L19

4102

10.31.2013

Shortest paths,
negative weights
All pairs

abhi shelat

WHAT ABOUT NEGATIVE EDGE WEIGHTS?

XE Live Exchange Rates

Change / Remove a currency ...

Auto-refresh 15x
0 : 56

		USD	EUR	GBP	INR	AUD	CAD	ZAR	NZD	JPY
	1 USD	1.00000	0.72611	0.62261	61.3426	1.05366	1.04474	9.87360	1.21095	98.0247
	Inverse:	1.00000	1.37721	1.60613	0.01630	0.94907	0.95718	0.10128	0.82580	0.01020
	1 EUR	1.37721	1.00000	0.85747	84.4815	1.45111	1.43882	13.5980	1.66772	135.000
	Inverse:	0.72611	1.00000	1.16622	0.01184	0.68913	0.69501	0.07354	0.59962	0.00741
	1 GBP	1.60613	1.16622	1.00000	98.5241	1.69231	1.67799	15.8582	1.94494	157.440
	Inverse:	0.62261	0.85747	1.00000	0.01015	0.59091	0.59595	0.06306	0.51416	0.00635
	1 BMD	1.00000	0.72611	0.62261	61.3426	1.05366	1.04474	9.87360	1.21095	98.0247
	Inverse:	1.00000	1.37721	1.60613	0.01630	0.94907	0.95718	0.10128	0.82580	0.01020

Mid-market rates: 2013-10-29 15:53 UTC

Click on a currency code to learn about it.

WHERE DOES OLD ARGUMENT BREAK DOWN

$$w(p) \geq d_u + \delta(y, u)$$

FIRST IDEAS:

SSSP(G,s)

$$\text{SHORT}_{i,v} =$$

SSSP(G,s)

$$\text{SHORT}_{i,v} = \begin{cases} \infty & i = 0 \\ 0 & v = s \\ \min_{x \in V} & \left\{ \begin{array}{l} \text{SHORT}_{i-1,v} \\ \text{SHORT}_{i-1,x} + w(x,v) \end{array} \right\} \end{cases}$$

MAX LEN OF A SIMPLE PATH:

BELLMAN-FORD(G,s)

BELLMAN-FORD(G, s)

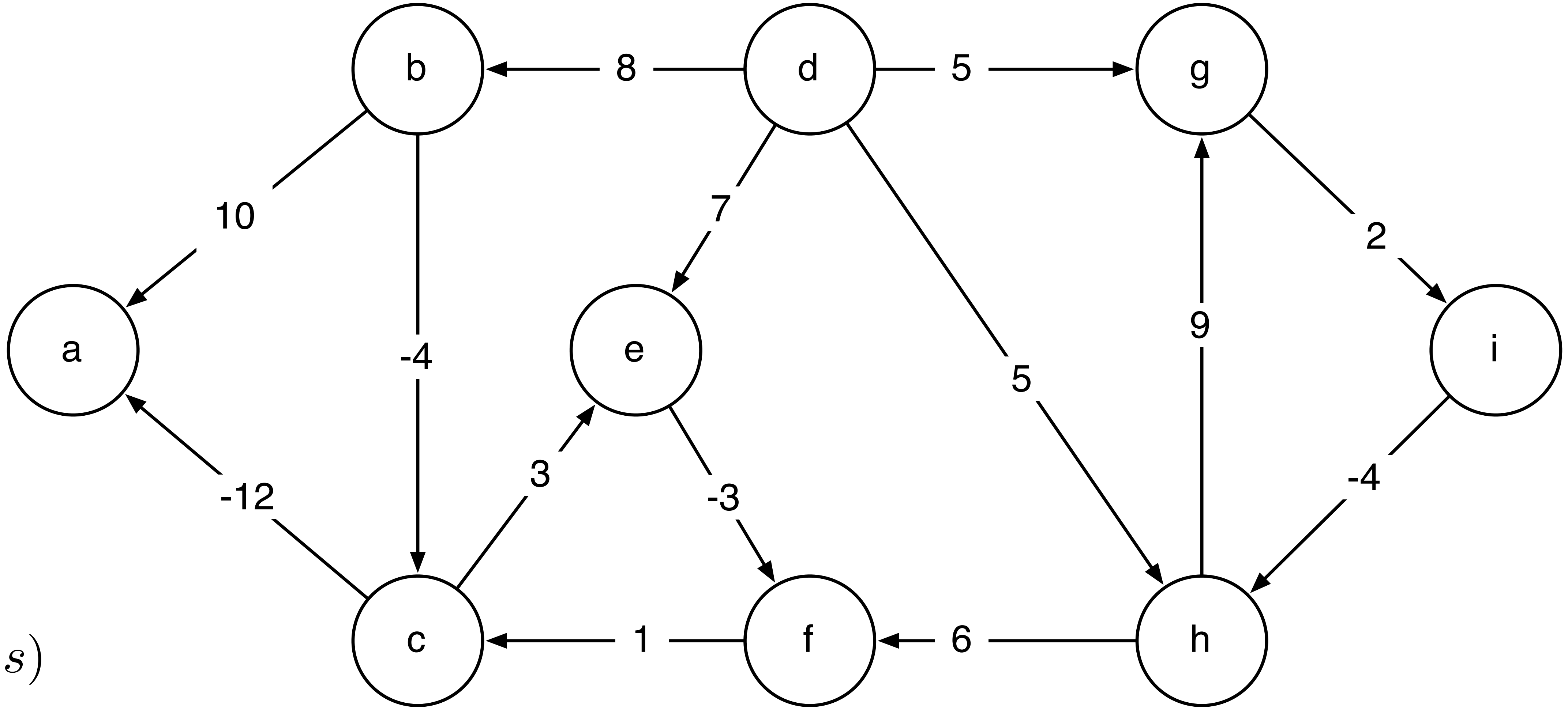
$\text{SHORT}_{0,s} \leftarrow 0$

$\forall v \in V - \{s\}$, $\text{SHORT}_{0,v} \leftarrow \infty$

for $i = 1, \ldots, V - 1$

  do for each $v \in V - \{s\}$

    do $\text{SHORT}_{i,v} = \min_{x \in Adj(v)} \left\{ \begin{array}{l} \text{SHORT}_{i-1,v} \\ w(x, v) + \text{SHORT}_{i-1,x} \end{array} \right\}$



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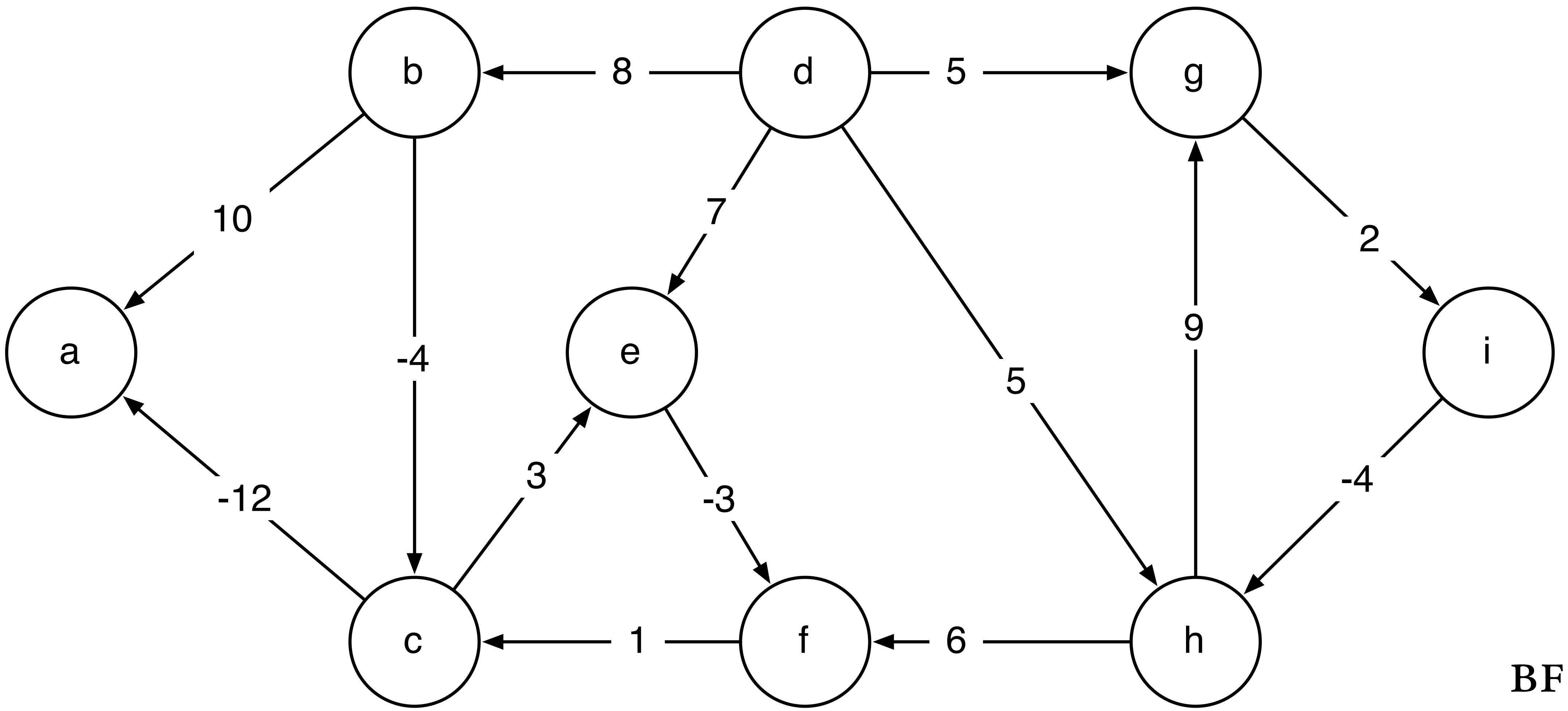

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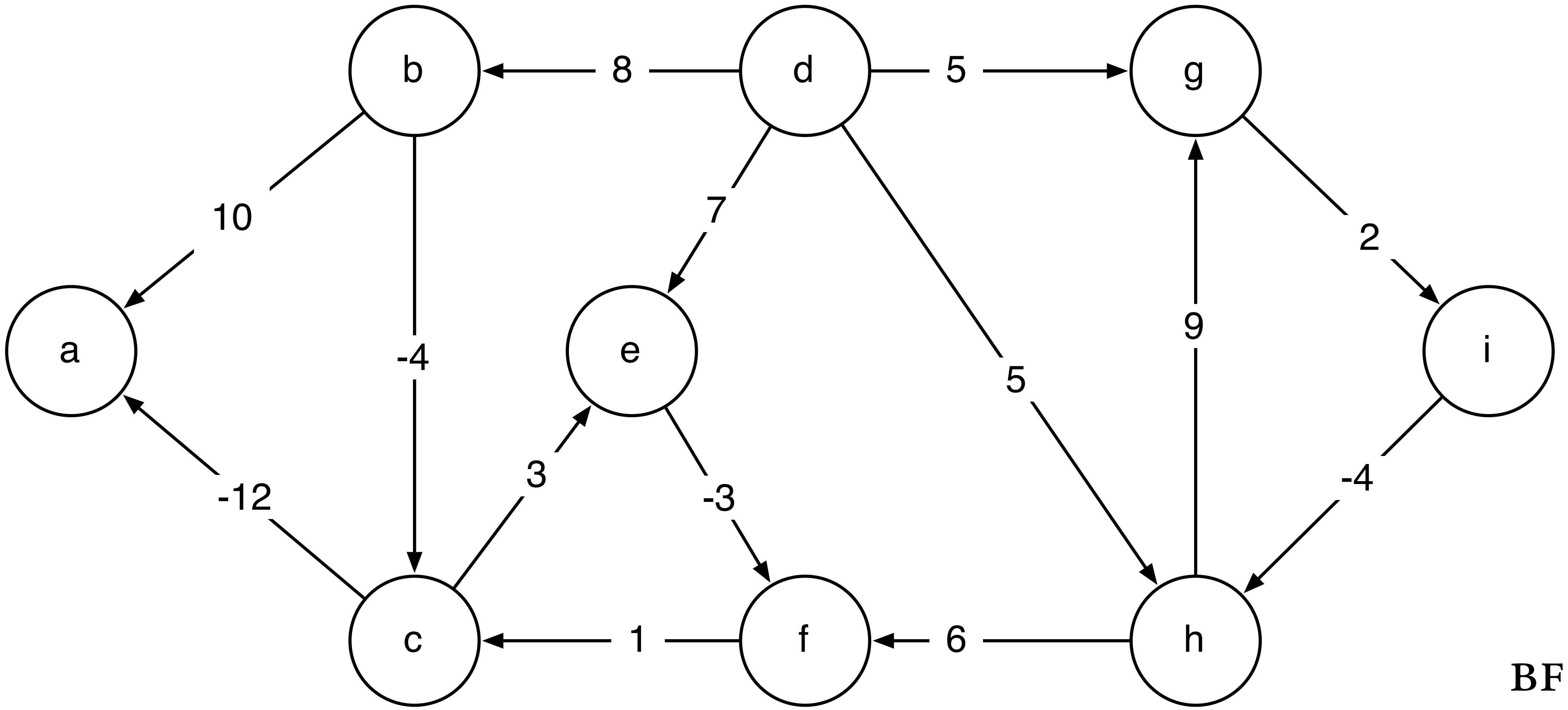
BF(G,d)

	0	1	2	3	4	5	6	7
A								
B								
C								
D	0							
E								
F								
G								
H								
I								



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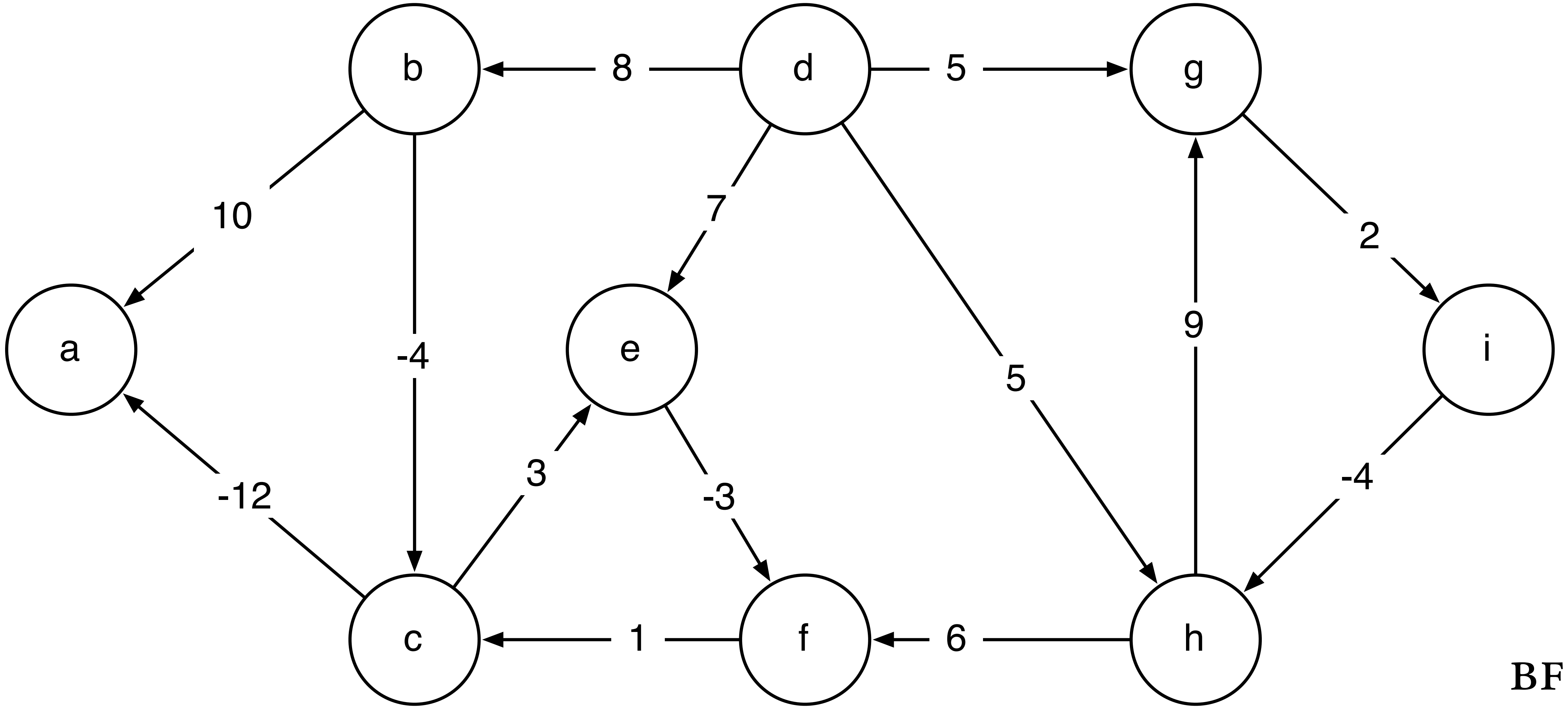
BF(G,d)

	0	1	2	3	4	5	6	7
A	∞							
B	∞							
C	∞							
D	0							
E	∞							
F	∞							
G	∞							
H	∞							
I	∞							



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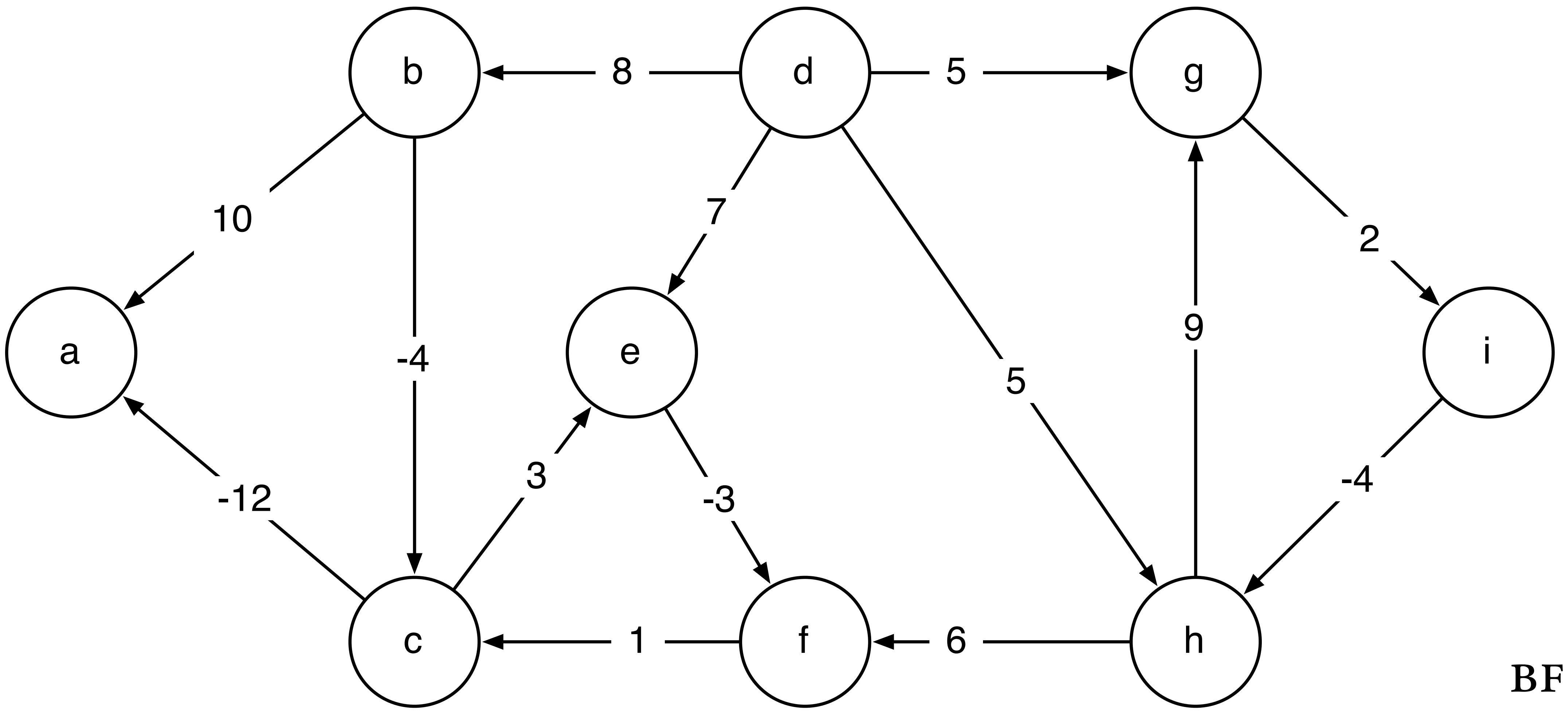
BF(G,d)

	0	1	2	3	4	5	6	7
A	∞							
B	∞	8						
C	∞							
D	0	0						
E	∞	7						
F	∞							
G	∞	5						
H	∞	5						
I	∞							



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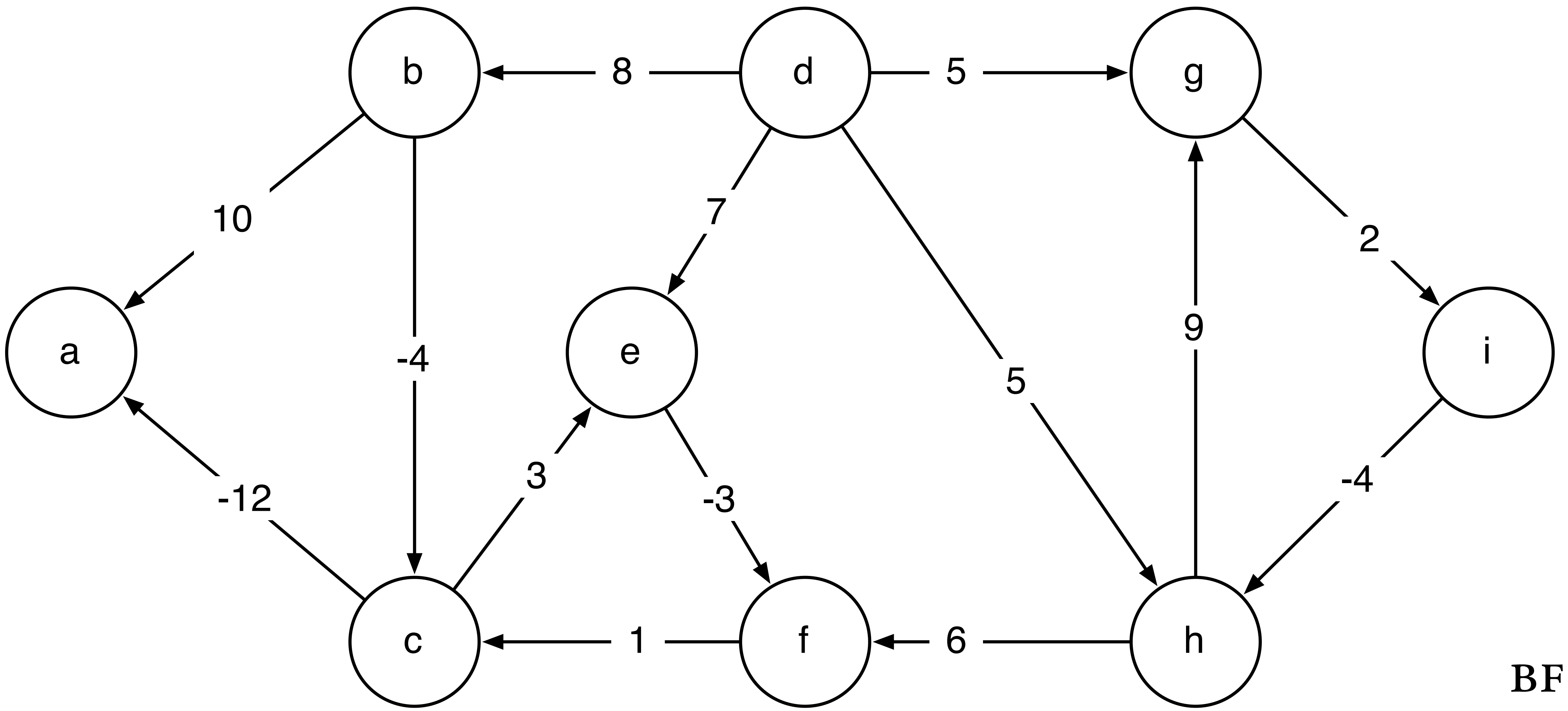
BF(G,d)

	0	1	2	3	4	5	6	7
A	∞		18					
B	∞	8	8					
C	∞		4					
D	0	0	0					
E	∞	7	7					
F	∞		4					
G	∞	5	5					
H	∞	5	5					
I	∞		7					



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BF(G,d)

	0	1	2	3	4	5	6	7
A	∞		18	-8				
B	∞	8	8	8				
C	∞		4	4				
D	0	0	0	0				
E	∞	7	7	7				
F	∞		4	4				
G	∞	5	5	5				
H	∞	5	5	3				
I	∞		7	7				

OPTIMIZATION

BELLMAN-FORD(G, s)

$\text{SHORT}_{0,s} \leftarrow 0$
$\forall v \in V - \{s\},\ \text{SHORT}_{0,v} \leftarrow \infty$
for $i = 1, \ldots, V - 1$
  do for each $e = (x, y) \in E$
    do $\text{SHORT}_{i,y} = \min \left\{ \begin{array}{l} \text{SHORT}_{i-1,y} \\ \text{SHORT}_{i,y} \\ w(x,y) + \text{SHORT}_{i-1,x} \end{array} \right\}$

BELLMAN-FORD(G, s)

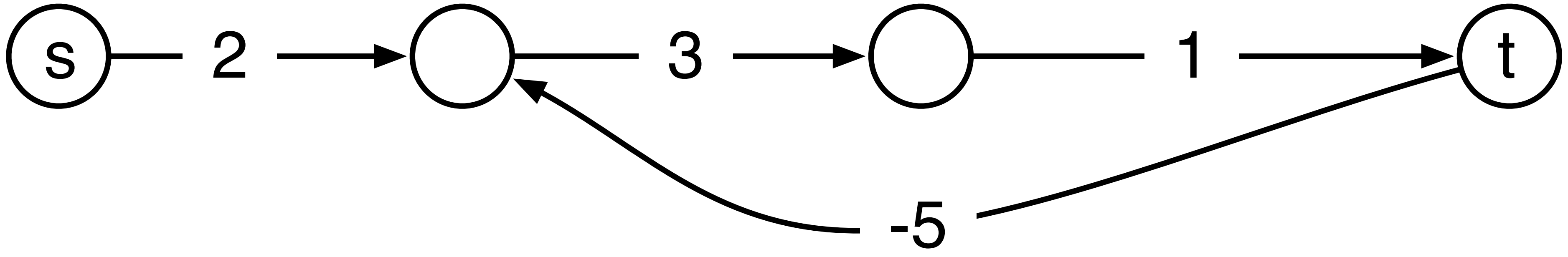
$d_s \leftarrow 0$
$\forall v \in V - \{s\},\ d_v \leftarrow \infty$
for $i = 1, \ldots, V - 1$
  do for each $e = (x, y) \in E$
    do $d_y \leftarrow \min \{\ d_y, w(x,y) + d_x\ \}$

RUNNING TIME

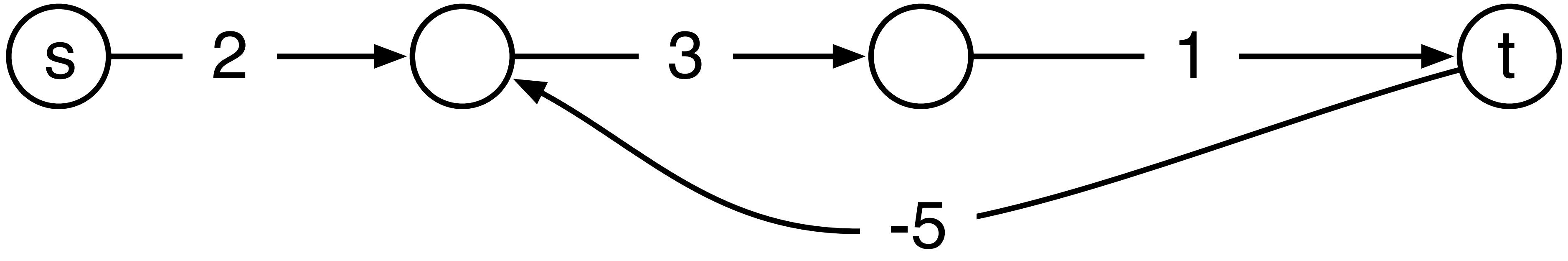
BELLMAN-FORD(G, s)

$d_s \leftarrow 0$
$\forall v \in V - \{s\},\ d_v \leftarrow \infty$
for $i = 1, \ldots, V - 1$
    do for each $e = (x, y) \in E$
        do $d_y \leftarrow \min\left\{\ d_y, w(x, y) + d_x\ \right\}$

NEGATIVE CYCLES?

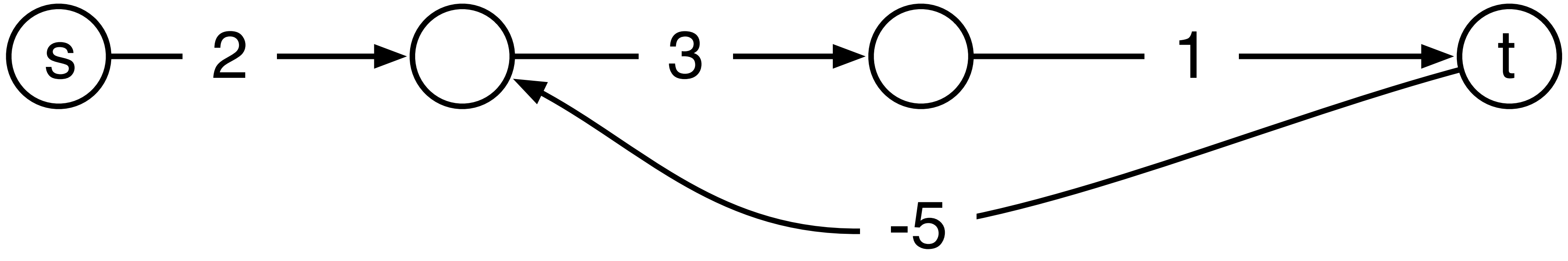


NEGATIVE CYCLES?



S	0			
A				
B				
T				

NEGATIVE CYCLES?



S	0	0	0	0
A	2	2	2	1
B		5	5	5
T			6	6

APPLICATIONS OF BF

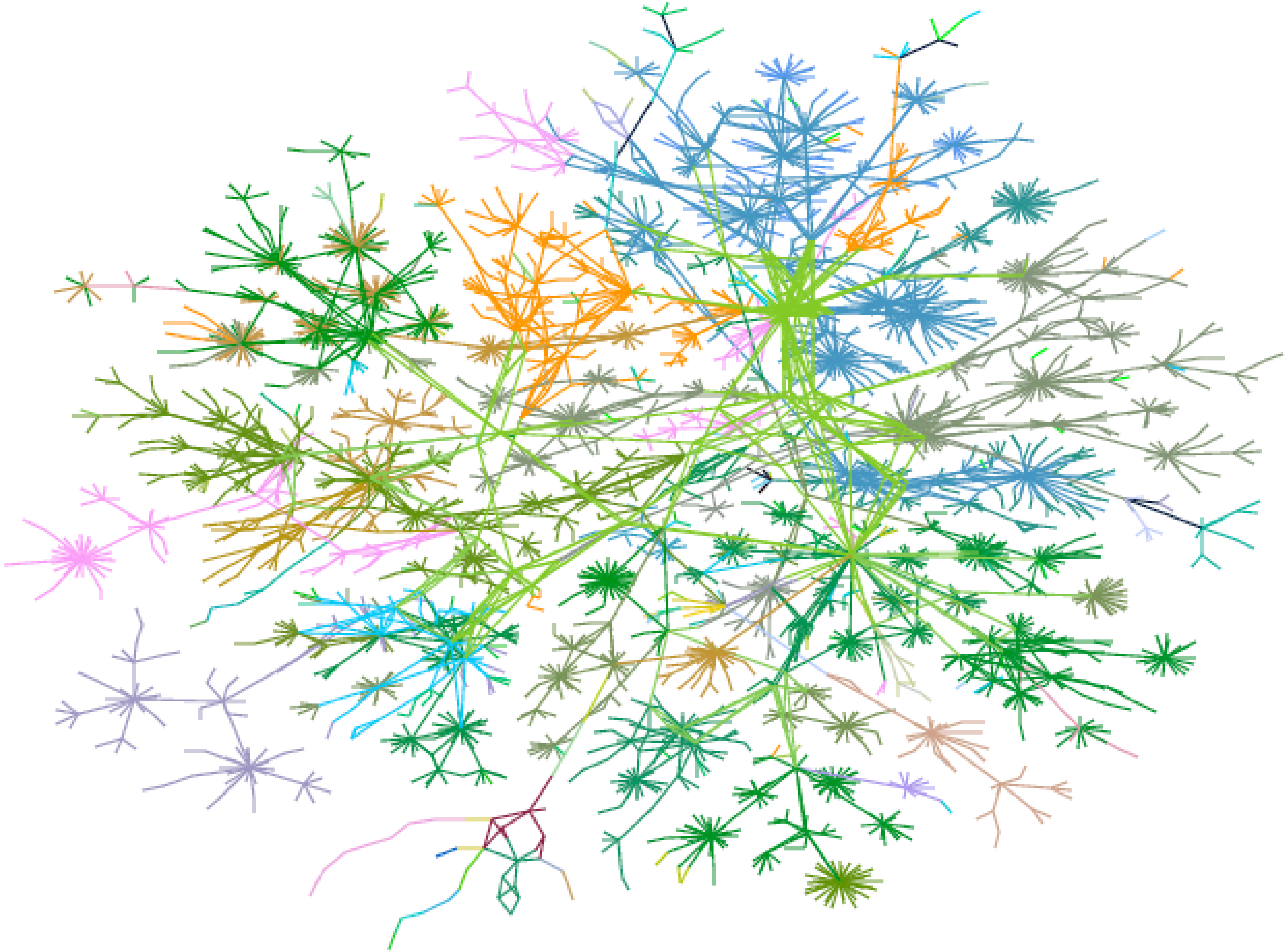
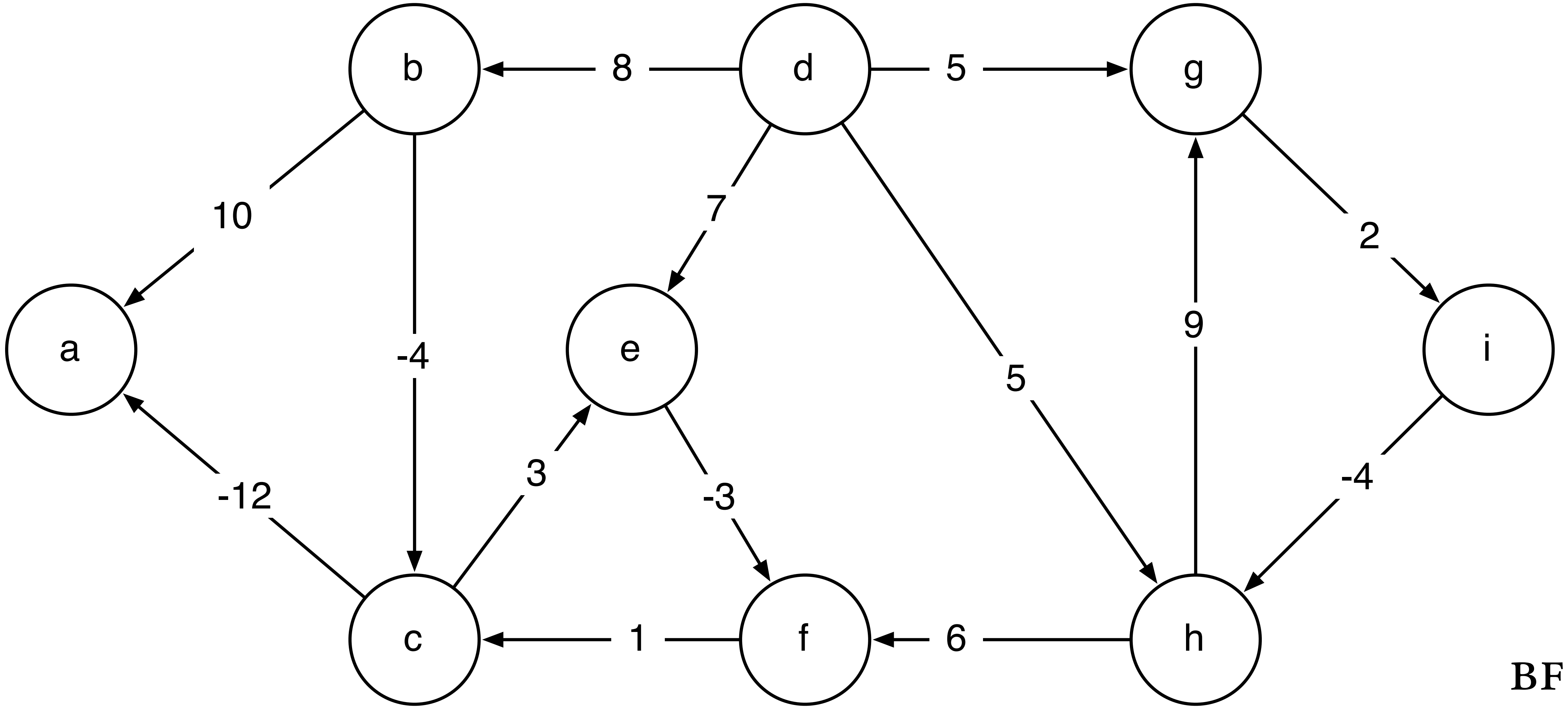


image: cheswick et al

Figure 3: Lucent's intranet as of 1 October 1999.



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	0	1	2	3	4	5	6	7
A	∞							
B	∞	8						
C	∞							
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E	∞	7						
F	∞							
G	∞	5						
H	∞	5						
I	∞							

WHAT HAPPENS WHEN
B CHANGES...

DISTANCE VECTOR

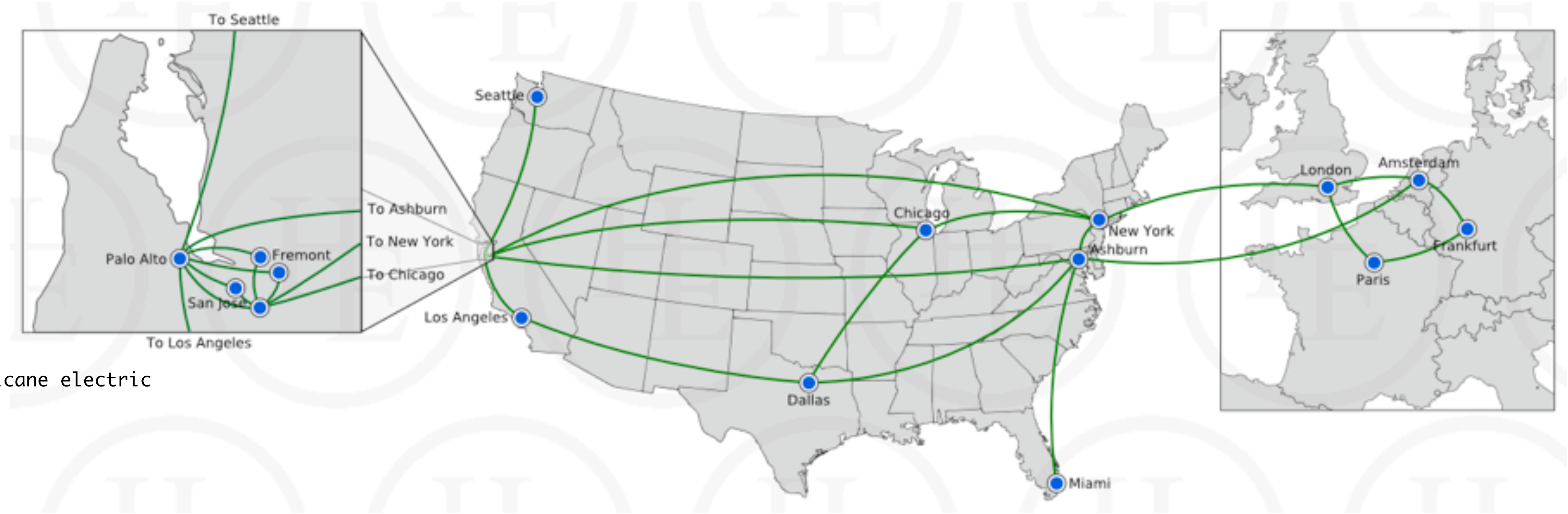
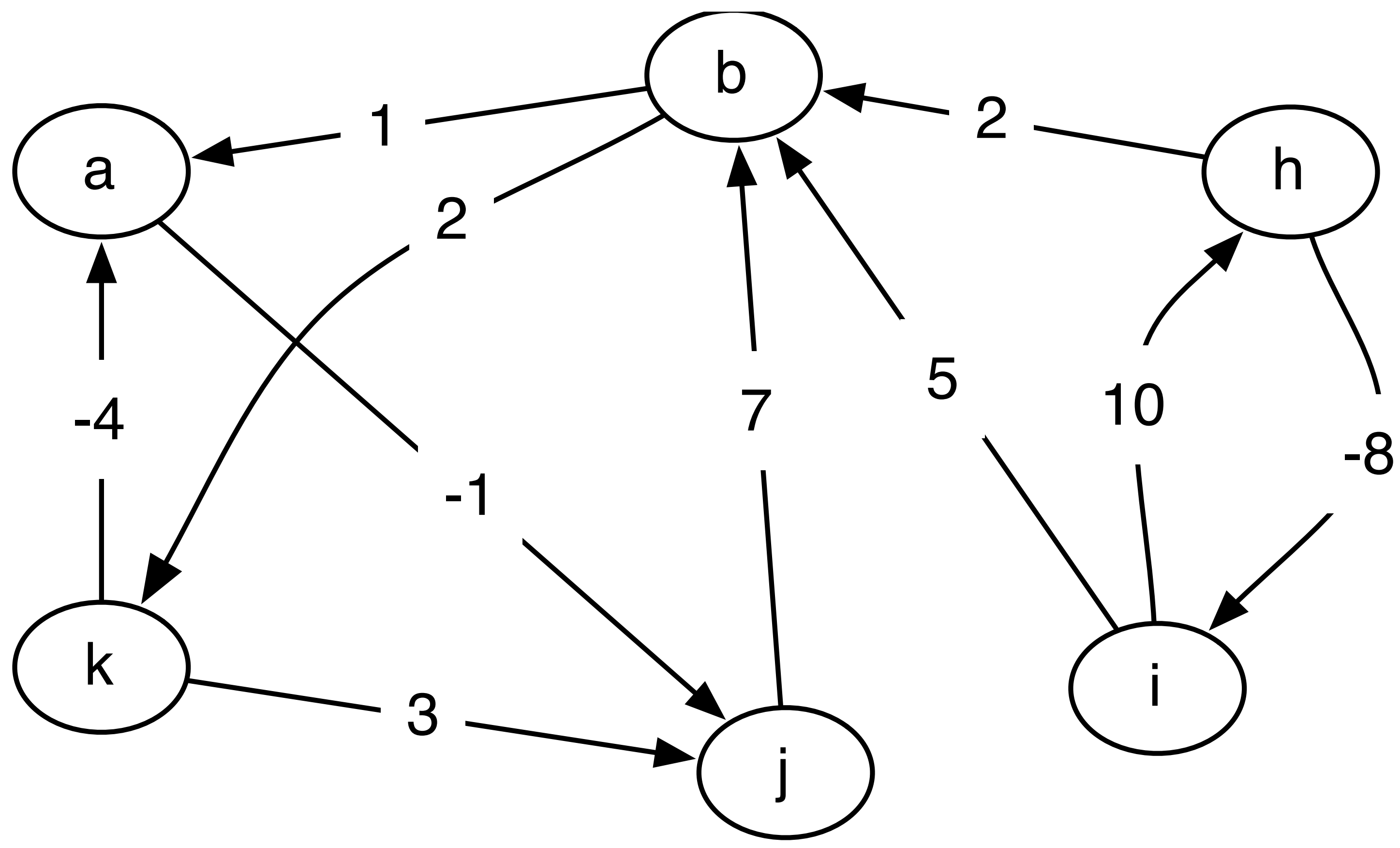


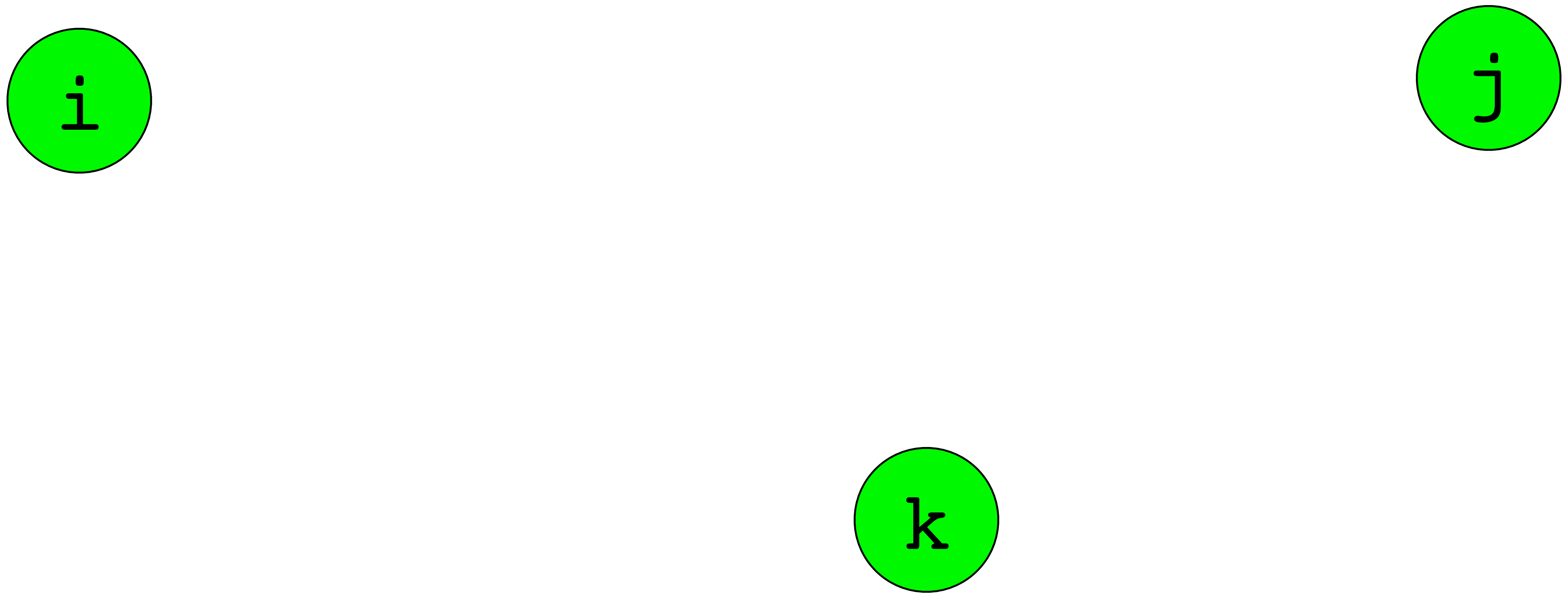
image: hurricane electric

ALL-PAIRS SHORTEST PATH



$$\mathrm{ASHORT}_{\mathrm{i,j,k}} =$$

$$\mathrm{ASHORT}_{\mathtt{i},\mathtt{j},\mathtt{k}} =$$



$$\mathrm{ASHORT}_{\mathrm{i,j,k}} =$$

$$
\text{ASHORT}_{i,j,k} = \begin{cases} w_{i,j} & k = 0 \\ \min \begin{cases} \text{ASHORT}_{i,j,k-1} \\ \text{ASHORT}_{i,k,k-1} + \text{ASHORT}_{k,j,k-1} \end{cases} & k \geq 1 \end{cases}
$$

FLOYD-WARSHALL(G,W)

```
int graph[128][128], n; // a weighted graph and its size
  void floydWarshall() {
    for( int k = 0; k < n; k++ )
    for( int i = 0; i < n; i++ )
    for( int j = 0; j < n; j++ )
      graph[i][j] = min( graph[i][j], graph[i][k] + graph[k][j] );
  }
  int main {
    // initialize the graph[][] adjacency matrix and n
    // graph[i][i] should be zero for all i.
    // graph[i][j] should be "infinity" if edge (i, j) does not exist
    // otherwise, graph[i][j] is the weight of the edge (i, j)
    floydWarshall();
    // now graph[i][j] is the length of the shortest path from i to j
  }
```

Max flow

Min Cut

“Consider a rail network connecting two cities by way of a number of intermediate cities, where each link of the network has a number assigned to it representing its capacity. Assuming a steady state condition, find a maximal flow from one given city to the other.”

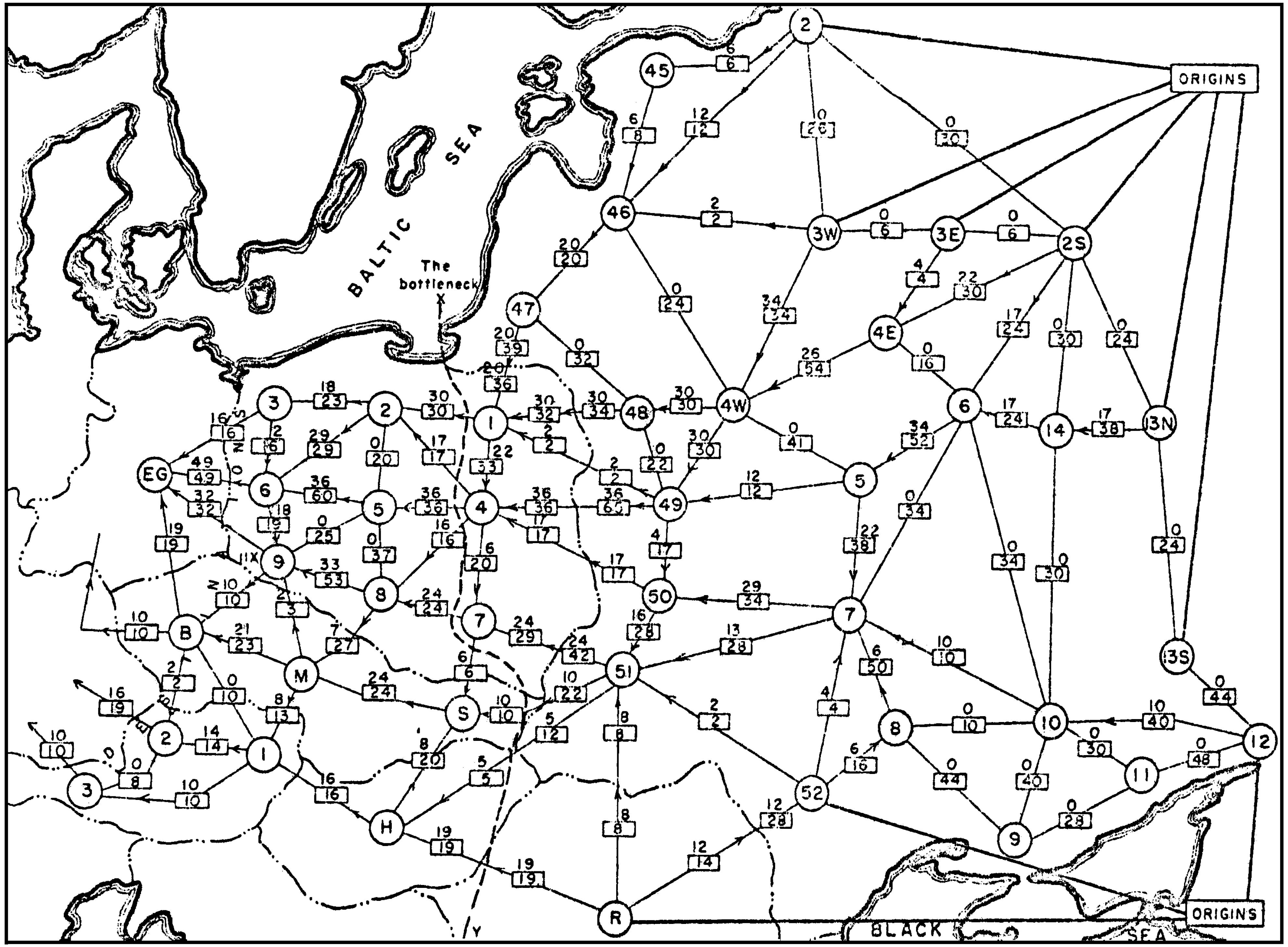


Figure 4 From Harris and Ross [3]: Schematic diagram of the railway network of the Western Soviet Union and East European countries, with a maximum flow of value 163,000 tons from Russia to Eastern Europe and a cut of capacity 163,000 tons indicated as 'The bottleneck'

courtesy Alexander Schrijver

FLOW NETWORKS

$$G = (V, E)$$

SOURCE + SINK:

CAPACITIES:

FLOW NETWORKS

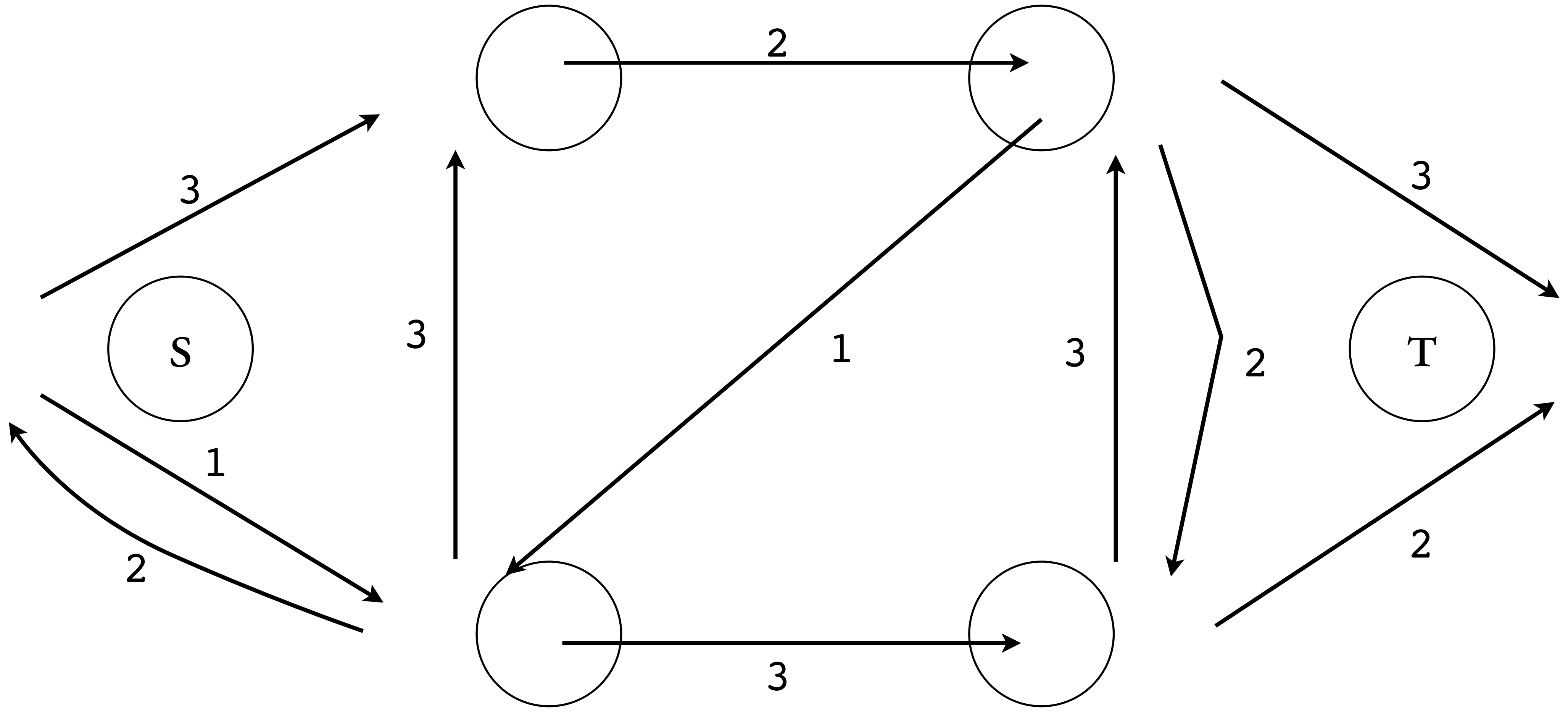
$$G = (V, E)$$

SOURCE + SINK: NODE S, AND T

CAPACITIES: $c(u, v)$

ASSUMED TO BE 0 IF NO (U,V) EDGE

EXAMPLE



FLOW

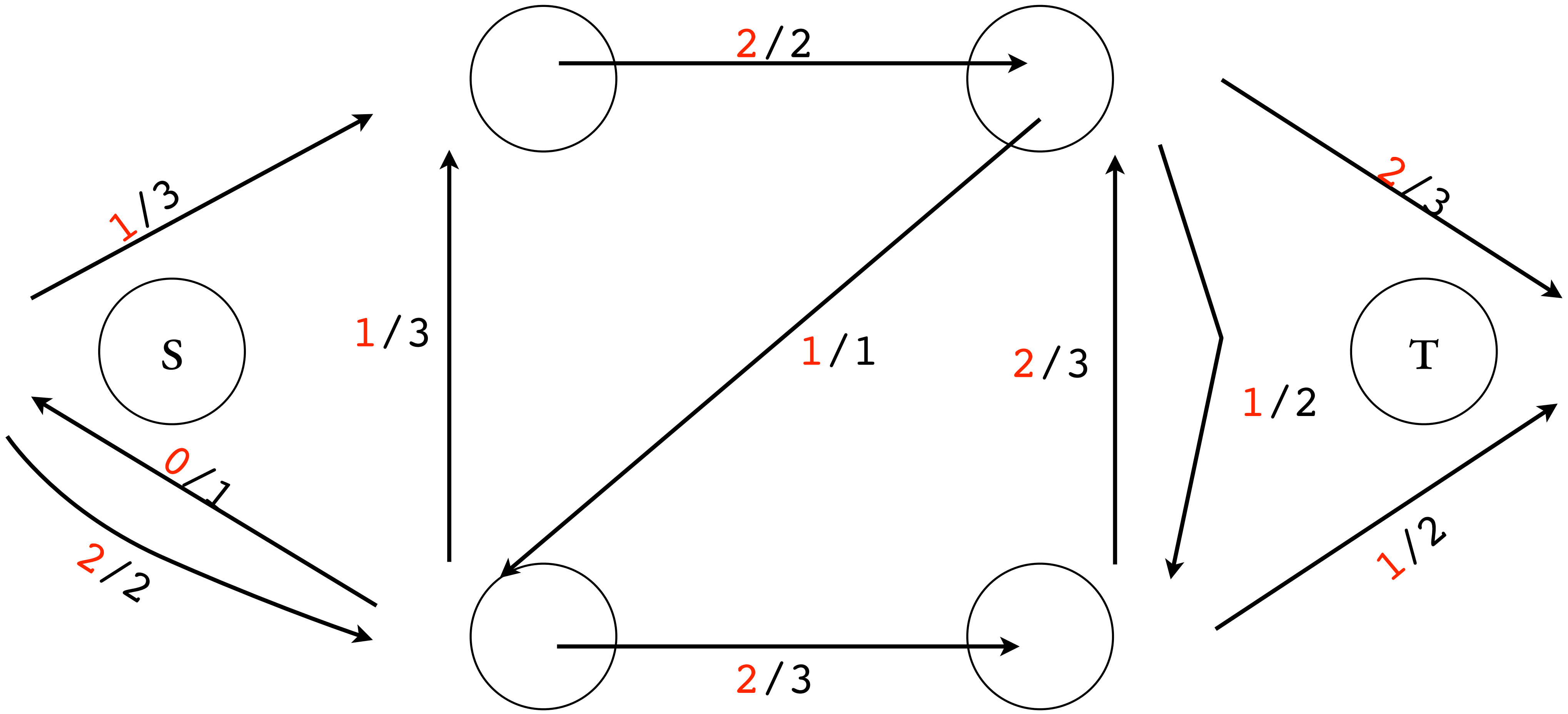
MAP FROM EDGES TO NUMBERS:

CAPACITY CONSTRAINT:

FLOW CONSTRAINT:

$$|f| =$$

EXAMPLE



MAX FLOW PROBLEM

GIVEN A GRAPH G, COMPUTE

GREEDY SOLUTION?