1 https://anonize.org

2

Introducing ANONIZE.

3 Open app, register using your UVA id

4 https://goo.gl/hZoQFG

L23

4102

4.14.2016

abhi shelat

Gabriel García Marquez

Love in the Time of Tindera

We have a group of suitors and reviewers

2>1>3

2>3>1

1>3>2

Each has preferences over the other group

1>3>2

1>2>2

3>2>1

2>1>3

2>3>1

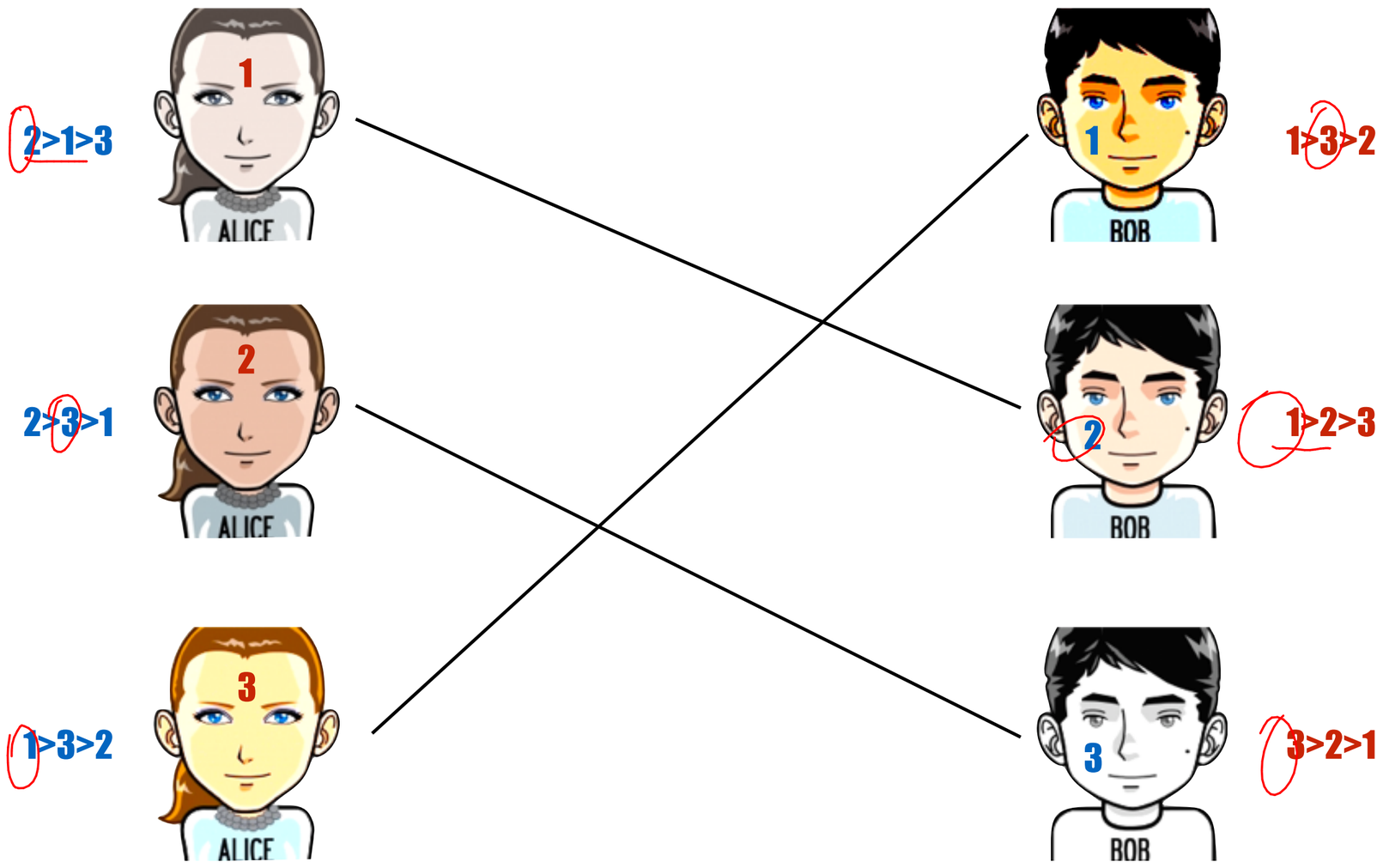
1>3>2

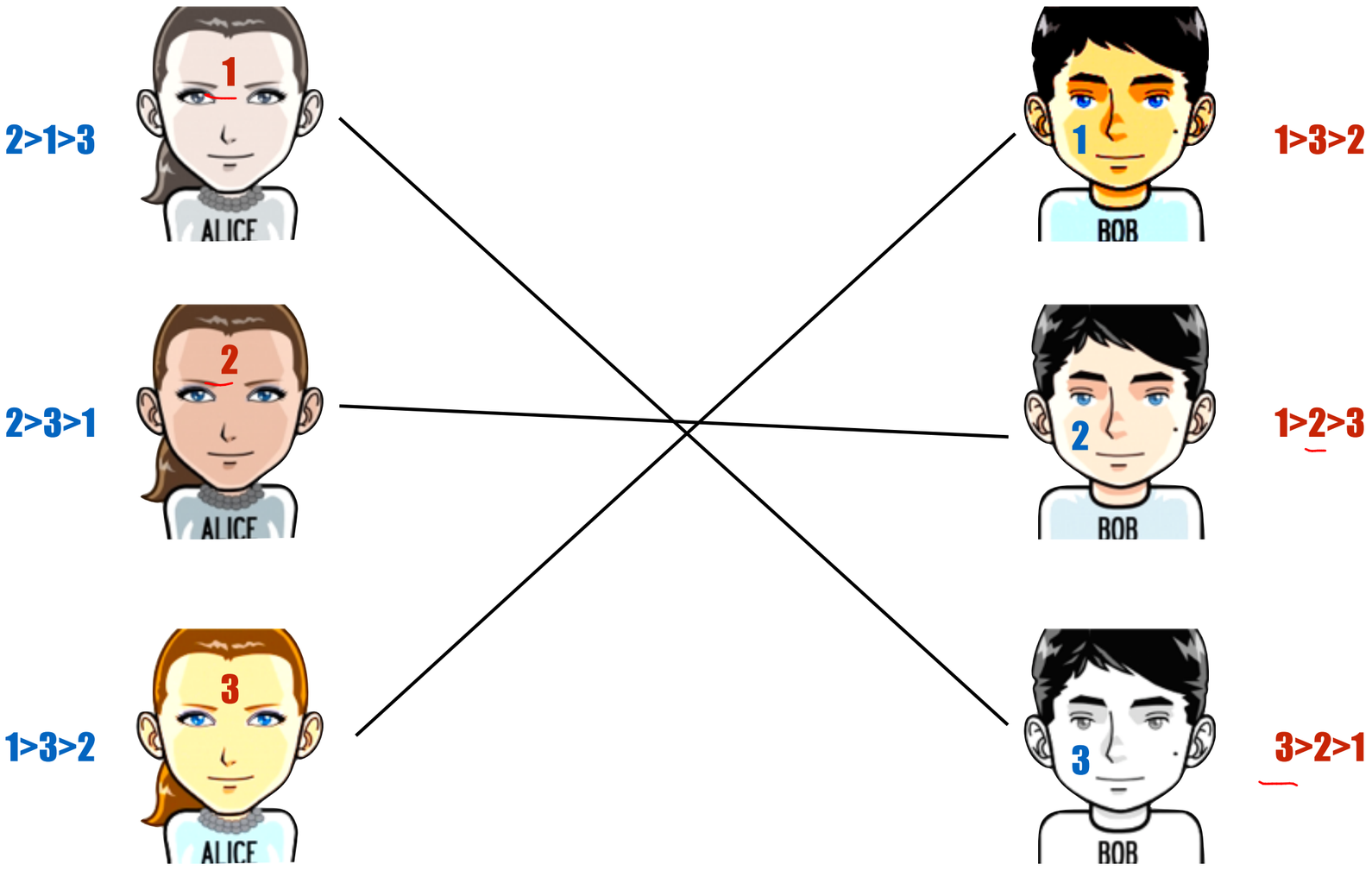
We seek a **stable matching** between the two

1>3>2

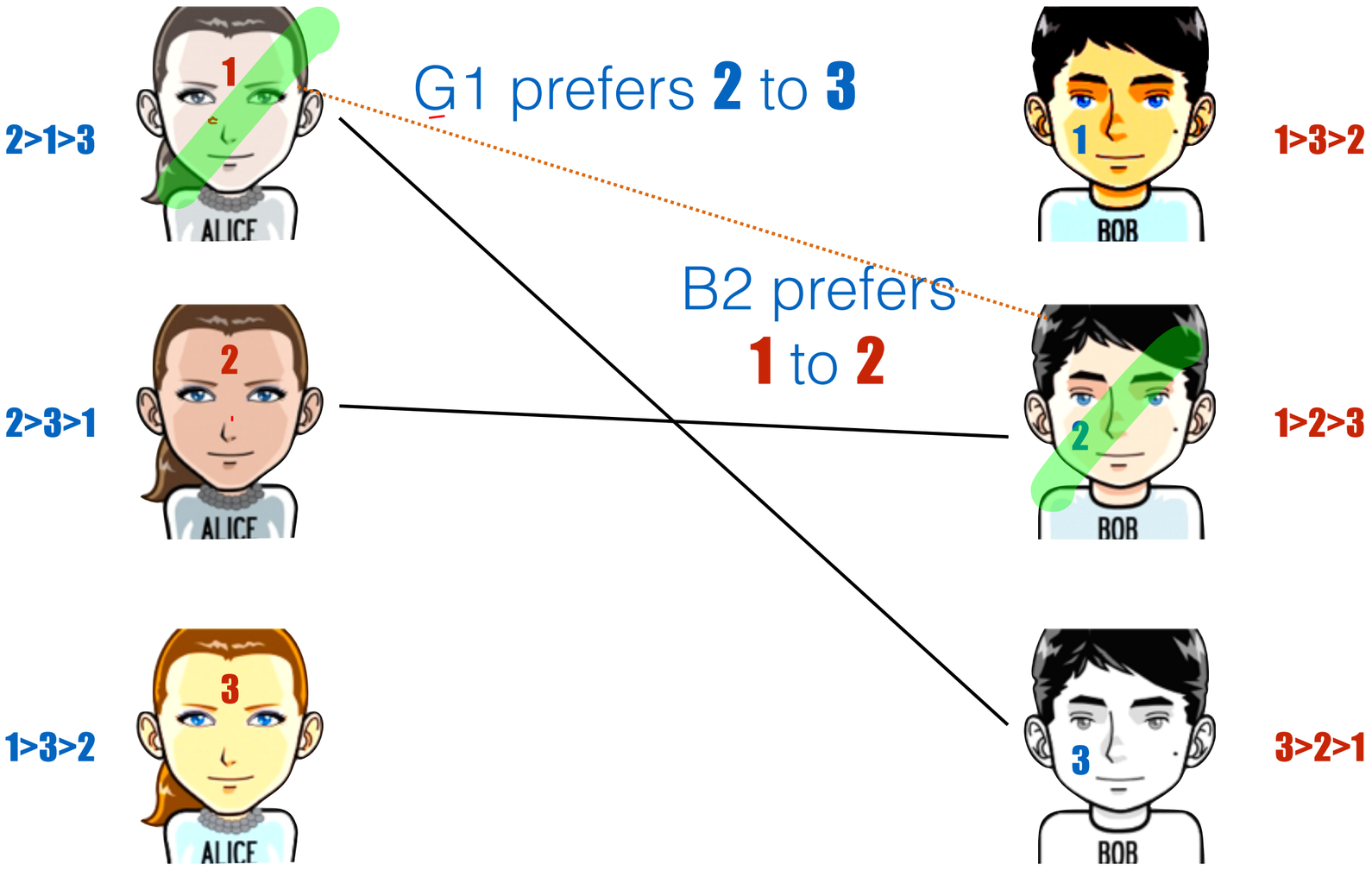
1>2>2

3>2>1


1
2>1>3
ALICE
1
1>3>2
BOB
2
2>3>1
ALICE
2
1>2>3
BOB
3
1>3>2
ALICE
3
3>2>1
BOB



Unstable Matching



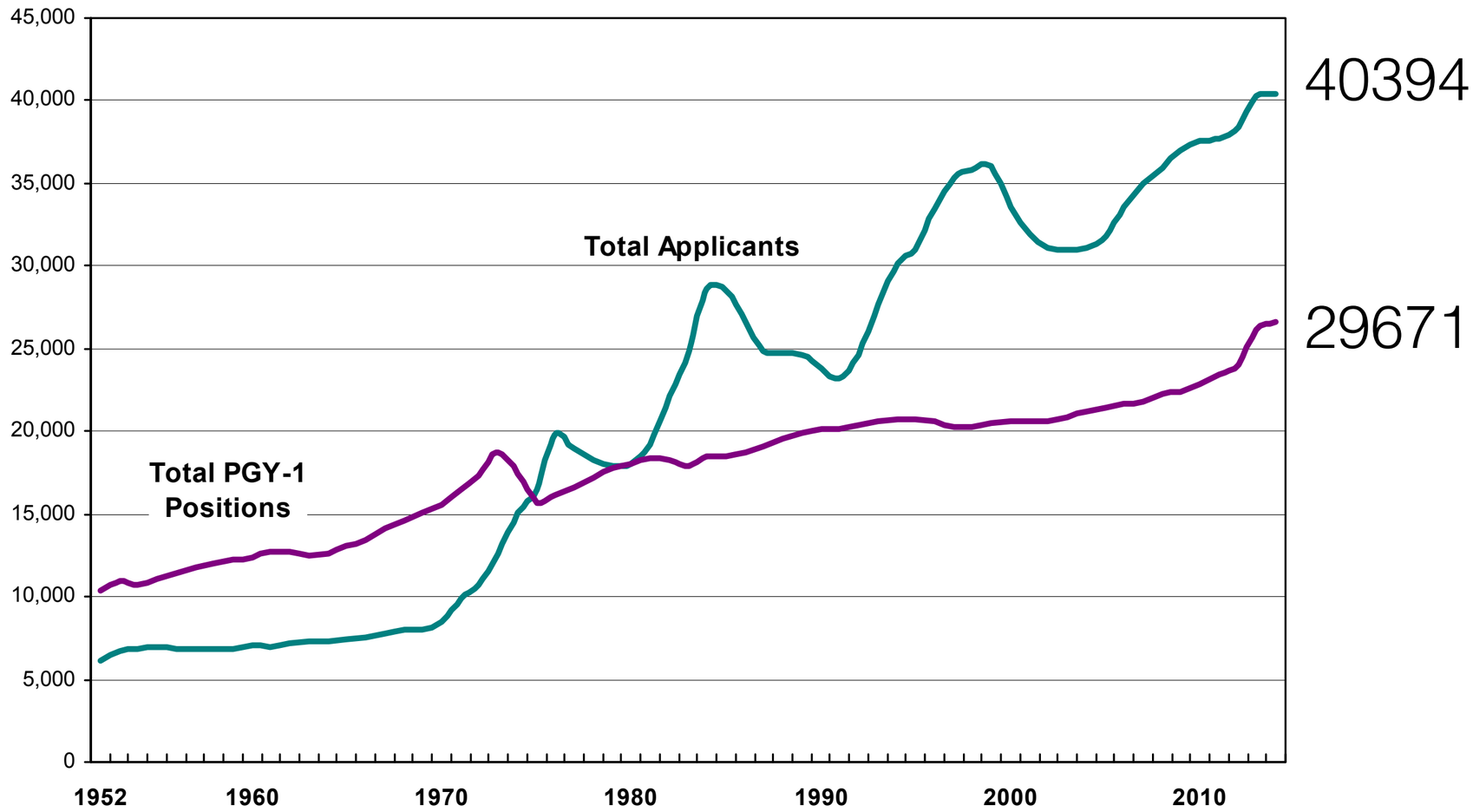
Unstable Matching

Stable Matching

Stable matching has many practical applications

THE MATCH℠
NATIONAL RESIDENT MATCHING PROGRAM®

Figure 1 **Applicants and 1st Year Positions in The Match, 1952 - 2014**



Applicant Type	Matched		
	2013 Graduates	Prior Year Graduates[1]	Total
CMG	2571	74	2645
IMG	146	353	499
USMG	23	2	25
TOTAL	**2740**	**429**	**3169**

UNIVERSITAS OSLOENSIS
MDCCCXI

NUS
National University
of Singapore

SINGAPORE UNIVERSITY OF
TECHNOLOGY AND DESIGN
Established in collaboration with MIT

NANYANG
TECHNOLOGICAL
UNIVERSITY

GreekYearbook
www.GreekYearbook.com

University of Virginia
Chi Omega Bid Day 2012

Definition: matchings

M= $\{m_1, m_2, m_3 \ldots m_n\}$ suitors

W= $\{w_1, w_2, w_3 \ldots w_n\}$ reviewers

S= $\{(m_i, w_j)_k\}_k$ → a set of pairs, each pair is ~~including~~ exactly one element from M and 1 element from W

* each $m_i \in M$ appears in at most 1 pair
* each $w_j \in W$ " " " " " "

Definition: matchings

$$M = \{m_1, \ldots, m_n\}$$
$$W = \{w_1, \ldots, w_n\}$$

$$S = \{(m_{i_1}, w_{j_1}), \ldots, (m_{i_k}, w_{i_k})\}$$

Each m_i (w_i) appears only once in a pairing.

→ A matching is perfect if every m_i appears.

$m_i \in M$

$|S| = n$

M

W

Image credits: Julia Nikolaeva

Definition: preferences

$$M = \{m_1, \ldots, m_n\}$$

Each element in M has preferences over W.

$w_i \prec_{m_i} w_j \prec_{m_i} w_1 \prec_{m_i} w_2 \ldots \prec_{m_i} w_n$

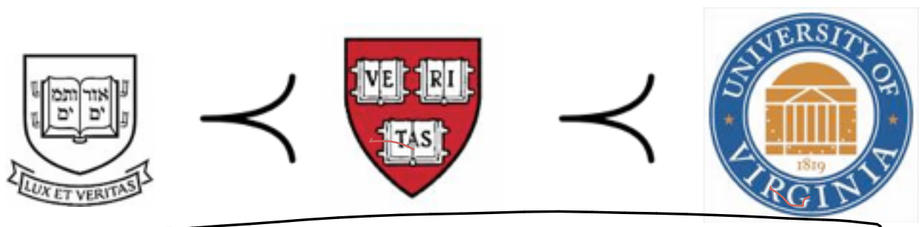
Image credits: Julia Nikolaeva

Example: preferences

$$M = \{m_1, \ldots, m_n\}$$

m_i has a preference relation $\prec_{m_i}$ on the set W

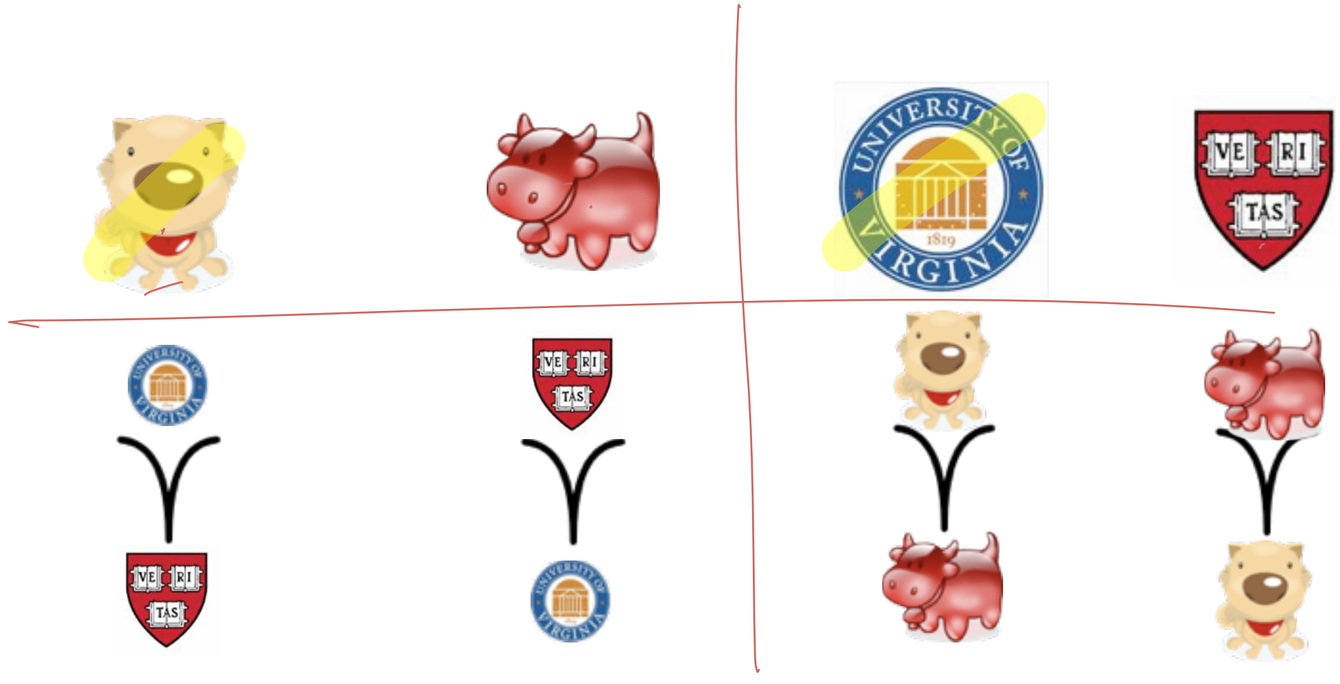
$$w_1 \prec_{m_i} w_4 \prec_{m_i} w_2 \prec_{m_i} w_8 \cdots w_n$$



M

W

(D, V) (C, H)

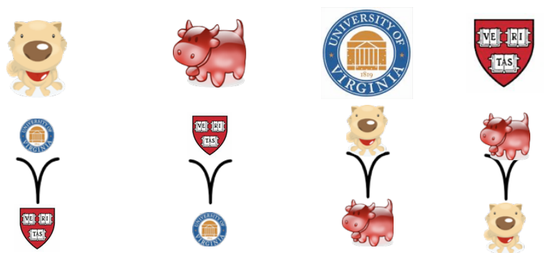


$$S = \left\{ \left(\quad \right) \quad \left(\quad \right) \right\}$$

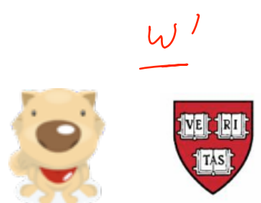
consid this matching

UNSTABLE MATCH

Def: instability



$$S = \left\{ \left(\quad \right) \quad \left(\quad \right) \right\}$$

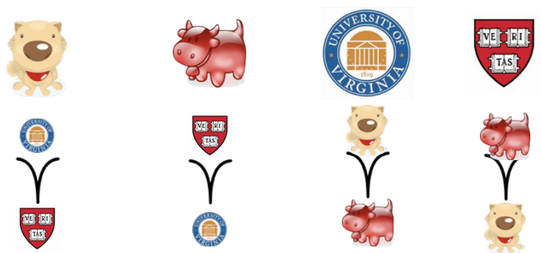


INSTABILITY: $\exists$ an unmatched pair $(m^*, w^*) \notin S$ such that

$(m^*, w') \in S$ $(m', w^*) \in S$, but

w^* prefers m^* to w' AND m^* prefers w^* to m'

Def: instability



$$S = \left\{ \left(\quad \right) \quad \left(\quad \right) \right\}$$

$$\left(\quad \right) \quad (m^*, w^*) \notin S$$

$$w' \prec_{m^*} w^*$$

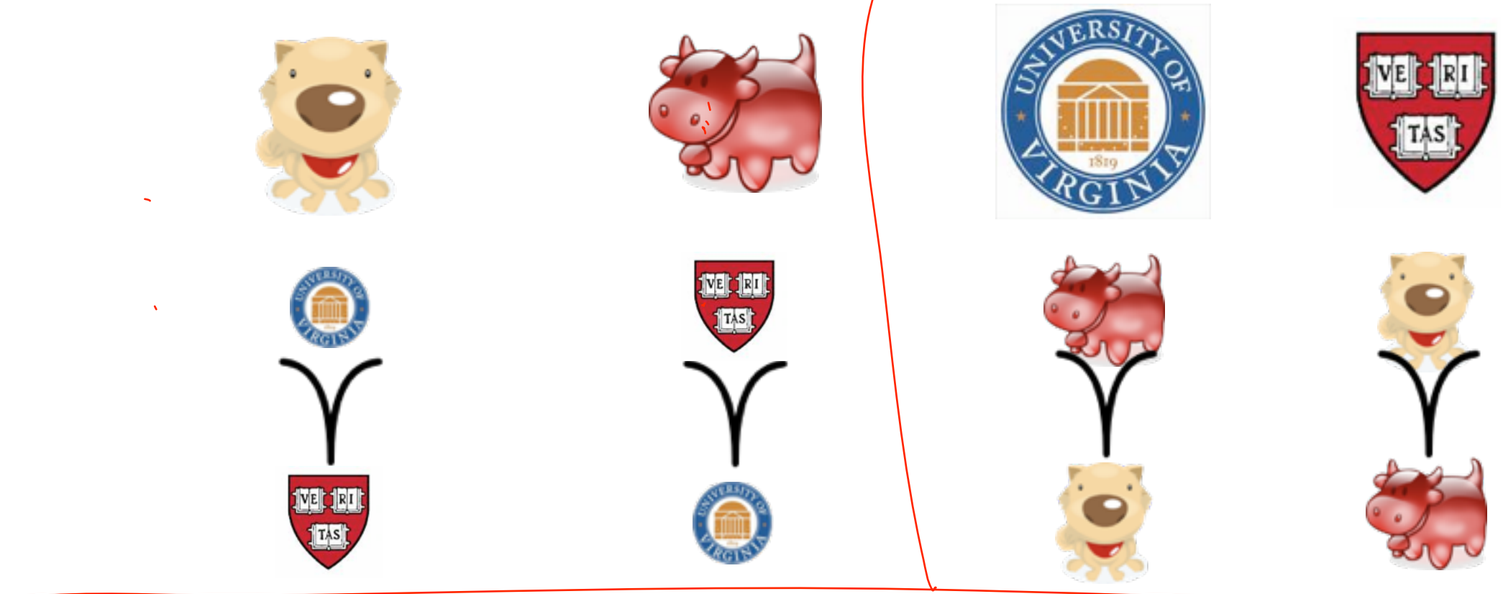
$$m' \prec_{w^*} m^*$$

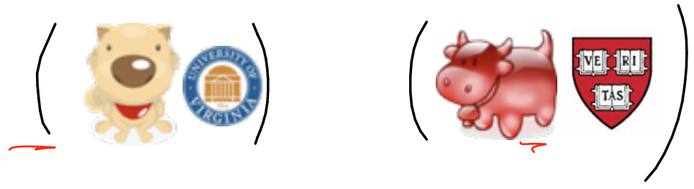
M = { (s_1,r_1), (s_2,r_2), … (s_n,r_n) }

is a stable matching if

No unmatched pair (s*,r*) prefer
each other to their partners in M

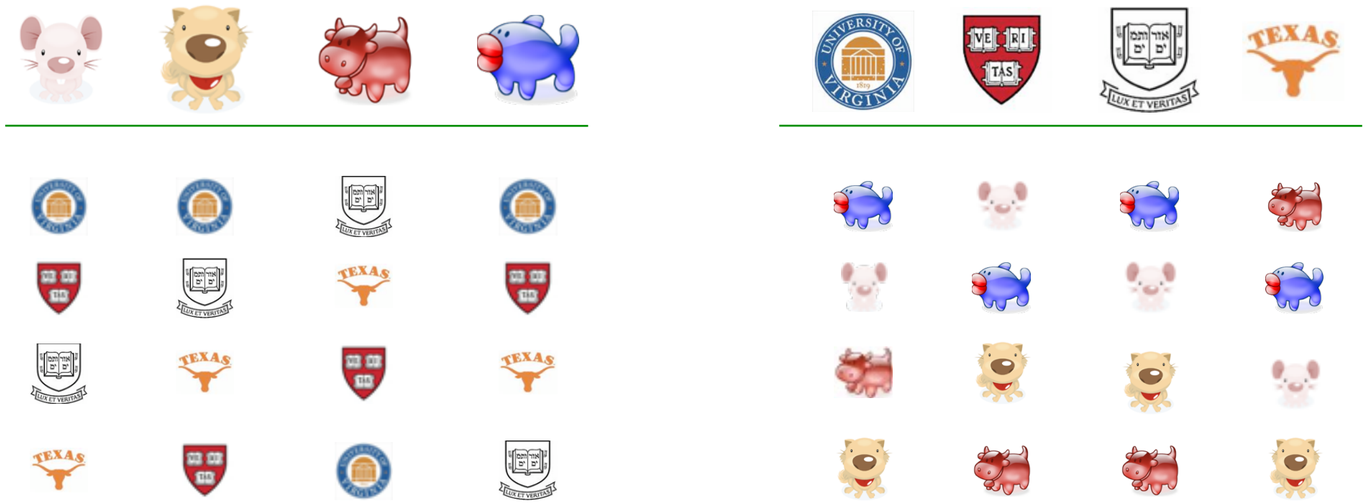
Example 2





STABLE, even though some players are match with their least favorite partners.

Prove: for every input



there exists a stable matching.

proposal algorithm

INITIALIZE ALL plays to be unmatched

While $\exists$ an unmatched $m \in M$

Let w be the highest preferred element in W that m has not yet asked.

If w is unmatched, MAKEPAIR(m,w)

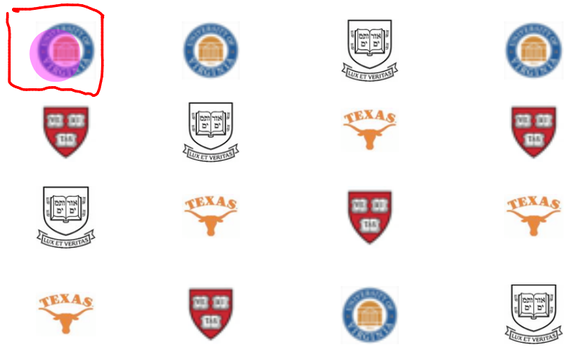
Else if (m',w) is a pair AND $m \underset{w}{\succ} m'$

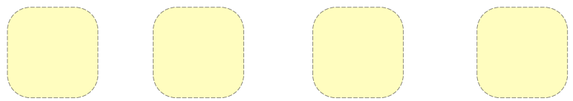
Break(m',w)

MAKE(m,w)

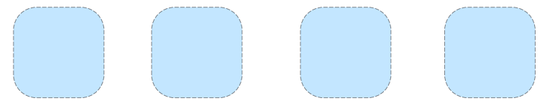
```
StableMatch(M, W, ≺_m, ≺_w)
 1  Initialize all m, w to be free
 2  while ∃free(m) and hasn't proposed to all W
 3        do Pick such an m
 4           Let w ∈ W be highest-ranked to whom m has not yet proposed
 5           if free(w)
 6              then Make a new pair (m, w)
 7           elseif (m', w) is paired and m' ≺_w m
 8                do Break pair (m', w) and make m' free
 9                   Make pair (m, w)
10  return Set of pairs
```

S

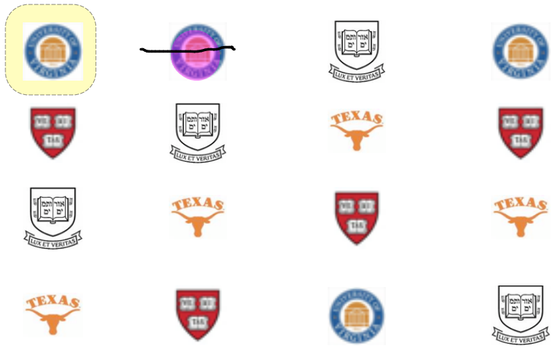


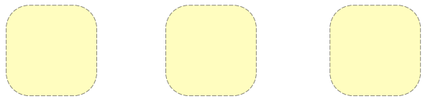


R

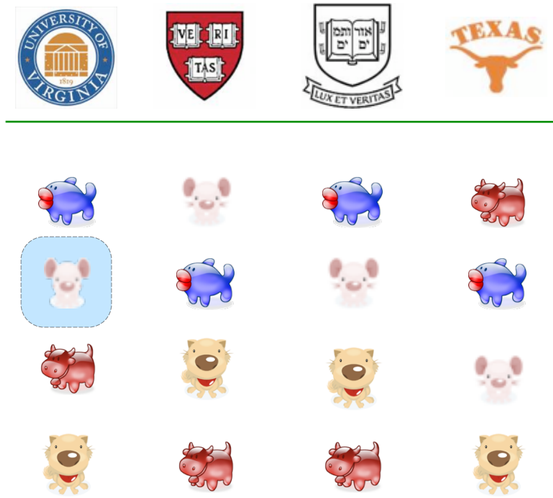


S

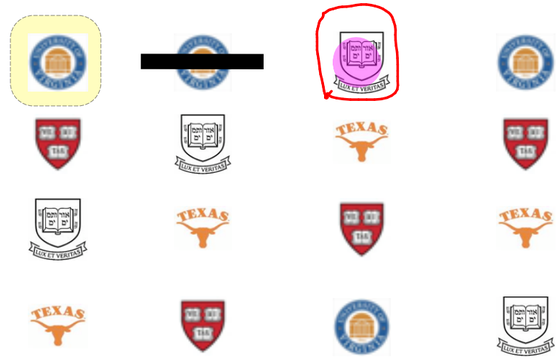




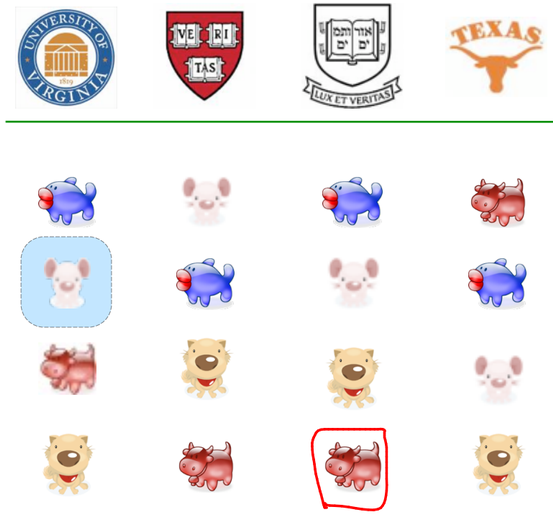
R



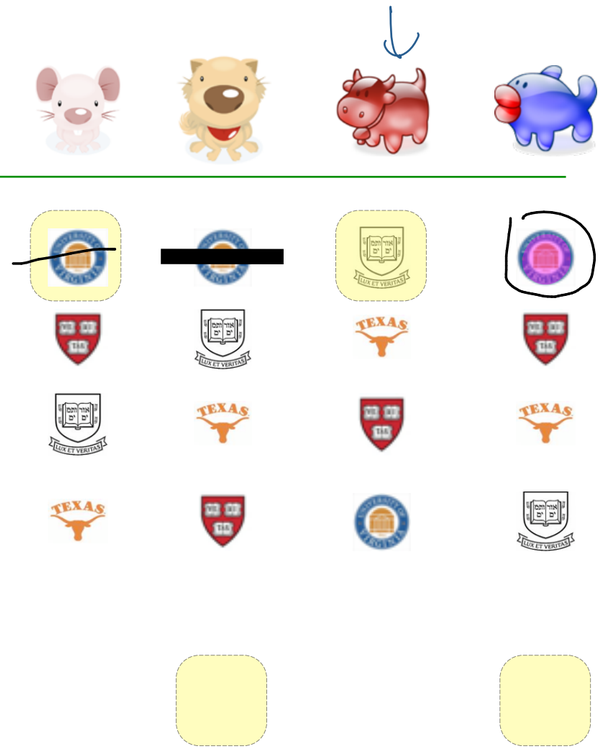
S



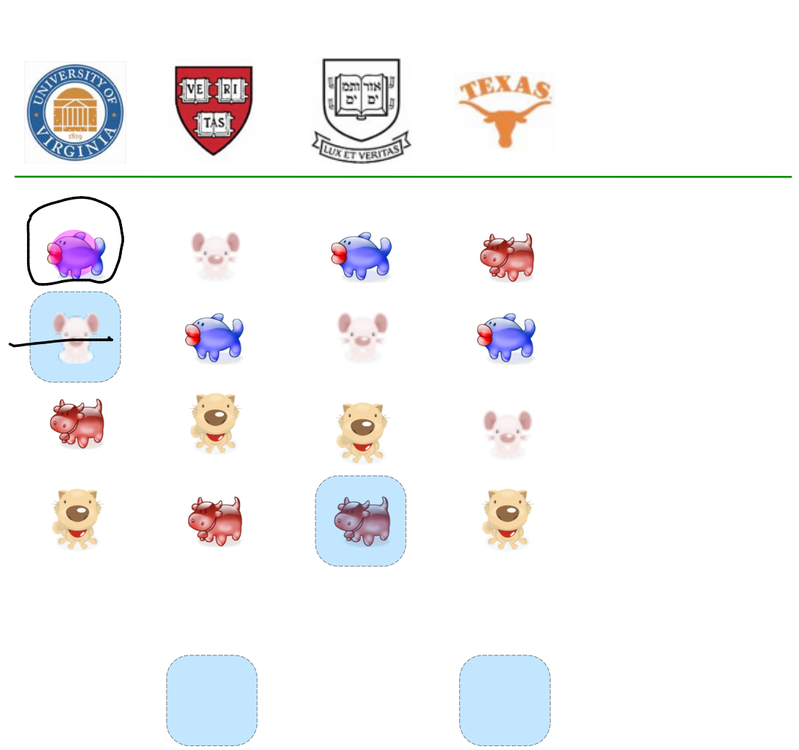
R



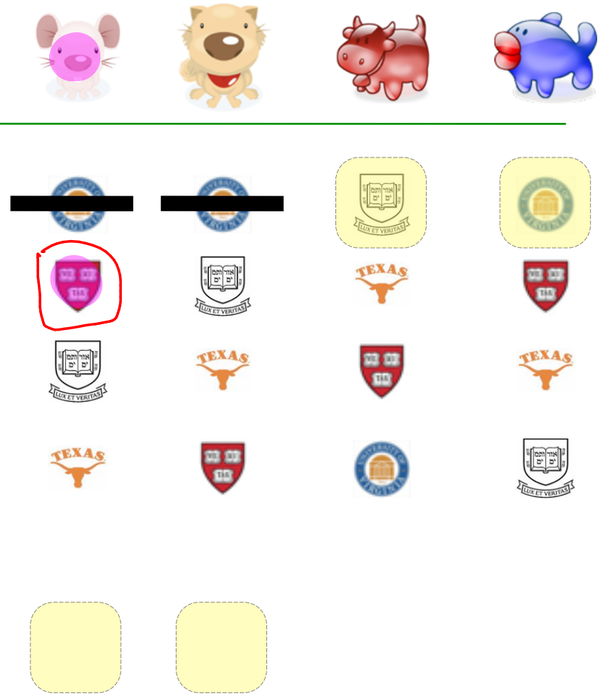
S



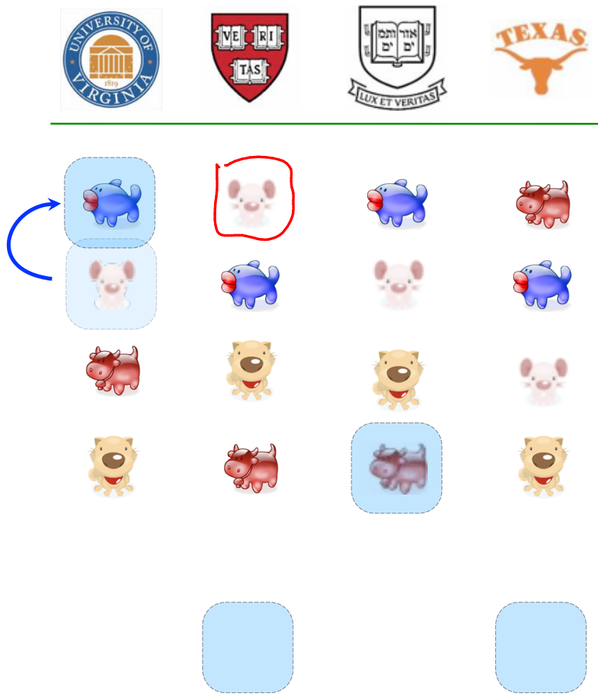
R



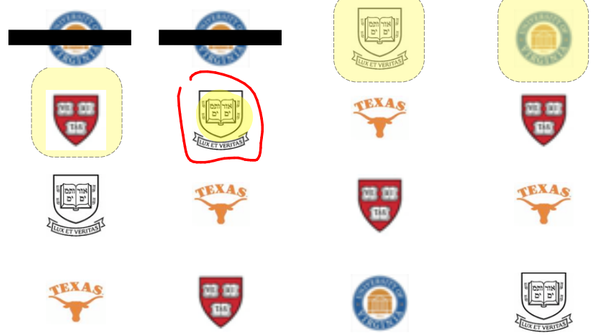
S

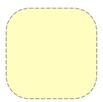


R



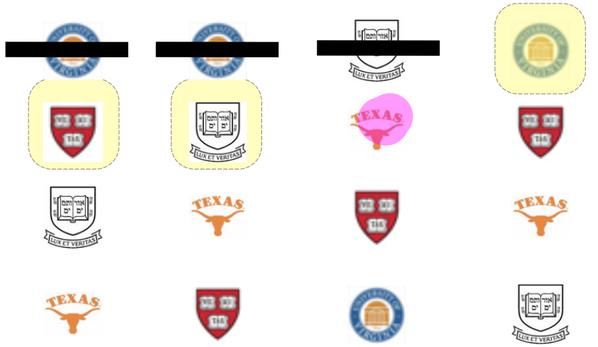
S





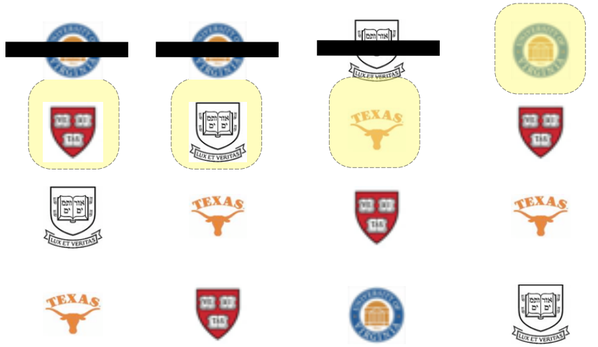
R

S

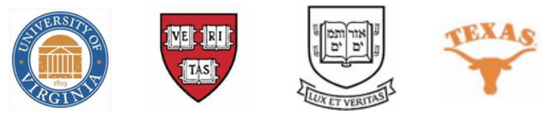


R

S



R



Proposal algorithm ends

proposal algorithm ends

$O(n^2)$ steps

each m proposes at most once to each w.

each m proposes at most n times.

size of M is n.

n

$n^2 - n$ steps

10^5 ~ 10^{10}

output is a matching

① EACH w is unmatched at the time it is matched in lines IF + ELSE.

② EACH m is unmatched @ time of proposal.

output is perfect

If there is an unmatched $m \in M$, then there is also an unmatched w.

$\Rightarrow$ output has size n.

output is perfect

if $\exists m$ who is free, then

$\exists w$ who has not been asked

Prove

output is stable

Suppose for the sake of reaching a contradiction that output has an instability.

That means that there exists $(m^*, w^*) \notin S$ and

(m^*, w'), $(m', w^*) \in S$ such that $m' \underset{w^*}{<} m^*$ $w' \underset{m^*}{<} w^*$

→ Consider the moment when (m^*, w') was made into a match.

Since $w' <_{m^*} w^*$, then m^* must have already proposed to w^*.

Either m^* proposed to w^* and w^* was in a match $(\hat{m}, w^*)$ or

at a later point $\hat{m}$ proposed and broke (m^*, w^*) to form $(\hat{m}, w^*)$

In either case, $\hat{m} \underset{w^*}{>} m^*$. AND either $m' = \hat{m}$ or

$m' \underset{w^*}{>} \hat{m} \underset{w^*}{>} m^*$ BUT THIS CONTRADICTS

☒

output is stable

spse not. $\exists (m^*, w), (m, w^*) \in S \qquad w \prec_{m^*} w^* \qquad m \prec_{w^*} m^*$

output is stable

spse not. $\exists (m^*, w), (m, w^*) \in S \qquad w \prec_{m^*} w^* \;\; m \prec_{w^*} m^*$

m* last proposal was to w

but $w \prec_{m^*} w^*$ and so m* must have already asked w*

and must have been rejected by $m^* \prec_{w^*} m'$

then either $m' \prec_{w^*} m$ or m'=m

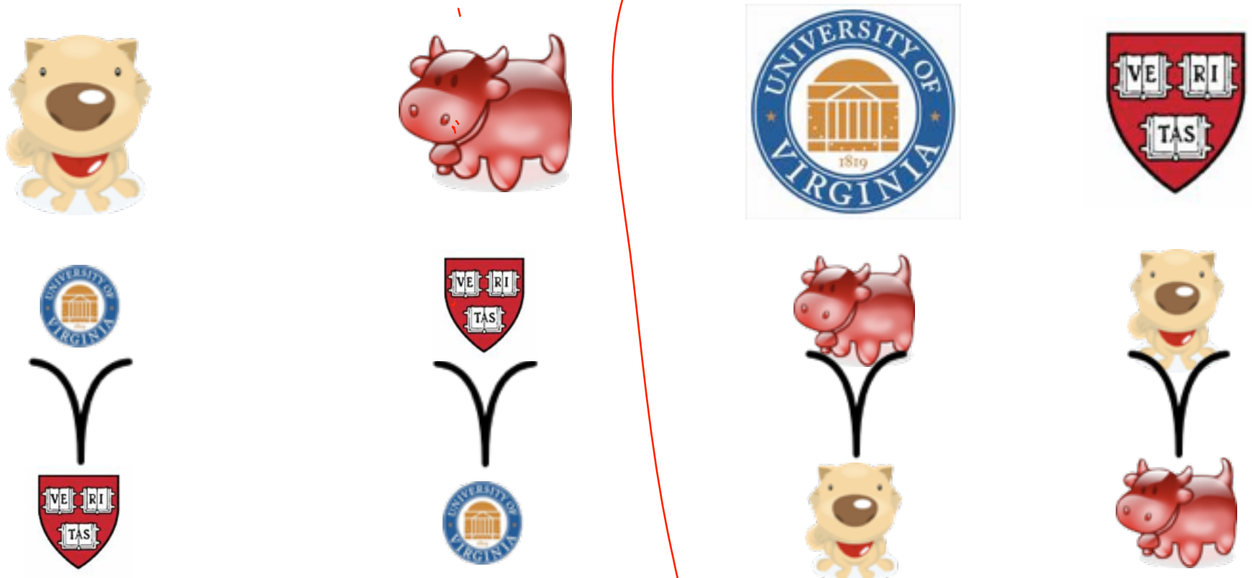
which contradicts assumption $m \prec_{w^*} m^*$

Proposer wins

(D,U) (B,H) stable

(B,U) (D,H) stable.

Proposer wins



Remarkable theorem

w is valid for m: $\exists$ a stable matching S s.t $(m,w) \in S$ and (m, w^*) is NOT in ANY stable matching for $w^* \underset{m}{>} w$

best(m): $\{ w \mid w \text{ is valid for } m \}$

$S^* = \{ (m, best(m)) \}$ "Best matching for M"

—GS returns S^*. (no matter which order the proposals are made)

GS is Suitor-optimal.

GS matching vs R-opt

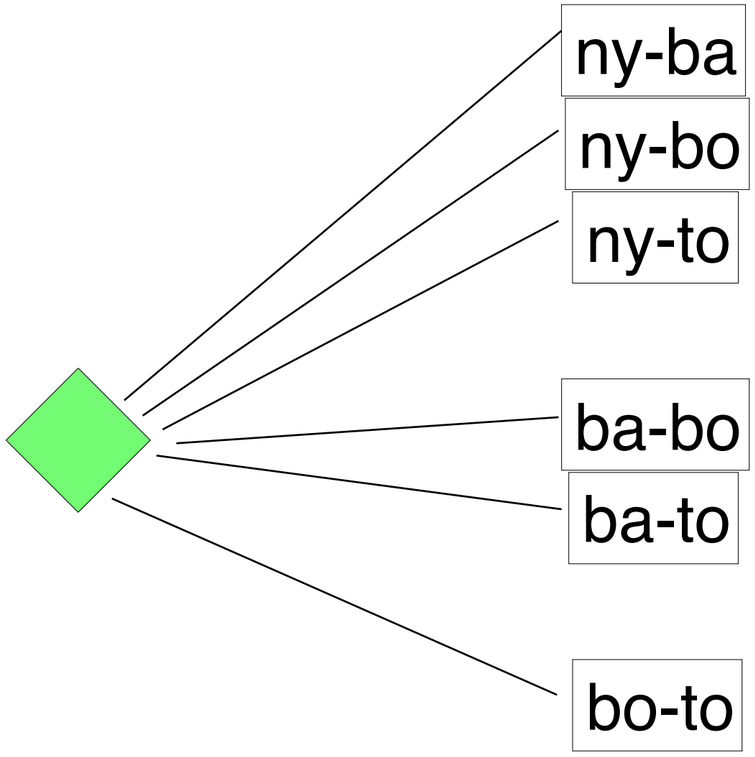
THE MATCH℠
NATIONAL RESIDENT MATCHING PROGRAM®

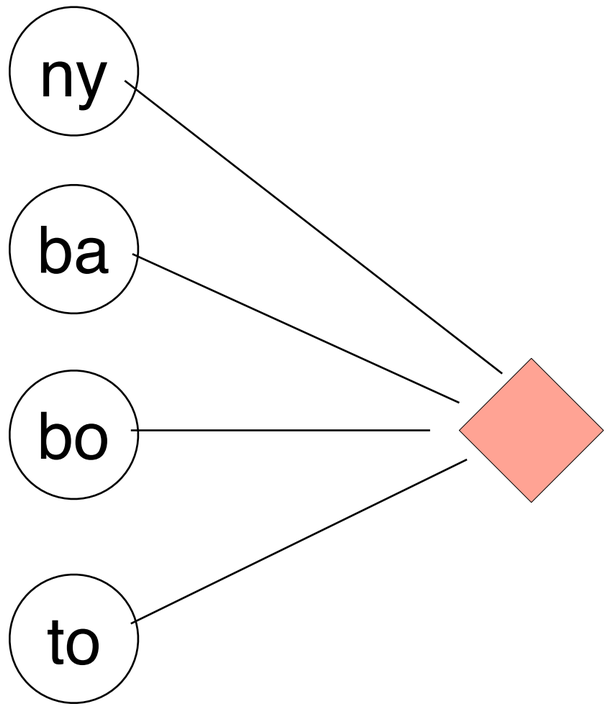
Baseball elimination

				Against			
	W	L	Left	A	P	N	M
ATL	83	71	8	-	1	6	1
PHL	80	79	3	1	-	0	2
NY	78	78	6	6	0	-	0
MONT	77	82	3	1	2	0	-

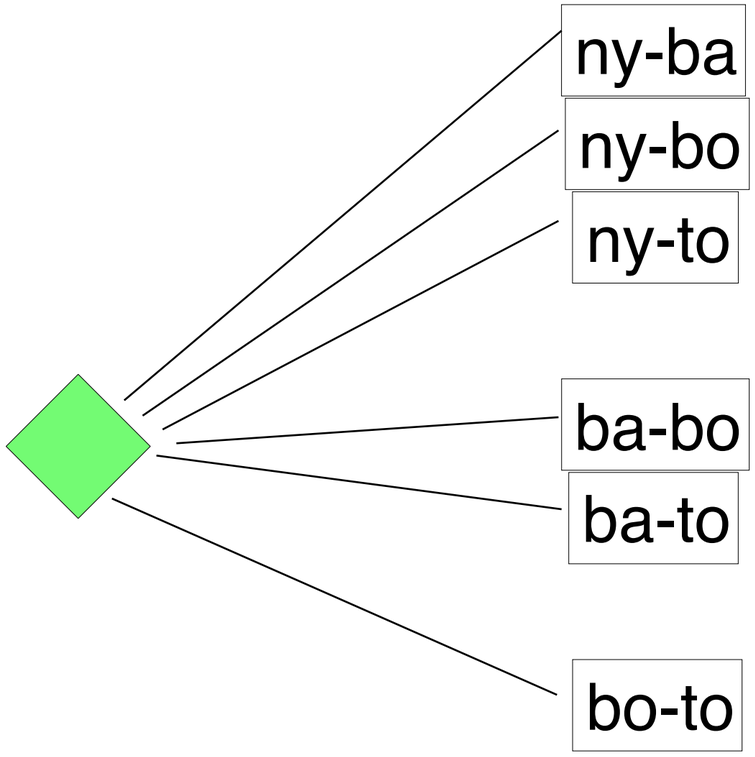
Baseball elimination

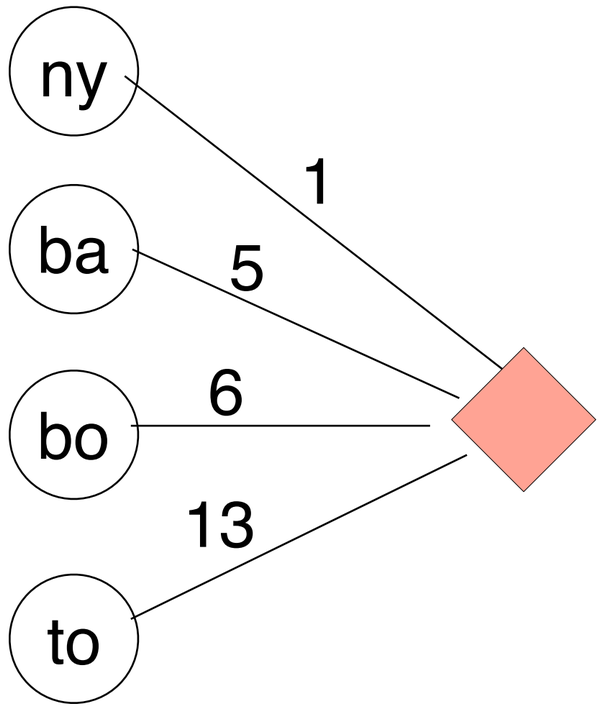
				Against				
	W	L	Left	N	B	Bo	T	D
NY	75	59	28		3	8	7	3
BAL	71	63	28	3		2	7	4
BOS	69	66	27	8	2			
TOR	63	72	27	7	7			
DET	49	86	27	3	4			



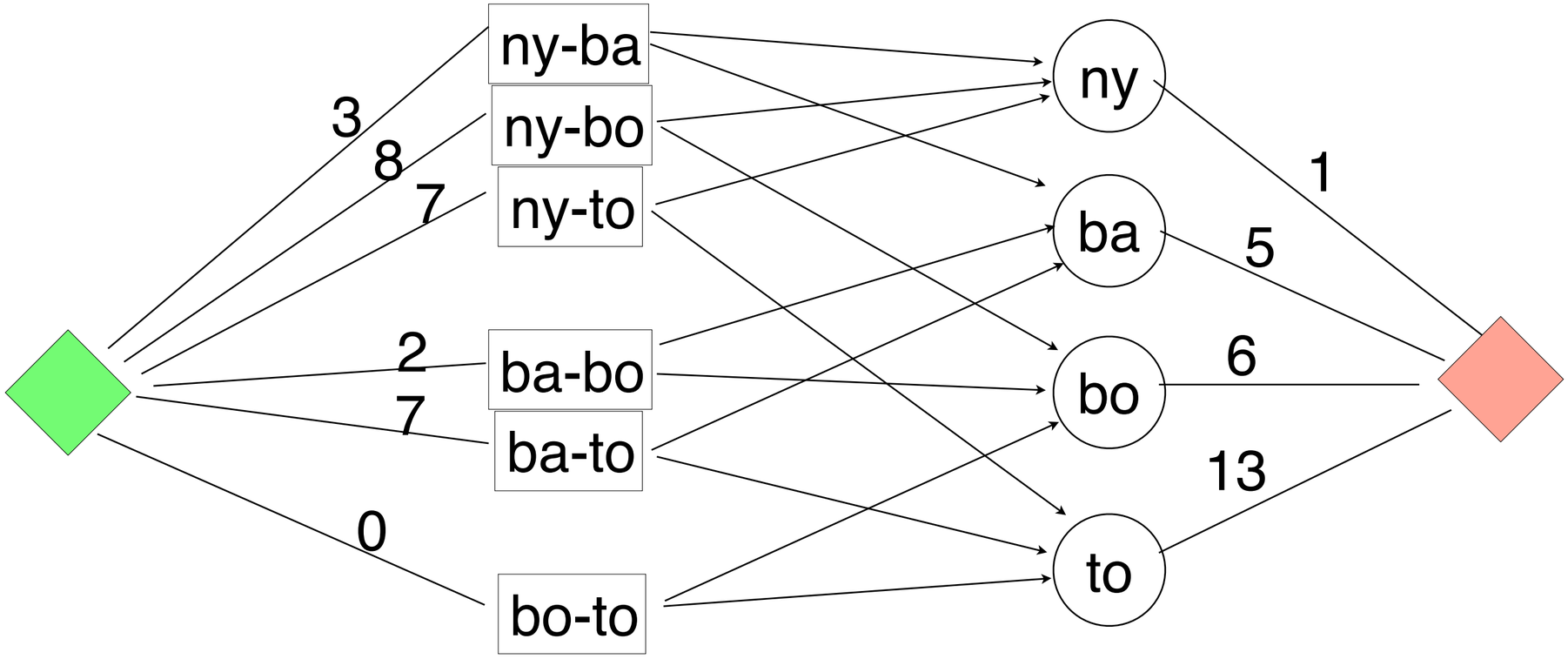


	W	L	Left	N	B	Bo	T	D
NY	75	59	28		3	8	7	3
BAL	71	63	28	3		2	7	4
BOS	69	66	27	8	2			
TOR	63	72	27	7	7			
DET	49	86	27	3	4			





	W	L	Left	N	B	Bo	T	D
NY	75	59	28		3	8	7	3
BAL	71	63	28	3		2	7	4
BOS	69	66	27	8	2			
TOR	63	72	27	7	7			
DET	49	86	27	3	4			



	W	L	Left	N	B	Bo	T	D
NY	75	59	28		3	8	7	3
BAL	71	63	28	3		2	7	4
BOS	69	66	27	8	2			
TOR	63	72	27	7	7			
DET	49	86	27	3	4			