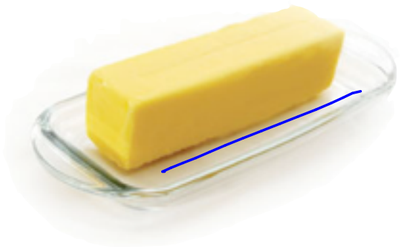
L24

4102

4.19.2016

abhi shelat

Guns and butter



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Guns and butter

$$\max x + 100y$$

$$\begin{aligned} 4x - y &\leq 8 \\ 2x + y &\leq 10 \\ 5x - 2y &\geq -2 \\ x, y &\geq 0 \end{aligned}$$

http://i16.photobucket.com/albums/b20/safebuy/ak47/ak47-electric_lg.jpg

http://2.bp.blogspot.com/_NX4zcMNX4VE/Sb8MQfffllI/AAAAAAAAAL0/eu4J0dfFhJE/s400/gourmet-butter.jpg

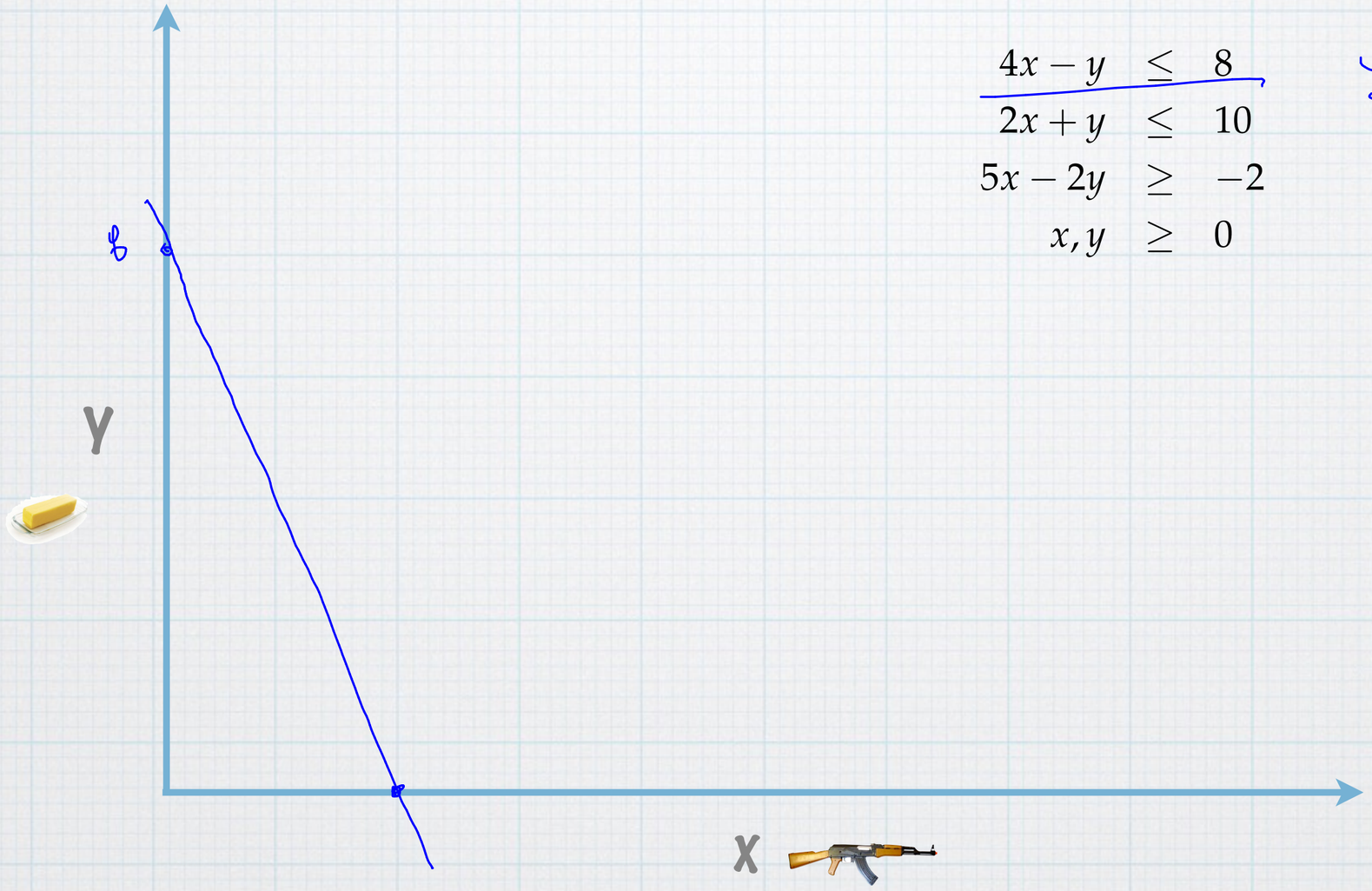
Linear program

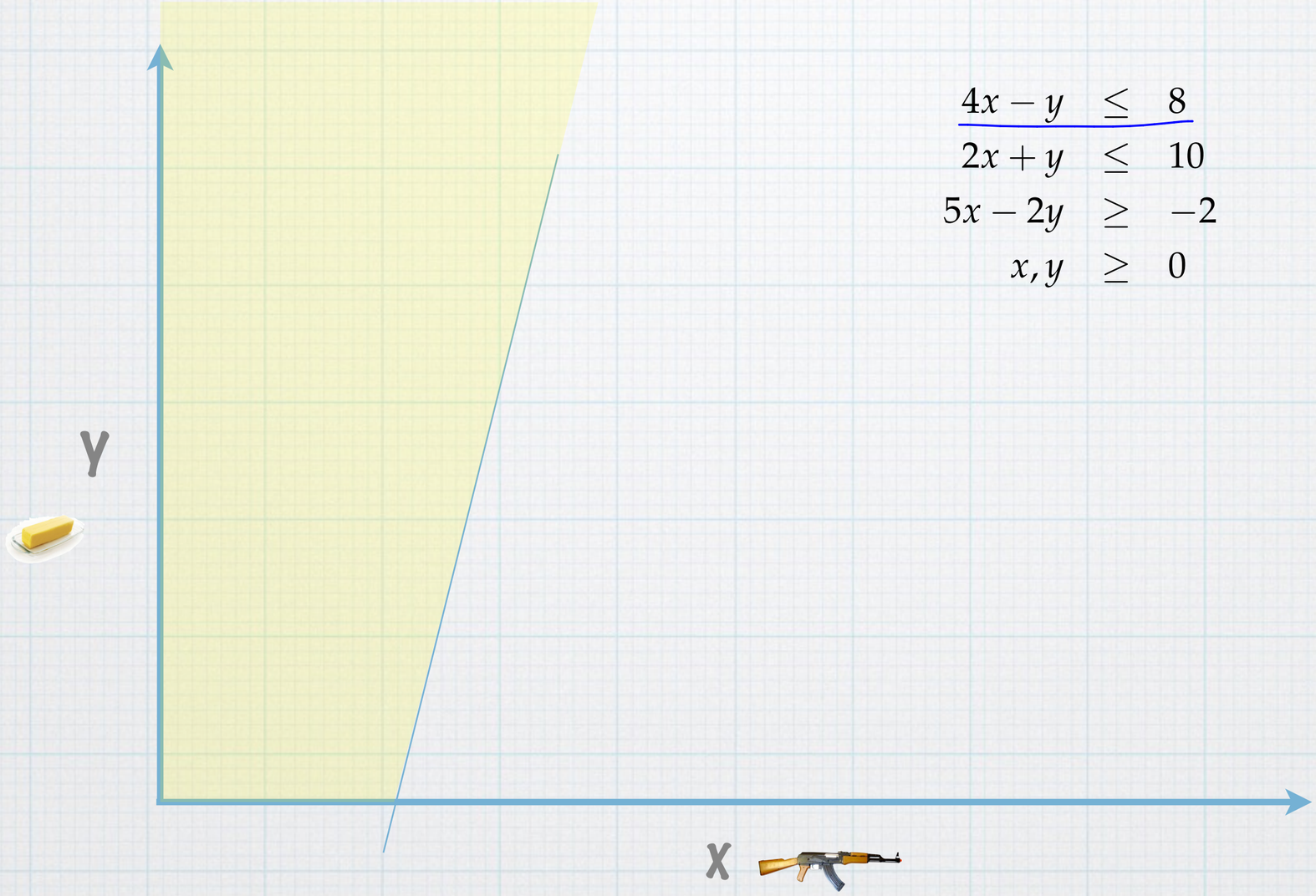
$$\max x + y$$

$$\begin{aligned} 4x - y &\leq 8 \\ 2x + y &\leq 10 \\ 5x - 2y &\geq -2 \\ x, y &\geq 0 \end{aligned}$$

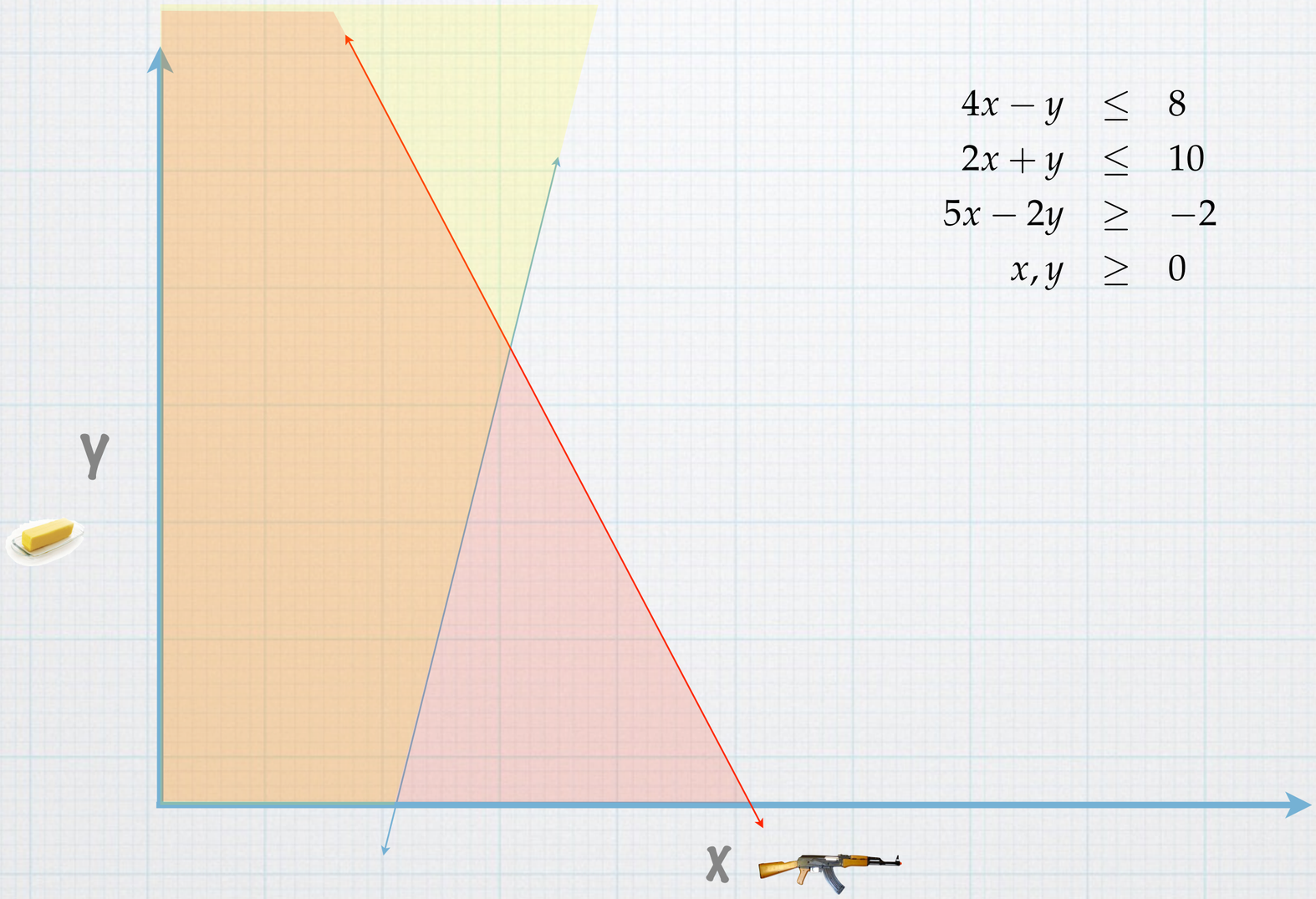
$$
\begin{aligned}
4x - y &\leq 8 \\
2x + y &\leq 10 \\
5x - 2y &\geq -2 \\
x, y &\geq 0
\end{aligned}
$$

$y \geq 8 - 4x$

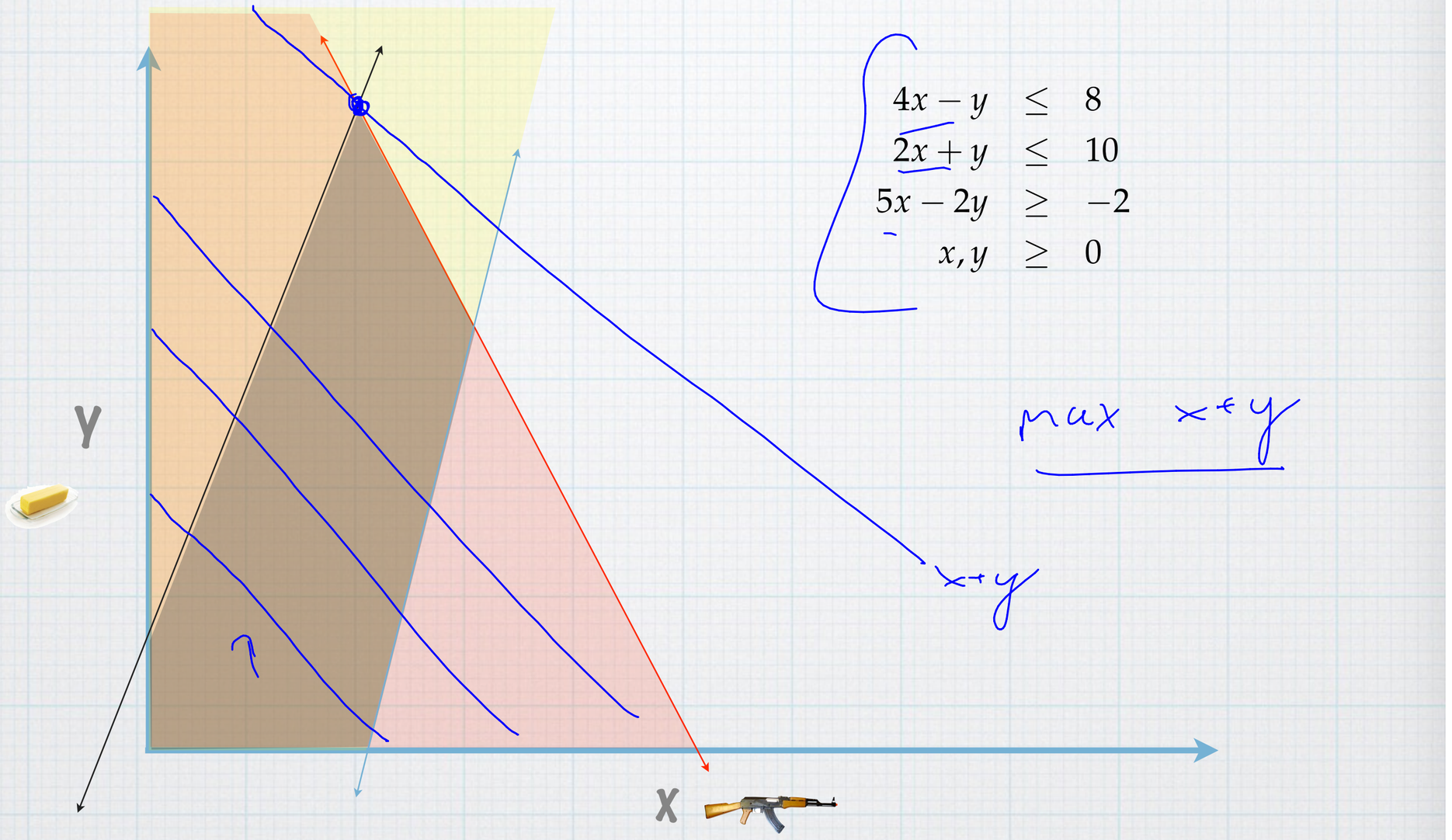




$$\begin{aligned} 4x - y &\leq 8 \\ 2x + y &\leq 10 \\ 5x - 2y &\geq -2 \\ x, y &\geq 0 \end{aligned}$$



$$
\begin{aligned}
4x - y &\leq 8 \\
2x + y &\leq 10 \\
5x - 2y &\geq -2 \\
x, y &\geq 0
\end{aligned}
$$

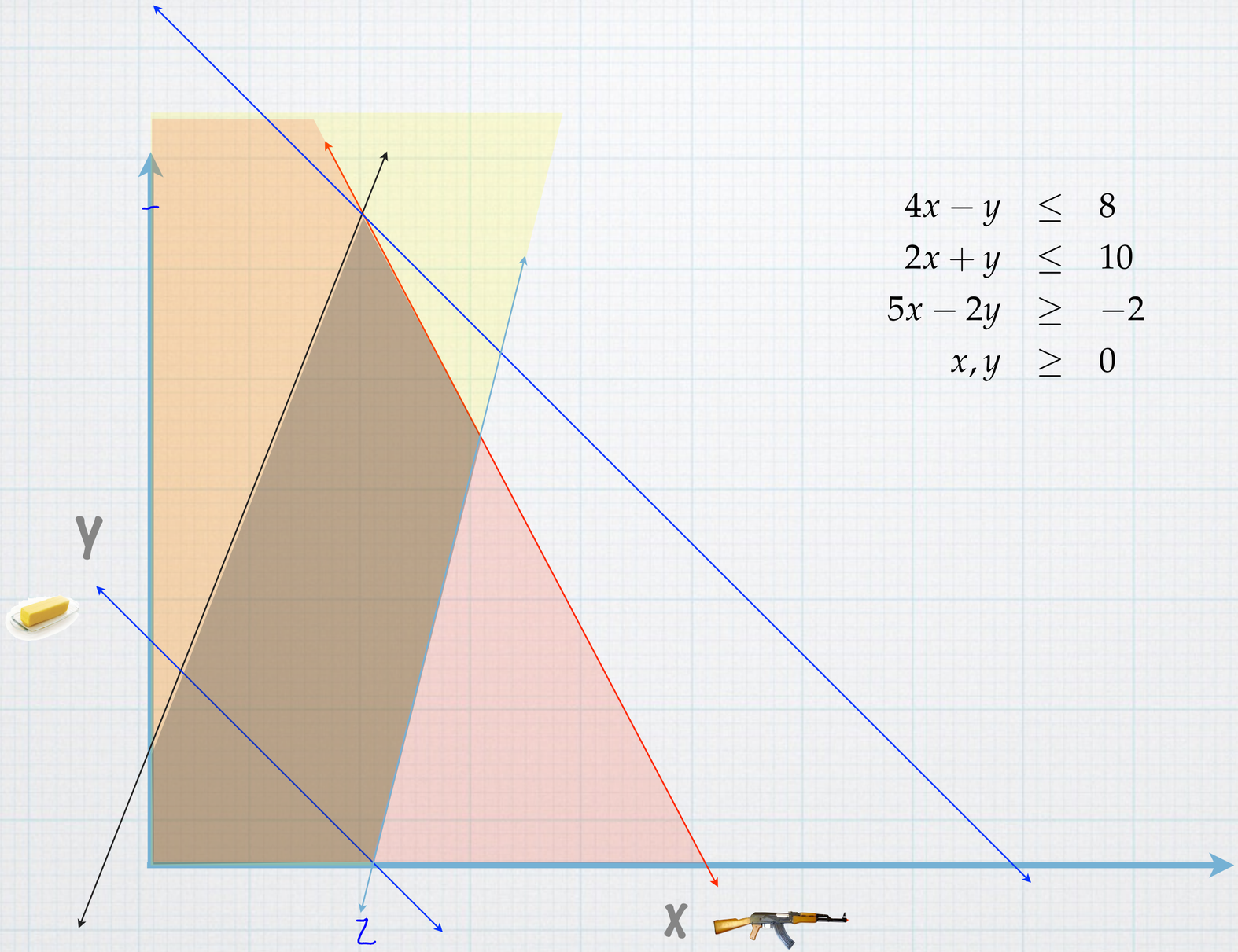


$$\begin{cases} 4x - y \leq 8 \\ 2x + y \leq 10 \\ 5x - 2y \geq -2 \\ x, y \geq 0 \end{cases}$$

$\max \; x+y$

$x+y$

LP.
generalization
to
many
variable



$$
\begin{aligned}
4x - y &\leq 8 \\
2x + y &\leq 10 \\
5x - 2y &\geq -2 \\
x, y &\geq 0
\end{aligned}
$$

Certificate of optimality

$$\max x + y$$

$$\begin{aligned} 4x - y &\leq 8 \\ 2x + y &\leq 10 \quad \cdot 7 \\ 5x - 2y &\geq -2 \quad \cdot -1 \\ x, y &\geq 0 \end{aligned}$$

$$14x + 7y \leq 70$$
$$-5x + 2y \leq 2$$
$$9x + 9y \leq 72$$

$$x + y \leq 8$$

Spidman

→ Keller

Dantzig

KHACHIYAN

KARMARKAR

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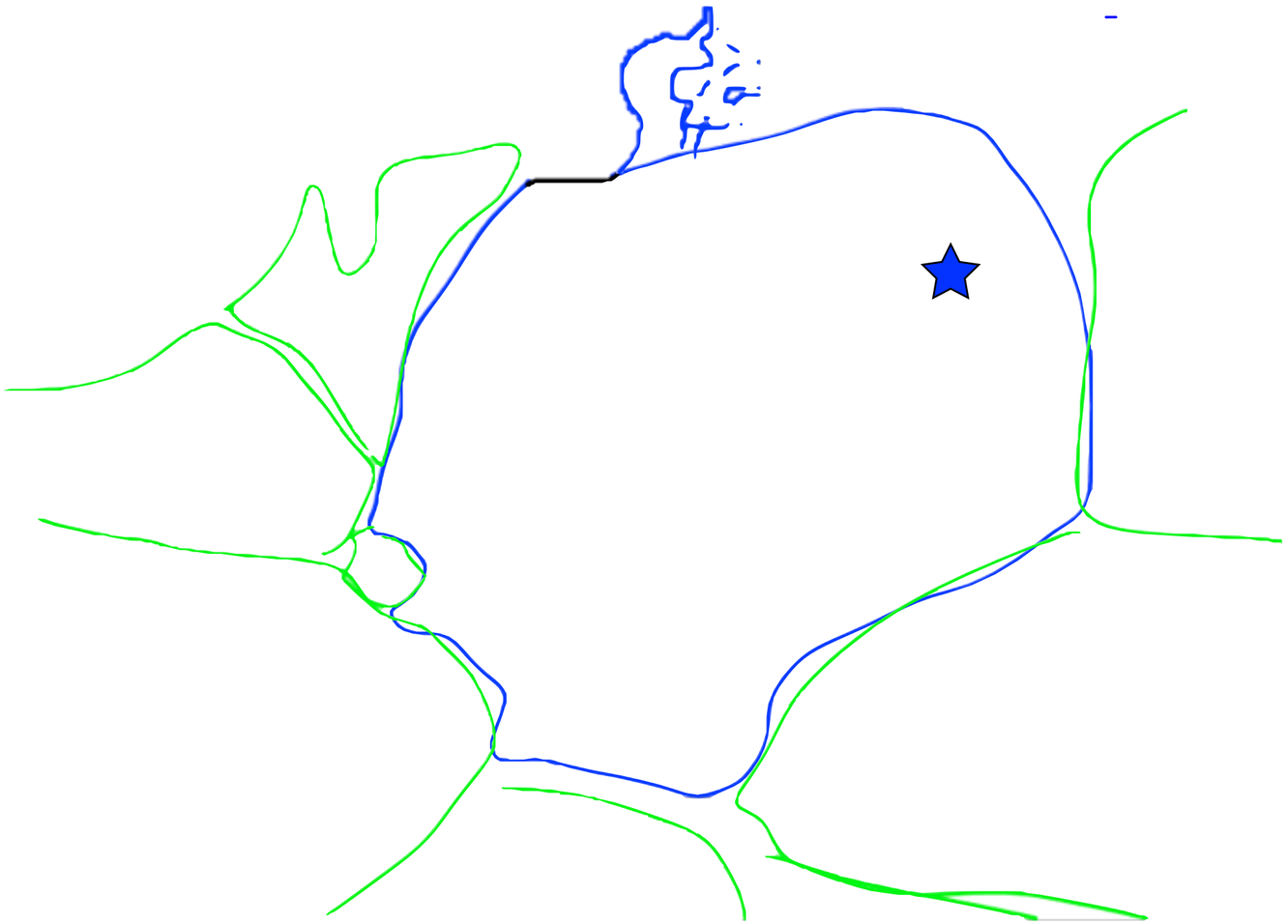
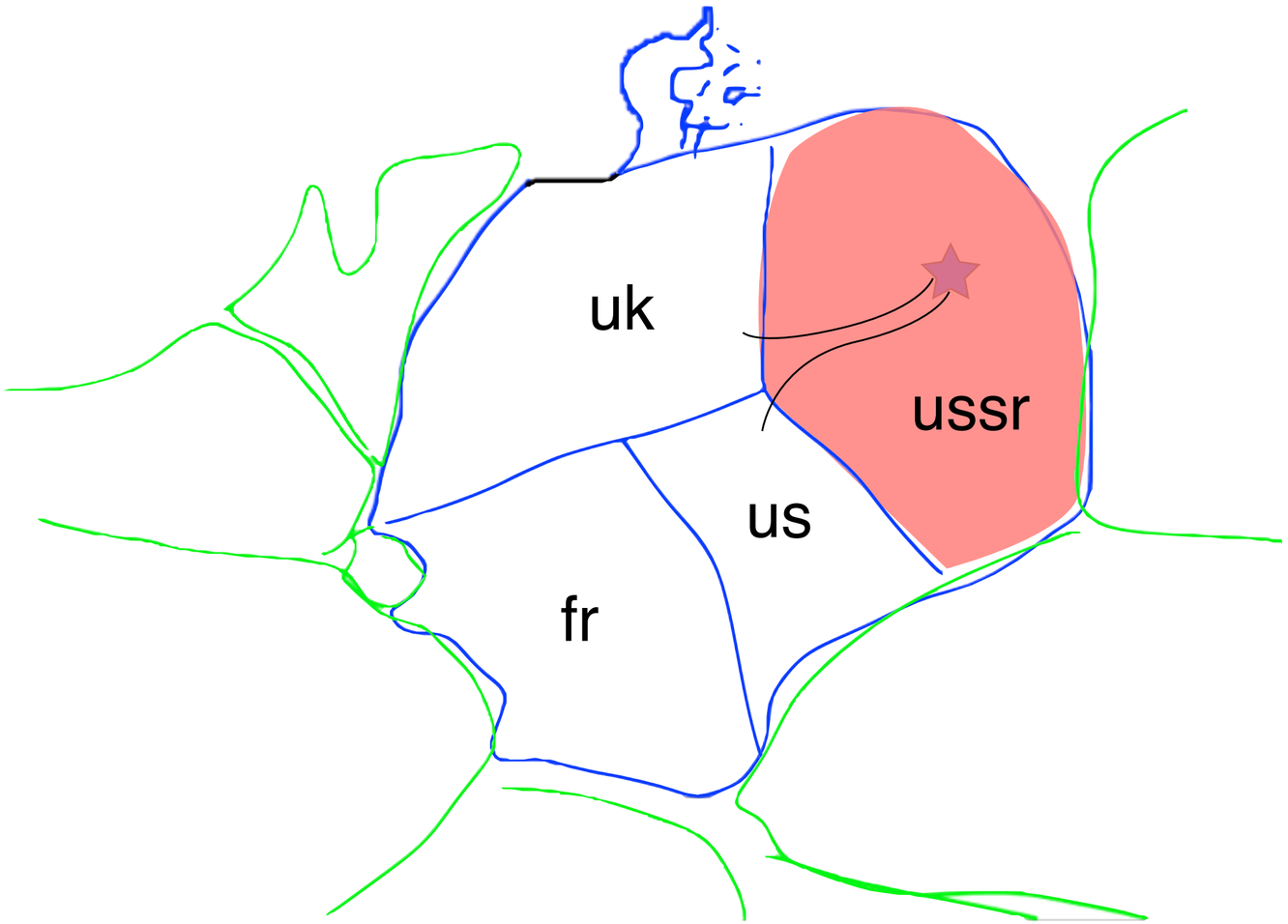
45

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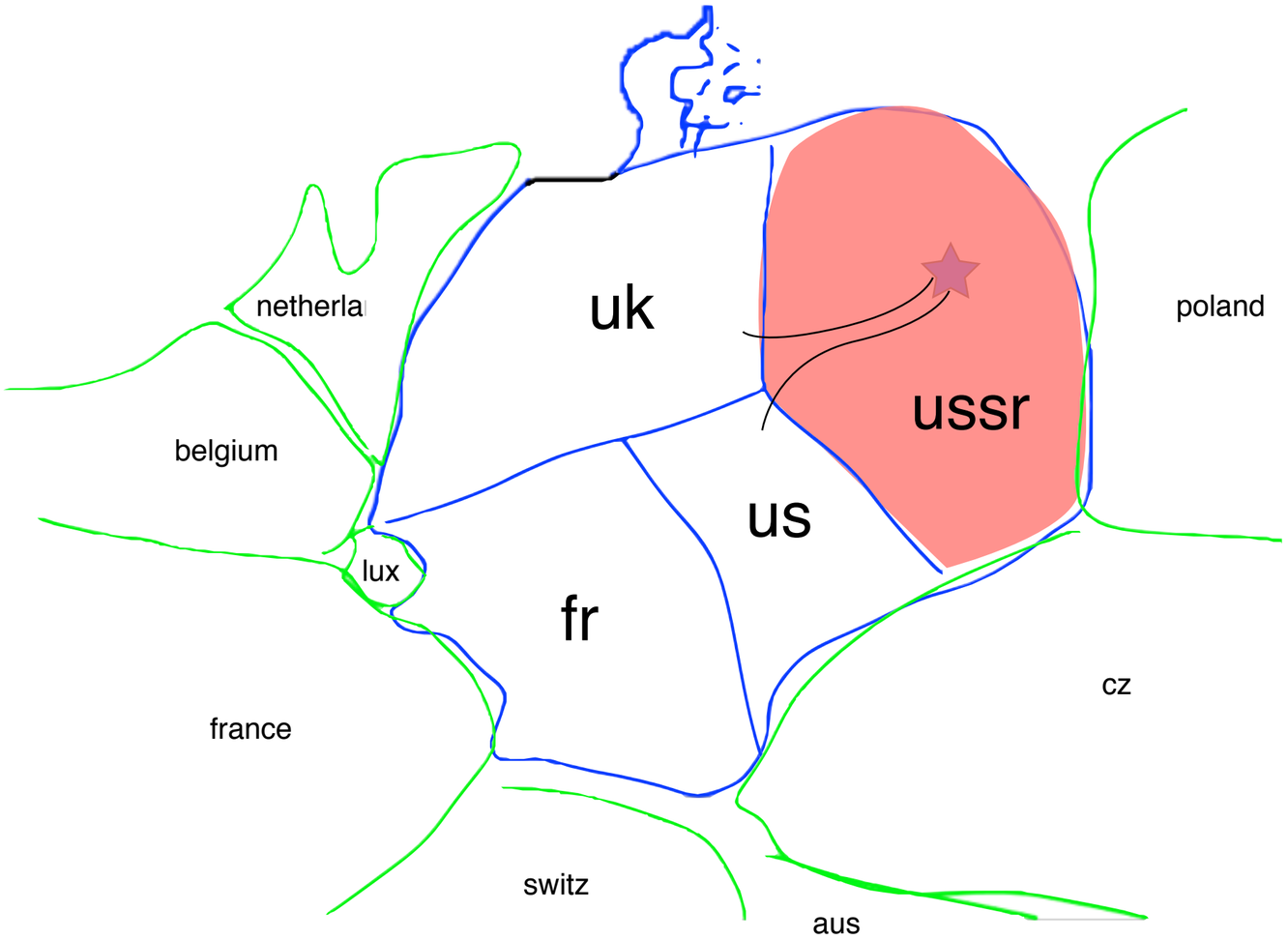
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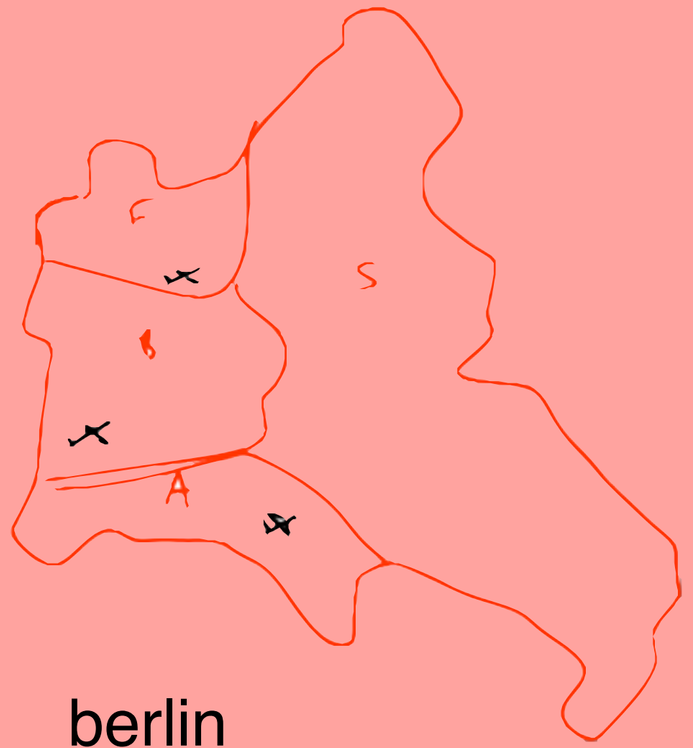


berlin

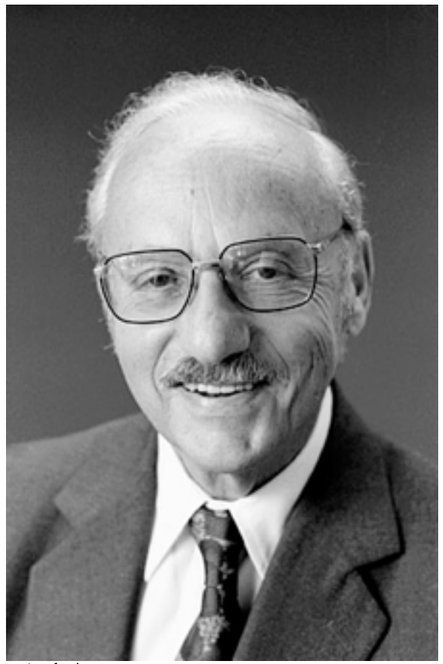


image:stamford

image:history of air cargo

linear programming saved Berlin

Stigler diet

calories	3000
protein	70g
calcium	.8g
iron	19mg
vitamin A	5000iu
thiamine	1.8mg
riboflavin	2.7mg
niacin	18mg
ascorbic acid	75mg

TABLE A. NUTRITIVE VALUES OF COMMON FOODS PER DOLLAR OF EXPENDITURE, AUGUST 15, 1939

Commodity	Unit	Price Aug. 15, 1939 (cents)	Edible Weight per $1.00 (grams)	Calories (1,000)	Protein (grams)	Calcium (grams)	Iron (mg.)	Vitamin A (1,000 I.U.)	Thiamine (mg.)	Riboflavin (mg.)	Niacin (mg.)	Ascorbic Acid (mg.)
**1. Wheat Flour (Enriched)	10 lb.	36.0	12,600	44.7	1,411	2.0	365		55.4	33.3	441	
2. Macaroni	1 lb.	14.1	3,217	11.6	418	.7	54		3.2	1.9	68	
3. Wheat Cereal (Enriched)	28 oz.	24.2	5,280	11.8	377	14.4	175		14.4	8.8	114	
4. Corn Flakes	8 oz.	7.1	3,104	11.4	232	.1	56		15.5	2.3	68	
5. Corn Meal	1 lb.	4.6	9,861	36.0	897	1.7	99	80.0	17.4	7.9	196	
6. Hominy Grits	24 oz.	8.5	8,065	28.6	680	.8	80		10.6	1.6	110	
7. Rice	1 lb.	7.5	6,048	21.2	400	.6	41		2.0	4.8	60	
8. Rolled Oats	1 lb.	7.1	6,389	25.3	907	5.1	541		37.1	8.9	64	
9. White Bread (Enriched)	1 lb.	7.9	5,742	15.6	488	2.5	115		13.8	8.5	126	
10. Whole Wheat Bread	1 lb.	9.1	4,985	12.2	464	2.7	125		13.9	6.4	160	
11. Rye Bread	1 lb.	9.2	4,930	12.4	489	1.1	82		9.9	3.0	66	
12. Pound Cake	1 lb.	24.8	1,829	8.0	130	.4	81	18.9	2.8	3.0	17	
13. Soda Crackers	1 lb.	15.1	3,004	12.5	288	.5	50					
14. Milk	1 qt.	11.0	8,967	6.1	310	10.5	18	16.8	4.0	16.0	7	177
**15. Evaporated Milk (can)	14½ oz.	6.7	6,085	8.4	422	15.1	9	26.0	3.0	23.5	11	60
16. Butter	1 lb.	30.8	1,473	10.8	9	.2	3	44.2		.2	2	
*17. Oleomargarine	1 lb.	16.1	2,817	20.6	17	.6	6	55.6	.2			
18. Eggs	1 doz.	32.6	1,857	2.9	238	1.0	58	18.6	2.8	6.6	1	
**19. Cheese (Cheddar)	1 lb.	24.2	1,874	7.4	448	16.4	19	28.1	.8	10.3	4	
20. Cream	½ pt.	14.1	1,689	3.5	49	1.7	8	16.9	.6	2.5		17
21. Peanut Butter	1 lb.	17.9	2,534	15.7	661	1.0	48		9.6	8.1	471	
22. Mayonnaise	½ pt.	16.7	1,198	8.6	18	.2	9	2.7	.4	.5		
23. Crisco	1 lb.	20.3	2,254	20.1								
24. Lard	1 lb.	9.8	4,628	41.7				.2		.5	5	
25. Sirloin Steak	1 lb.	39.6	1,145*	4.9	166	.1	54	.2	2.1	2.9	69	
26. Round Steak	1 lb.	36.4	1,246*	2.2	214	.1	32	.4	2.5	2.4	87	
27. Rib Roast	1 lb.	29.2	1,553*	3.4	213	.1	33			2.0		
28. Chuck Roast	1 lb.	22.6	2,007*	3.6	309	.2	46	.4	1.0	4.0	120	
29. Plate	1 lb.	14.6	3,107*	8.5	404	.2	62		.9			
**30. Liver (Beef)	1 lb.	26.8	1,692*	2.2	333	.2	130	169.2	6.4	50.8	316	525
31. Leg of Lamb	1 lb.	27.8	1,643*	3.1	245	.1	20		2.6	3.9	86	
32. Lamb Chops (Rib)	1 lb.	36.6	1,259*	3.3	140	.1	15		1.7	2.7	54	
33. Pork Chops	1 lb.	30.7	1,477*	3.5	196	.2	30		17.4	2.7	60	
34. Pork Loin Roast	1 lb.	24.2	1,874*	4.4	249	.3	37		18.2	3.6	79	
35. Bacon	1 lb.	25.6	1,772*	10.4	152	.2	23		1.8	1.8	71	
36. Ham—smoked	1 lb.	27.4	1,655*	6.7	212	.2	31		9.9	3.3	50	
37. Salt Pork	1 lb.	16.0	2,835*	18.8	104	.1	26		1.4	1.8		
38. Roasting Chicken	1 lb.	30.3	1,497*	1.8	184	.1	30	.1	.9	1.8	68	46
39. Veal Cutlets	1 lb.	42.3	1,072*	1.7	156	.1	24		1.4	2.4	57	
40. Salmon, Pink (can)	16 oz.	15.0	3,489	5.8	705	6.6	45	3.5	1.0	4.9	209	
41. Apples	1 lb.	4.4	9,078	5.8	27	.5	56	7.3	3.6	2.7	5	544
42. Bananas	1 lb.	6.1	4,982	4.9	60	.4	30	17.4	2.5	3.5	28	498
43. Lemons	1 doz.	26.0	2,380	1.0	21	.6	14		.5		4	952
44. Oranges	1 doz.	30.9	4,439	2.2	40	1.1	18	11.1	3.6	1.3	10	1,998
*45. Green Beans	1 lb.	7.1	5,750	2.4	138	3.7	80	69.0	4.3	5.8	37	862
**46. Cabbage	1 lb.	3.7	8,949	2.6	125	4.0	36	7.2	9.0	4.5	26	5,369
47. Carrots	1 bunch	4.7	6,080	2.7	73	2.8	43	188.5	6.1	4.3	89	608
48. Celery	1 stalk	7.3	3,915	.9	51	3.0	23	.9	1.4	1.4	9	313
49. Lettuce	1 head	8.2	2,247	.4	27	1.1	22	112.4	1.8	3.4	11	449
*50. Onions	1 lb.	5.6	11,844	5.8	166	3.8	59	16.6	4.7	5.9	21	1,184

*51. Potatoes	15 lb.	34.0	16,810	14.3	336	1.8	118	6.7	29.4	7.1	198	2,522
**52. Spinach	1 lb.	8.1	4,598	1.1	106	—	138	918.4	5.7	13.8	33	2,755
**53. Sweet Potatoes	1 lb.	5.1	7,649	9.6	138	2.7	54	290.7	8.4	5.4	85	1,912
54. Peaches (can)	No. 2½	16.8	4,894	3.7	20	.4	10	21.5	.5	1.0	31	196
55. Pears (can)	No. 2½	20.4	4,030	3.0	8	.5	8	.8	.8	.8	5	81
56. Pineapple (can)	No. 2½	21.3	5,993	2.4	16	.4	8	2.0	2.8	.8	7	399
57. Asparagus (can)	No. 2	27.7	1,045	.4	33	.3	12	16.3	1.4	2.1	17	272
58. Green Beans (can)	No. 2	10.0	5,386	1.0	54	2.0	65	53.9	1.6	4.3	32	481
59. Pork and Beans (can)	16 oz.	7.1	6,389	7.5	564	4.0	134	8.5	8.3	7.7	56	
60. Corn (can)	No. 2	10.4	5,452	5.2	136	.2	16	12.0	1.6	2.7	42	218
61. Peas (can)	No. 2	13.8	4,109	2.3	136	.6	45	84.9	4.9	2.5	37	370
62. Tomatoes (can)	No. 2	8.6	6,283	1.3	63	.7	38	53.2	3.4	2.5	36	1,253
63. Tomato Soup (can)	10½ oz.	7.6	3,917	1.6	71	.6	45	57.9	3.5	2.4	37	802
*64. Peaches, Dried	1 lb.	15.7	2,889	8.5	87	1.7	173	86.9	1.2	4.3	55	57
*65. Prunes, Dried	1 lb.	9.0	4,284	12.8	99	2.5	154	85.7	3.9	4.3	65	257
66. Raisins, Dried	15 oz.	9.4	4,524	13.5	104	2.5	136	4.5	6.3	1.4	24	136
67. Peas, Dried	1 lb.	7.9	5,742	20.0	1,367	4.2	345	2.9	28.7	18.4	162	
**68. Lima Beans, Dried	1 lb.	8.9	5,007	17.4	1,055	3.7	459	5.1	26.9	38.2	93	
**69. Navy Beans, Dried	1 lb.	8.9	7,888	26.9	1,691	11.4	792		38.4	24.6	217	
70. Coffee	1 lb.	22.4	2,025	—	—	—	—		4.0	5.1	50	
71. Tea	½ lb.	17.4	652	—	—	—	—			2.3	42	
72. Cocoa	8 oz.	8.6	2,657	8.7	237	3.0	72		2.0	11.9	40	
73. Chocolate	8 oz.	16.2	1,409	8.0	77	1.3	39		.9	3.4	14	
74. Sugar	10 lb.	51.7	8,773	54.9	—	—	—					
75. Corn Sirup	24 oz.	13.7	4,966	14.7	—	.5	74				5	
76. Molasses	18 oz.	13.6	3,752	9.0	—	10.3	244		1.9	7.5	146	
77. Strawberry Preserves	1 lb.	20.5	2,213	6.4	11	.4	7	.2	.2	.4	3	

* Quantities including inedible portions.

TABLE B. NUTRITIVE VALUES OF COMMON FOODS PER DOLLAR OF EXPENDITURE, AUGUST 15, 1944

Commodity	Price Aug. 15, 1944 (cents)	Calories (1,000)	Protein (grams)	Calcium (grams)	Iron (mg.)	Vitamin A (1,000 I.U.)	Thiamine (mg.)	Riboflavin (mg.)	Niacin (mg.)	Ascorbic Acid (mg.)
1. Wheat Flour	64.6	24.9	786	1.1	203		30.9	18.6	246	
3. Wheat Cereal	23.2	12.5	398	15.0	183		15.0	9.2	119	
5. Corn Meal	6.3	26.3	655	1.2	72	22.6	12.7	5.8	77	
8. Rolled Oats	9.9	18.1	651	3.7	245		26.6	6.4	46	
15. Evaporated Milk	10.0	5.6	283	10.1	6	17.4	2.0	15.7	7	40
40. Cabbage	4.9	2.0	94	5.0	27	5.4	6.8	3.4	20	4,034
51. Potatoes	80.1	6.1	145	.8	50	2.8	12.5	3.0	84	1,071
52. Spinach	11.6	.8	74	—	96	641.3	4.0	9.6	23	1,924
53. Sweet Potatoes	12.3	4.0	57	1.1	22	120.5	3.5	2.2	34	793
69. Navy Beans	10.8	14.7	924	6.2	433		21.0	13.4	119	
74. Sugar	67.0	26.9	—	—	—					
78. Pancake Flour[1]	12.2	16.0	479	18.1	46		3.7	1.9	41	
79. Beets[2]	7.3	2.2	85	1.1	70	132.3	2.9	6.3	29	895
80. Liver (Pork)[3]	21.9	2.7	408	.2	518	145.0	10.4	51.8	472	580

[1] Unit: 20 oz.; edible weight: 4,647 g. [2] Unit: 1 bunch; edible weight: 4,071 g. [3] Unit: 1 lb.; edible weight: 2,071 g.

UVA diet

	x_1 Brownie	x_2 Dumpling	x_3 Espresso	x_4 Roots
cost	5	2	3	8
cals	400	200	150	500
choc	3	2	0	0
sugar	2	2	4	4
fat	2	4	0	5

requirements: 500 calories, 6 oz choc, 10 oz sugar, 8 oz fat

MIN $5x_1 + 2x_2 + 3x_3 + 8x_4$

subject to

a_{11} a_{12} a_{13} a_{14}

$400x_1 + 200x_2 + 150x_3 + 500x_4 \geq 500$

$3x_1 + 2x_2 + 0x_3 + 0x_4 \geq 6$

$2x_1 + 2x_2 + 4x_3 + 4x_4 \geq 10$

$2x_1 + 4x_2 + 5x_4 \geq 8$

LP

shortest paths as LP

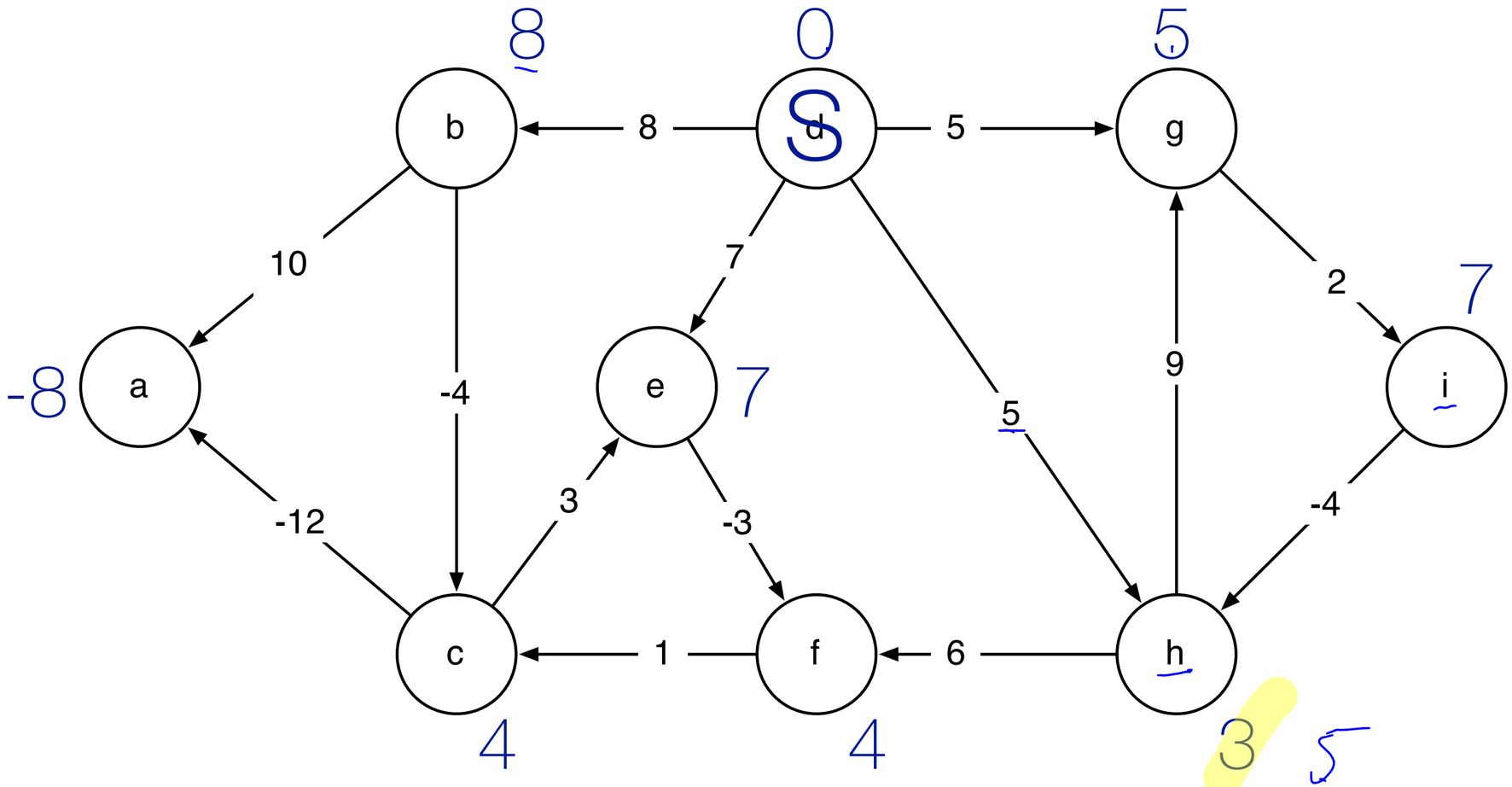
inputs: $G = (V, E)$, s (source), and $\ell(e) \to \mathbb{N}^+$

output: d_v for each $v \in V$ $\quad d_v$ = length of the shortest path from $s \rightsquigarrow v$

shortest paths as LP

$$d_t = \sum_{v \in V} d_v$$

$$\begin{cases} \max d_t \\ d_y - d_x \le l(x,y) \qquad \forall e = (x,y) \in E \\ d_s = 0 \end{cases}$$



$d_y \leq d_x + l(x,y)$

$$\max d_t$$

$$d_y - d_x \leq l(x,y) \qquad \forall e = (x,y) \in E$$

$$d_s = 0$$

dt = 30

max flow as lp

input: (G, c, s, t) $G = (V, E)$ $c : E \to \mathbb{Z}_+$

$$\max \sum_v f(s,v) - \sum_v f(v,s) \quad \text{s.t.}$$

$$f(e) \leq c(e) \quad \text{for every } e \in E$$

$$\sum_{v \in V} f(u,v) - \sum_{v \in V} f(v,u) = 0 \quad \text{for } u \in V - \{s,t\}$$

$$f(u,v) \geq 0$$

max flow as lp

$$\max \sum_v f(s,v) - \sum_v f(v,s)$$

$$f(u,v) \leq c(u,v) \qquad \text{for (u,v) in E}$$

$$\sum_u f(u,v) = \sum_w f(v,w) \quad \forall v$$

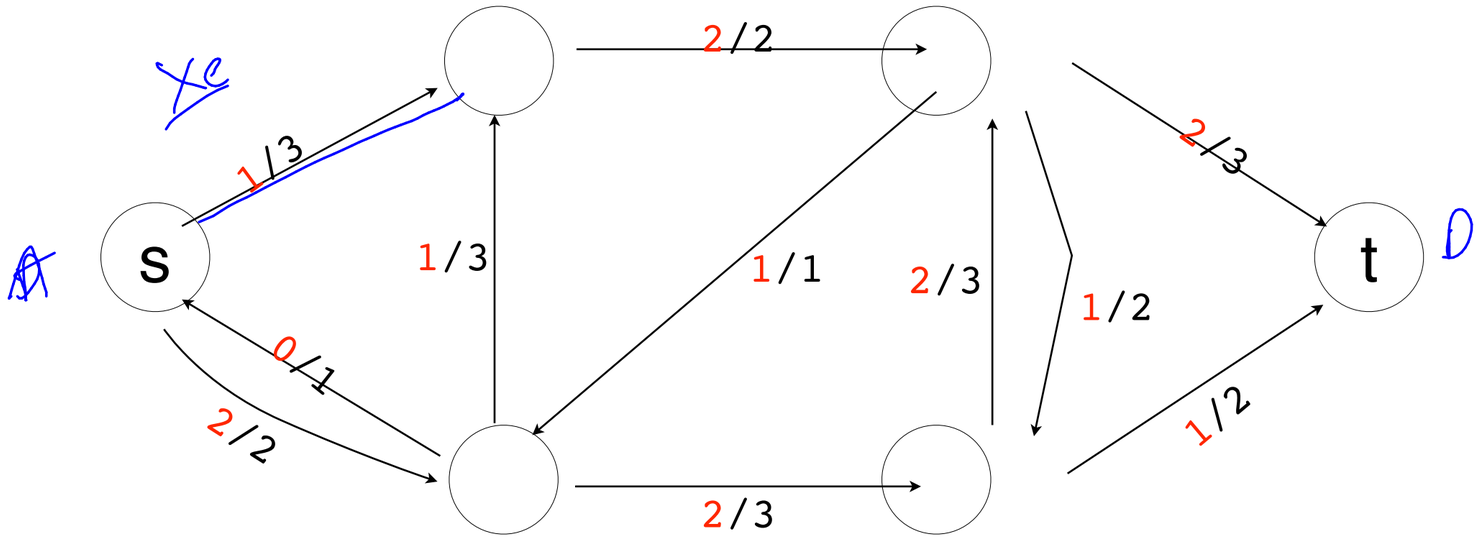
$$f(u,v) \geq 0 \qquad \text{for (u,v) in E}$$

min-cost flow as lp

input: (G, c, s, t) $G = (V, E)$ $c : E \to \mathbb{Z}_+$ $x : E \to \mathbb{Z}_+$ d

price

d → max-flow value.



min-cost flow as lp

$\min \quad f_e \cdot x_e$

price for each edge

flow on that edge

s.t.

$$\sum_{v \in V} f(s,v) - \sum_{v \in V} f(v,s) = d$$

(same constraints as previous flow)

min-cost flow as lp

$$\min_{e} x_e \cdot f(e)$$

$$f(e) \leq c(e)$$

$$f(e) \geq 0$$

$$\sum_{u} f(u,v) = \sum_{w} f(v,w)$$

$$\sum_{v} f(s,v) - \sum_{v} f(v,s) = d$$

LP

standard form

$\underline{x} = (x_1 \dots x_n)$

$c = (c_1 \dots c_n)$ given

$$\begin{cases} \max\limits_{x} \sum x_i c_i \\ \sum a_{ij} x_i \le b_i \\ x_i \ge 0 \end{cases}$$

max $x^t c$

getting to standard form

$$\min_{x} \sum x_i c_i \longrightarrow \max_{x} \sum -x_i \cdot c_i$$

$$\sum a_{ij} x_i \leq b_i$$

$$x_i \geq 0$$

getting to standard form

$$\max_{x} \sum x_i c_i$$

$\sum a_{ij} x_i \geq b_i$ ⟶ $\sum -a_{ij} x_i \leq -b_i$

$x_i \geq 0$

?

getting to standard form

$$\max_{x} \sum x_i c_i$$

$$\sum a_{ij} x_i = b_i$$

$$x_i \geq 0$$

$$\sum a_{ij} x_i \leq b_i$$

$$\sum -a_{ij} x_i \leq -b_i$$

getting to standard form

$$\max_{x} \sum x_i c_i$$

$$\sum a_{ij} x_i \leq b_i$$

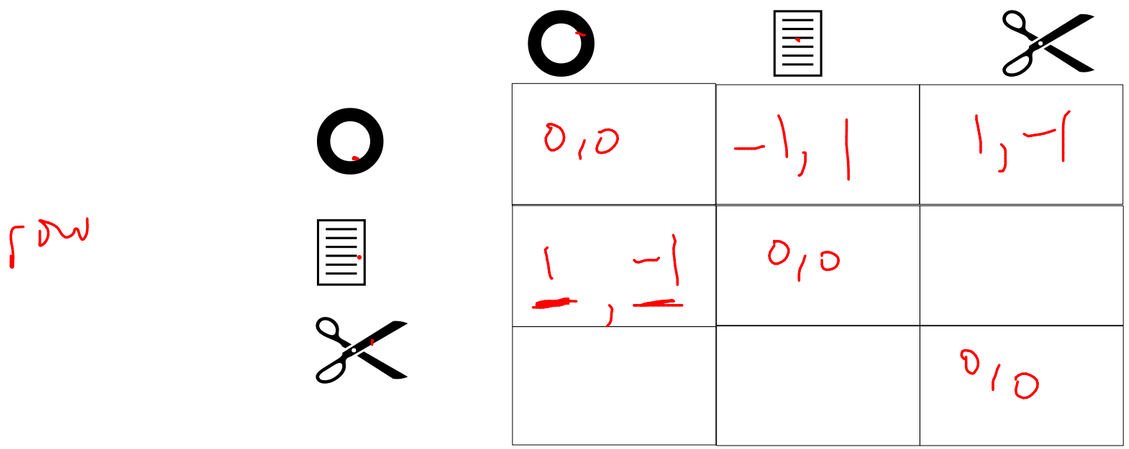
(non-negative)
ie, what if there
is
no constraint
on x?

$\underline{x} \rightarrow (x^{'} - x^{''})$

$x^{'} \geq 0$

$x^{''} \geq 0$

zero-sum games



zero-sum games

	COLIN	
ROWENA	3,-3	-1,1
	-2,2	1,-1

zero-sum games

		COLIN	
ROWENA		3,-3	-1,1
		-2,2	1,-1

zero-sum games

$3r_1c_1 + (-1)r_1 \cdot c_2 + (-2)r_2 \cdot c_1 + r_2c_2$

↑ value of the game.

COLIN

ROWENA

	c_1	c_2
r_1	3	-1
r_2	-2	1

zero-sum games

COLIN

ROWENA		
r_1	3	-1
r_2	-2	1

$$\sum_{i,j} G_{ij} r_i c_j$$

zero-sum games

rowena \ colin	L	R
r_1	3	-1
r_2	-2	1

rowena announces her strategy first:

$$\max_{z} \min \{ 3r_1 - 2r_2 ,\ -r_1 + r_2 \}$$

→ what Colin can get from the game.

$$\max z$$

$$z \leq 3r_1 - 2r_2$$

$$z \leq -r_1 + r_2$$

$$r_1 + r_2 = 1$$

zero-sum games

		colin
rowena	3	-1
	-2	1

rowena announces her strategy first:

(r_1, r_2) $\qquad \min\{\ 3r_1 - 2r_2,\ -r_1 + r_2\ \}$

zero-sum games

	colin	
rowena	3	-1
	-2	1

rowena announces her strategy first:

$$\max z$$

$$z \leq 3r_1 - 2r_2$$

$$z \leq -r_1 + 2r_2$$

$$r_1 + r_2 = 1$$

$$r_1, r_2 \geq 0$$

zero-sum games

rowena \ colin	c_1	c_2
t	3	-1
b	-2	1

colin announces his strategy first:

(c_1, c_2)

$$\max\{3c_1 - c_2,\ -2c_1 + c_2\}$$

zero-sum games

		colin
rowena	3	-1
	-2	1

colin announces his strategy first:

pick (c_1, c_2) so as to min $\max\{3c_1 - c_2,\ -2c_1 + c_2\}$

min w

$w \geq 3c_1 - c_2$

$w \geq -2c_1 + c_2$

zero-sum games

		colin
rowena	3	-1
	-2	1

colin announces his strategy first:

$$\min w$$

$$\begin{aligned} -3c_1 + c_2 + w &\geq 0 \\ 2c_1 - c_2 + w &\geq 0 \\ c_1 + c_2 &= 1 \\ c_1, c_2 &\geq 0 \end{aligned}$$

zero-sum games

rowena \ colin	
3	-1
-2	1

$$\max z$$

$$\begin{aligned} -3r_1 + 2r_2 + z &\leq 0 \\ r_1 - r_2 + z &\leq 0 \\ r_1 + r_2 &= 1 \\ r_1, r_2 &\geq 0 \end{aligned}$$

zero-sum games

colin \ rowena	
3	-1
-2	1

$$\max z$$

$$\begin{aligned} -3r_1 + 2r_2 + z &\leq 0 \\ r_1 - r_2 + z &\leq 0 \\ r_1 + r_2 &= 1 \\ r_1, r_2 &\geq 0 \end{aligned}$$

$$\min w$$

$$\begin{aligned} -3c_1 + c_2 + w &\geq 0 \\ 2c_1 - c_2 + w &\geq 0 \\ c_1 + c_2 &= 1 \\ c_1, c_2 &\geq 0 \end{aligned}$$

zero-sum games

		2/7	5/7
3/7	rowena	3	-1
4/7		-2	1

colin

value of the game is : 1/7

$$\max_x \min_y \sum_{i,j} G_{ij} x_i y_j = \min_y \max_x \sum_{i,j} G_{ij} x_i y_j$$

LIAR

=

Poker

https://www.inf.ethz.ch/personal/fukudak/cdd_home/

Welcome to the cdd and cddplus Homepage

Last update: May 15, 2015

Currently, the C-library version cddlib of cdd packages is the only one being updated, while standalone codes cdd and cddplus are still useful. To know what cdd, cddplus and cddlib are, please read

- cddplus readme
- cddlib readme

Manuals (html version):

- cdd/cdd+ manual
- cddlib manual

Get source codes:

- cdd/cddpuls directory click here
- cdd package cdd-061a.tar.gz
- cddplus package cdd+-077a.tar.gz (to be compled with g++ 4.1. With more recent g++, try patch) NEW . With g++ 3.1, use cdd+-077.tar.gz
- cddlib package cddlib-094h.tar.gz NEW

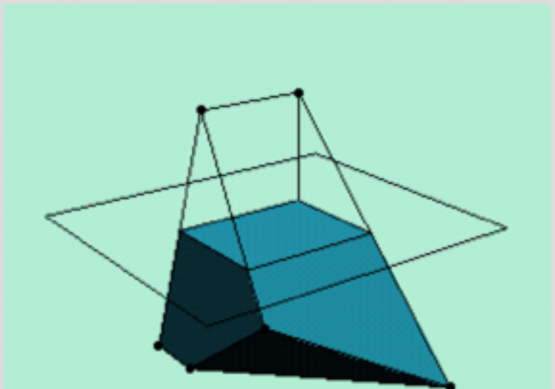
To know the implementation:

- ``The double description revisited'' gzipped ps file

To learn the fundamental concepts of Convex Hull, Vornonoi, Delaunay, etc.:

- ``Polyhedral Computation FAQ'' (still experimental) html version or pdf file

Links to cdd/cdd+/cddlib users and more. NEW



```
H-representation
begin
24 13 rational
0 0 0 0 0 0 1/2 5/12 1/3 1/4 1/6 1/12 -1
0 0 0 0 0 7/22 5/22 5/33 1/11 1/22 1/66 0 -1
0 0 0 0 0 7/44 1/11 1/22 1/55 1/220 0 0 -1
0 0 0 0 14/99 7/99 1/33 1/99 1/495 0 0 0 -1
0 0 0 0 7/99 7/264 1/132 1/792 0 0 0 0 -1
0 0 0 1/11 1/33 1/132 1/924 0 0 0 0 0 -1
0 0 0 1/22 1/99 1/792 0 0 0 0 0 0 -1
0 0 1/11 1/55 1/495 0 0 0 0 0 0 0 -1
0 0 1/22 1/220 0 0 0 0 0 0 0 0 -1
0 1/6 1/66 0 0 0 0 0 0 0 0 0 -1
0 1/12 0 0 0 0 0 0 0 0 0 0 -1
1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 0
-1 1 1 1 1 1 1 1 1 1 1 1 0
0 1 0 0 0 0 0 0 0 0 0 0 0
0 0 1 0 0 0 0 0 0 0 0 0 0
0 0 0 1 0 0 0 0 0 0 0 0 0
0 0 0 0 1 0 0 0 0 0 0 0 0
0 0 0 0 0 1 0 0 0 0 0 0 0
0 0 0 0 0 0 1 0 0 0 0 0 0
0 0 0 0 0 0 0 1 0 0 0 0 0
0 0 0 0 0 0 0 0 1 0 0 0 0
0 0 0 0 0 0 0 0 0 1 0 0 0
0 0 0 0 0 0 0 0 0 0 1 0 0
0 0 0 0 0 0 0 0 0 0 0 1 0
end
maximize
0 0 0 0 0 0 0 0 0 0 0 0 1
```

```
* Compiled for Rational Exact Arithmetic with GMP
*cdd LP Result
*cdd input file : 12.ine  (24 x 13)
*LP solver: Dual Simplex
*LP status: a dual pair (x, y) of optimal solutions found.
*maximization is chosen.
*Objective function is
 0 + 0 X[1] + 0 X[2] + 0 X[3] + 0 X[4] +
 0 X[5] + 0 X[6] + 0 X[7] + 0 X[8] + 0 X[9] +
 0 X[10] + 0 X[11] + 1 X[12]
*LP status: a dual pair (x, y) of optimal solutions found.
begin
  primal_solution
  1 :  280/1643
  2 :  4217/14787
  3 :  130/477
  4 :  280/1643
  5 :  120/1643
  6 :  140/4929
  7 :  0
  8 :  0
  9 :  0
  10 :  0
  11 :  0
  12 :  70/4929
  dual_solution
  24 :  383/29574
  21 :  599/73935
  20 :  74/14787
  22 :  1003/98580
  23 :  173/14787
  1 :  74/4929
  3 :  99/1643
  5 :  264/1643
  7 :  462/1643
  9 :  1540/4929
  11 :  280/1643
  12 :  70/4929
  optimal_value : 70/4929
end
*number of pivot operations = 8
*Computation starts     at Tue Apr 19 12:54:03 2016
*            terminates at Tue Apr 19 12:54:03 2016
*Total processor time = 0 seconds
*                     = 0h 0m 0s
closing the file 12.lps
closing the file 12.ddl
```

how to “evaluate” an lp

$$\max c^T \vec{x}$$

$$A\vec{x} \leq \vec{b}$$

$$\vec{x} \geq 0$$

definitions

feasible point:

vertex:

neighbor of vertex v:

simplex

init:

while

do:

$$\max 2x_1 + 5x_2$$

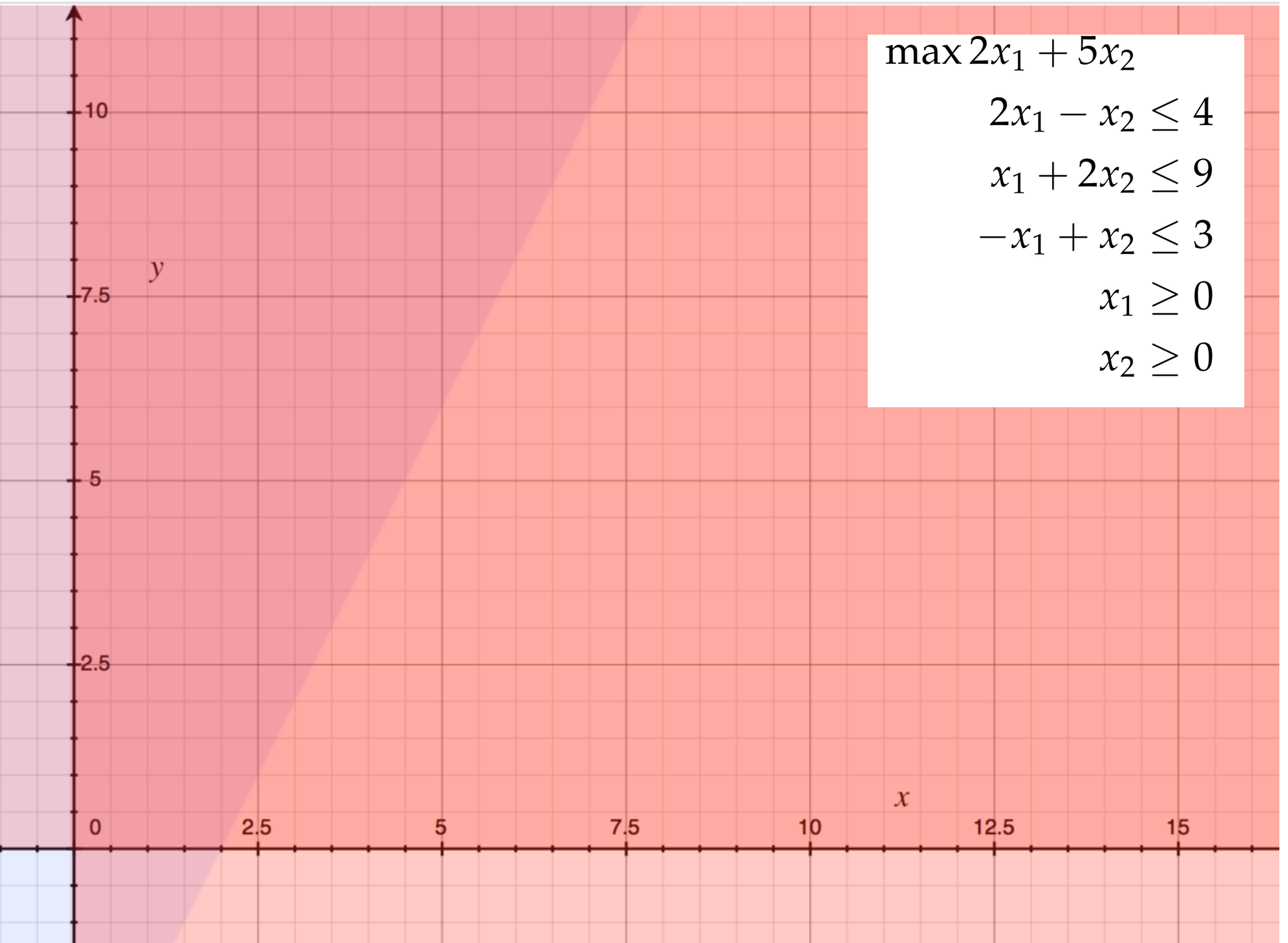
$$2x_1 - x_2 \leq 4$$

$$x_1 + 2x_2 \leq 9$$

$$-x_1 + x_2 \leq 3$$

$$x_1 \geq 0$$

$$x_2 \geq 0$$



$$\max 2x_1 + 5x_2$$

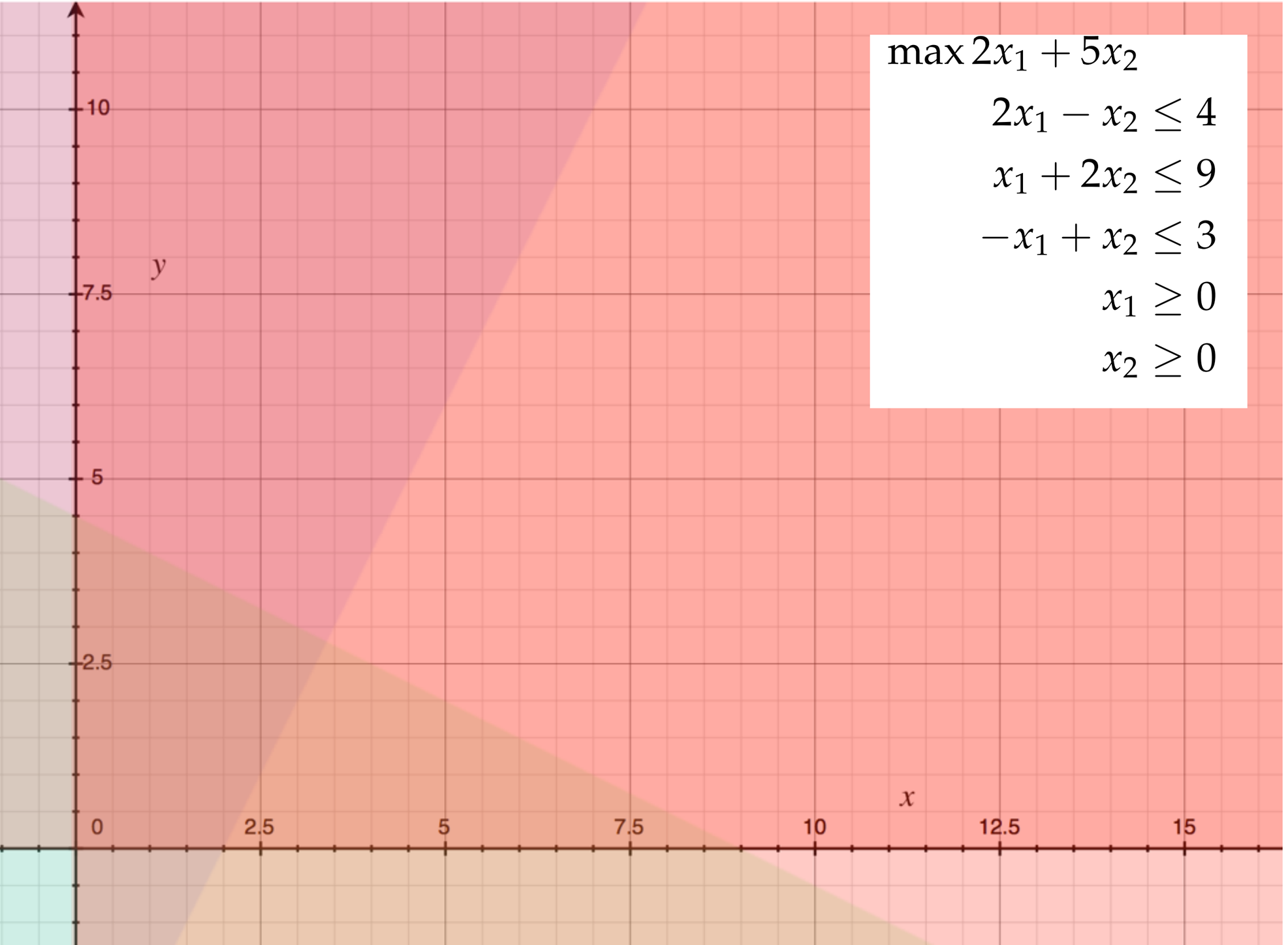
$$2x_1 - x_2 \leq 4$$

$$x_1 + 2x_2 \leq 9$$

$$-x_1 + x_2 \leq 3$$

$$x_1 \geq 0$$

$$x_2 \geq 0$$



$$\max 2x_1 + 5x_2$$

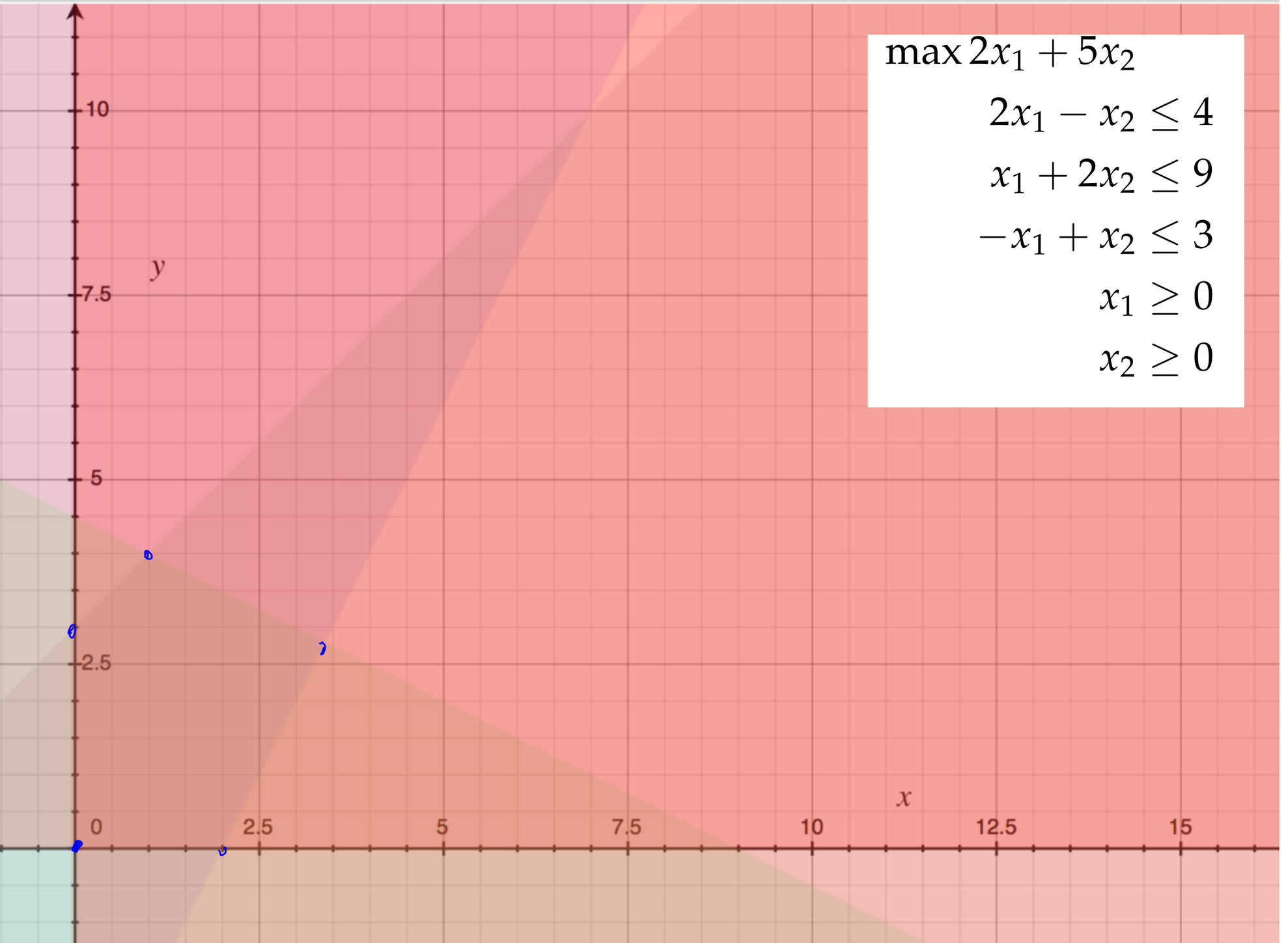
$$2x_1 - x_2 \leq 4$$

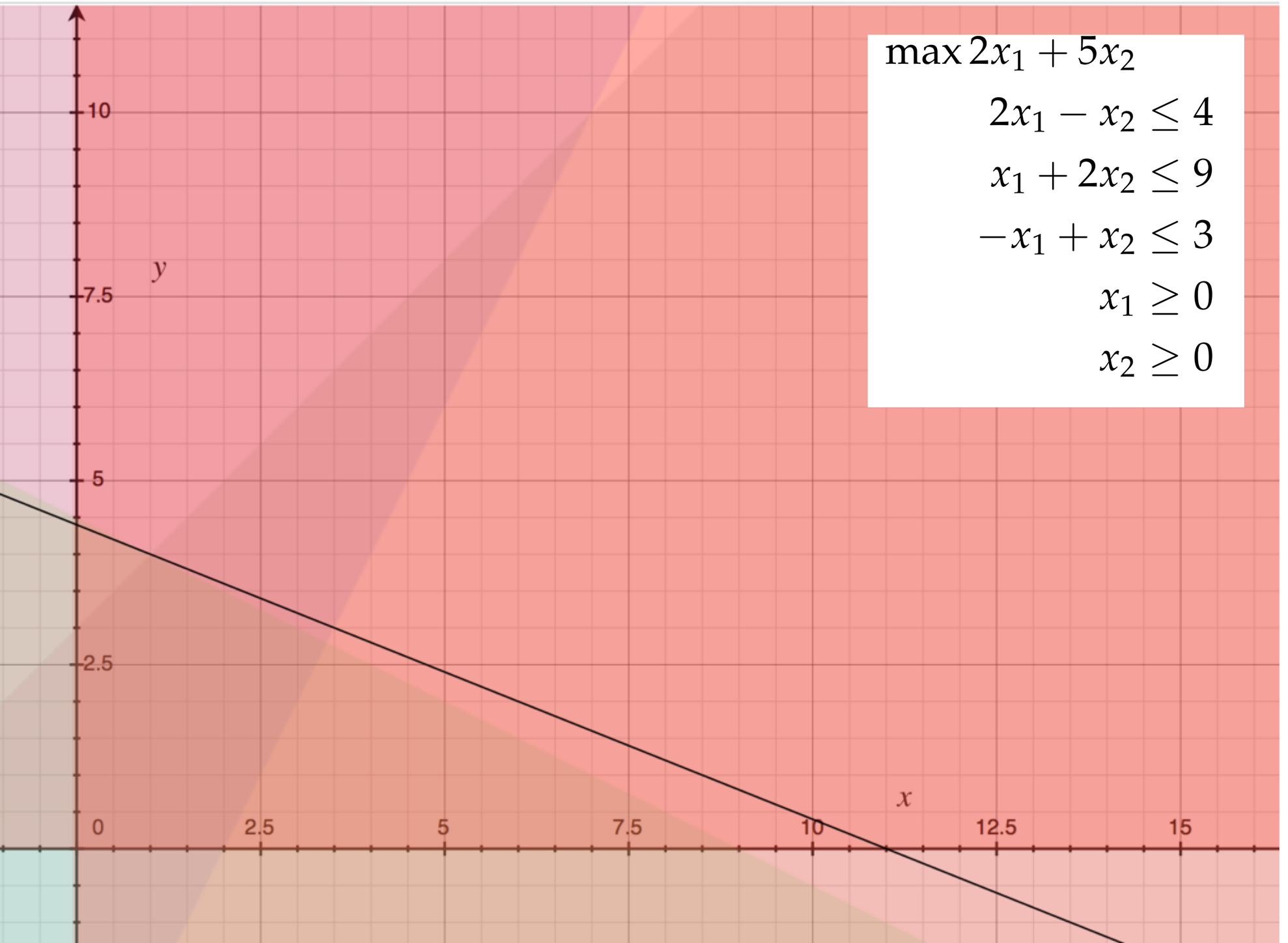
$$x_1 + 2x_2 \leq 9$$

$$-x_1 + x_2 \leq 3$$

$$x_1 \geq 0$$

$$x_2 \geq 0$$


$\max 2x_1 + 5x_2$
$2x_1 - x_2 \leq 4$
$x_1 + 2x_2 \leq 9$
$-x_1 + x_2 \leq 3$
$x_1 \geq 0$
$x_2 \geq 0$
y
x
10
7.5
5
2.5
0
2.5
5
7.5
10
12.5
15



$$\max 2x_1 + 5x_2$$

$$2x_1 - x_2 \leq 4$$

$$x_1 + 2x_2 \leq 9$$

$$-x_1 + x_2 \leq 3$$

$$x_1 \geq 0$$

$$x_2 \geq 0$$