L19

4102

3.22.2016

abhi shelat

userid:

Explain how to find a minimum spanning tree:

(Try to just recall from memory, to test how much you understood. Then, look at your notes if you need to.)

Explain why your method works:

Give 2-3 sentences explaining at a high level.

SINGLE SOURCE SHORTEST PATH

SSSP

weights are > 0

$\Theta(E \log V)$

DIJKSTRA$(G = (V, E), s)$

for all $v \in V$
  do $d_u \leftarrow \infty$
    $\pi_u \leftarrow$ NIL
$d_s \leftarrow 0$
$Q \leftarrow$ MAKEQUEUE(V) ▷ use d_u as key
while $Q \neq \emptyset$
  do $u \leftarrow$ EXTRACTMIN(Q)
    for each $v \in Adj(u)$
      do if $d_v > d_u + w(u, v)$
        then $d_v \leftarrow d_u + w(u, v)$
          $\pi_v \leftarrow u$
          DECREASEKEY(Q, v)

breadth first search

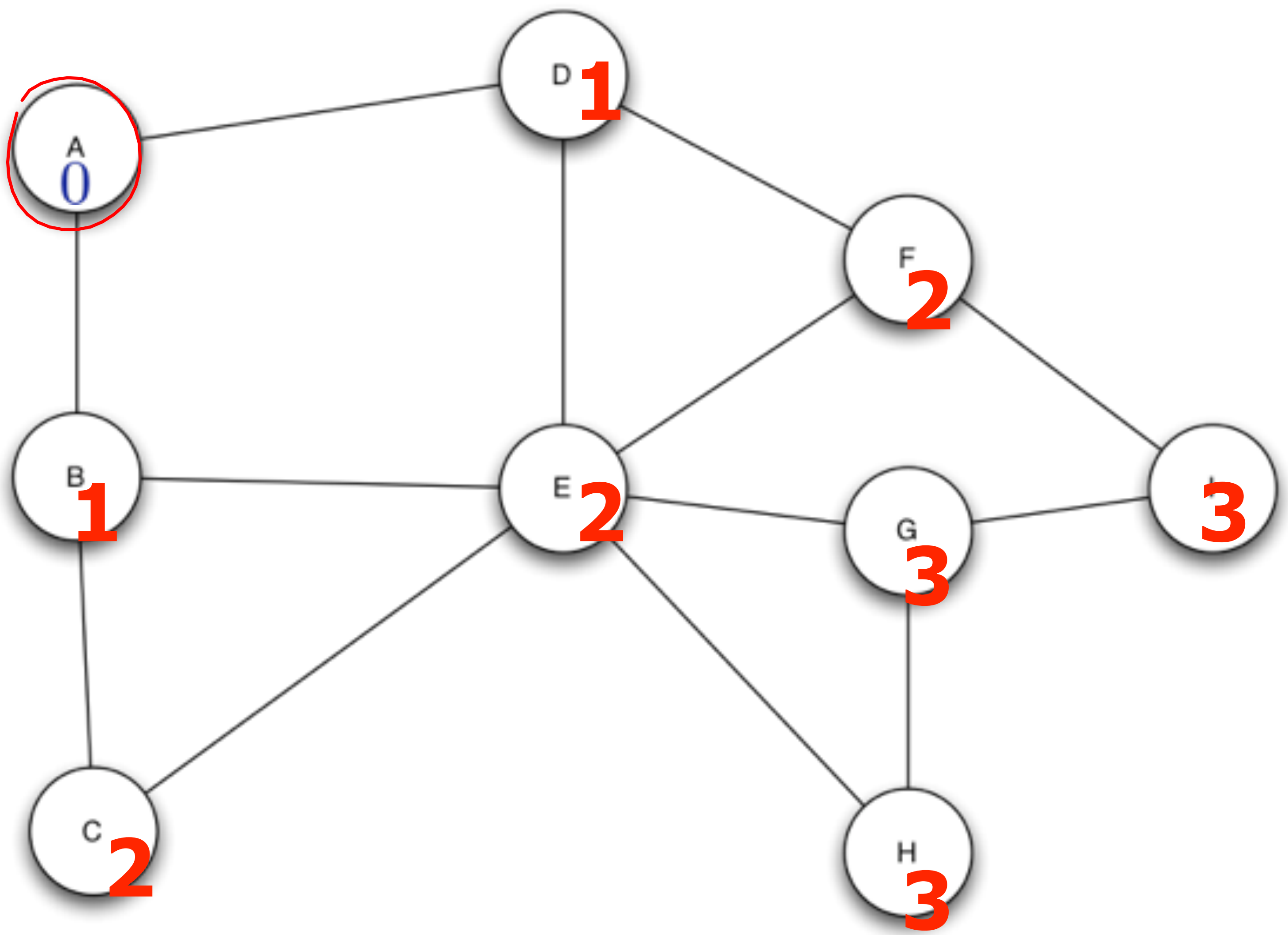
input: $G = (V, E), s$

output: $\forall v \in V \quad d_v = \delta(s, v)$

$w(e) = 1$

smallest # of edges from s to v

bfs(G, a)



Q

a
b
d
c
e

breadth first search

BFS(V, E, s)

for each $u \in V - \{s\}$

  do $d[u] \leftarrow \infty$

$d[s] \leftarrow 0$

$Q \leftarrow \emptyset$

ENQUEUE(Q, s)

while $Q \neq \emptyset$

  do $u \leftarrow$ DEQUEUE(Q)

    for each $v \in Adj[u]$

      do if $d[v] = \infty$

        then $d[v] \leftarrow d[u] + 1$

          ENQUEUE(Q, v)

$\Theta(E+V)$

```
BFS(V, E, s)
for each u ∈ V − {s}
    do d[u] ← ∞
d[s] ← 0
Q ← ∅
ENQUEUE(Q, s)
while Q ≠ ∅
    do u ← DEQUEUE(Q)
       for each v ∈ Adj[u]
           do if d[v] = ∞
                 then d[v] ← d[u] + 1
                      ENQUEUE(Q, v)
```

```
DIJKSTRA(G = (V, E), s)
for all v ∈ V
    do d_u ← ∞
       π_u ← NIL
d_s ← 0
Q ← MAKEQUEUE(V)    ▷ use d_u as key
while Q ≠ ∅
    do u ← EXTRACTMIN(Q)
       for each v ∈ Adj(u)
           do if d_v > d_u + w(u, v)
                 then d_v ← d_u + w(u, v)
                      π_v ← u
                      DECREASEKEY(Q, v)
```

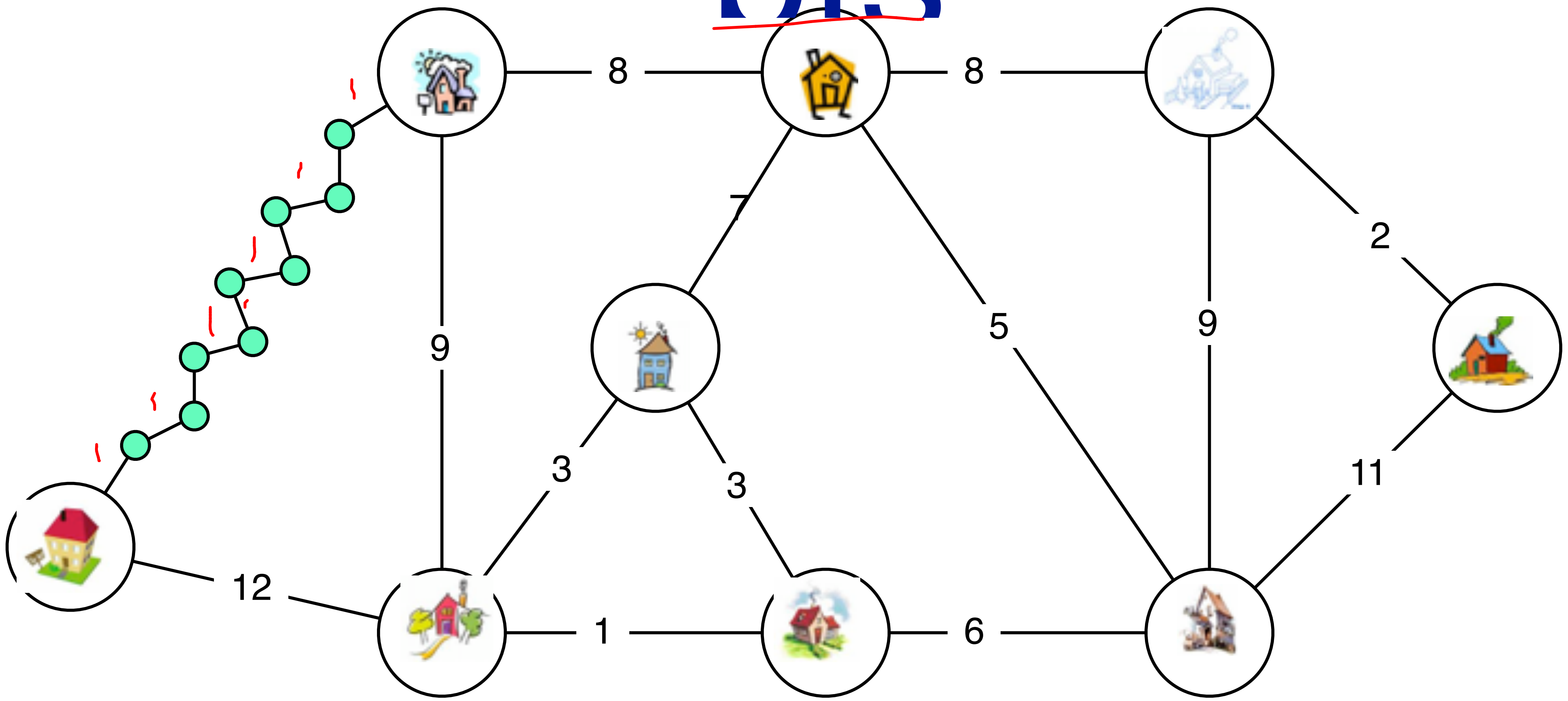
BFS theorem

Thm: At the end of the BFS algorithm, all of the nodes have been assigned

$$d_v = \delta(s, v).$$

↑ length of the SP when $w(e) = 1$.

bfs



shortest paths

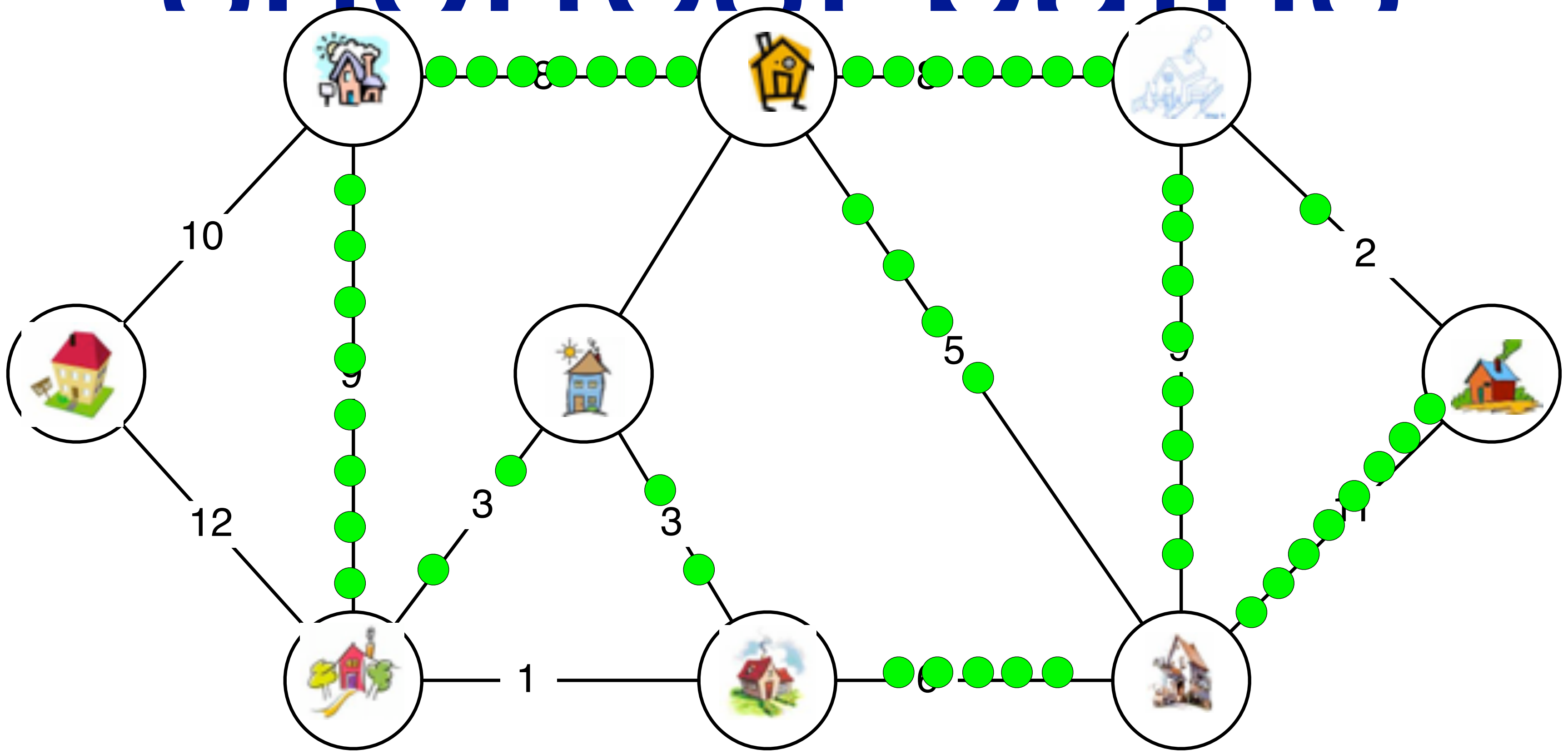


image:www.princegeorgeva.org, thefranciscofamily.org, www.rightdriveacademy.co.uk, www.ccscambridge.org, www.drawingcoach.com, www.pastoral.org.uk, www.daasgallery.com

What about Negative edge weights?

Mid-market rates as of **2016-03-31 17:38 UTC**

Currency code ▲▼	Currency name ▲▼	Units per USD	USD per Unit
USD	US Dollar	1.0000000000	1.0000000000
EUR	Euro	0.8783121137	1.1385474303
GBP	British Pound	0.6956087704	1.4375896950
INR	Indian Rupee	66.1909310706	0.0151078098
AUD	Australian Dollar	1.3050318080	0.7662648480
CAD	Canadian Dollar	1.2997506294	0.7693783541
SGD	Singapore Dollar	1.3478961522	0.7418969172
CHF	Swiss Franc	0.9590451582	1.0427037678
MYR	Malaysian Ringgit	3.8700000000	0.2583979328
JPY	Japanese Yen	112.5375383115	0.0088859239
CNY	Chinese Yuan Renminbi	6.4492409303	0.1550570076
NZD	New Zealand Dollar	1.4480018872	0.6906068347
THB	Thai Baht	35.1005319022	0.0284895968
HUF	Hungarian Forint	275.7012427385	0.0036271146
AED	Emirati Dirham	3.6730000000	0.2722570106
HKD	Hong Kong Dollar	7.7563973683	0.1289258341
MXN	Mexican Peso	17.3168505322	0.0577472213
ZAR	South African Rand	14.7201431400	0.0679341220
PHP	Philippine Peso	45.9250000000	0.0217746326

EUR

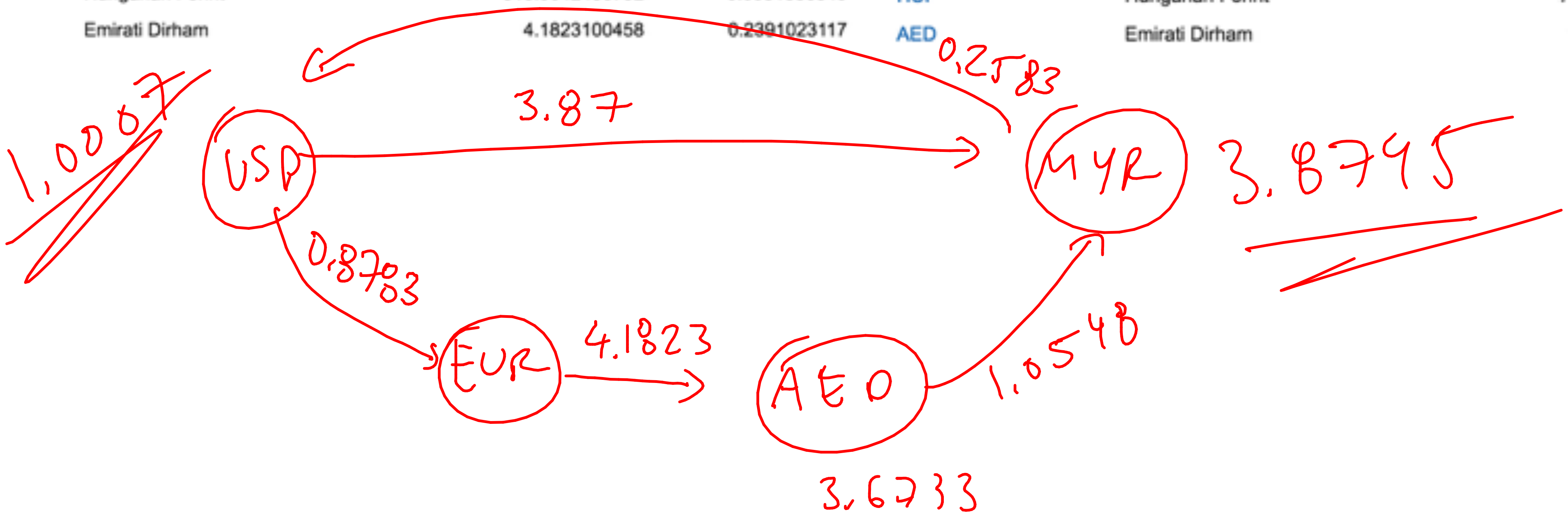
Mid-market rates as of 2016-03-31 17:40 UTC

Currency code ▲▼	Currency name ▲▼	Units per EUR	EUR per Unit
USD	US Dollar	1.1386632306	0.8782227907
EUR	Euro	1.0000000000	1.0000000000
GBP	British Pound	0.7921136388	1.2624451227
INR	Indian Rupee	75.3658843112	0.0132686030
AUD	Australian Dollar	1.4859561878	0.6729673514
CAD	Canadian Dollar	1.4796754127	0.6758238945
SGD	Singapore Dollar	1.5347639238	0.6515660060
CHF	Swiss Franc	1.0917416715	0.9159676012
MYR	Malaysian Ringgit	4.4140052400	0.2265516114
JPY	Japanese Yen	128.1388820287	0.0078040325
CNY	Chinese Yuan Renminbi	7.3411003512	0.1362193612
NZD	New Zealand Dollar	1.6484648003	0.6066250246
THB	Thai Baht	39.9627318192	0.0250233143
HUF	Hungarian Forint	313.9042436792	0.0031856849
AED	Emirati Dirham	4.1823100458	0.2391023117

XE Currency Table: AED - Emirati Dirham

Mid-market rates as of 2016-03-31 17:39 UTC

Currency code ▲▼	Currency name ▲▼	Units per AED	AED per Unit
USD	US Dollar	0.2722570106	3.6730000000
EUR	Euro	0.2391289974	4.1818433177
GBP	British Pound	0.1893997890	5.2798369266
INR	Indian Rupee	18.0207422309	0.0554916100
AUD	Australian Dollar	0.3552996418	2.8145257760
CAD	Canadian Dollar	0.3538334124	2.8261887234
SGD	Singapore Dollar	0.3669652245	2.7250538559
CHF	Swiss Franc	0.2610686193	3.8304105746
MYR	Malaysian Ringgit	1.0548325619	0.9480177576
JPY	Japanese Yen	30.6399242607	0.0326371564
CNY	Chinese Yuan Renminbi	1.7555154332	0.5696332719
NZD	New Zealand Dollar	0.3941937299	2.5368237088
THB	Thai Baht	9.5553789460	0.1046530970
HUF	Hungarian Forint	75.0637936939	0.0133220019
AED	Emirati Dirham	1.0000000000	1.0000000000



Mid-market rates as of **2016-03-31 17:40 UTC**

Currency code ▲▼	Currency name ▲▼	Units per EUR	EUR per Unit
USD	US Dollar	1.1386632306	0.8782227907
EUR	Euro	1.0000000000	1.0000000000
GBP	British Pound	0.7921136388	1.2624451227
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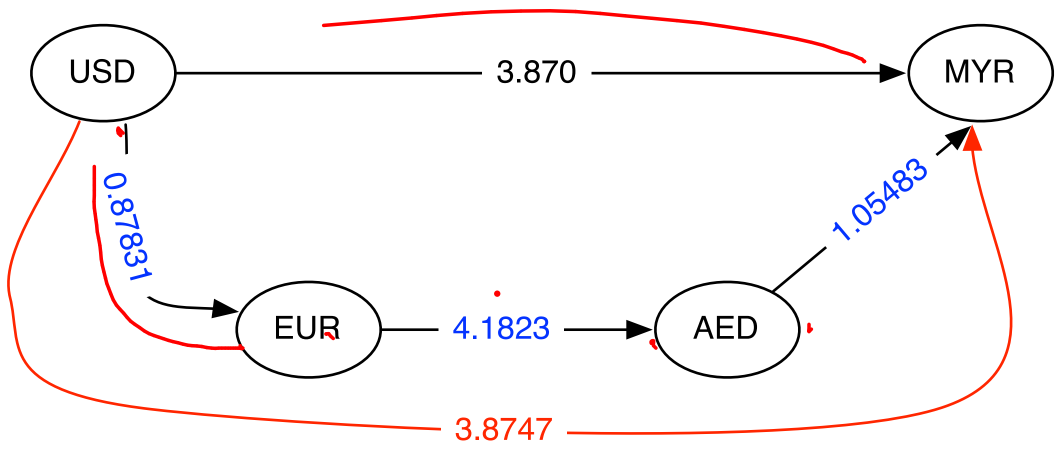
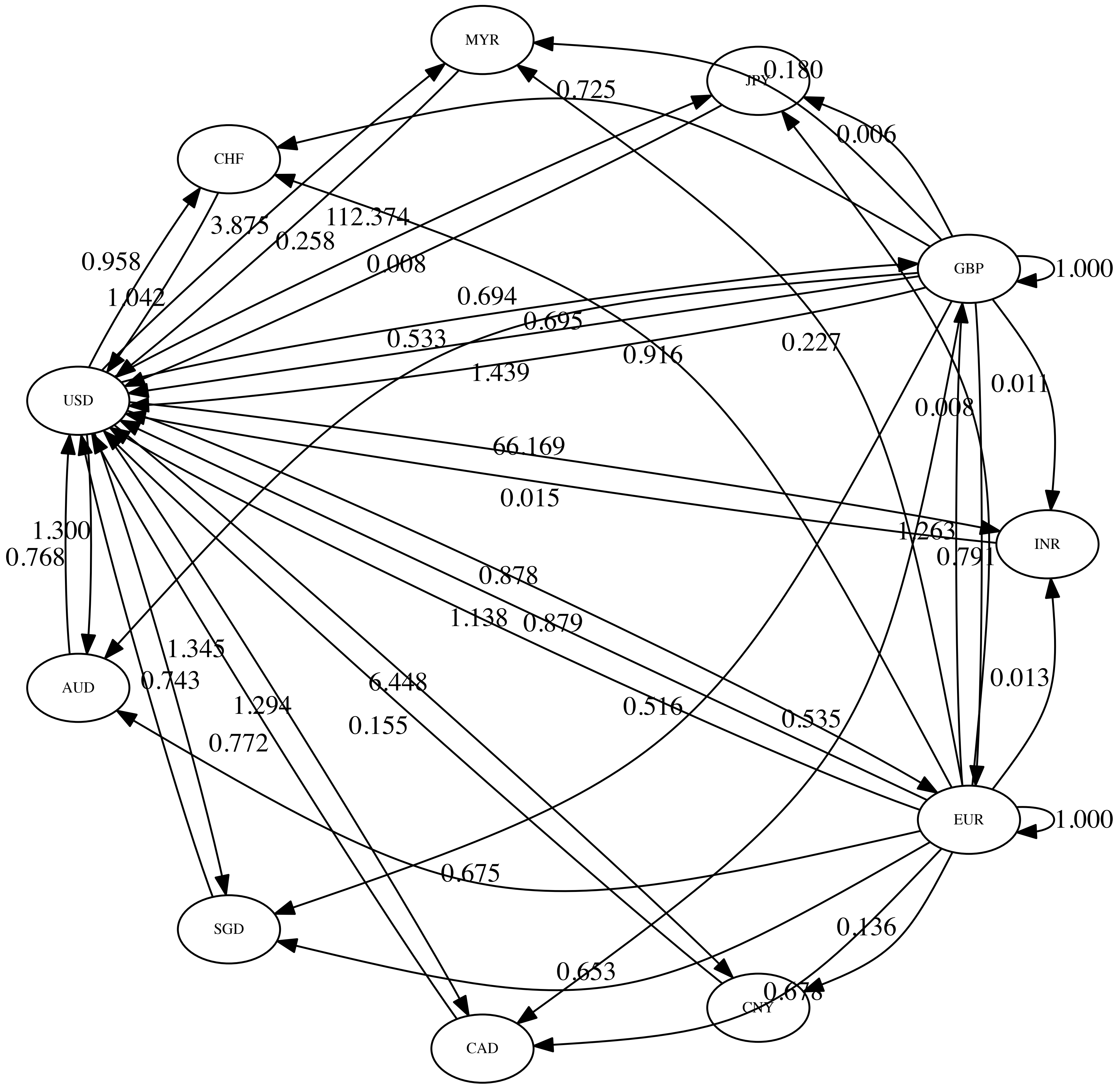
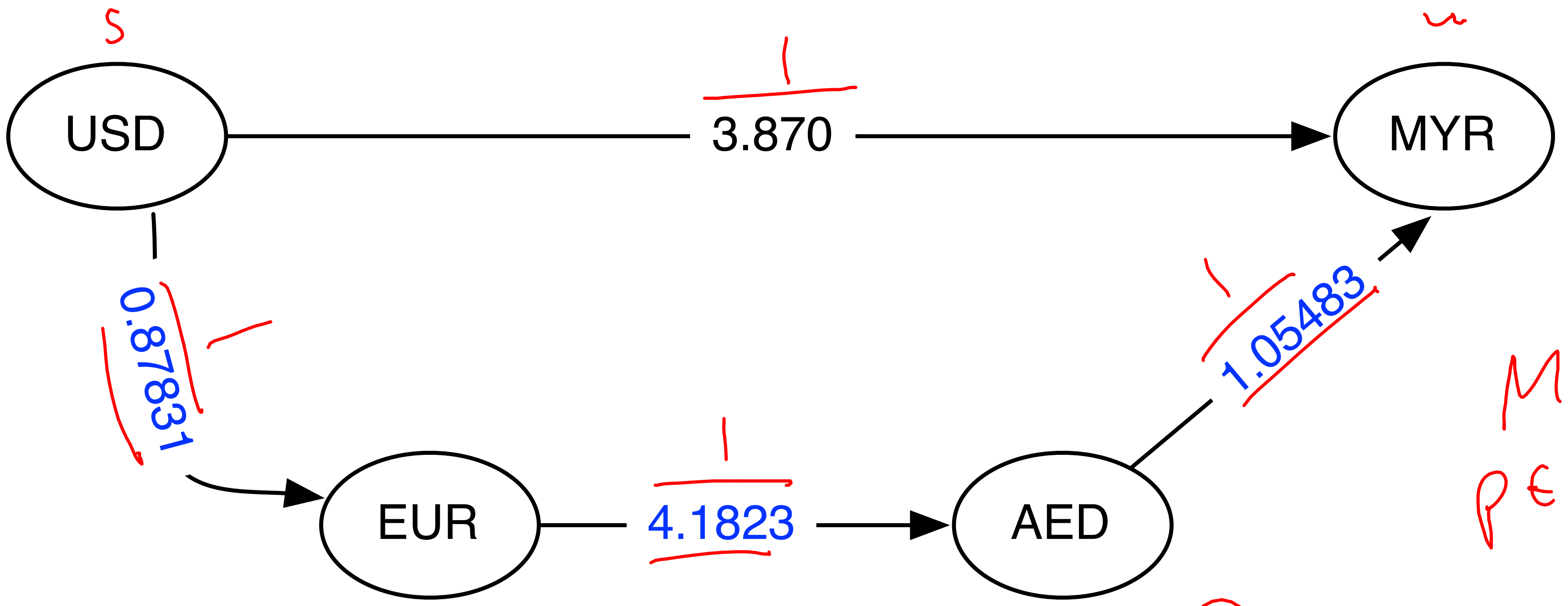



MYR
JPY
CHF
GBP
USD
INR
AUD
EUR
SGD
CNY
CAD
0.180
0.725
0.006
3.875
112.374
0.258
0.958
0.008
1.000
0.694
1.042
0.695
0.533
0.916
0.227
1.439
0.011
0.008
66.169
0.015
1.263
1.300
0.791
0.768
0.878
1.138
0.879
1.345
0.013
0.743
6.448
1.294
0.516
0.535
0.155
0.772
1.000
0.675
0.136
0.653
0.678



$$\max_{p \in \{s \leadsto u\}} \prod_{e \in p} w(e)$$

if $u.e \cdot e.a \cdot a.m > u.m$, then

$$\log\left(\frac{1}{u.e} \cdot \frac{1}{e.a} \cdot \frac{1}{a.m}\right) < \log\left(\frac{1}{u.m}\right)$$

$$\log\left(\frac{1}{u.e}\right) + \log\left(\frac{1}{e.a}\right) + \log\left(\frac{1}{a.m}\right) < \log\left(\frac{1}{u.m}\right)$$

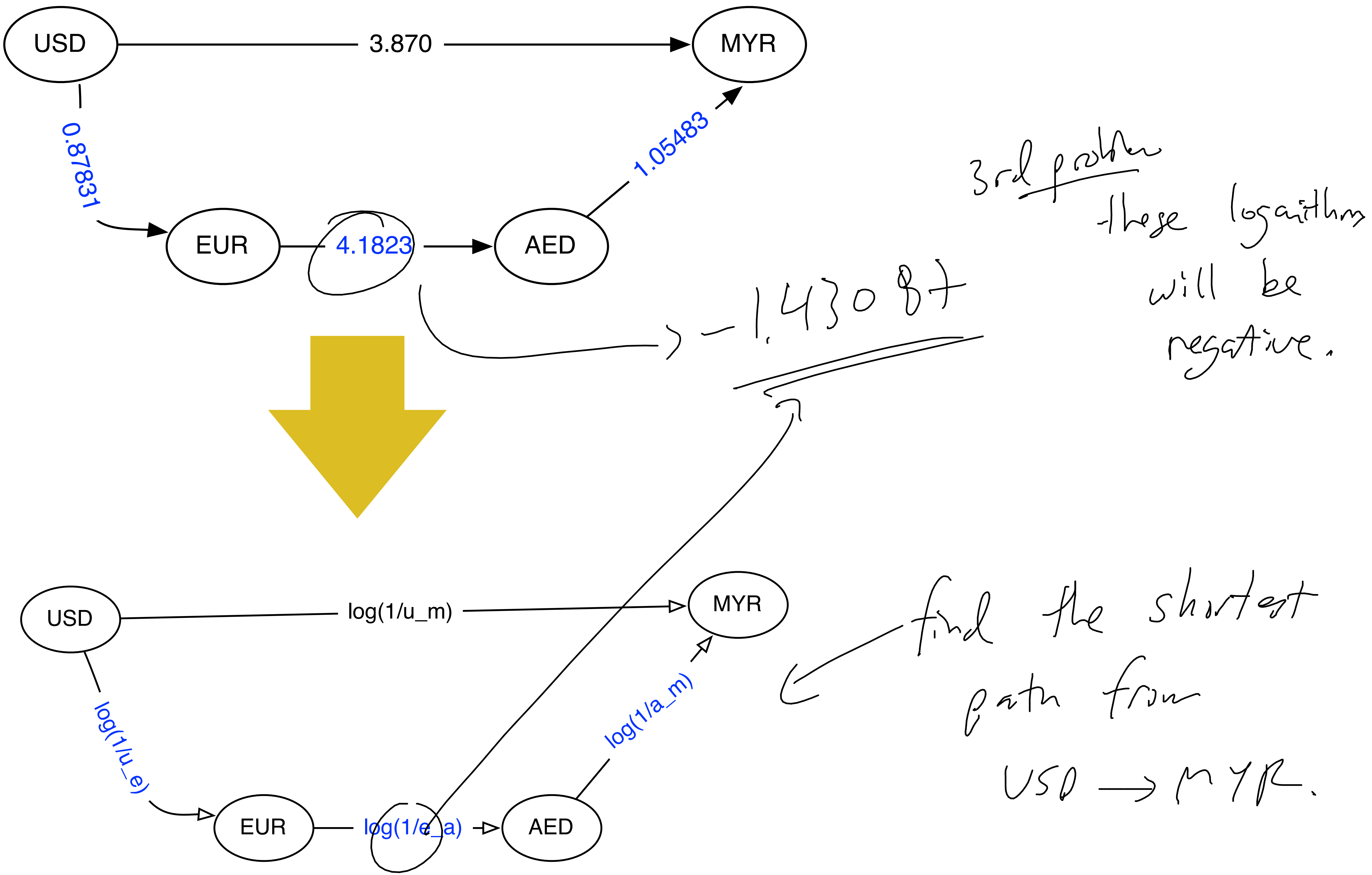
(1) with inverse exchange rates, this problem is equivalent to

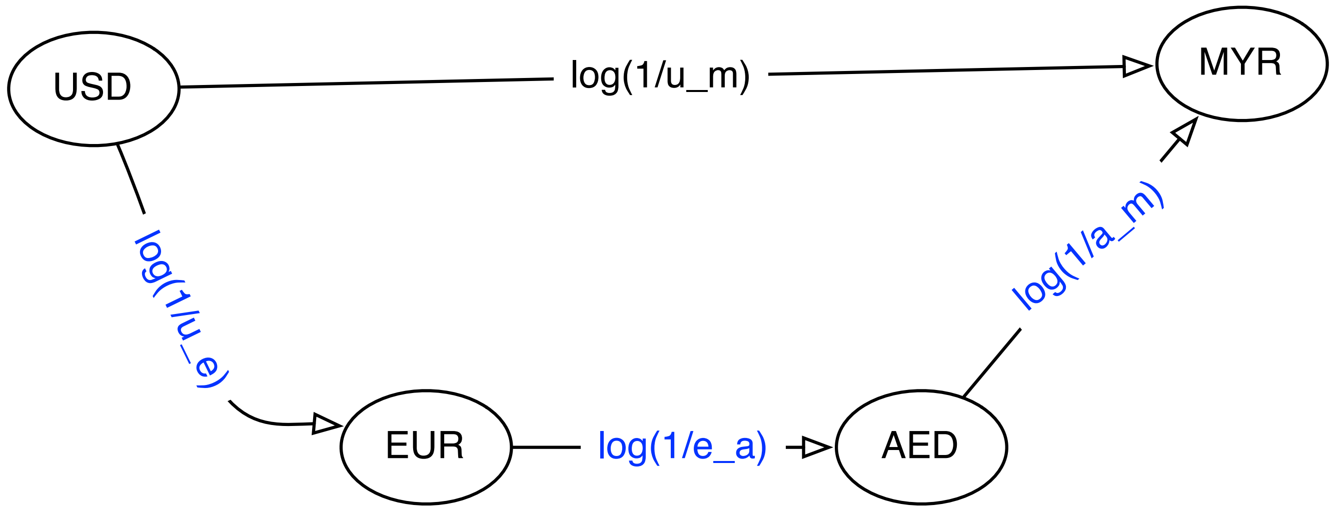
$$\min_{p \in \{s \leadsto u\}} \prod_{e \in p} \frac{1}{w(e)}$$

(2) By taking logs of weight, the problem is equivalent to

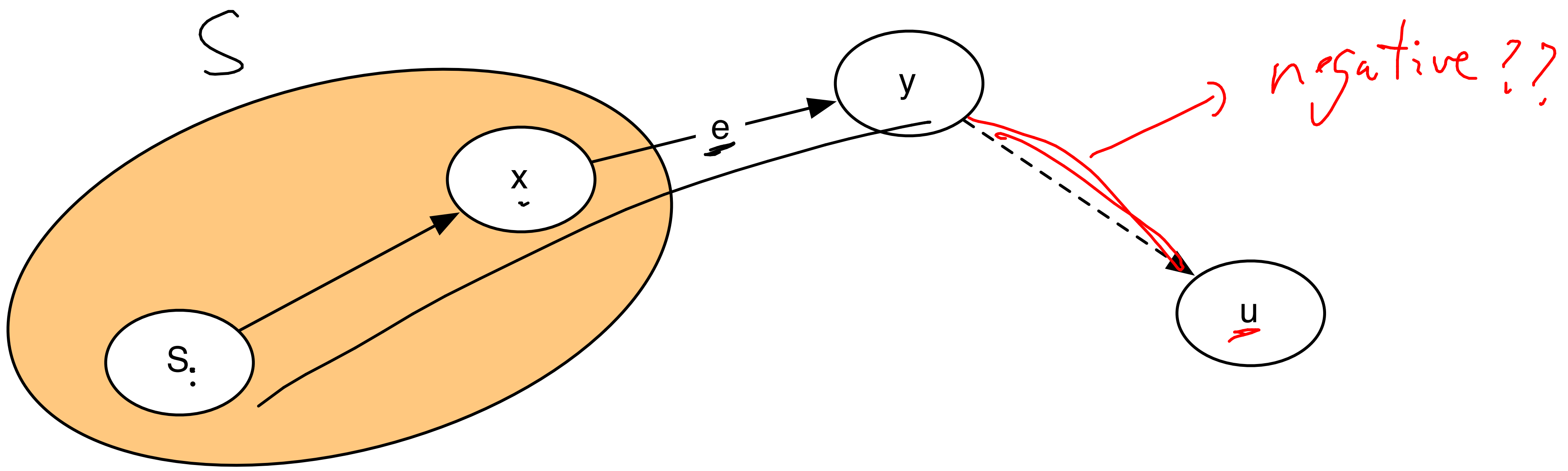
$$\min_{p \in \{s \leadsto u\}} \sum_{e \in p} \log\left(\frac{1}{w(e)}\right)$$

← this is SSSP


USD
3.870
MYR
0.87831
1.05483
EUR
4.1823
AED
-1.43087
3rd problem
these logarithm will be negative.
USD
log(1/u_m)
MYR
log(1/u_e)
log(1/a_m)
EUR
log(1/e_a)
AED
find the shortest path from USD → MYR.


USD
MYR
EUR
AED
log(1/u_m)
log(1/u_e)
log(1/e_a)
log(1/a_m)

where does old argument break down



Consider any path from $s \leadsto u$.

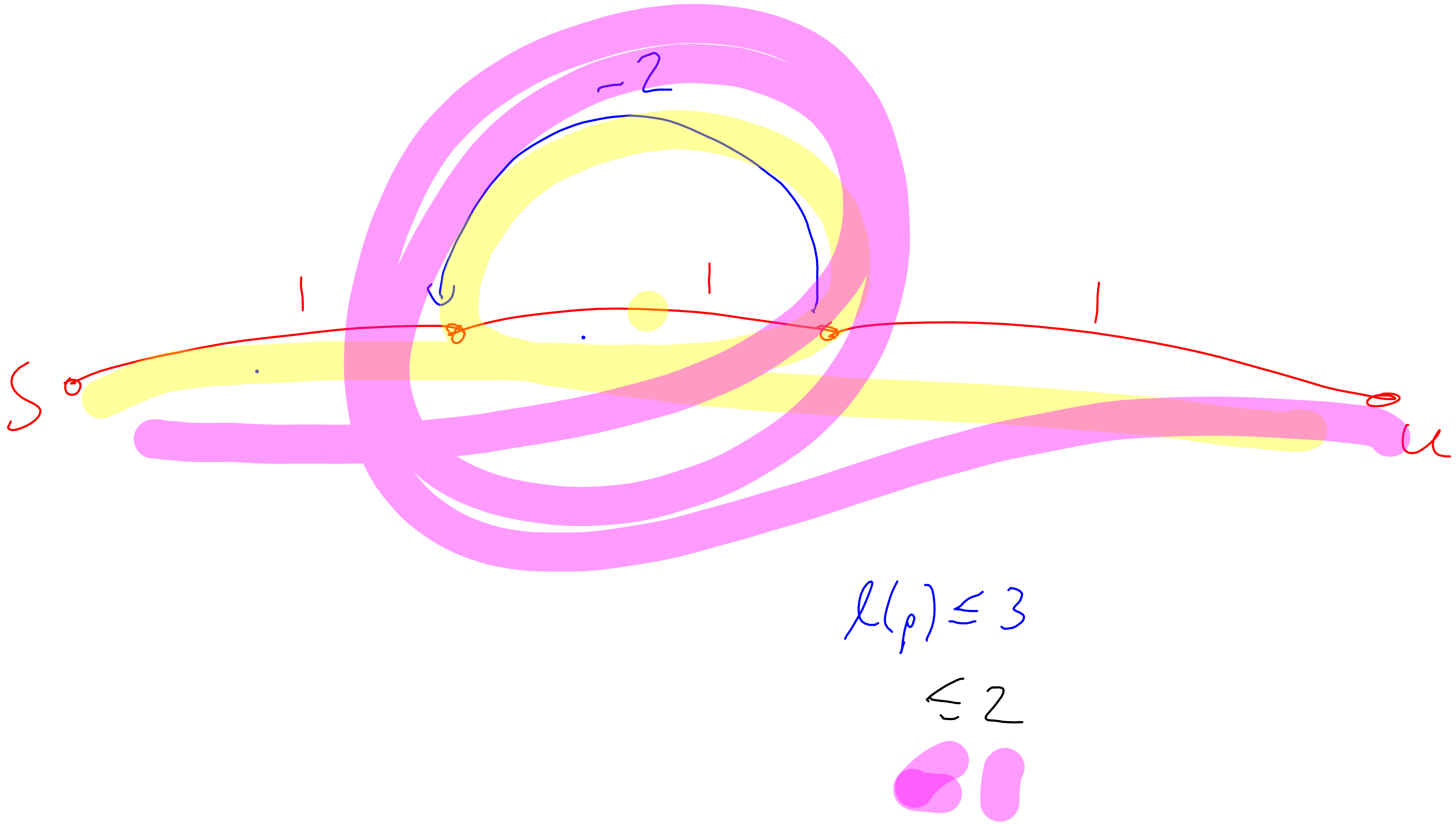
$$\ell(p) = \delta(s,x) + w(x,y) + \ell(y \leadsto u)$$

$$\geq d_y + \ell(y \leadsto u)$$

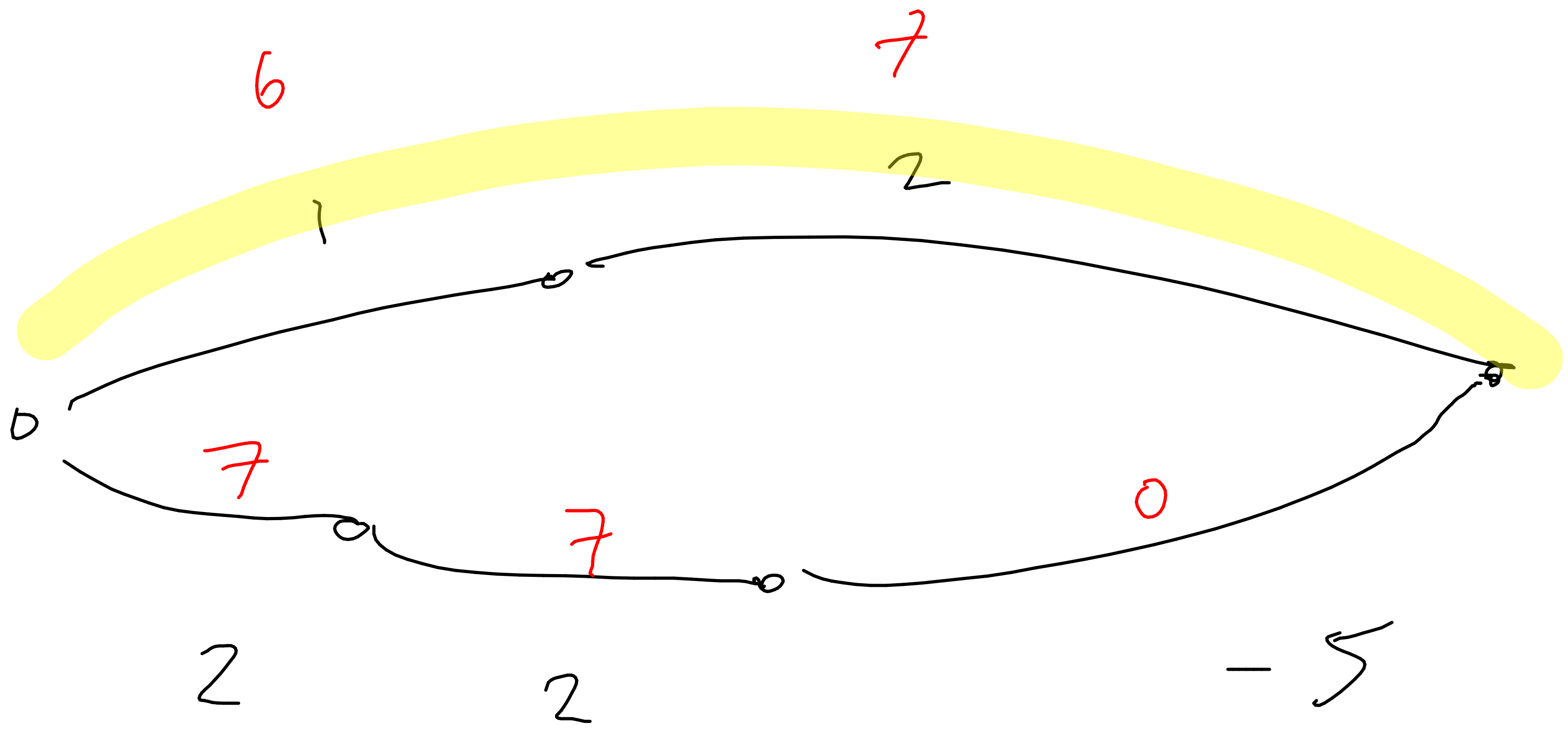
$$\geq d_u + \ell(y \leadsto u) \geq d_u$$

this no longer true if $\ell(y \leadsto u) < 0$

2nd problem: cycles



first ideas: Add to each edge



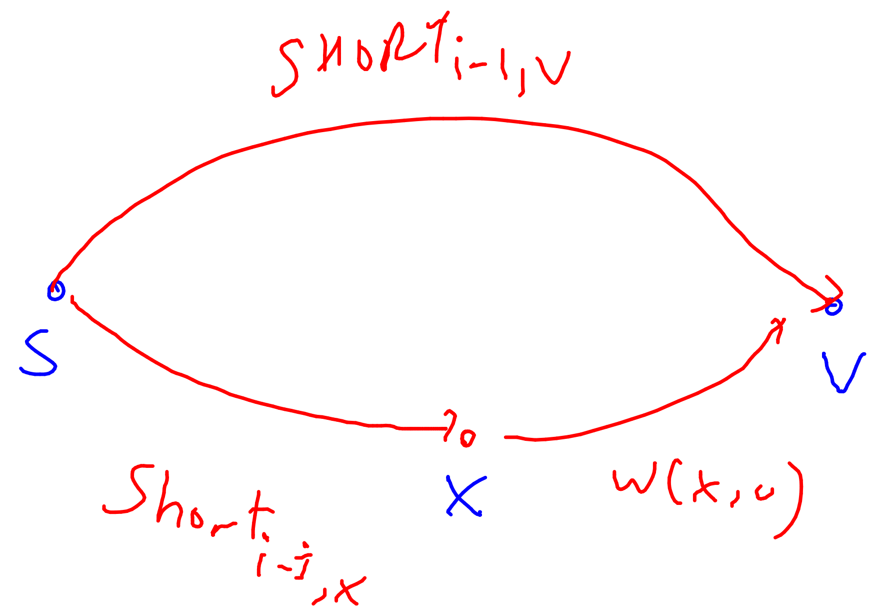
wrong b/c we add the offset too many times in the bottom path

sssp(G,s)

$\text{SHORT}_{i,v}$ = length of the shortest path from $s \rightsquigarrow v$ that uses $\le i$ edges.

$$\text{Short}_{i,v} = \min \begin{cases} \text{Short}_{i-1,v} \\ \text{Short}_{i-1,x} + w(x,v) \quad \text{for all } x \in V \end{cases}$$

(actually only for the neighbors of x)



for all $x \in V$ in the graph

sssp(G,s)

$$\text{SHORT}_{i,v} = \begin{cases} \infty & i = 0 \\ 0 & v = s \\ \min_{x \in V} & \left\{ \begin{array}{l} \text{SHORT}_{i-1,v} \\ \text{SHORT}_{i-1,x} + w(x,v) \end{array} \right\} \end{cases}$$

max len of a simple path:

① max length of a simple path

is $V-1$

Bellman-Ford(G,s)

Initialize $Short_{i,v}$ for $i = 0$

for $i = 1$ to $V - 1$

update $SHORT_{i,v}$ using equation.

BELLMAN-FORD(G, s)

$\text{SHORT}_{0,s} \leftarrow 0$

$\forall v \in V - \{s\}$, $\text{SHORT}_{0,v} \leftarrow \infty$

for $i = 1, \ldots, V - 1$

do for each $v \in V - \{s\}$

do $\text{SHORT}_{i,v} = \min_{x \in Adj(v)} \begin{cases} \text{SHORT}_{i-1,v} \\ w(x, v) + \text{SHORT}_{i-1,x} \end{cases}$

instead, we can just iterate over the edges of the graph

Rewrite loop as indexing over the edges.

BELLMAN-FORD(G, s)

$\text{SHORT}_{0,s} \leftarrow 0$

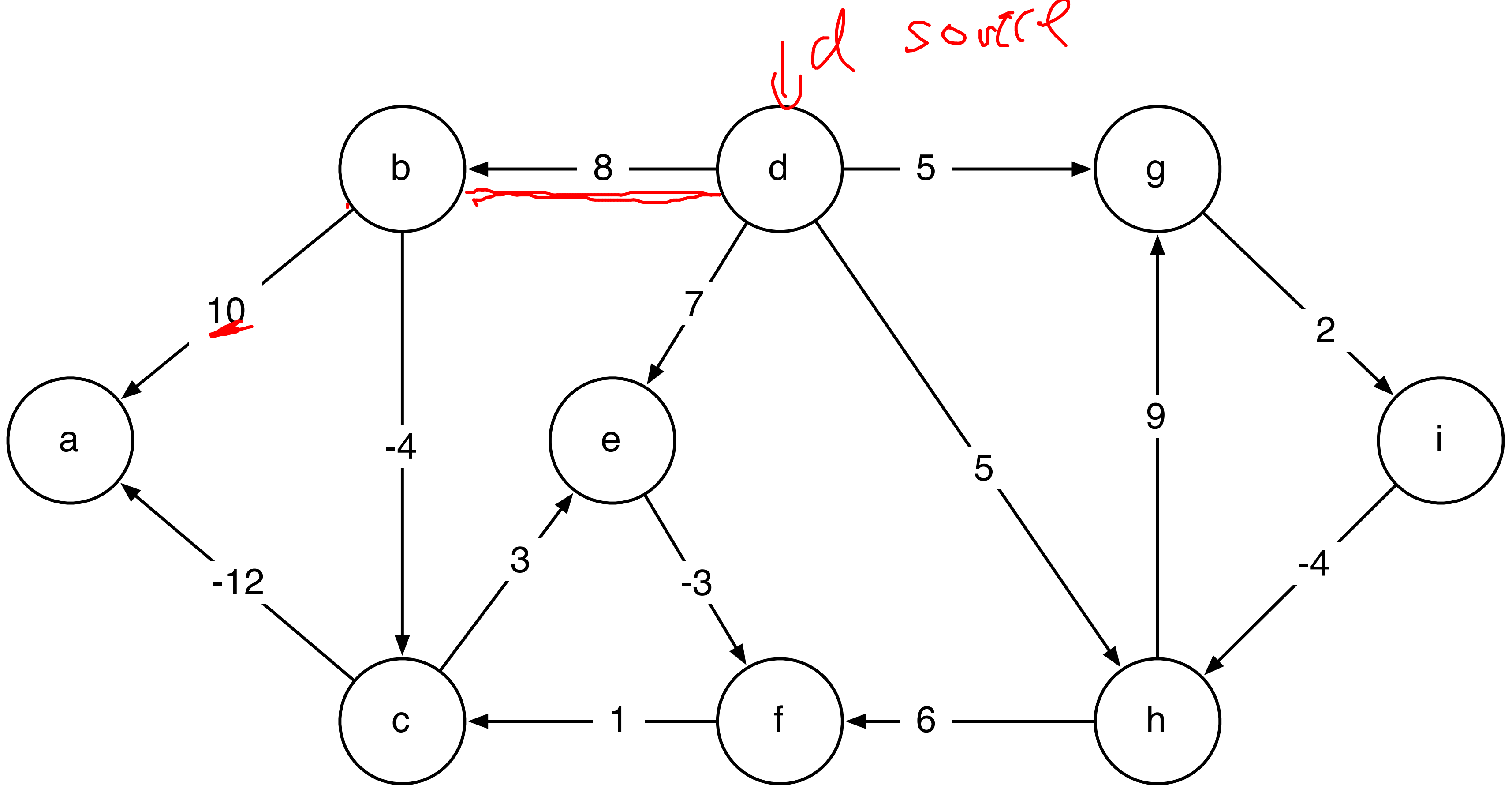
$\forall v \in V - \{s\}$, $\text{SHORT}_{0,v} \leftarrow \infty$

for $i = 1, \ldots, V - 1$

  do for each $e = (x, y) \in E$

    do $\text{SHORT}_{i,y} = \min \left\{ \begin{array}{l} \text{SHORT}_{i-1,y} \\ \text{SHORT}_{i,y} \\ w(x, y) + \text{SHORT}_{i-1,x} \end{array} \right\}$

$\Theta(V \cdot E)$



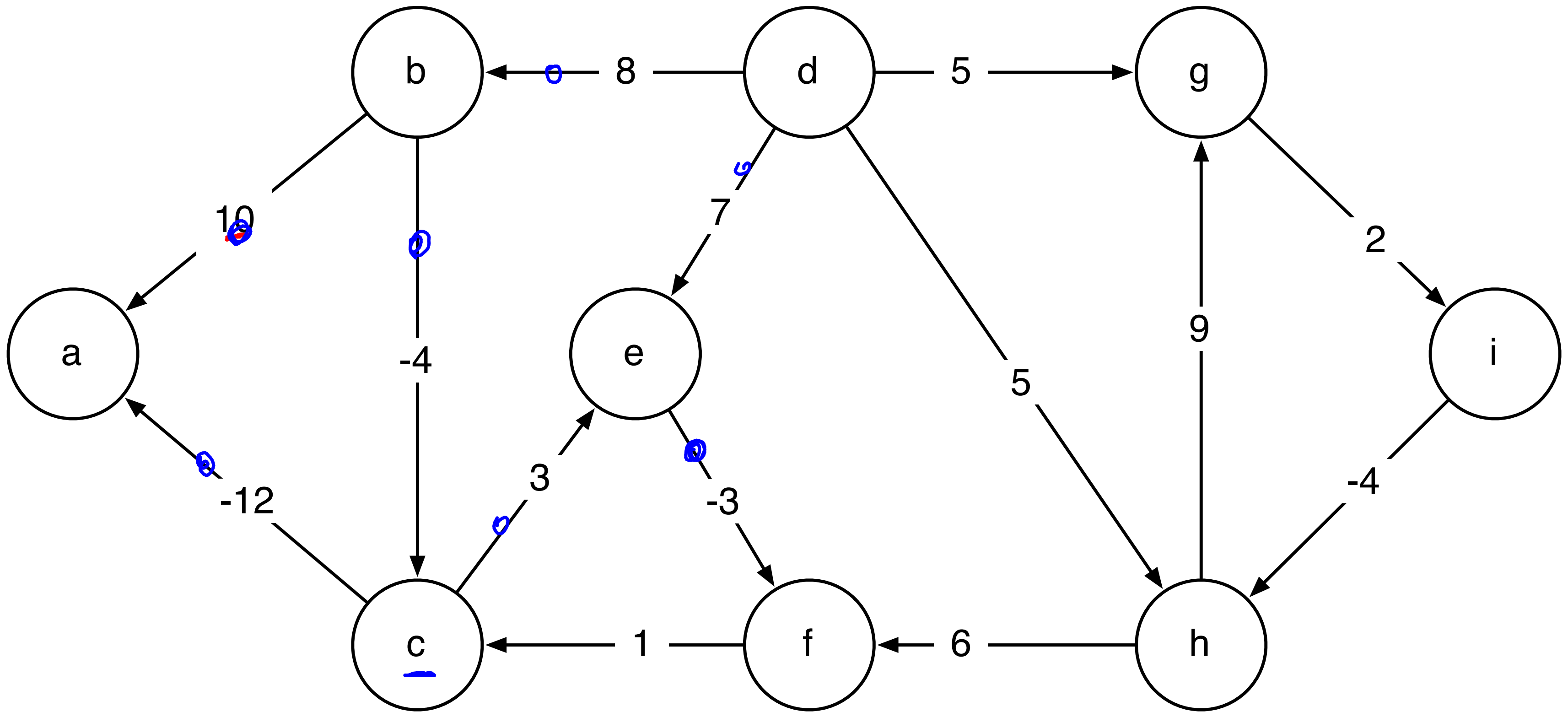
$$\text{short}_{1,b} = \min \begin{cases} \text{short}_{0,b} \\ \text{short}_{0,d} + 8 \end{cases}$$

bf(G,d)

	0	1	2	3	4	5	6	7
a	∞	∞						
b	∞	8						
c	∞							
d	0							
e	∞	7						
f	∞							
g	∞	5						
h	∞	5						
i	∞							

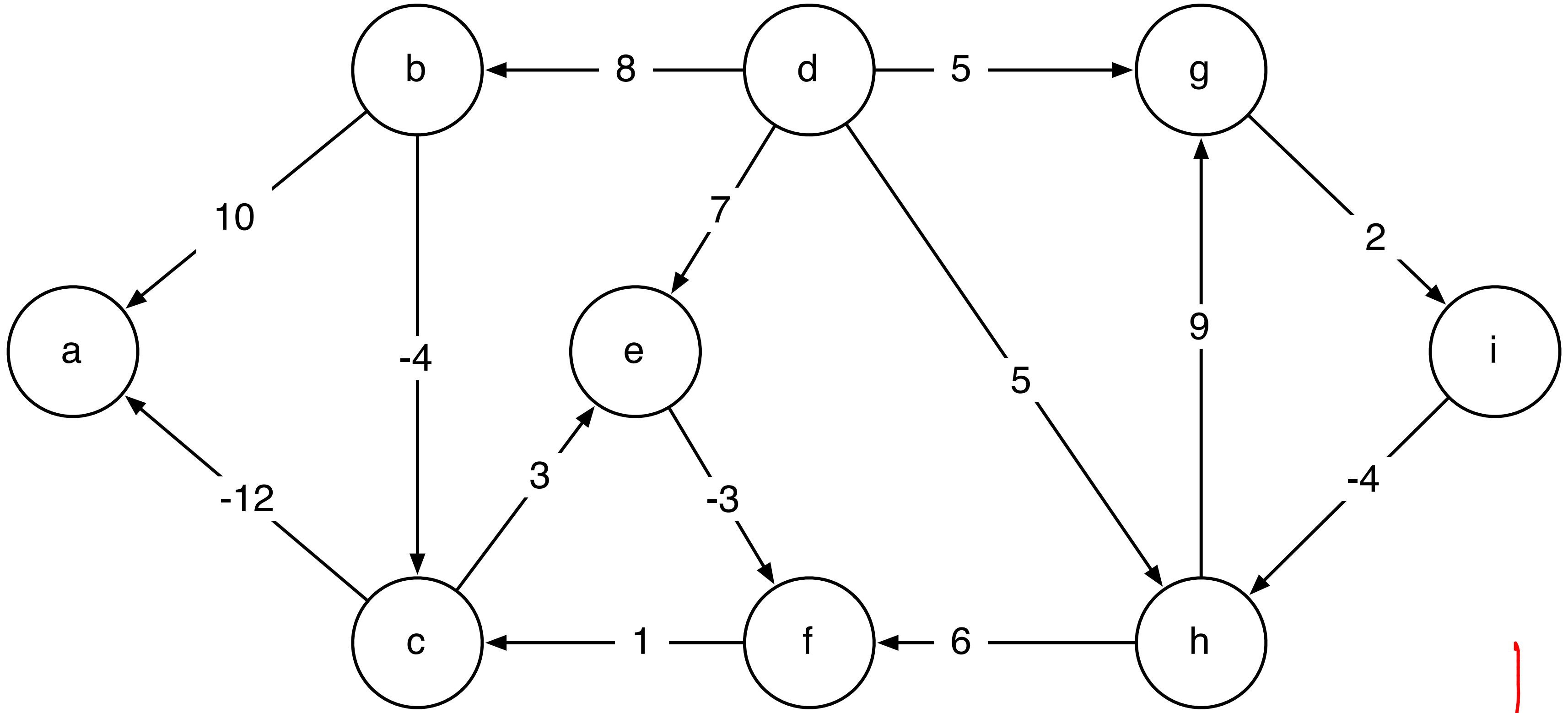
$$\text{SHORT}_{i,v} = \begin{cases} \infty & i = 0 \\ 0 & v = s \\ \min_{x \in V} & \left\{ \begin{array}{l} \text{SHORT}_{i-1,v} \\ \text{SHORT}_{i-1,x} + w(x,v) \end{array} \right\} \end{cases}$$

$$\text{short}_{1,a} = \min \begin{cases} \text{short}_{0,b} + 10 \\ \text{short}_{0,a} \end{cases}$$



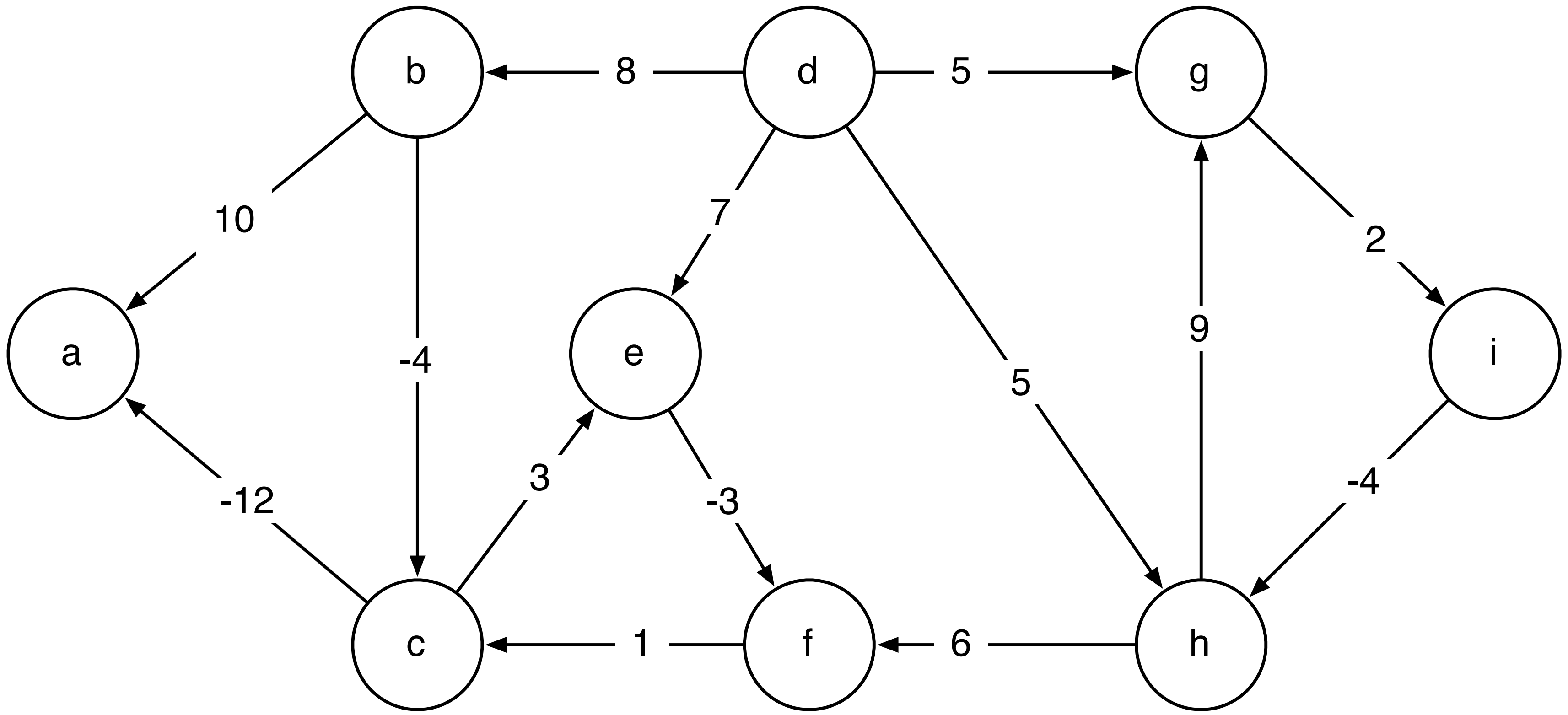
$$\text{SHORT}_{i,v} = \begin{cases} \infty & i = 0 \\ 0 & v = s \\ \min_{x \in V} & \left\{ \begin{array}{l} \text{SHORT}_{i-1,v} \\ \text{SHORT}_{i-1,x} + w(x,v) \end{array} \right\} \end{cases}$$

		1						
a			18					
b		8	8					
c			4					
d	0	0	0					
e		7						
f			4					
g		5						
h		5						
i								



$$\text{SHORT}_{i,v} = \begin{cases} \infty & i = 0 \\ 0 & v = s \\ \min_{x \in V} & \left\{ \begin{array}{l} \text{SHORT}_{i-1,v} \\ \text{SHORT}_{i-1,x} + w(x,v) \end{array} \right\} \end{cases}$$

		18					
	8	8					
		4					
0	0	0					
	7	7					
		4					
	5	5					
	5	5					
		7					



$$\text{SHORT}_{i,v} = \begin{cases} \infty & i = 0 \\ 0 & v = s \\ \min_{x \in V} & \left\{ \begin{array}{l} \text{SHORT}_{i-1,v} \\ \text{SHORT}_{i-1,x} + w(x,v) \end{array} \right\} \end{cases}$$

		18	-8				
	8	8	8				
		4	4				
0	0	0	0				
	7	7	7				
		4	4				
	5	5	5				
	5	5	3				
		7	7				

Optimization

$\text{BELLMAN-FORD}(G, s)$

$\text{SHORT}_{0,s} \leftarrow 0$

$\forall v \in V - \{s\}$, $\text{SHORT}_{0,v} \leftarrow \infty$

for $i = 1, \ldots, V-1$

  do for each $e = (x, y) \in E$

    do $\text{SHORT}_{i,y} = \min \left\{ \begin{array}{l} \text{SHORT}_{i-1,y} \\ \text{SHORT}_{i,y} \\ w(x,y) + \text{SHORT}_{i-1,x} \end{array} \right\}$

$\text{BELLMAN-FORD}(G, s)$

$d_s \leftarrow 0$

$\forall v \in V - \{s\}$, $d_v \leftarrow \infty$

for $i = 1, \ldots, V-1$

  do for each $e = (x, y) \in E$

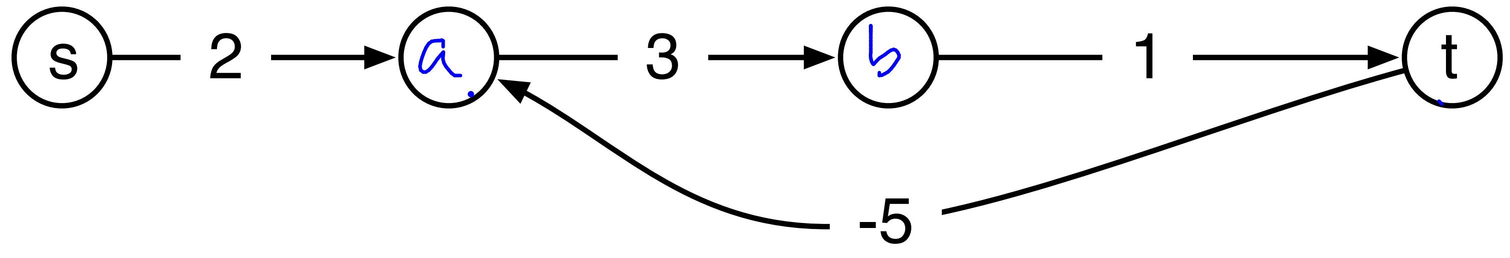
    do $d_y \leftarrow \min\{\, d_y, w(x,y) + d_x \,\}$

running time

BELLMAN-FORD(G, s)

$d_s \leftarrow 0$
$\forall v \in V - \{s\}$, $d_v \leftarrow \infty$
for $i = 1, \ldots, V - 1$
    do for each $e = (x, y) \in E$
        do $d_y \leftarrow \min\{\ d_y, w(x, y) + d_x\ \}$

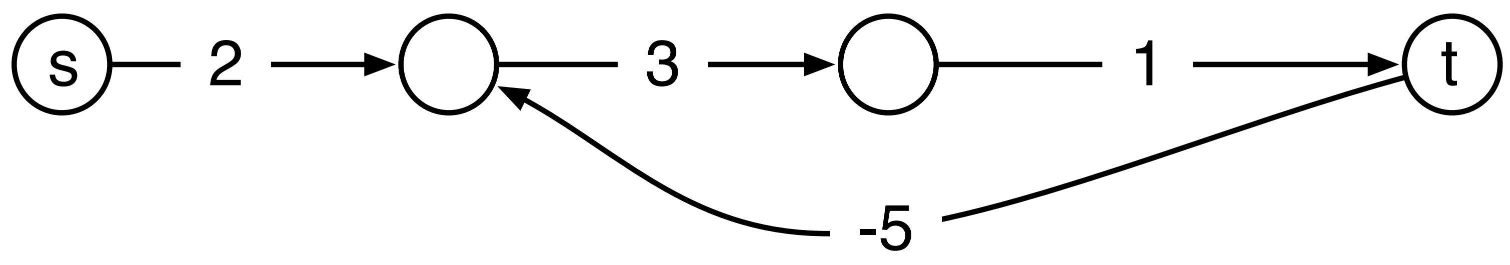
negative cycles?



			3	
s	0	0	0	
a	2	2	2	1
b		5	5	
t			6	

← on the V^{th} iteration, some value decreased.

negative cycles?



s	0	0	0	0
a	2	2	2	1
b		5	5	5
t			6	6

applications of BF

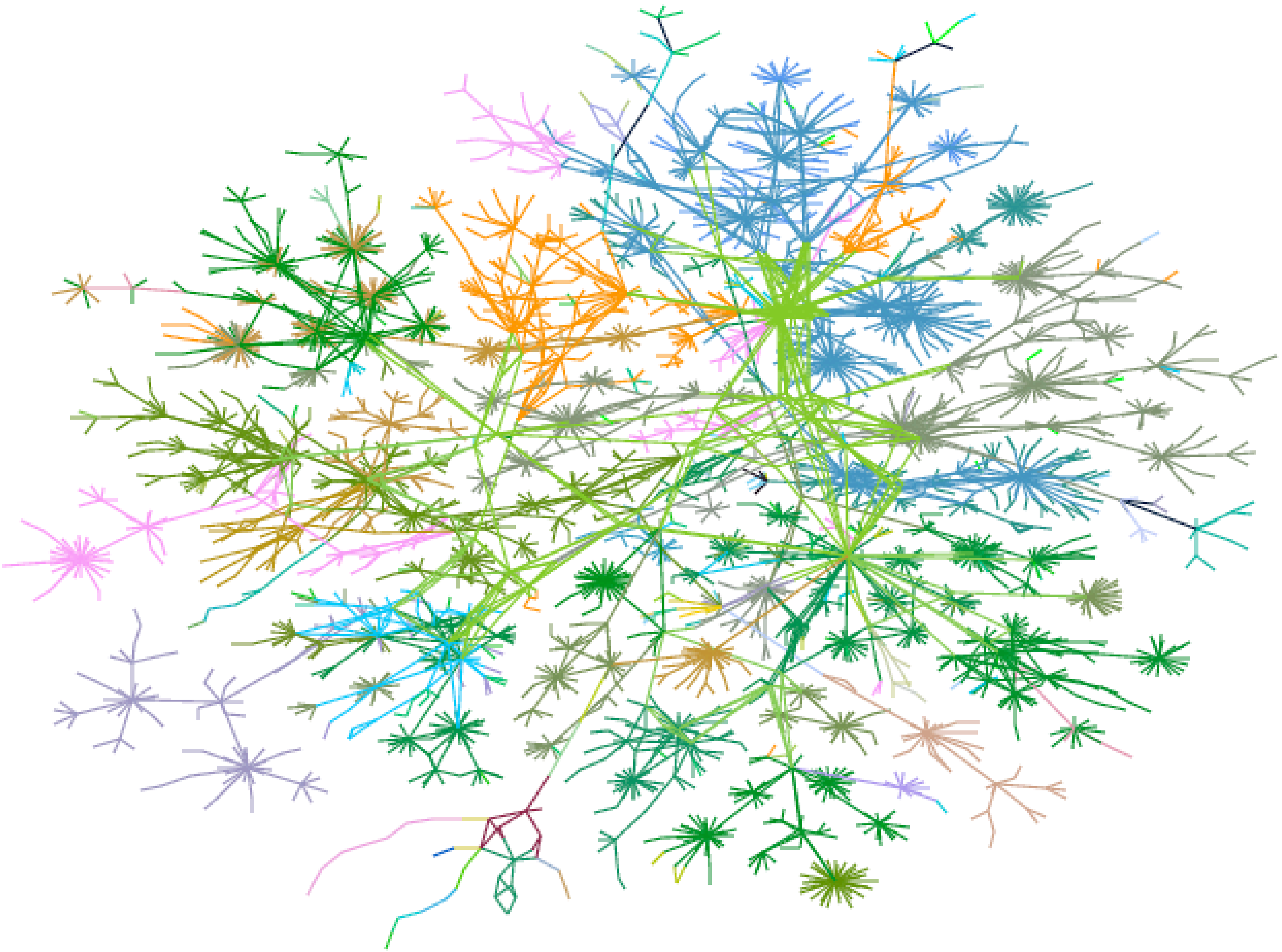
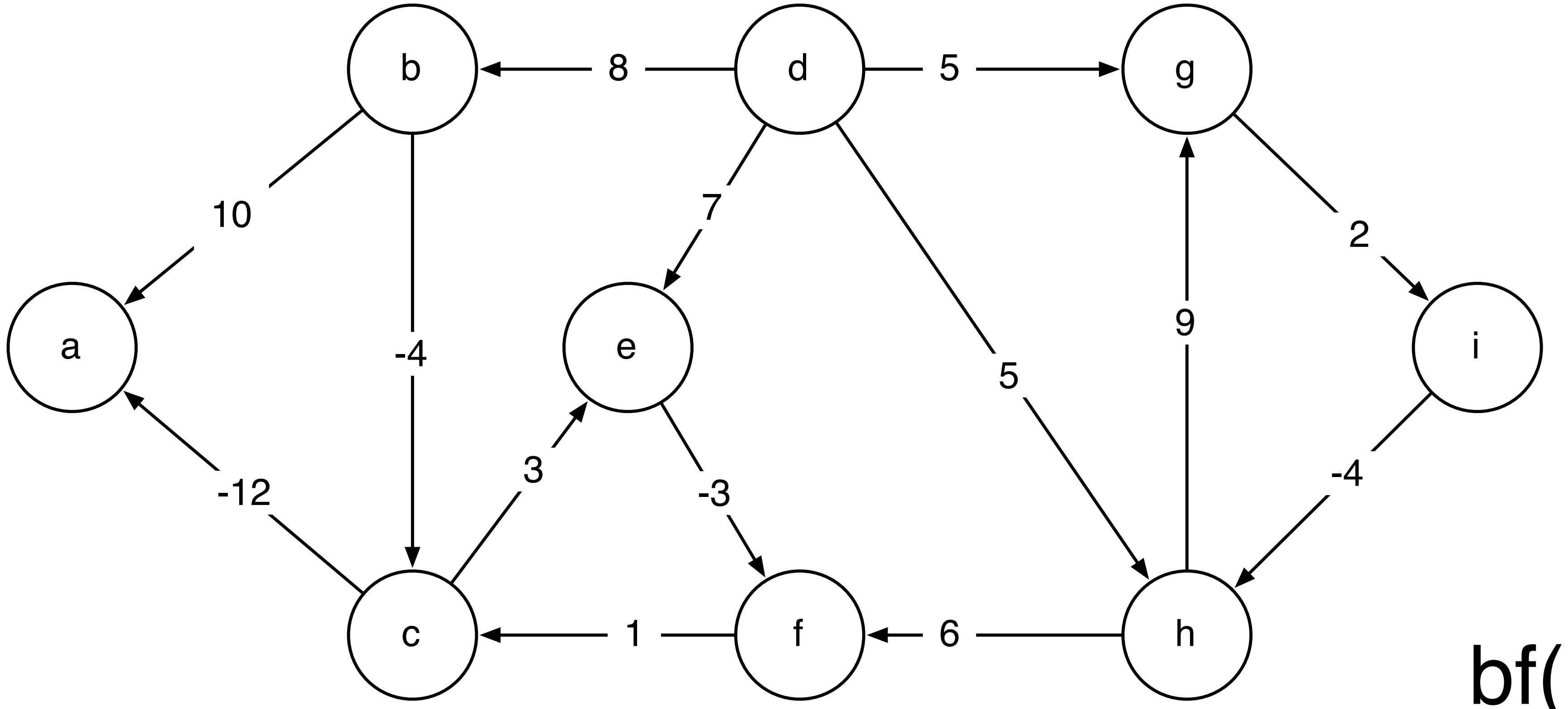


image: cheswick et al

Figure 3: Lucent's intranet as of 1 October 1999.



what happens when
B changes...

bf(G,d)

	0	1	2	3	4	5	6	7
a	∞							
b	∞	8						
c	∞							
d	0	0						
e	∞	7						
f	∞							
g	∞	5						
h	∞	5						
i	∞							

distance vector

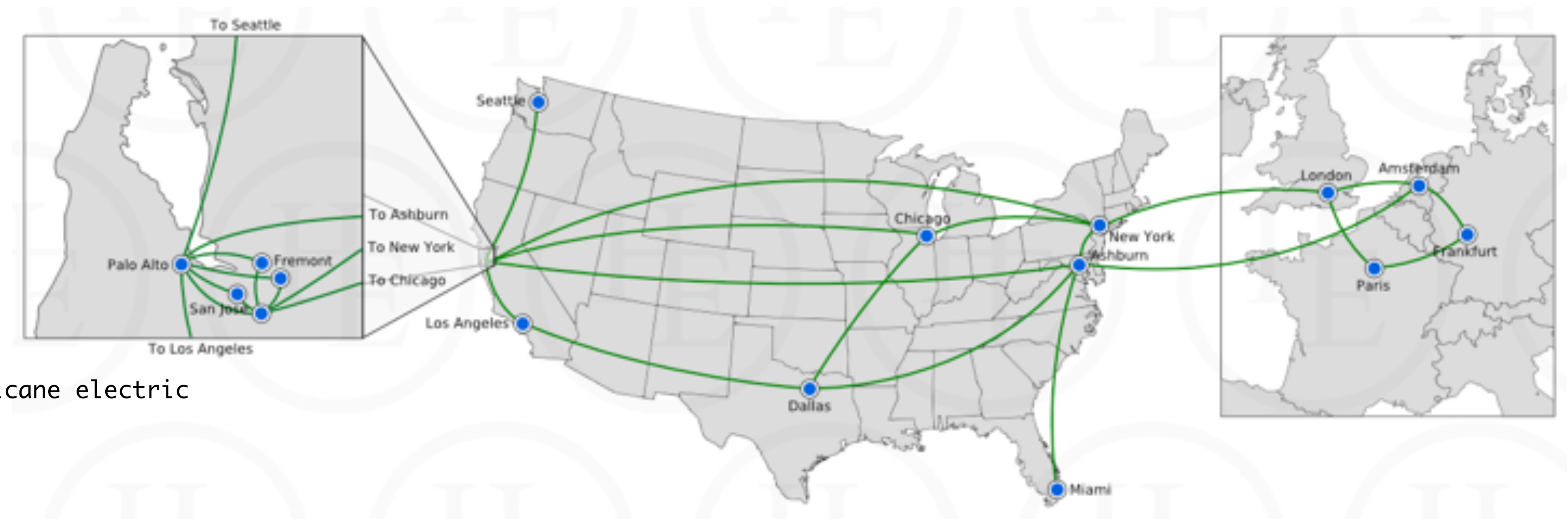
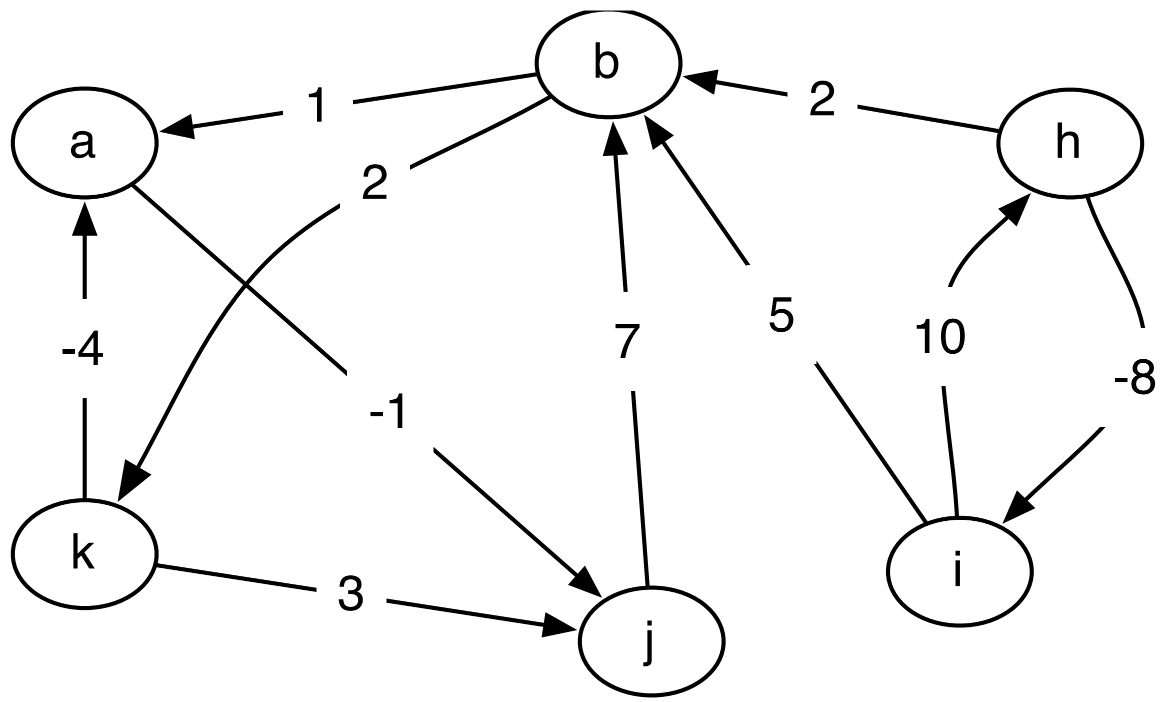


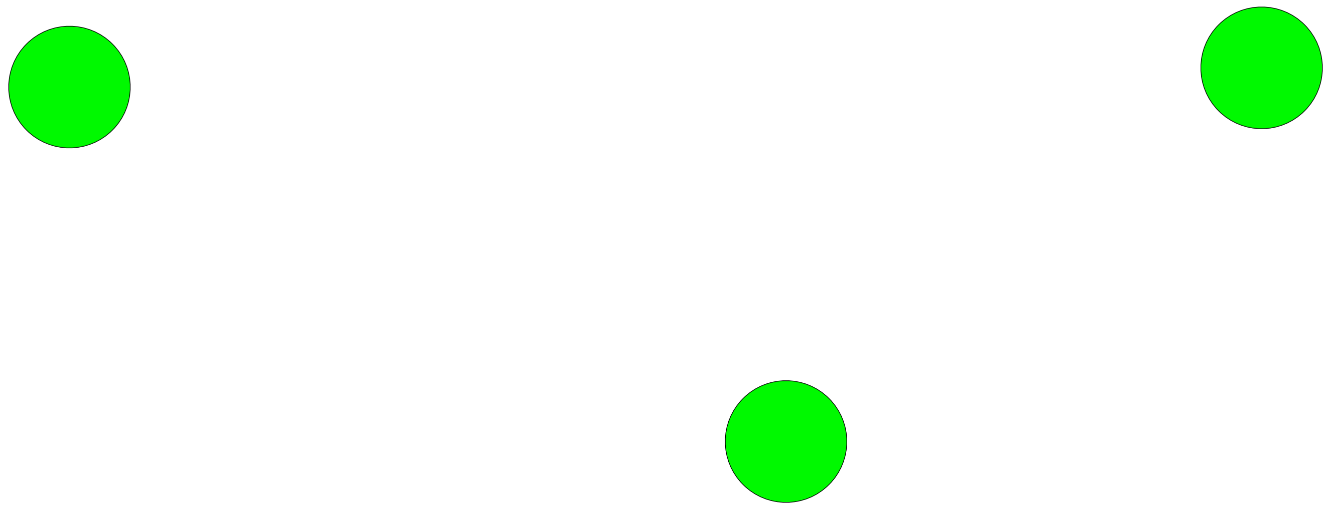
image: hurricane electric

all-pairs shortest path



$$\mathrm{ASHORT}_{i,j,k} =$$

$$\mathrm{ASHORT}_{\mathtt{i},\mathtt{j},\mathtt{k}} =$$



$$\mathrm{ASHORT}_{i,j,k} =$$

$$\text{ASHORT}_{i,j,k} = \left\{ \begin{array}{ll} w_{i,j} & k = 0 \\ \min \left\{ \begin{array}{l} \text{ASHORT}_{i,j,k-1} \\ \text{ASHORT}_{i,k,k-1} + \text{ASHORT}_{k,j,k-1} \end{array} \right. & k \geq 1 \end{array} \right\}$$

Floyd-Warshall(G,W)