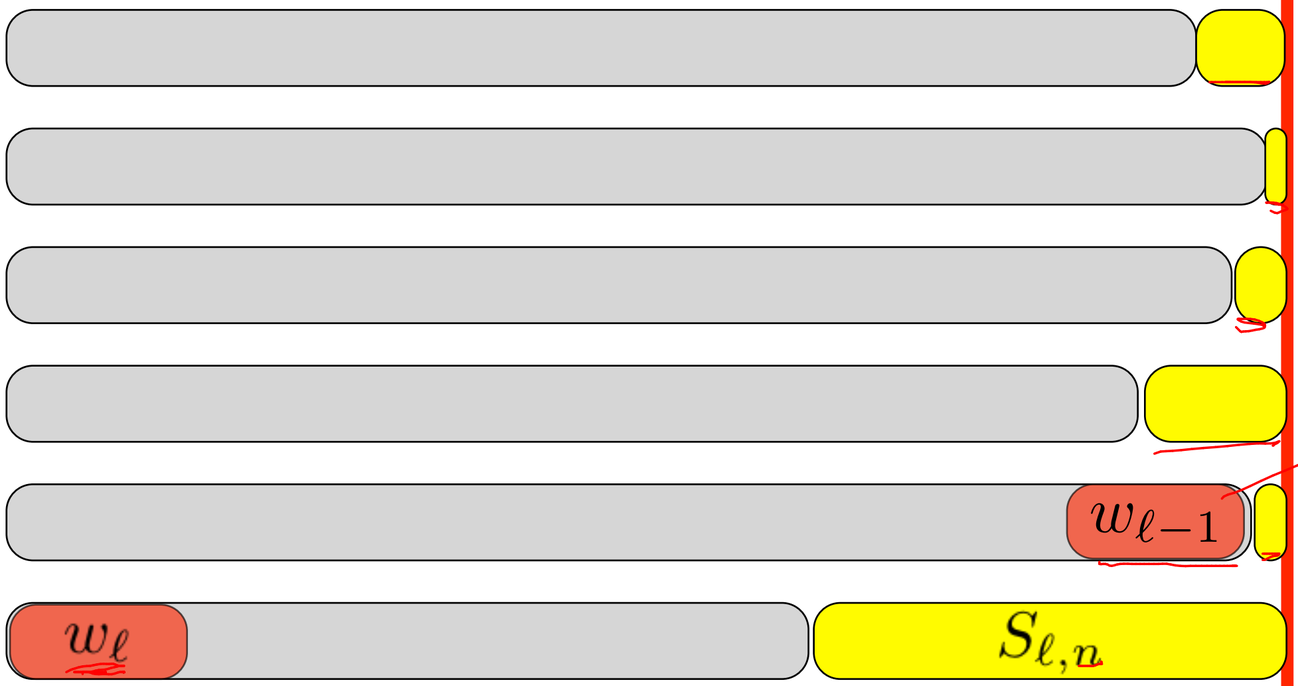
length of line i

such that $c_i \leq M \quad \forall i$

$$\min \sum (M - c_i)^2$$

MIN (SLACK)^2

imagine optimal



last line

fwoll is w_ℓ

slack when line starts with w_ℓ

$$\mathrm{BEST}_n = \mathrm{BEST}_{\ell-1} + S_{\ell,n}^2$$

$$\text{BEST}_n = \min \begin{cases} \text{BEST}_0 + S^2_{1,n} \\ \text{BEST}_1 + S^2_{2,n} \\ \text{BEST}_2 + S^2_{3,n} \\ \cdots \\ \text{BEST}_{\ell-1} + S^2_{\ell,n} \\ \\ \cdots \\ \\ \text{BEST}_{n-1} + S^2_{n,n} \end{cases}$$

typesetting algorithm

make table for $S_{i,j}$

for `i=1` to `n`

best[`i`] = min{ best[`j`] + s[`j+1`][`i`]^2 }

typesetting algorithm

make table for $S_{i,j}$

for `i=1` to `n`

best[i] = min{ best[j] + s[j+1][i]^2 }

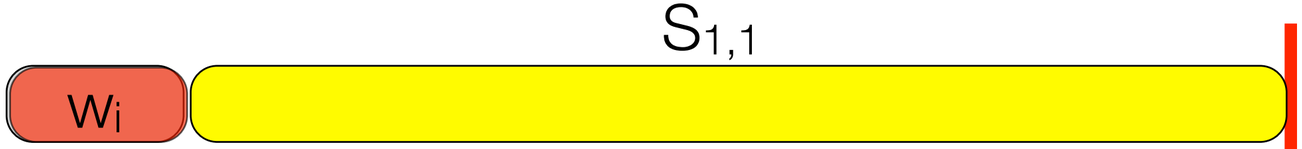
```
// compute best_0,...,best_n
int best[] = new int[n+1];
int choice[] = new int[n+1];
best[0] = 0;
for(int i=1;i<=n;i++) {
    int min = infty;
    int ch  = 0;
    for(int j=0;j<i;j++) {
        int t = best[j] + S[j+1][i]*S[j+1][i];
        if (t<min) { min = t; ch = j;}
    }
    best[i] = min;
    choice[i] = ch;
}
```

how to compute $S_{i,j}$

$S_{i,j}$

slack when line starts with w_i and ends w_j

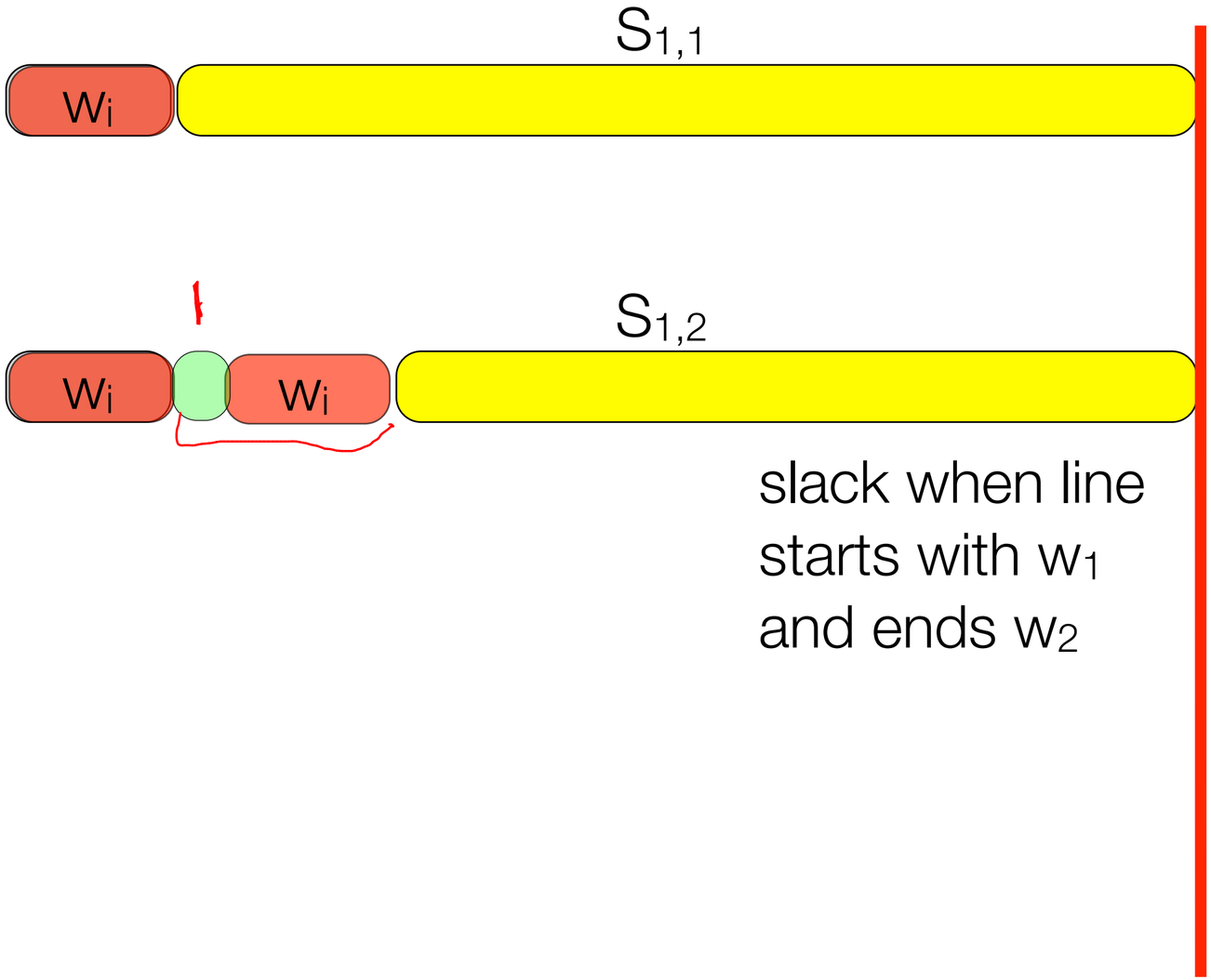
Simplest case



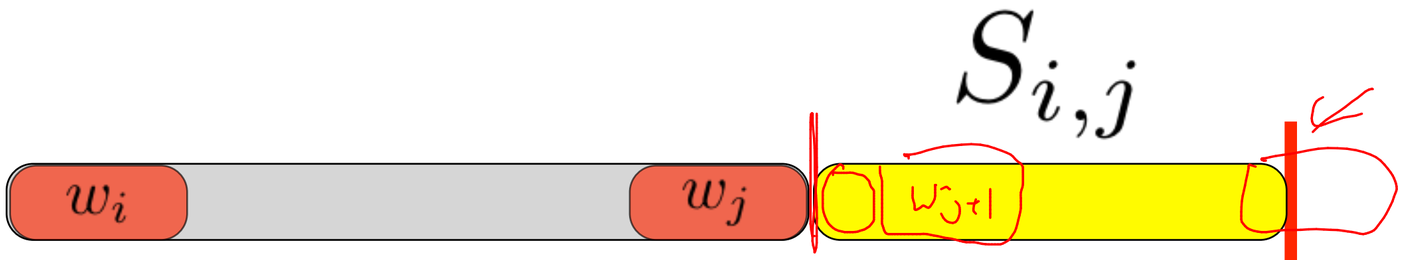
$S_{i,i} = M - |w_i|$

slack when line starts with w_i and ends w_i

Simplest case



how to compute $S_{i,j}$



slack when line starts with w_i and ends w_j

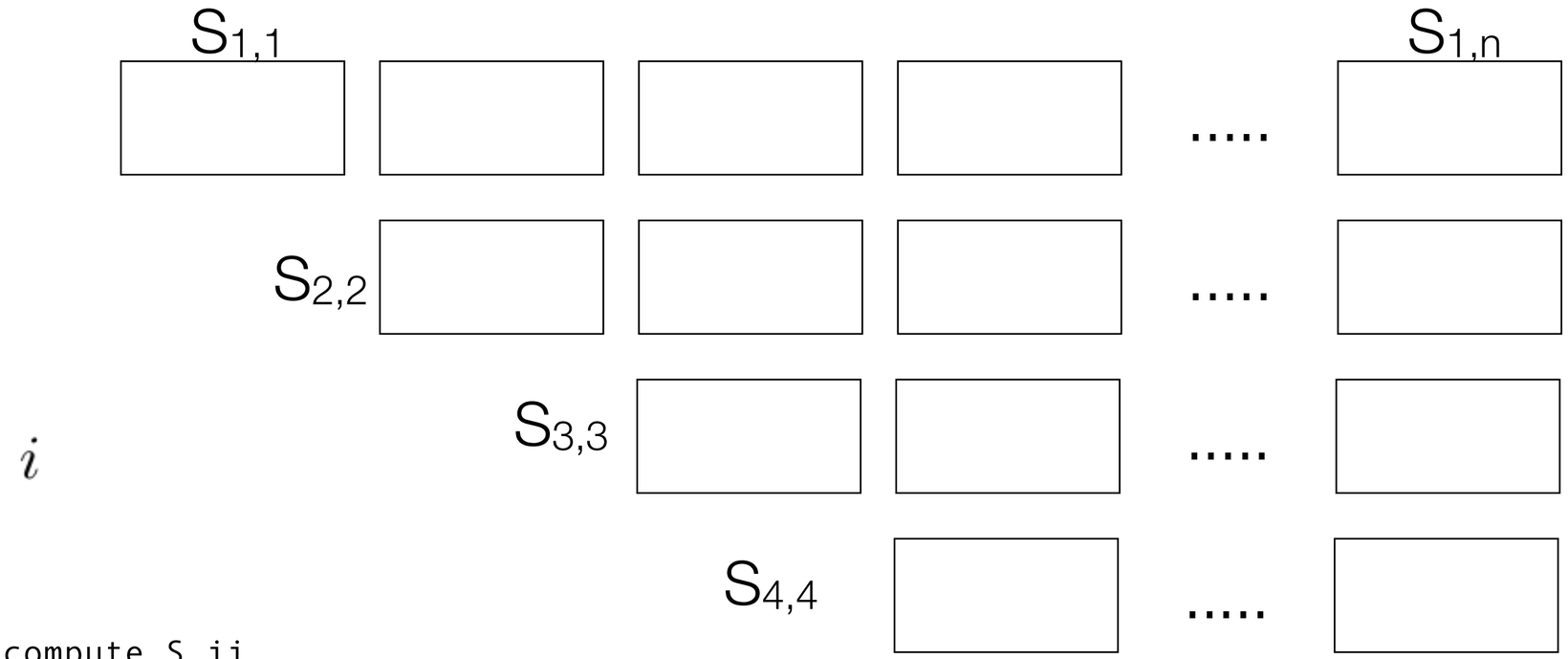
$S_{i,j} \geq M$, then set $S_{i,j} = \infty$

$\sim M^2$

$$S_{i,i} = M - |w_i|$$

$$S_{i,j} = S_{i,j-1} - 1 - |w_j|$$

How to compute S




```
// compute S_ij
int S[][] = new int[n+1][n+1];
for(int i=1;i<=n;i++) {
    S[i][i] = M - lens[i];
    for(int j=i+1; j<=n; j++) {
        S[i][j] = S[i][j-1] - lens[j] - 1;
        if (S[i][j]<0) {
            while(j<=n) { S[i][j++] = infty; }
        }
    }
}
```

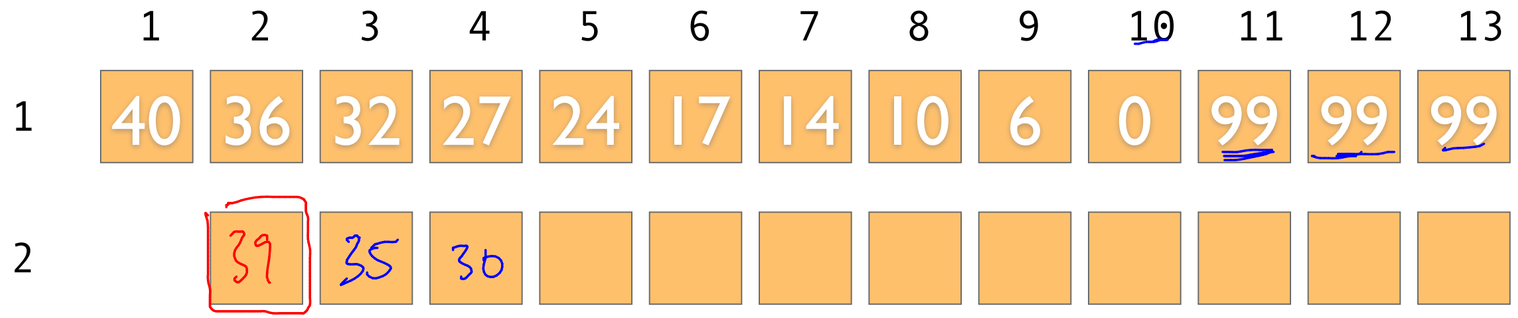
2nd equation

Example

```
It was the best of times, it was the worst of times; it was the age o
wisdom, it was the age of foolishness; it was the epoch of belief, it
was the epoch of incredulity; it was the season of
```

```
 2  3  3  4  2  6  2  3  3  5  2  6  2  3  3  3  2  7  2  3  3
 3  2 12  2  3  3  5  2  7  2  3  3  5  2 12  2  3  3  6  2
```

first step: make $S_{i,j}$



2 3 3 4 2 6 2 3 3 5 2 6 2 3 3 3 2 7 2 3 3
3 2 12 2 3 3 5 2 7 2 3 3 5 2 12 2 3 3 6 2

$M = 42$

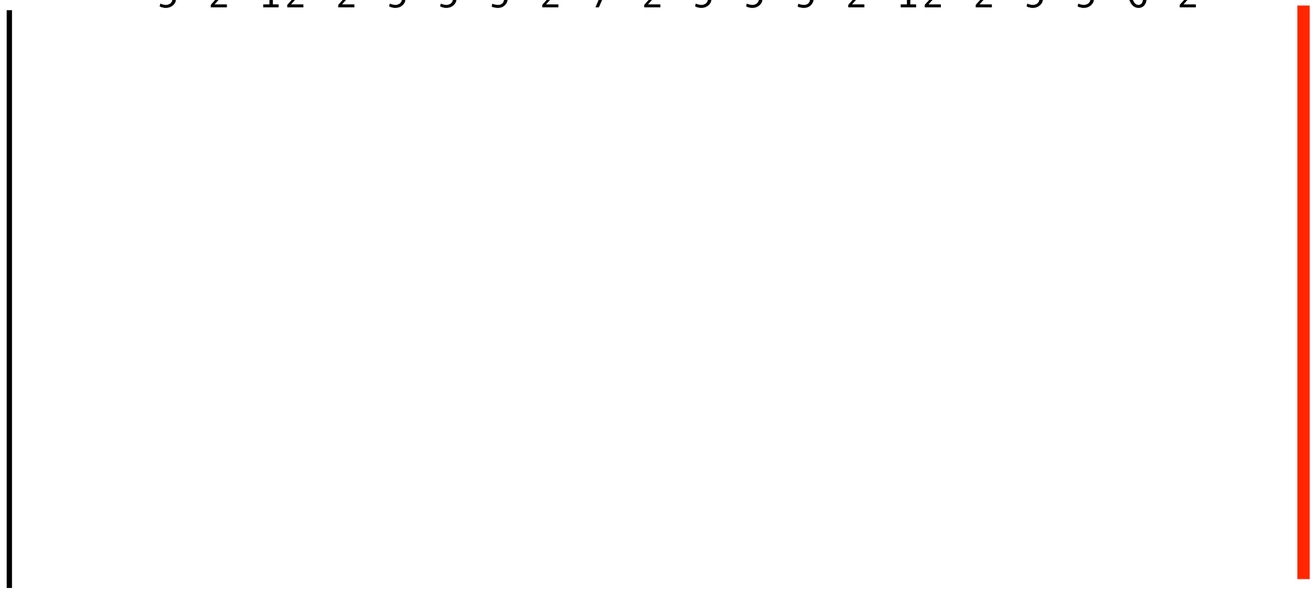
$$S_{i,i} = M - |w_i|$$

$$S_{i,j} = S_{i,j-1} - 1 - |w_j|$$

first step: make $S_{i,j}$

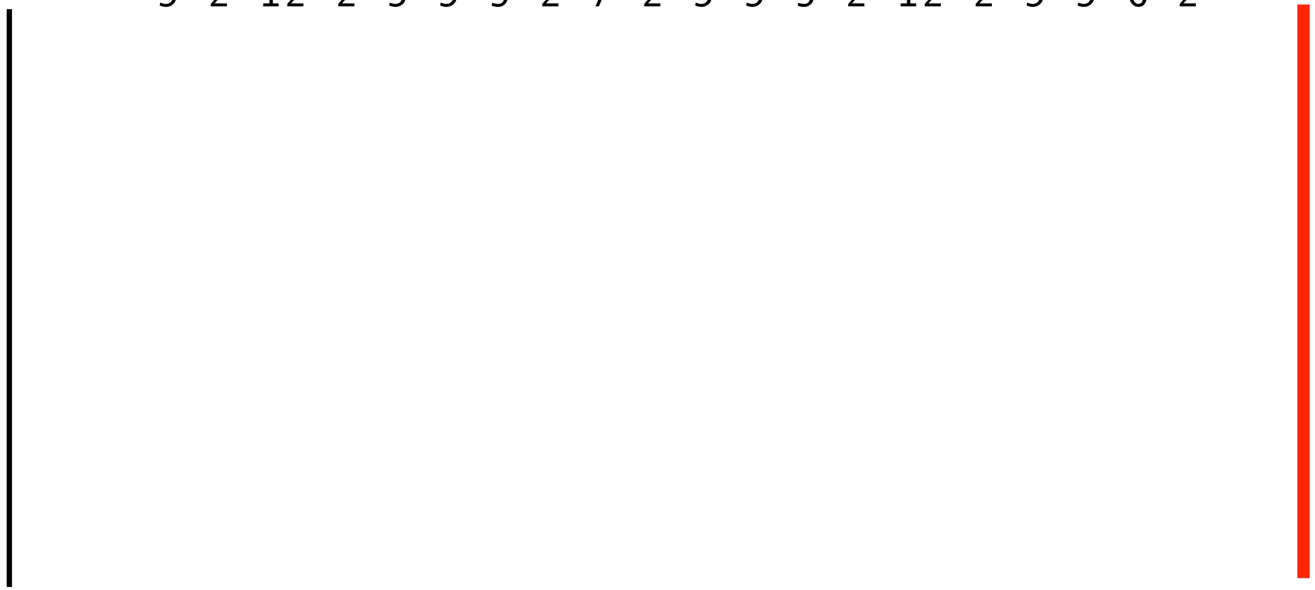
	1	2	3	4	5	6	7	8	9	10	11	12	13
1	40	36	32	27	24	17	14	10	6	0	99	99	99
2		39	35	30	27	20	17	13	9	3	0	99	99

2 3 3 4 2 6 2 3 3 5 2 6 2 3 3 3 2 7 2 3 3
3 2 12 2 3 3 5 2 7 2 3 3 5 2 12 2 3 3 6 2



	1	2	3	4	5	6	7	8	9	10	11	12	13
1	40	36	32	27	24	17	14	10	6	0	99	99	99
2		39	35	30	27	20	17	13	9	3	0	99	99
3													

2 3 3 4 2 6 2 3 3 5 2 6 2 3 3 3 2 7 2 3 3
3 2 12 2 3 3 5 2 7 2 3 3 5 2 12 2 3 3 6 2



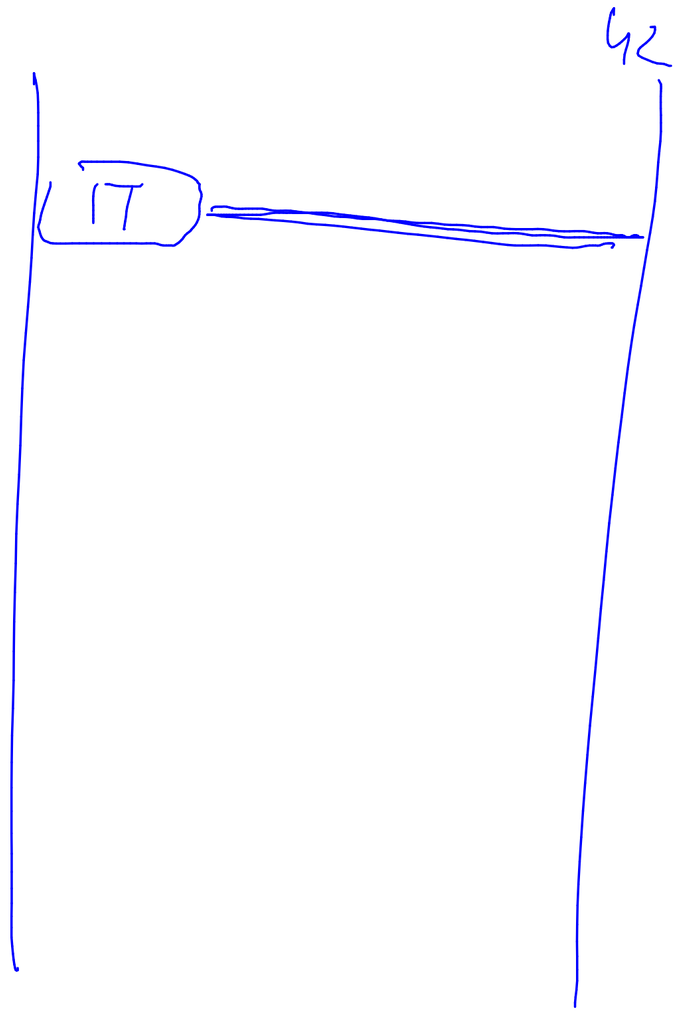
second step: compute

	0	1	2	3	4	5	6	7	8	9	10	...
best	0	1600										

$Best_1 = \min \{ Best_0 + (S_{1,1})^2 = 1600$

$$\mathrm{BEST}_i = \min_{j=0}^{i-1} \left\{ \mathrm{BEST}_j + S^2_{j+1,i} \right\}$$

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	40	36	32	27	24	17	14	10	6	0	99	99	99
2		39	35	30	27	20	17	13	9	3	0	99	99



second step: compute

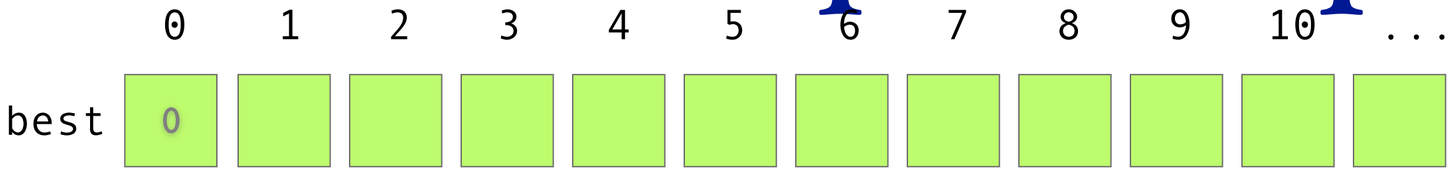
	0	1	2	3	4	5	6	7	8	9	10	...
best	0	1600										

$Best_2 =$

$$\mathrm{BEST}_i = \min_{j=0}^{i-1} \left\{ \mathrm{BEST}_j + S^2_{j+1,i} \right\}$$

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	40	36	32	27	24	17	14	10	6	0	99	99	99
2		39	35	30	27	20	17	13	9	3	0	99	99

second step: compute



$$\mathrm{BEST}_i = \min_{j=0}^{i-1} \left\{ \mathrm{BEST}_j + S^2_{j+1,i} \right\}$$

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	40	36	32	27	24	17	14	10	6	0	99	99	99
2		39	35	30	27	20	17	13	9	3	0	99	99

$$\text{BEST}_i = \min_{j=0}^{i-1} \left\{ \text{BEST}_j + S_{j+1,i}^2 \right\}$$

	0	1	2	3	4	5	6	7	8	9	10	11	12	13
best	0	1600												
choice	0	0	0	0	0	0	0	0	0	0	0			

42 characters

aaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaa

It was

$Best_2 = \min \begin{cases} Best_0 + (S_{1,2})^2 \rightarrow 0 + 36^2 = 1296 \\ Best_1 + (S_{2,2})^2 \rightarrow 1600 + 39^2 > 2000 \end{cases}$

It ——————

was ——————

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	40	36	32	27	24	17	14	10	6	0	99	99	99
2		39	35	30	27	20	17	13	9	3	0	99	99

$$\text{BEST}_i = \min_{j=0}^{i-1} \left\{ \text{BEST}_j + S^2_{j+1,i} \right\}$$

	0	1	2	3	4	5	6	7	8	9	10	11	12	13
best	0	1600	1296											
choice	0	0	0	0	0	0	0	0	0	0	0			

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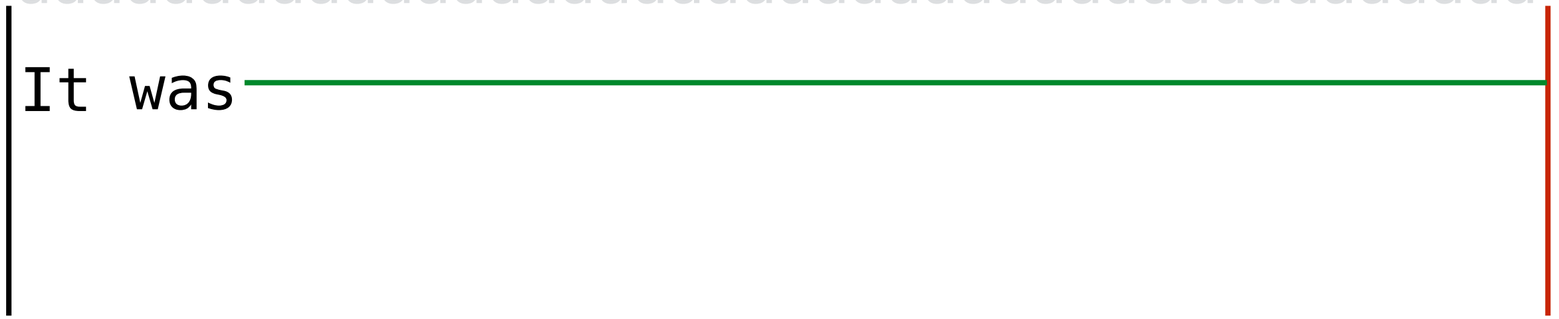

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	40	36	32	27	24	17	14	10	6	0	99	99	99
2		39	35	30	27	20	17	13	9	3	0	99	99

$$\text{BEST}_i = \min_{j=0}^{i-1} \left\{ \text{BEST}_j + S^2_{j+1,i} \right\}$$

	0	1	2	3	4	5	6	7	8	9	10	11	12	13
best	0	1600	1296	1024										
choice	0	0	0	0	0	0	0	0	0	0	0			

aaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaa

It was the

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	40	36	32	27	24	17	14	10	6	0	99	99	99
2		39	35	30	27	20	17	13	9	3	0	99	99

$$\text{BEST}_i = \min_{j=0}^{i-1} \left\{ \text{BEST}_j + S_{j+1,i}^2 \right\}$$

	0	1	2	3	4	5	6	7	8	9	10	11	12	13
best	0	1600	1296	1024	729	576	289	196	100	36	0			
choice	0	0	0	0	0	0	0	0	0	0	0			

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```
It was the best
```

$$\text{BEST}_i = \min_{j=0}^{i-1} \left\{ \text{BEST}_j + S^2_{j+1,i} \right\}$$

	0	1	2	3	4	5	6	7	8	9	10	11	12	13
best	0	1600	1296	1024	729	576	289	196	100	36	0			
choice	0	0	0	0	0	0	0	0	0	0	0			

aaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaa

It was the best of

$$\text{BEST}_i = \min_{j=0}^{i-1} \left\{ \text{BEST}_j + S^2_{j+1,i} \right\}$$

	0	1	2	3	4	5	6	7	8	9	10	11	12	13
best	0	1600	1296	1024	729	576	289	196	100	36	0			
choice	0	0	0	0	0	0	0	0	0	0	0			

aaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaa

It was the best of times, it was the worst of

* { of

9 } worst of

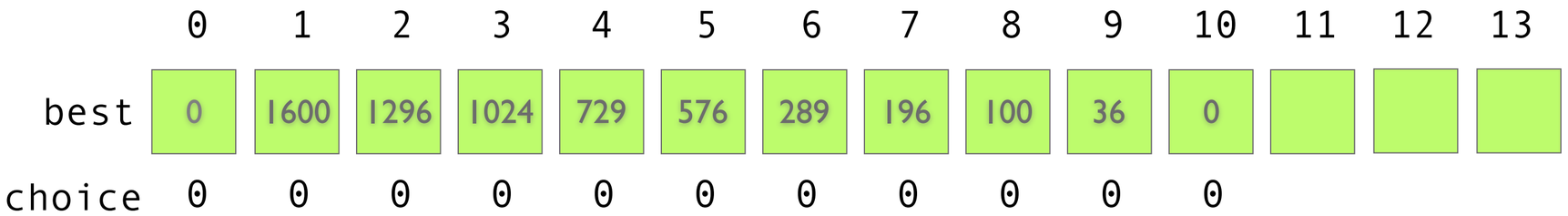
$$Best_{11} = \min \begin{cases} * \; Best_{10} + (S_{11,11})^2 \\ Best_9 + (S_{10,11})^2 \\ \vdots \\ Best_0 + (S_{1,11})^2 \end{cases}$$

$Best_{10} + (S_{11,11})^2 \rightarrow 0 + (40)^2 = 1600$

$Best_9 + (S_{10,11})^2 \rightarrow 36 + (42-8)^2 = 36 + 34^2 = 36 + 1156 = 1192$

$\rightarrow 60^2$

$$\text{BEST}_i = \min_{j=0}^{i-1} \left\{ \text{BEST}_j + S^2_{j+1,i} \right\}$$



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It was the best of times, it was the worst of

$\text{Best}_{11} = \min \{$

$$\text{BEST}_i = \min_{j=0}^{i-1} \left\{ \text{BEST}_j + S^2_{j+1,i} \right\}$$

	0	1	2	3	4	5	6	7	8	9	10	11	12	13
best	0	1600	1296	1024	729	576	289	196	100	36	0			
choice	0	0	0	0	0	0	0	0	0	0	0			

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It was the best of times, it was the worst of

$$\text{BEST}_{11} = \min \begin{cases} \text{BEST}_{10} + S^2_{11,11} \\ \text{BEST}_9 + S^2_{10,11} \\ \text{BEST}_8 + S^2_{9,11} \\ \text{BEST}_7 + S^2_{8,11} \\ \text{BEST}_6 + S^2_{7,11} \\ \ldots \end{cases}$$

$$\text{BEST}_i = \min_{j=0}^{i-1} \left\{ \text{BEST}_j + S^2_{j+1,i} \right\}$$

	0	1	2	3	4	5	6	7	8	9	10	11	12	13
best	0	1600	1296	1024	729	576	289	196	100	36	0			
choice	0	0	0	0	0	0	0	0	0	0	0			

aaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaa

It was the best of times, it was

the worst of

$$\text{BEST}_{11} = \min \begin{cases} \text{BEST}_{10} + S^2_{11,11} \\ \text{BEST}_9 + S^2_{10,11} \\ \text{BEST}_8 + S^2_{9,11} \\ \text{BEST}_7 + S^2_{8,11} \\ \text{BEST}_6 + S^2_{7,11} \\ \ldots \end{cases}$$

$$\text{BEST}_i = \min_{j=0}^{i-1} \left\{ \text{BEST}_j + S^2_{j+1,i} \right\}$$

	0	1	2	3	4	5	6	7	8	9	10	11	12	13
best	0	1600	1296	1024	729	576	289	196	100	36	0	818		
choice	0	0	0	0	0	0	0	0	0	0	0	6		

aaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaa

```
It was the best of times,
it was the worst of
```

$$\text{BEST}_{11} = \min \begin{cases} \text{BEST}_{10} + S^2_{11,11} \\ \text{BEST}_9 + S^2_{10,11} \\ \text{BEST}_8 + S^2_{9,11} \\ \text{BEST}_7 + S^2_{8,11} \\ \text{BEST}_6 + S^2_{7,11} \\ \dots \end{cases}$$

$289 + (S_{7,11})^2$ is the best

$$\text{BEST}_i = \min_{j=0}^{i-1} \left\{ \text{BEST}_j + S^2_{j+1,i} \right\}$$

	0	1	2	3	4	5	6	7	8	9	10	11	12	13
best	0	1600	1296	1024	729	576	289	196	100	36	0	818	545	
choice	0	0	0	0	0	0	0	0	0	0	0	6	6	

aaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaa

```
It was the best of times,
it was the worst of times, it
```

$$\text{BEST}_{13} = \min \begin{cases} \text{BEST}_{12} + S^2_{13,13} \\ \text{BEST}_{11} + S^2_{12,13} \\ \ldots \\ \text{BEST}_{7} + S^2_{8,13} \\ \text{BEST}_{6} + S^2_{7,13} \end{cases}$$

$$\text{BEST}_i = \min_{j=0}^{i-1} \left\{ \text{BEST}_j + S^2_{j+1,i} \right\}$$

	0	1	2	3	4	5	6	7	8	9	10	11	12	13
best	0	1600	1296	1024	729	576	289	196	100	36	0	818	545	
choice	0	0	0	0	0	0	0	0	0	0	0	6	6	

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```
It was the best of times,it
was the worst of times,it
```

$$\text{BEST}_{13} = \min \begin{cases} \text{BEST}_{12} + S^2_{13,13} \\ \text{BEST}_{11} + S^2_{12,13} \\ \ldots \\ \text{BEST}_{7} + S^2_{8,13} \\ \text{BEST}_{6} + S^2_{7,13} \end{cases}$$

```
d-172-25-159-219:typeset abhi$ java typeset charly 42
0 best: 0 ch 0
1 best: 1600 ch 0
2 best: 1296 ch 0
3 best: 1024 ch 0
4 best: 729 ch 0
5 best: 576 ch 0
6 best: 289 ch 0
7 best: 196 ch 0
8 best: 100 ch 0
9 best: 36 ch 0
10 best: 0 ch 0
11 best: 818 ch 6
12 best: 545 ch 6
13 best: 452 ch 7
14 best: 340 ch 7
15 best: 244 ch 8
16 best: 164 ch 8
17 best: 117 ch 9
18 best: 37 ch 9
19 best: 16 ch 10
20 best: 0 ch 10
21 best: 509 ch 14
22 best: 413 ch 15
23 best: 344 ch 15
24 best: 133 ch 17
25 best: 118 ch 17
26 best: 62 ch 18
27 best: 32 ch 19
28 best: 4 ch 20
29 best: 444 ch 23
30 best: 348 ch 23
31 best: 277 ch 24
32 best: 197 ch 24
33 best: 149 ch 24
34 best: 87 ch 26
35 best: 66 ch 26
36 best: 446 ch 31
37 best: 377 ch 31
38 best: 297 ch 32
39 best: 233 ch 32
```

120

```
0 best: 0 ch 0
1 best: 1600 ch 0        It
2 best: 1296 ch 0        It was
3 best: 1024 ch 0        It was the
4 best: 729 ch 0         It was the best
5 best: 576 ch 0         It was the best of
6 best: 289 ch 0         It was the best of times,
7 best: 196 ch 0         It was the best of times, it
8 best: 100 ch 0         It was the best of times, it was
9 best: 36 ch 0          It was the best of times, it was the
10 best: 0 ch 0          It was the best of times, it was the worst
11 best: 818 ch 6        It was the best of times,\nit was the worst of
12 best: 545 ch 6        It was the best of times,\nit was the worst of times,
13 best: 452 ch 7        It was the best of times, it\nwas the worst of times, it
14 best: 340 ch 7        It was the best of times, it\nwas the worst of times, it was
15 best: 244 ch 8        It was the best of times, it was\nthe worst of times, it was the
16 best: 164 ch 8        It was the best of times, it was\nthe worst of times, it was the age
17 best: 117 ch 9        It was the best of times, it was the\nworst of times, it was the age of
18 best: 37 ch 9         It was the best of times, it was the\nworst of times, it was the age of wisdom,
19 best: 16 ch 10        It was the best of times, it was the worst\nof times, it was the age of wisdom, it
20 best: 0 ch 10         It was the best of times, it was the worst\nof times, it was the age of wisdom, it was
21 best: 509 ch 14       It was the best of times, it\nwas the worst of times, it was\nthe age of wisdom, it was the
22 best: 413 ch 15       It was the best of times, it was\nthe worst of times, it was the\nage of wisdom, it was the age
23 best: 344 ch 15       It was the best of times, it was\nthe worst of times, it was the\nage of wisdom, it was the age of
24 best: 133 ch 17       It was the best of times, it was the\nworst of times, it was the age of\nwisdom, it was the age of foolishness,
25 best: 118 ch 17       It was the best of times, it was the\nworst of times, it was the age of\nwisdom, it was the age of foolishness, it
26 best: 62 ch 18        It was the best of times, it was the\nworst of times, it was the age of wisdom,\nit was the age of foolishness, it was
```

Gerrymander

Congressional District 5

nati

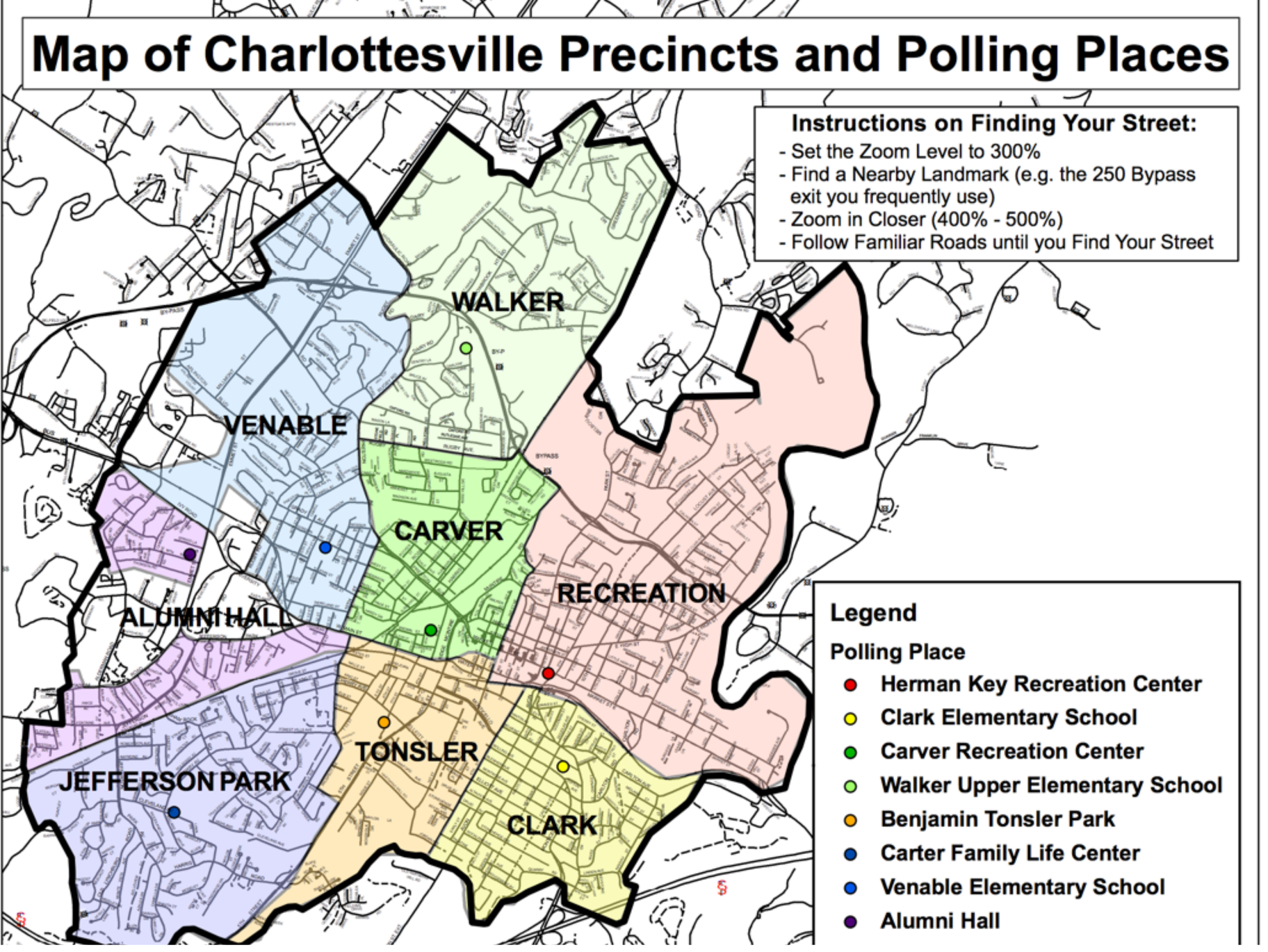
5

Nelson

9

Virgin

Map of Charlottesville Precincts and Polling Places



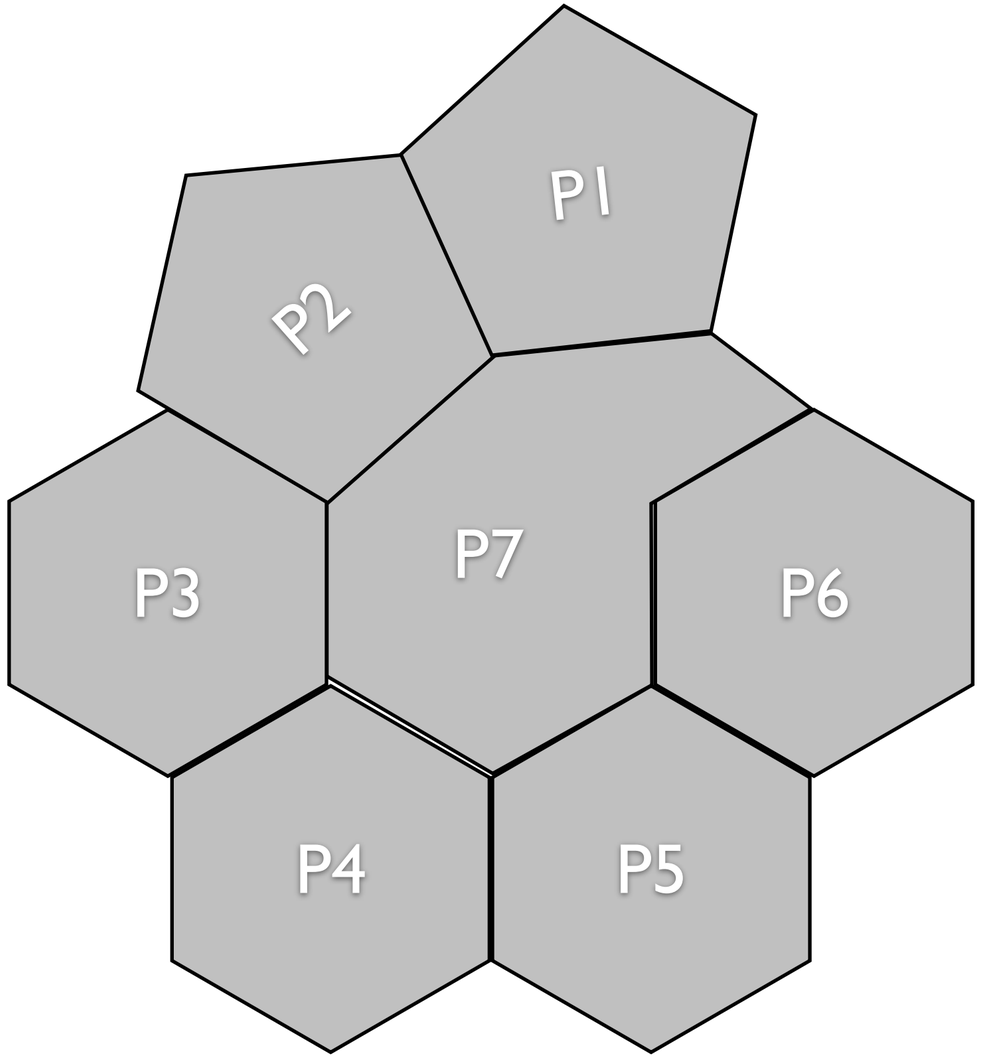
Instructions on Finding Your Street:

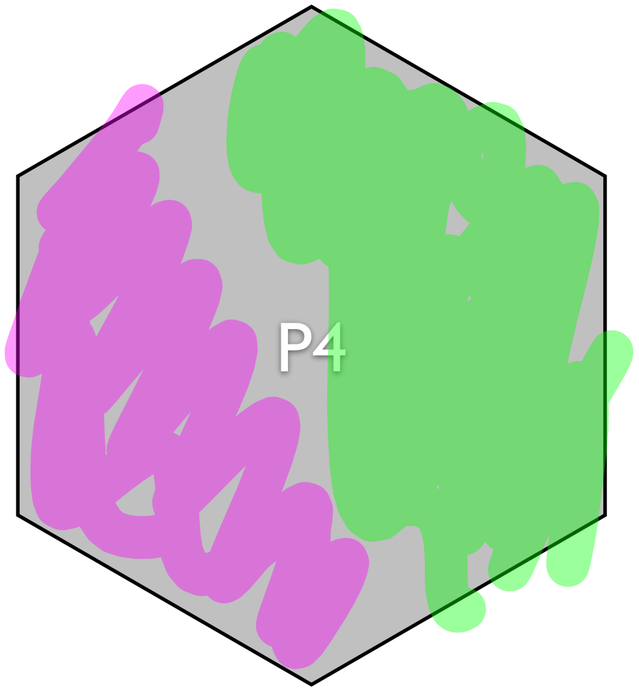
- Set the Zoom Level to 300%
- Find a Nearby Landmark (e.g. the 250 Bypass exit you frequently use)
- Zoom in Closer (400% - 500%)
- Follow Familiar Roads until you Find Your Street

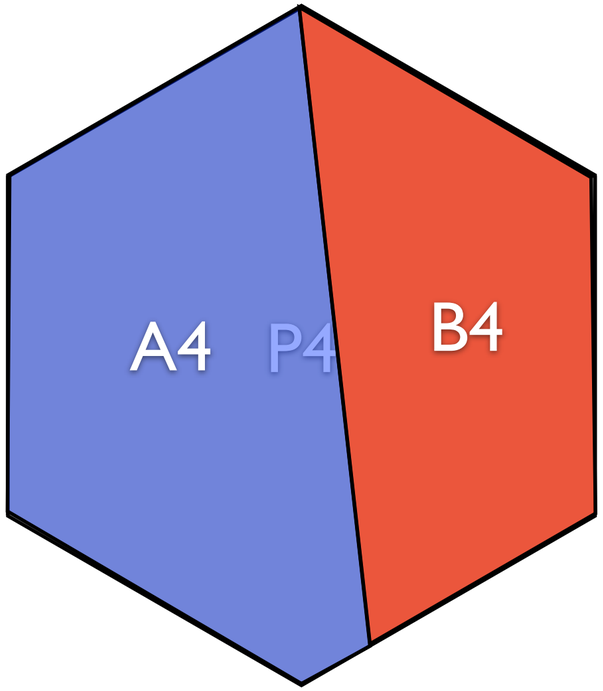
Legend

Polling Place

- Herman Key Recreation Center
- Clark Elementary School
- Carver Recreation Center
- Walker Upper Elementary School
- Benjamin Tonsler Park
- Carter Family Life Center
- Venable Elementary School
- Alumni Hall


P1
P2
P3
P7
P6
P4
P5


P4



P_4

A_4

party A

B_4

amount votes for B

2 districts

gerrymander problem

given: $A_1, \ldots A_n$, (n is even) M people in it. all precints have eq! population

an $A_i \to$ # for party A in precint i.

output: D_1, D_2. 2 districts, partition of the precincts.

$|D_1| = |D_2|$

$A(D_1) > \frac{M_n}{4}$

$A(D_2) > \frac{m}{4}$

if possible majority for party A in both districts

gerrymander problem

given: m $A_1, A_2, \ldots, A_n$ n is even

output: D_1, D_2

such that $|D_1| = |D_2|$

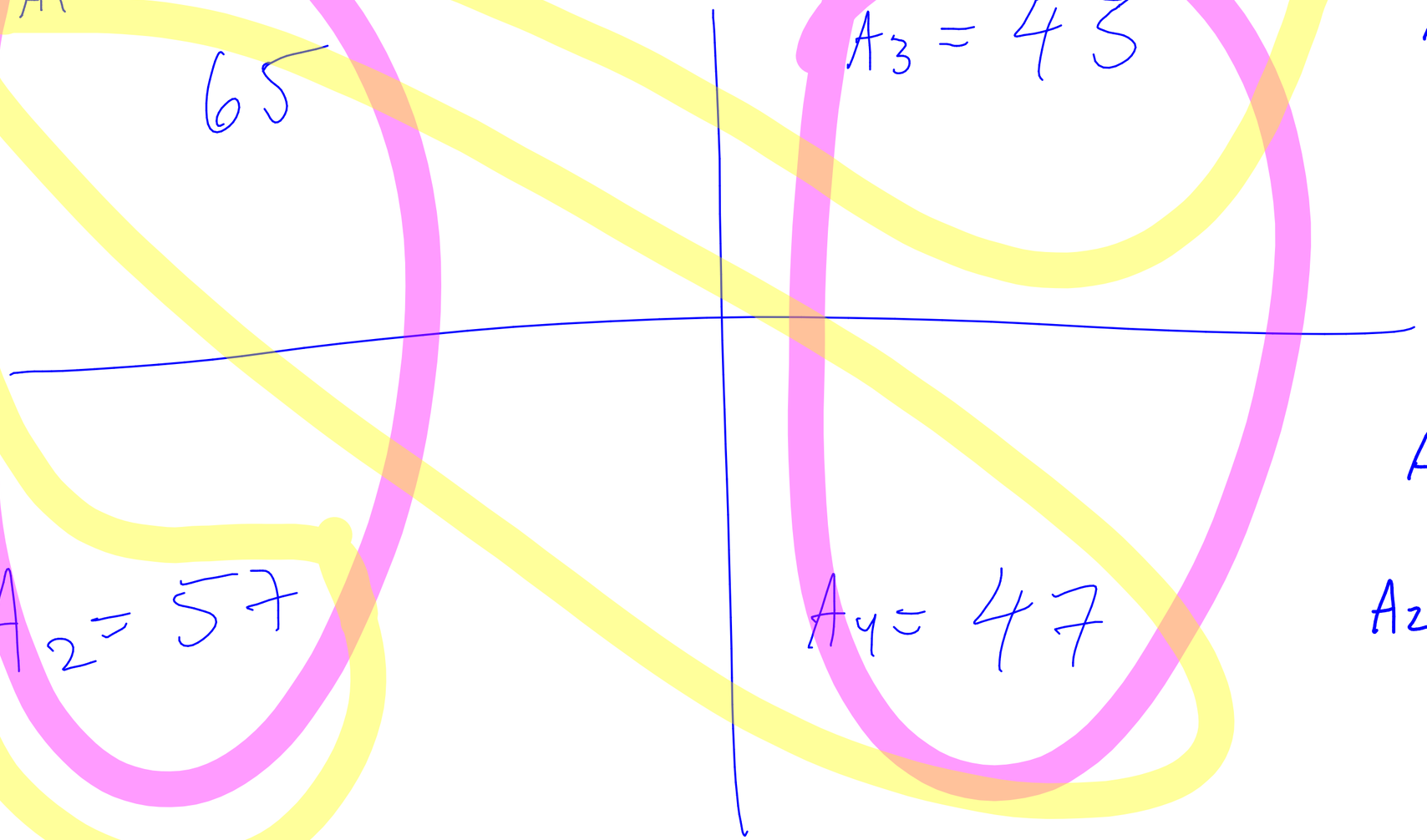
$$A(D_1) > \frac{mn}{4}$$

$$A(D_2) > \frac{mn}{4}$$

district 1 has $\frac{mn}{2}$

or “failure” if no such solution is possible

example



M = 100

A1+A2 = 122 (MAJ)

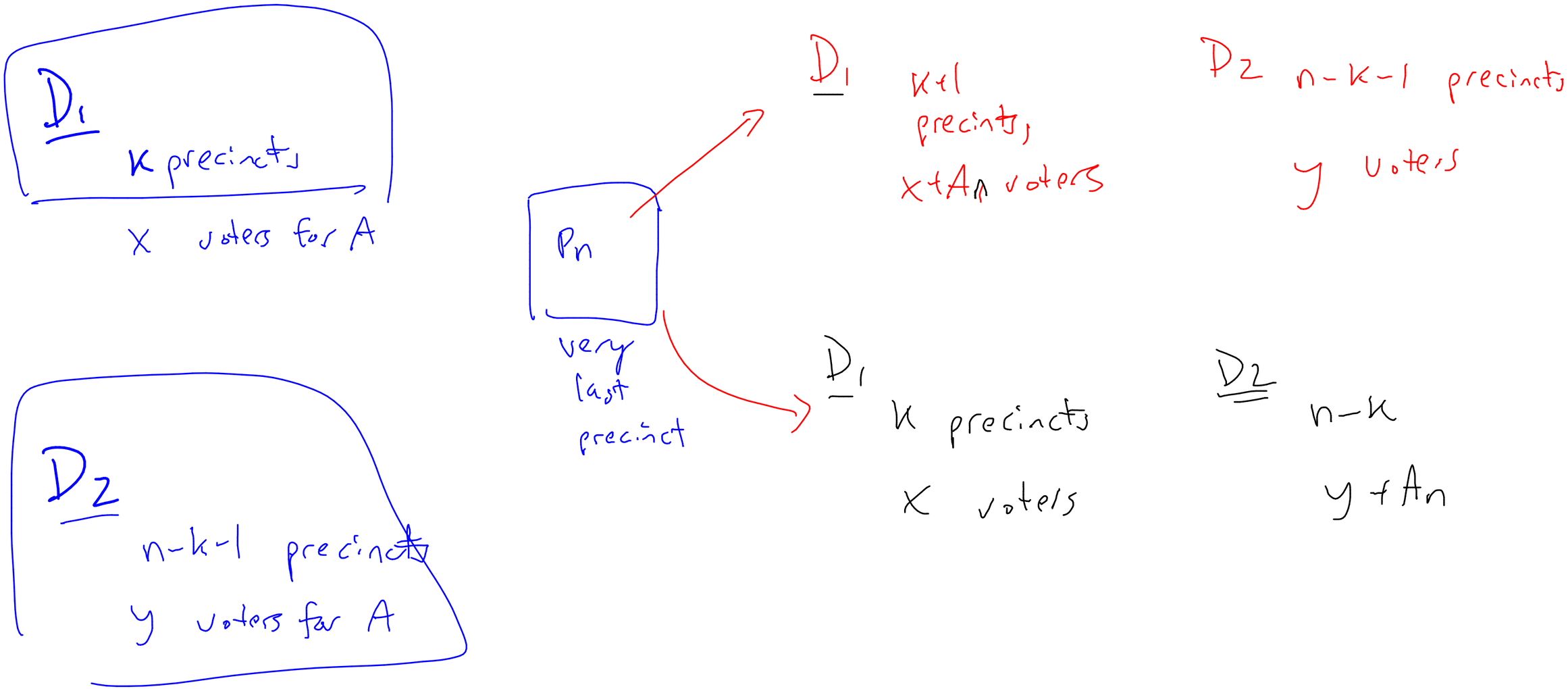
A3+A4 = 92 (MIN)

A1+A4 = 112 MAJ

A2+A3 = 102 MAJ

gerrymander

imagine very last precinct and how it is assigned:



gerrymander

$S_{j,k,x,y}$ = true if among the first j precincts
with k precincts to D_2
x vote for A in D_1
and y vote for A in D_2

false otherwise.

$S_{j,k,x,y}$ = $S_{j-1,k,x-A_j,y}$ OR $S_{j-1,k-1,x,y-A_j}$

"Assign P_j to D_1" "Assign P_j to D_2"

gerrymander

$S_{j,k,x,y}$ = there is a split of first **j** precincts in which |D1|=**k** and **x** people in D1 vote A **y** people in D2 vote A

$$S_{j,k,x,y} = S_{j-1,k-1,x-A_j,y} \vee S_{j-1,k,x,y-A_j}$$

gerrymander(P,A,m)

initialize array S[o,o,o,o] = TRUE

$S_{1,0,x,0} = S_{0000}$

$$S_{j,k,x,y} = S_{j-1,k-1,x-A_j,y} \vee S_{j-1,k,x,y-A_j}$$

gerrymander(P,A,m)

NP-complete

initialize array S[0,0,0,0] = true

for j=1,...,n

  for k=1,...,n/2

    for x=0,...,jm

      for y=0,...,jm

        fill table according to equation

search for true entry at S[n,n/2, >mn/4, >mn/4]

loop to j ??

min(j, n/2)

j·m, total # of voters

nm

$\Theta(n^4 M^2)$

100s

m = 10^6

$S_{n,\frac{n}{2},x,y}$

$> \frac{mn}{4}$ $> \frac{mn}{4}$