

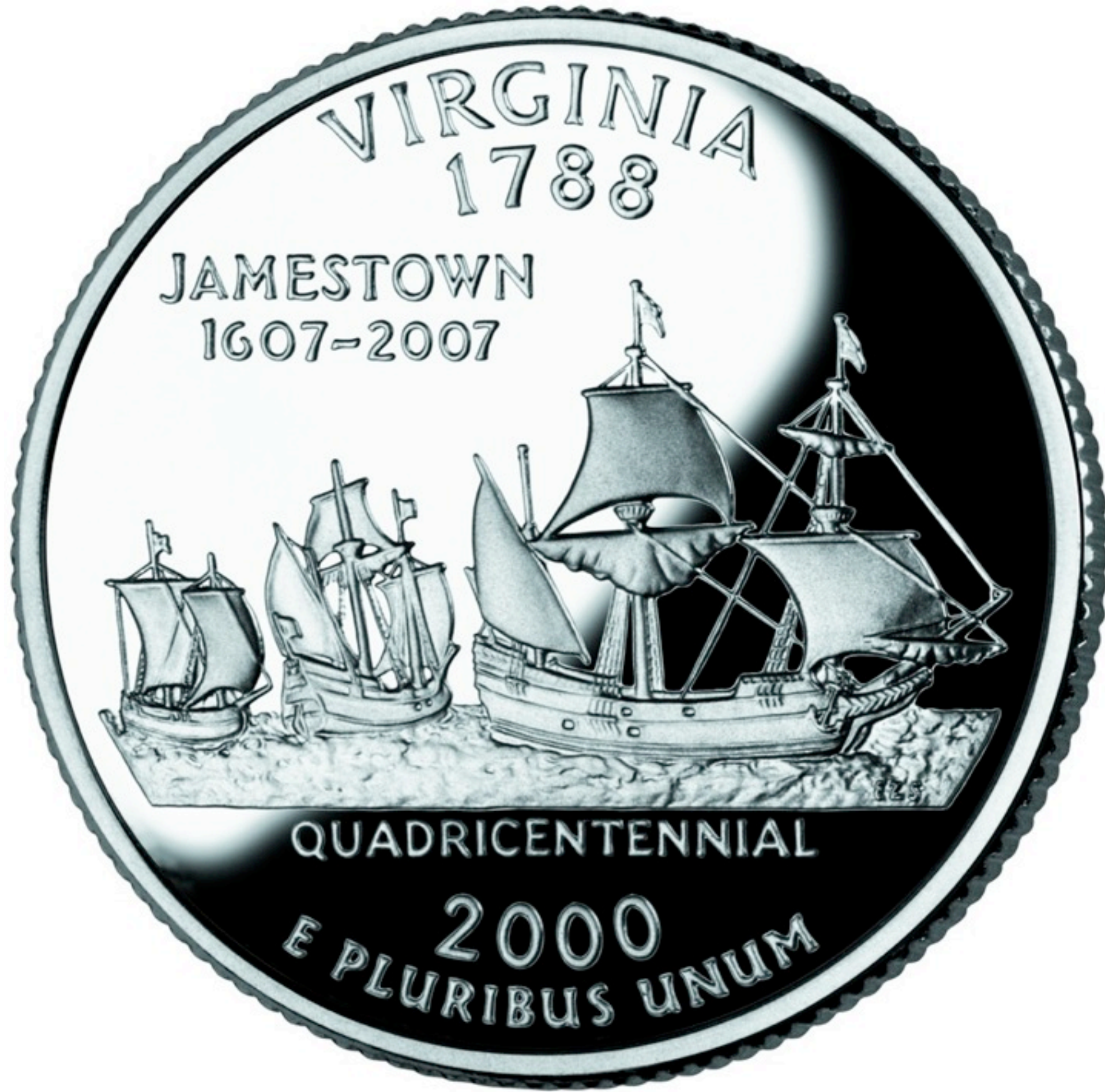
L28

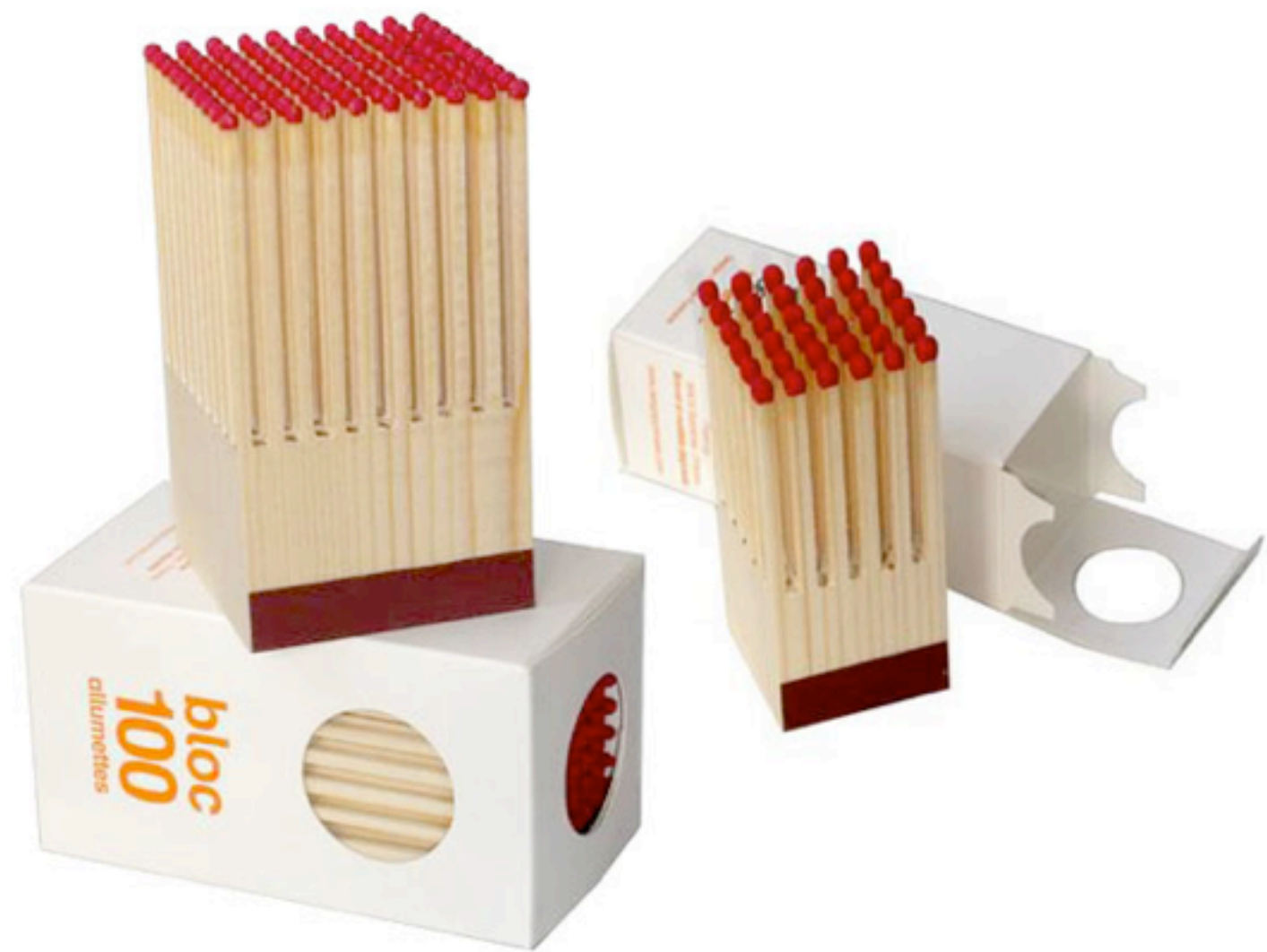
4102

12.05.2013

abhi

review,
crypto

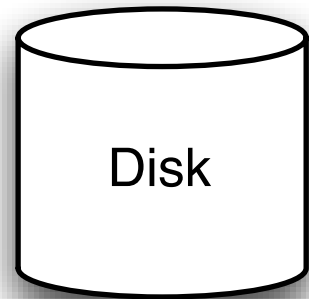




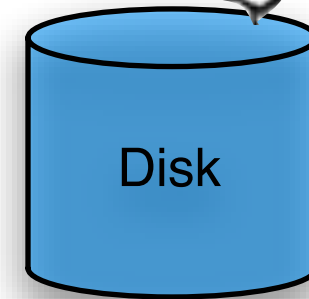
<http://kitsunenoir.com/blogimages/bloc-matches.jpg>

FINGERPRINTING

Alice



Bob



STRING MATCHING

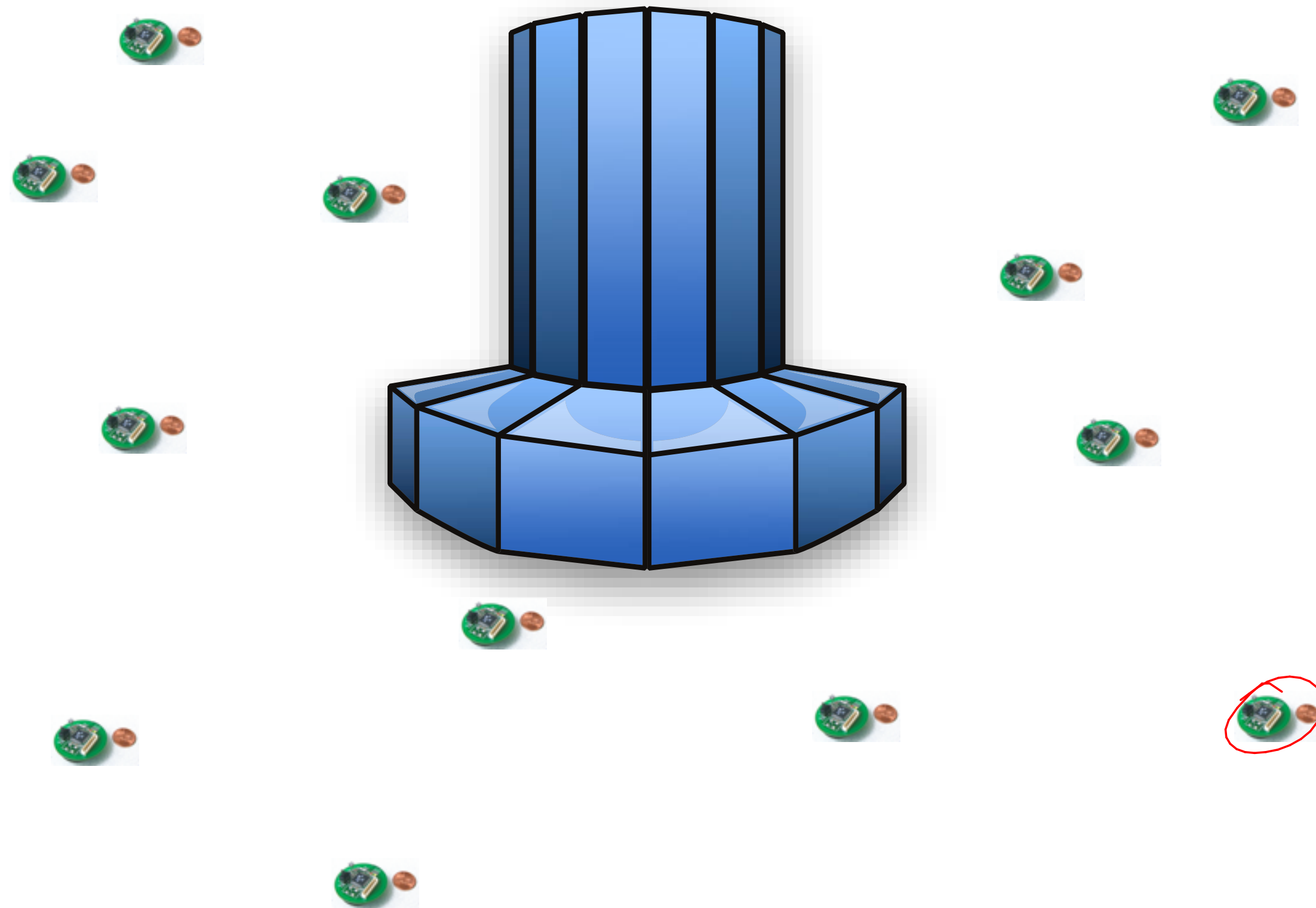
PATTERN



CORPUS



RELIABLE COMMUNICATION



GOAL:

DEVISE A RELIABLE METHOD FOR NODES TO SEND MESSAGE
TO THE SERVER WITH AS LITTLE COORDINATION AS POSSIBLE.

SIMPLE ALGORITHM

AT TIME T , FLIP A COIN THAT IS HEADS WITH PR $\frac{1}{n}$

IF HEADS, THEN BROADCAST. IF SUCCESS, THEN STOP.

ELSE WAIT AND TRY AGAIN.

REPEAT $cn \log n$ TIMES

ANALYZE THE SIMPLE ALGORITHM

$S_{i,t}$ = event that node i succeeds in sending its message at time t

$$\Pr[S_{i,t} = 1] = \left(\frac{t}{n}\right) \left[1 - \frac{t}{n}\right]^{n-1}$$

↑
heads for
 i

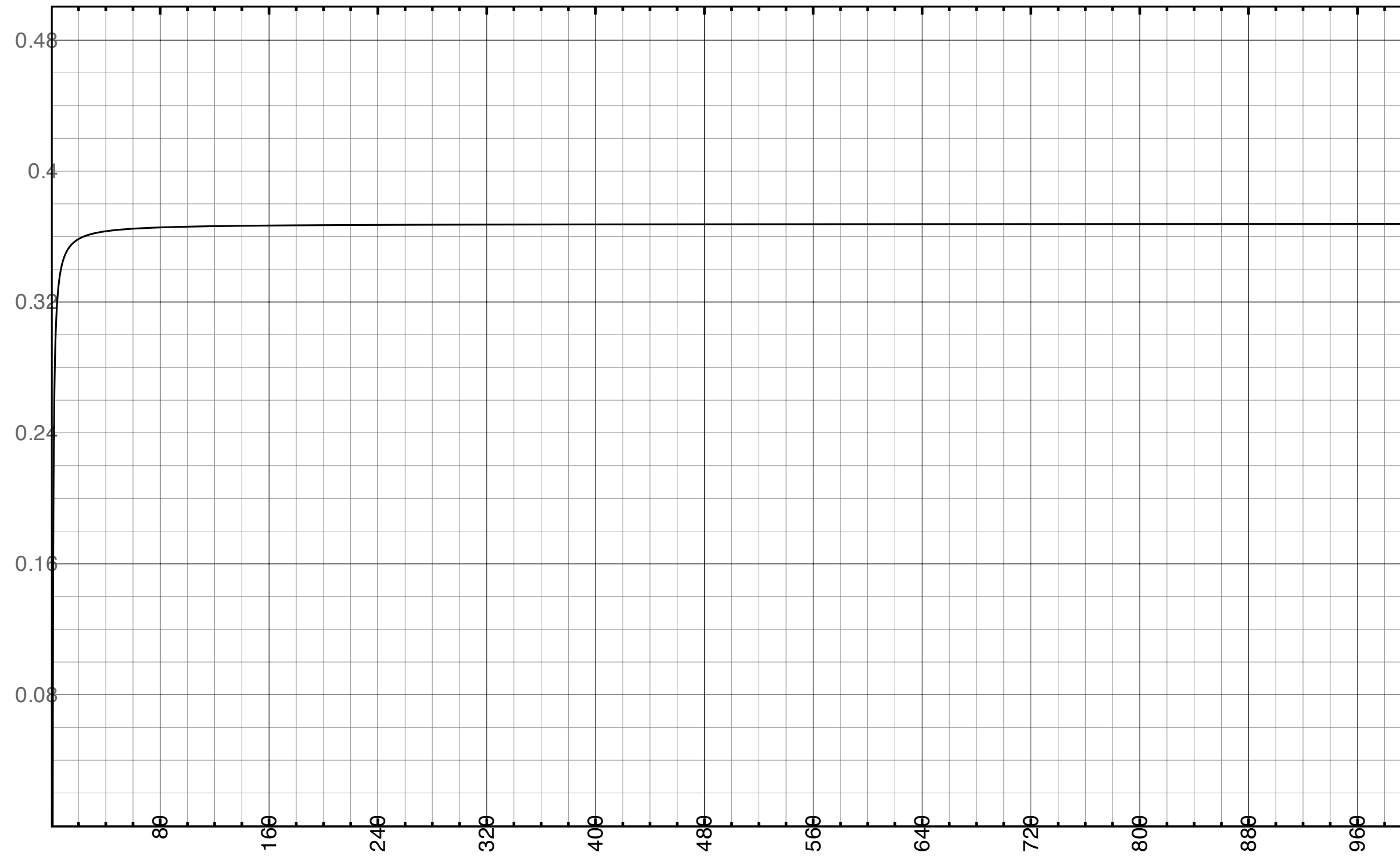
$$\Pr[S_{i,t} = 1] = \frac{1}{n} \underbrace{\left(1 - \frac{1}{n}\right)^{n-1}}_{e^{-1}} \sim \frac{1}{n \cdot e}$$

FACT: IF

$$f(n) = \left(1 - \frac{1}{n}\right)^n \quad \text{THEN}$$

FACT: IF

$$f(n) = \left(1 - \frac{1}{n}\right)^n \quad \text{THEN} \quad \sim \frac{1}{e}$$



APMA



$S_{i,t}$ = NODE i SUCCEEDS IN SENDING AT TIME t

$$\underline{\frac{1}{en}} \leq \underline{\Pr[S_{i,t} = 1]} \leq \underline{\underline{\frac{1}{2n}}}$$

FAILURE

$F_{i,t}$ = probability that i fails @ times $1, 2, 3, \dots, t$

FAILURE

$F_{i,t}$ = NODE i FAILS TO SEND AT TIMES $1, 2, \dots, t$

$$\Pr[\underline{F_{i,t}}] = \bigwedge_{j=1}^t \underline{\Pr[S_{i,j}]}$$

FAILURE

$F_{i,t}$ = NODE i FAILS TO SEND AT TIMES $1,2,\dots,t$

$$\Pr[F_{i,t}] = \bigwedge_{j=1}^t \Pr[\overline{S_{i,j}}] = \prod_{j=1}^t \Pr[\overline{S_{i,j}}]$$

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FAILURE

$F_{i,t}$ = NODE i FAILS TO SEND AT TIMES $1, 2, \dots, t$

$$\Pr[\underline{F_{i,t}}] = \bigwedge_{j=1}^t \Pr[\overline{S_{i,j}}] = \prod_{j=1}^t \Pr[\overline{S_{i,j}}] < \left(1 - \frac{1}{2n}\right)^t$$

FOR

$$t = \underline{O(n \ln n)}$$

$$\Pr[\underline{F_{i,t}}] = \underline{\underline{n^{-c}}}$$

$$\left(1 - \frac{1}{2n}\right)$$

ALL FAIL

$$F_t =$$

$$\Pr[F_t] =$$

ALL FAIL

$F_t =$ SOME NODE i FAILS TO SEND AT TIMES $1, 2, \dots, t$

$$\Pr[F_t] = \bigvee_{\underline{i=1}}^n \Pr[\underline{F_{i,t}}]$$

ALL FAIL

$F_t =$ SOME NODE i FAILS TO SEND AT TIMES $1, 2, \dots, t$

$$\Pr[F_t] = \bigvee_{i=1}^n \Pr[F_{i,t}] \leq \sum_{i=1}^n \Pr[F_{i,t}] \leq \sum_{i=1}^n n^{-c}$$

SUMMARY

AT TIME T, FLIP A COIN THAT IS HEADS WITH PR

$\frac{1}{n}$

IF HEADS, THEN BROADCAST. IF SUCCESS, THEN STOP.

ELSE WAIT AND TRY AGAIN.

REPEAT $O(n \ln n)$ TIMES

WITH PROBABILITY

EVERY NODE SUCCEEDS IN SENDING MESSAGE.

TOOLS WE USED

ANALYSIS OF

$$\left(1 - \frac{1}{n}\right)^n$$

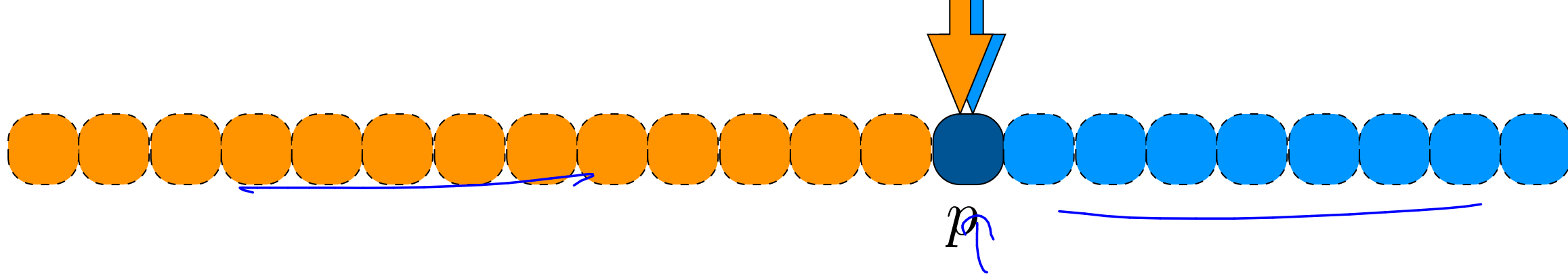
PROBABILITY THAT **MANY** INDEPENDENT EVENTS **ALL** OCCUR:

PROBABILITY THAT **ONE OUT OF N** EVENTS OCCURS:

SECOND EXAMPLE:

MEDIAN





SELECT $(i, A[1, \dots, n])$

PICK FIRST ELEMENT

PARTITION LIST ABOUT THIS ONE

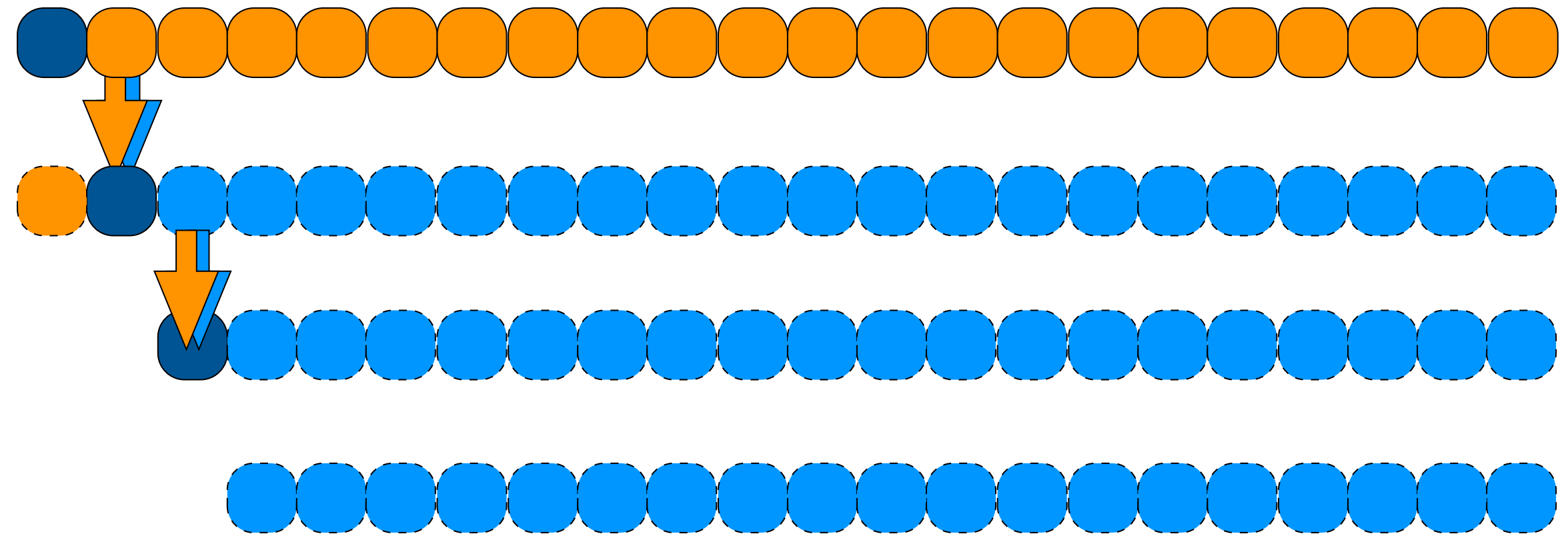
IF PIVOT IS POSITION i , RETURN PIVOT

ELSE IF PIVOT IS IN POSITION $> i$ **SELECT** $(i, A[1, \dots, p - 1])$

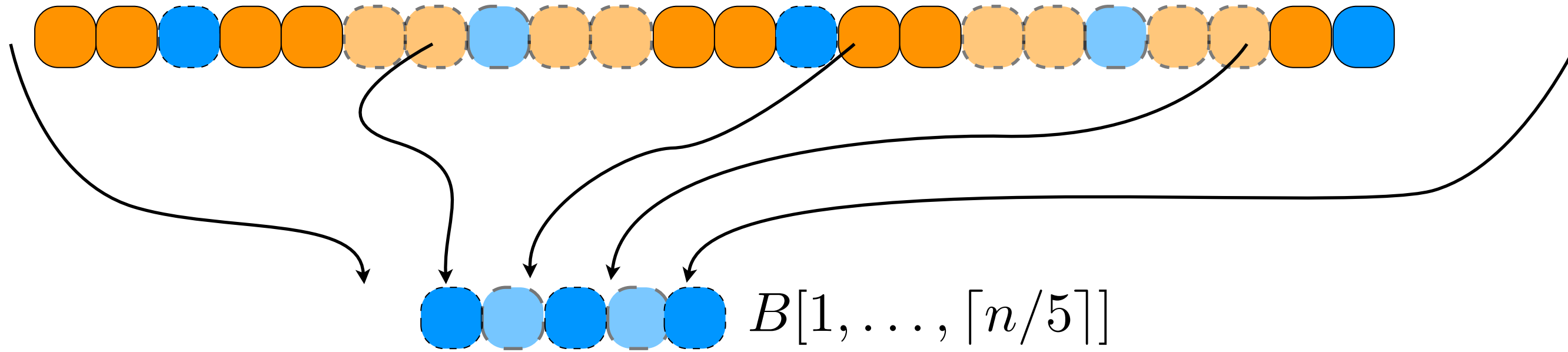
ELSE **SELECT** $((i - p - 1), A[p + 1, \dots, n])$



PROBLEM: WHAT IF WE ALWAYS PICK BAD PARTITIONS?

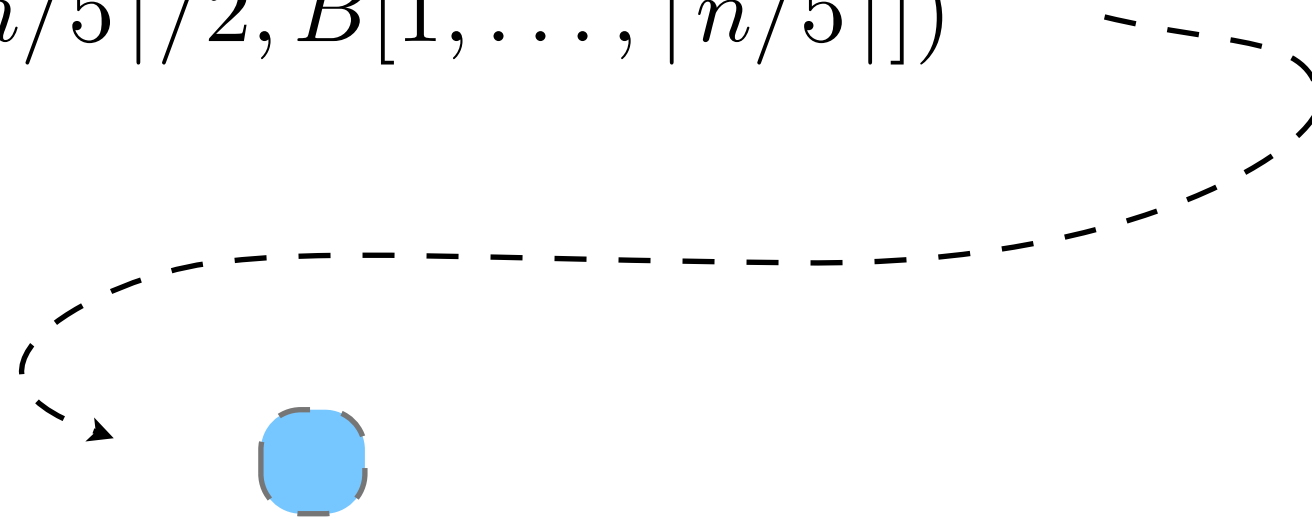


PARTITION ($A[1, \dots, n]$)



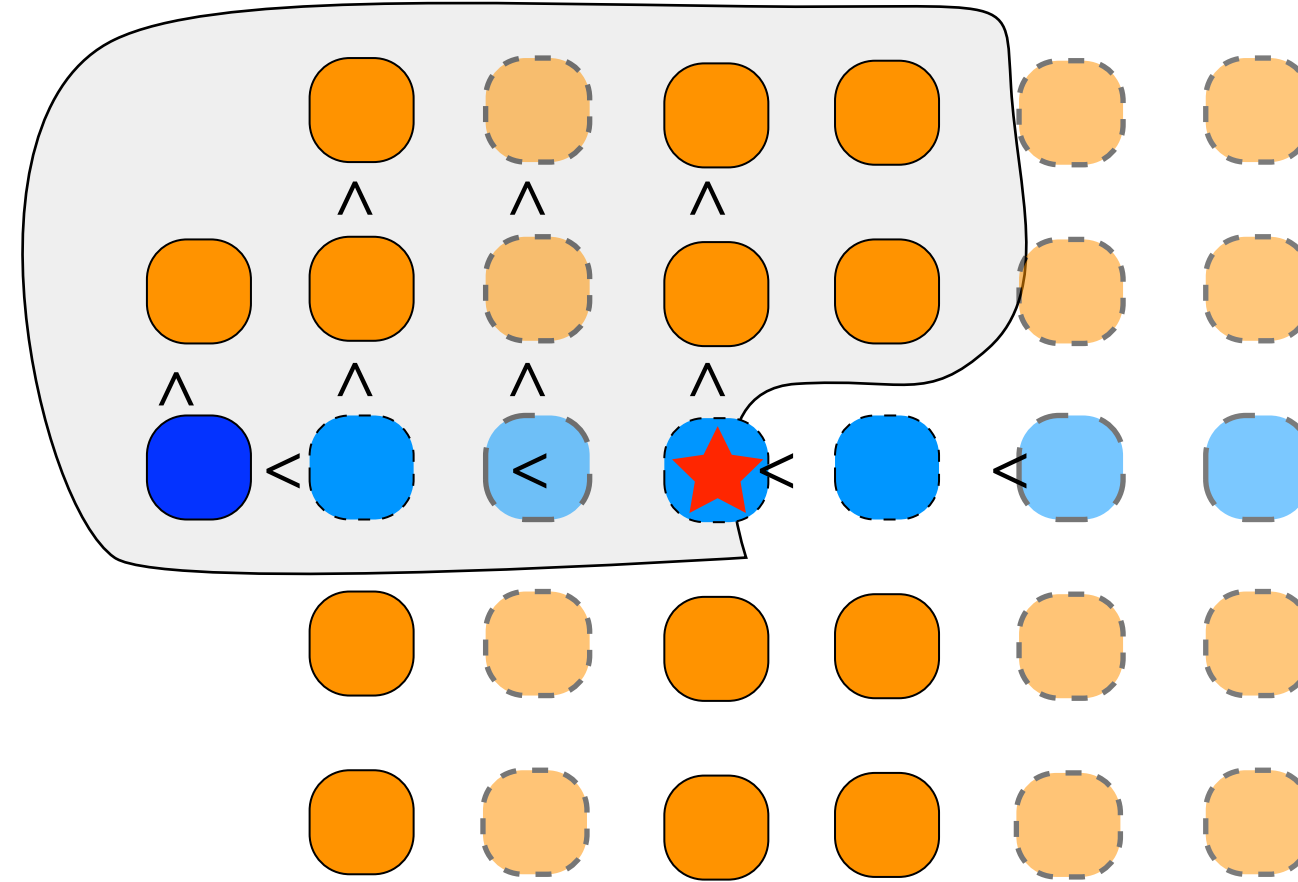
SELECT

($\lceil n/5 \rceil / 2, B[1, \dots, \lceil n/5 \rceil]$)



A NICE PROPERTY OF OUR PARTITION

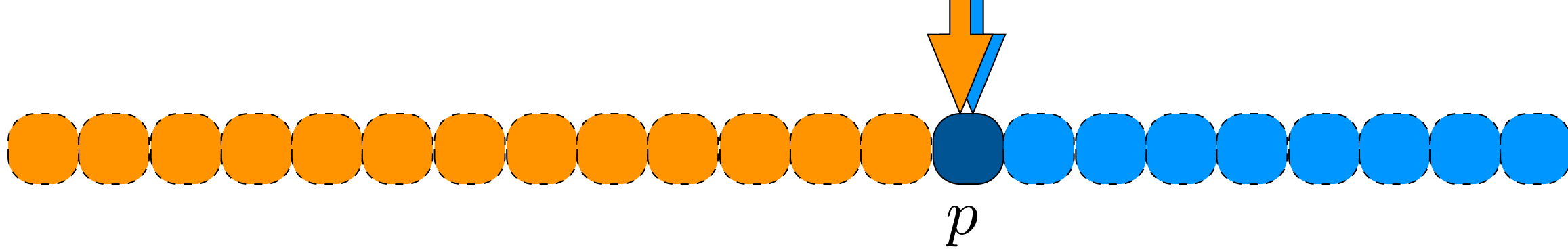
$$3 \left(\left\lceil \frac{1}{2} \lceil n/5 \rceil \right\rceil - 2 \right) \geq \frac{3n}{10} - 6$$



THIS IMPLIES THERE ARE

AT MOST $\frac{7n}{10} + 6$ NUMBERS

LARGER THAN ★
/SMALLER



SELECT $(i, A[1, \dots, n])$

~~PICK FIRST ELEMENT~~

PIVOT = PARTITION $(A[1, \dots, n])$

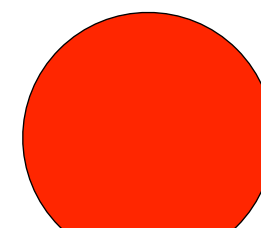
IF PIVOT IS POSITION i , RETURN PIVOT

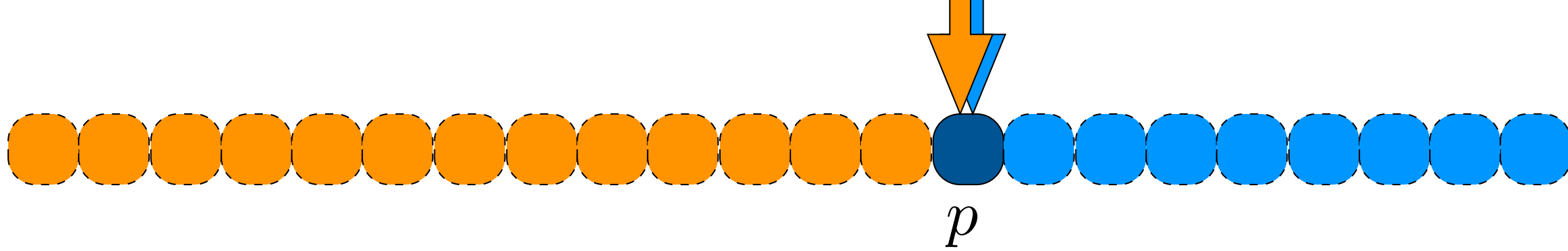
ELSE IF PIVOT IS IN POSITION $> i$ **SELECT** $(i, A[1, \dots, p - 1])$

ELSE **SELECT** $((i - p - 1), A[p + 1, \dots, n])$

$$S(n) = S(\lceil n/5 \rceil) + O(n) + S(7n/10 + 6)$$

$$\Theta(n)$$





RANDOMIZEDSELECT

$(i, A[1, \dots, n])$

PICK RANDOM PARTITION ELEMENT

PARTITION LIST ABOUT THIS ONE

IF PIVOT IS POSITION i , RETURN PIVOT

ELSE IF PIVOT IS IN POSITION $> i$ **SELECT** $(i, A[1, \dots, p - 1])$

ELSE **SELECT** $((i - p - 1), A[p + 1, \dots, n])$

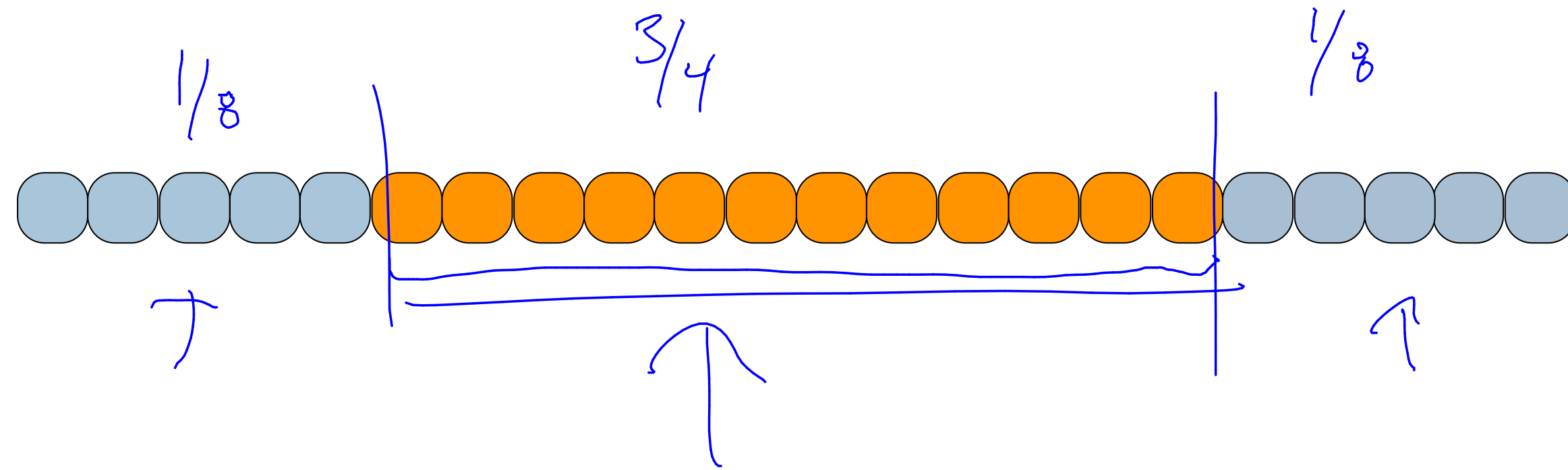
RANDOMIZEDSELECT

$(i, A[1, \dots, n])$

PICK RANDOM PARTITION ELEMENT

PARTITION LIST ABOUT THIS ONE

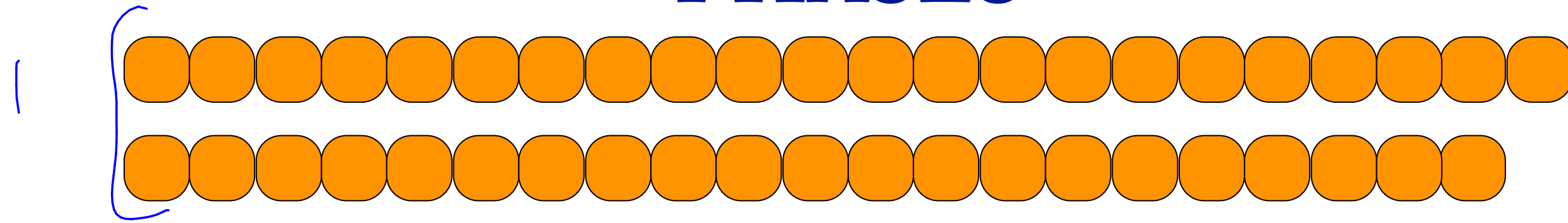
....



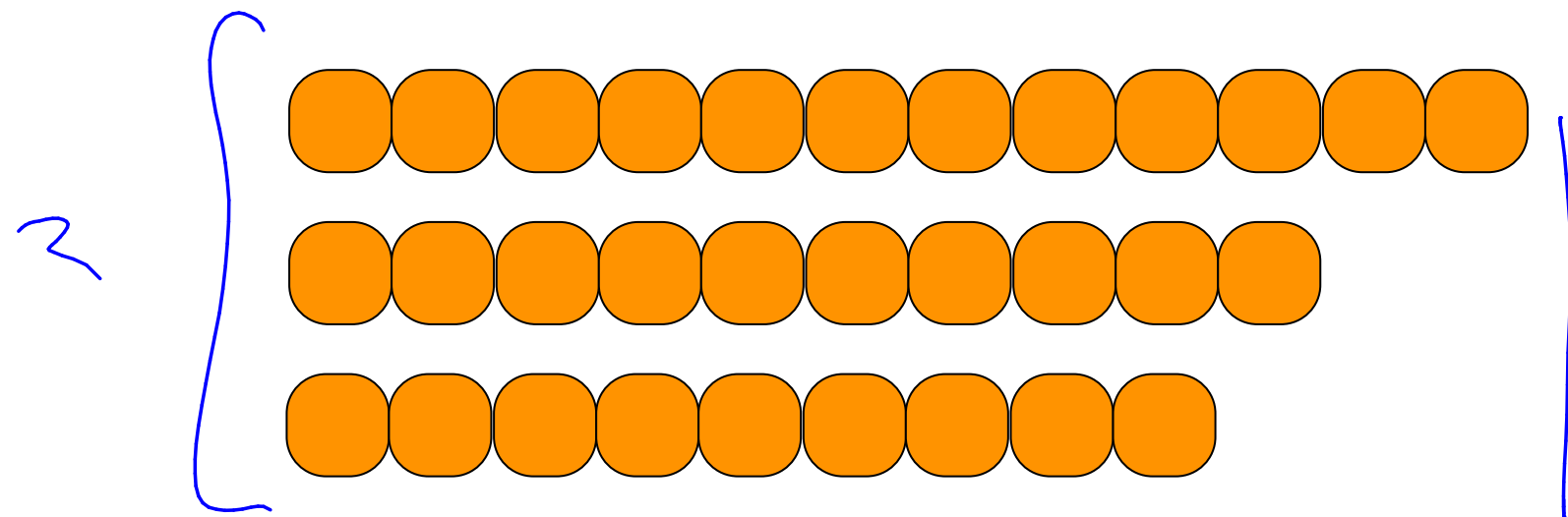
RUNNING TIME ANALYSIS

RECURSIVE CALLS

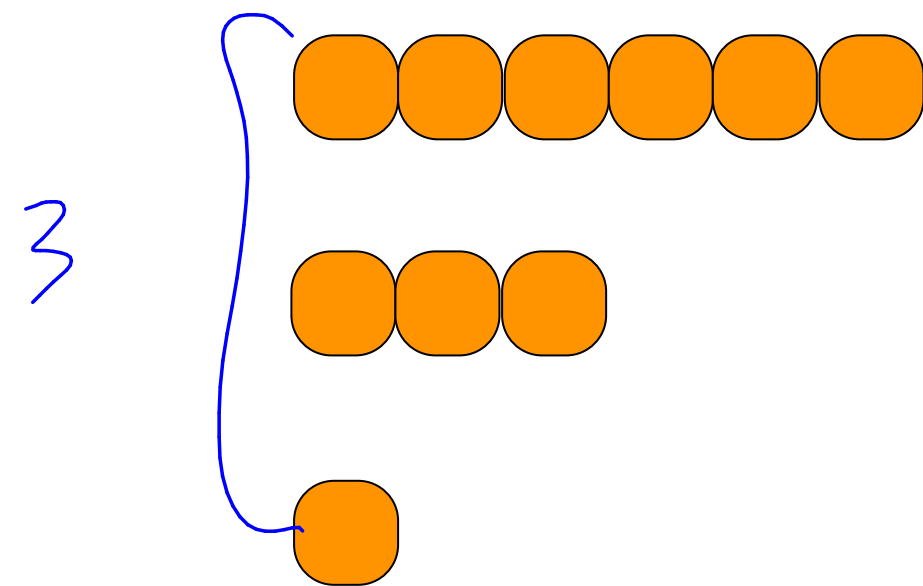
PHASES



$$n \cdot \left(\frac{3}{4}\right)^0$$

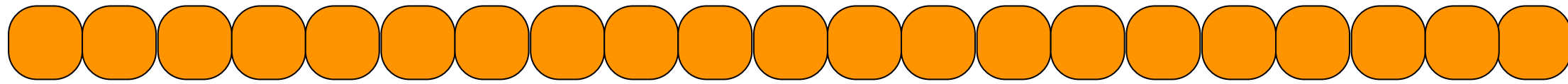


$$\frac{1}{n} \left(\frac{3}{4}\right)^j$$



PHASES

ALGORITHM IS IN **PHASE J** IF



SIZE OF INPUT LIST IS <

$$\underline{\left(\frac{3}{4}\right)^j n}$$

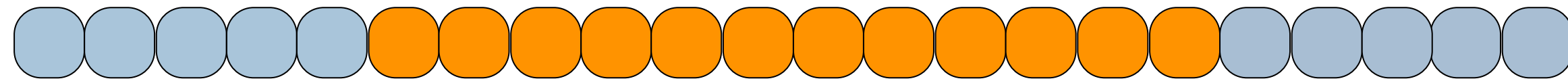
RANDOMIZEDSELECT

$(i, A[1, \dots, n])$

PICK RANDOM PARTITION ELEMENT

PARTITION LIST ABOUT THIS ONE

...



$X_j =$ **NUMBER OF** STEPS IN PHASE J

$E[X_j] =$

$X_j =$ NUMBER OF STEPS IN PHASE J

$$E[X_j] = \sum_{j=0}^{\infty} j \cdot \Pr[X_j = j]$$

$$\Pr[X_j = 1] =$$

$$\Pr[X_j = 2] =$$

$$\Pr[X_j = j] =$$

A handwritten diagram in blue ink. It shows a binomial distribution with parameters $n=4$ and $p=3/4$. The expression $\binom{4}{j} \left(\frac{3}{4}\right)^j$ is written inside a hand-drawn box. Below the box, there are two upward-pointing arrows, one under the $\binom{4}{j}$ term and one under the $\left(\frac{3}{4}\right)^j$ term. The bottom of the box is crossed out with several horizontal lines.

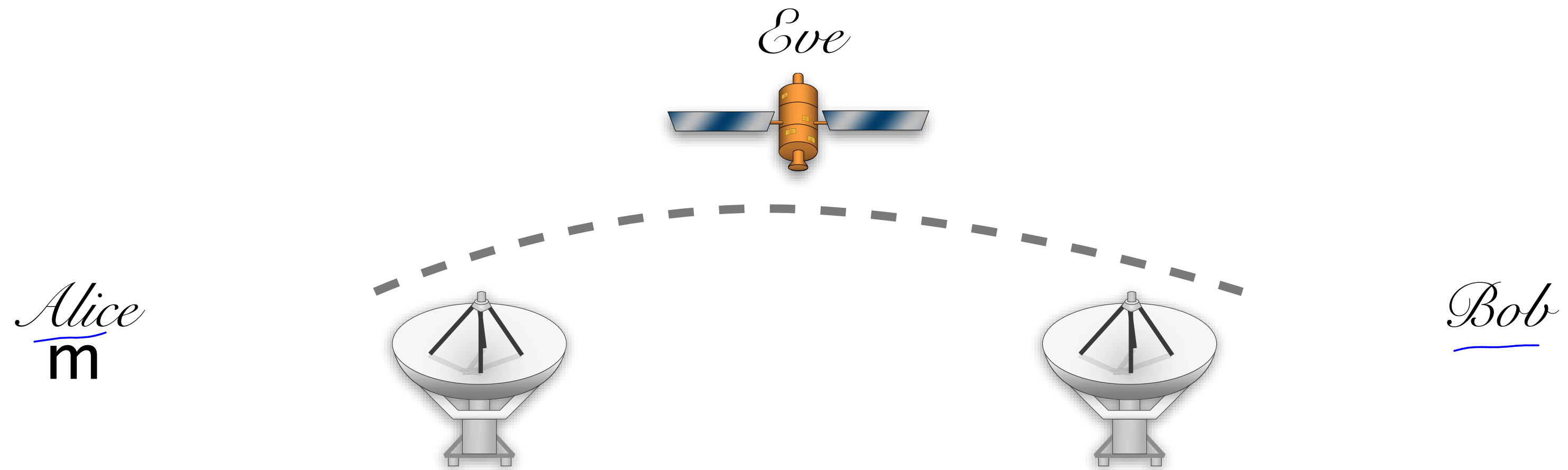
LINEARITY OF EXPECTATION

$$\forall X, Y, \quad \underline{E[X + Y]} = \underline{E[X]} + \underline{E[Y]}$$

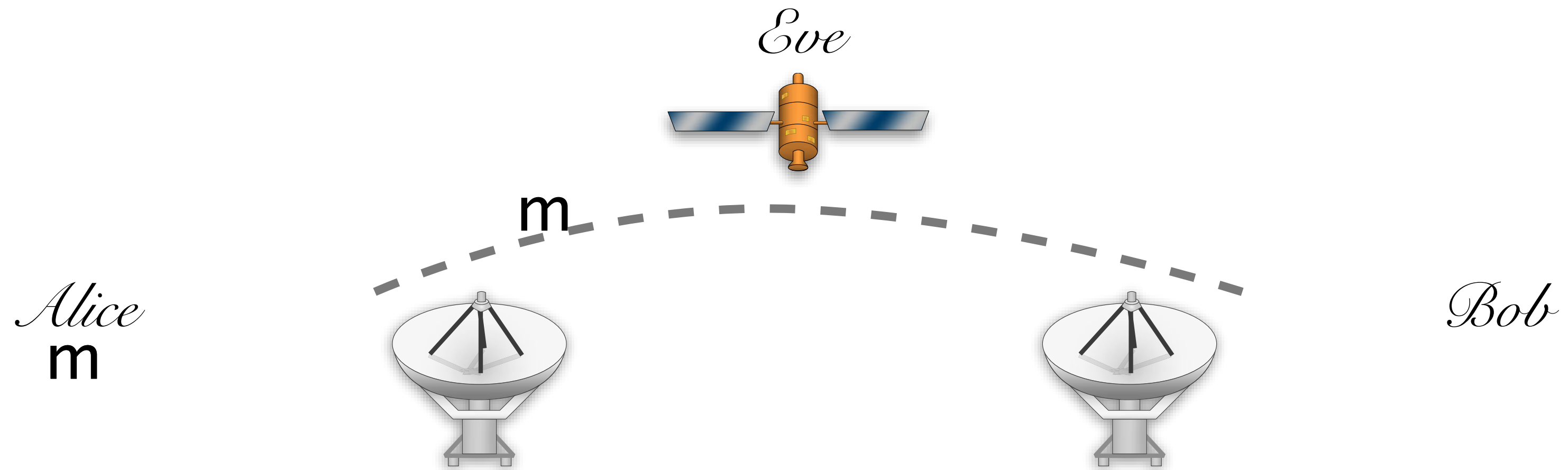
EXPECTED RUNNING TIME

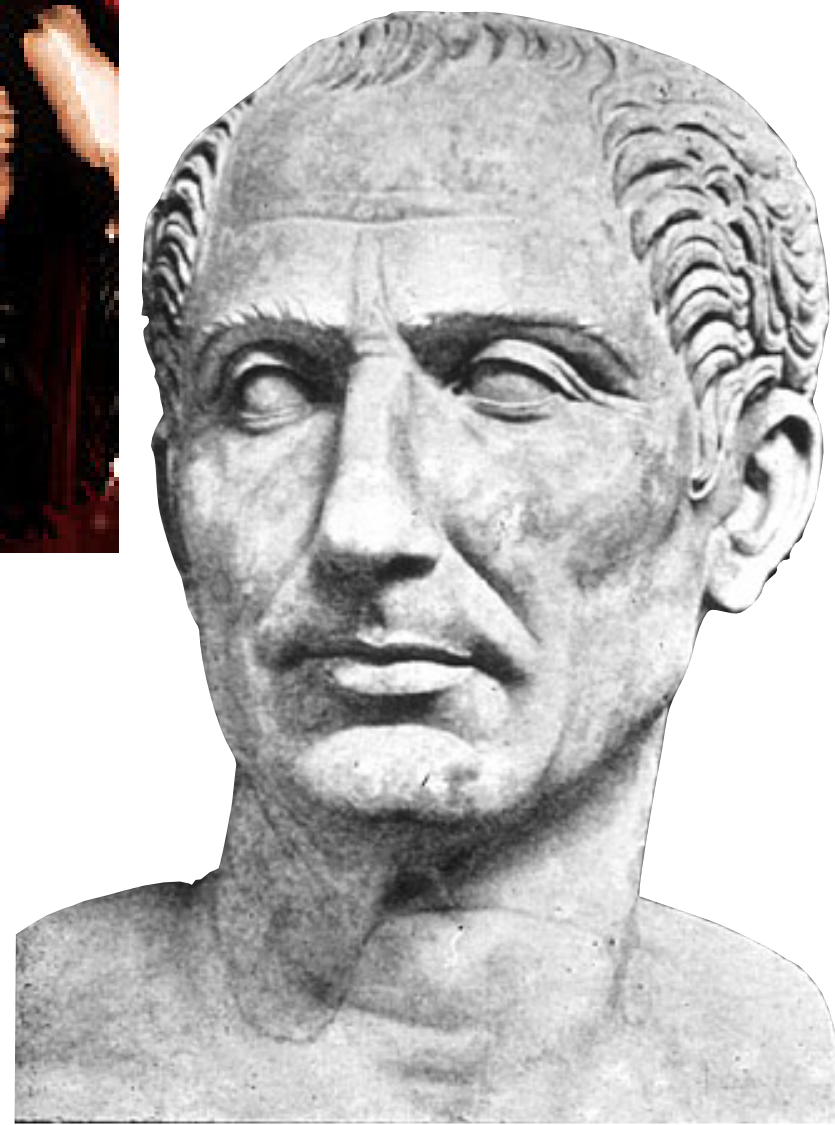
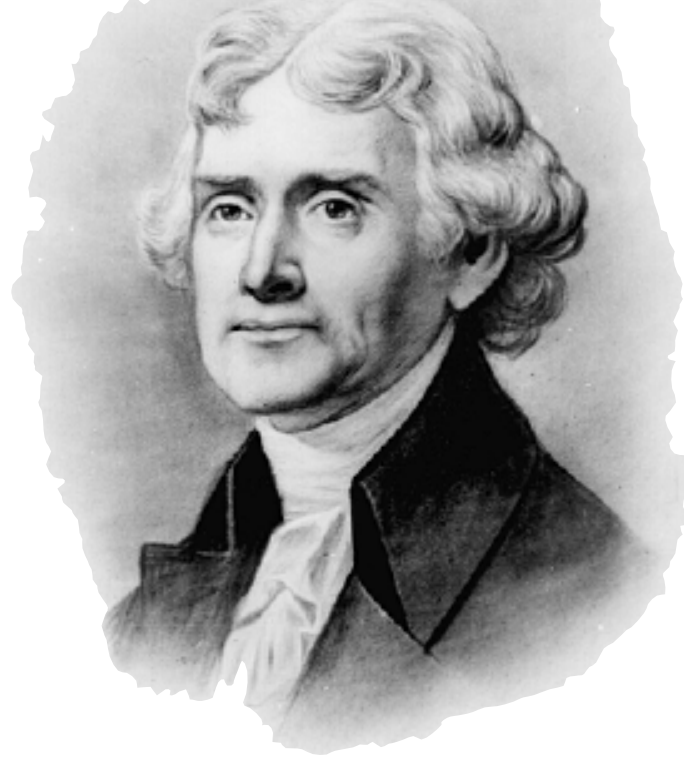
$$E[X] = \underline{\underline{\Theta(n)}}$$

PRIVATE COMMUNICATION



PRIVATE COMMUNICATION





Η ΓΕΩΓΡΑΦΙΑ ΠΟΤΑ ΕΥΘΥΣΑ ΘΥΡΑ ΔΕ ΜΕΤΑΝΙΣΤΑ ΔΕ ΜΕ ΔΙΤΑ ΔΕ ΦΕΝΑ // ΑΦ
 ΘΟΡΑ ΕΣΟΛΟ Γ // ΣΗΙ ΔΩ ΘΗΛΑ ΔΟΥΙΤΡΑ // ΑΦ ΕΣ 341 Ε // 08 ± Δ ΔΩ ΘΗ ΦΗ ΝΟ ± ΝΑ ΕΣ
 ΑΥΘΥΣΑ ΚΑΤΗ ΔΕ ΣΟΦΕΣ ΔΕ ΕΥΘΥΣΑ ΕΡΘΕ ΦΘΗΜΕΙ ΔΑ ΦΑ ΛΑ ΔΑ
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x p o r a t a r z b b = y o
 hio huy unum hunc y puz zow no. y hunc unum
 u t y @ a v z y x
 The original is kept in the Dept of
 Gilbert Hill

Cipher of Anthony de Sen. D. M. ... Bal:
 a b c d e f g h i k l m n o p q r s t u x y z
 o + + + a o f o i t n o p v s n f Δ ε ε γ θ γ
 Nullor . # . r . d . d . a . Doublets . -
 ad for was that if son use at of the from by p not when was
 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
 the n m is was say no my might read see write down, z say you
 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

SUBSTITUTION CIPHER



$$\mathcal{M} = \{A, B, \dots, Z\}^*$$

$$\mathcal{K} = \text{the set of permutations over } \{A, B, \dots, Z\}$$

$$\text{Gen} = k \text{ where } k \xleftarrow{r} \mathcal{K}.$$

$$\text{Enc}_k(m_1 m_2 \dots m_n) = c_1 c_2 \dots c_n \text{ where } c_i = k(m_i)$$

$$\text{Dec}_k(c_1 c_2 \dots c_n) = m_1 m_2 \dots m_n \text{ where } m_i = k^{-1}(c_i)$$

SIZE OF KEYSPACE IS

$$26! = 403291461126605635584000000$$

EOB TZSRWF XEASG ZV DWGYEZIPWQYOG NFKRXENPQERX ERDOFNIARX VZW VQDNHNEQENFP NFERWQDENZFX
URÉJRRF SNXEWAXEVAH RFENENRX NF ZAW DZFFRDERS XZDNREG XADO ERDOFNIARX OQKR URDZTR
NFSNXYRFXQUHR RFQUHNFP VZW NFXEQFDR QAEZTQERS ERHHRW TQDONFRX XRDAWR JNWRHRXX FREJZWLX
NFERWFRE UQFLNFP XQERHHNER WQSNZERHRKNXNZF QFS TZWR NF EONX DZAWXR (JR) NFEWZSADR XZTR ZV
EOB VAFSQTRFEQH DZFDREYEX ZV EONX XEASG RTYOQXNX JNHH UR YHQDRS ZF WNPZWZAX YWZZVX ZV
XRDAWNEG UQXRS ZF YWRDNXR SRVNFNENZFX QFS QXXATYENZFX

EOR TZSRWF XEASG ZV DWGYEZIPWQYOG NFKRXENPQERX ERDOFNIARX VZW VQDNHNEQENFP NFERWQDENZFX
UREJRRF SNXEWAXEVAH RFENENRX NF ZAW DZFFRDERS XZDNREG XADO ERDOFNIARX OQKR URDZTR
NFSNXYRFXQUHR RFQUHNFP VZW NFXEQFDR QAEZTQERS ERHHRW TQDONFRX XRDAWR JNWRHRXX FREJZWLX
NFERWFRE UQFLNFP XQERHHNER WQSNZERHRKNXNZF QFS TZWR NF EONX DZAWXR JR NFEWZSADR XZTR ZV
EOR VAFSQTRFEQH DZFDREYEX ZV EONX XEASG RTYOQXNX JNHH UR YHQDRS ZF WNPZWZAX YWZZVX ZV
XRDAWNEG UQXRS ZF YWRDNXR SRVNFNENZFX QFS QXXATYENZFX

FREQUENCY ANALYSIS

RSA

'78

① modular exponentiation

② greatest common divisors

③ Picking large primes \$

④ Euler's theorem

} p/c

public key encryption.

AMZ

PK

$$\text{Enc}(\overset{\text{AMZ}}{\text{PK}}, m) \rightarrow \underline{c}$$

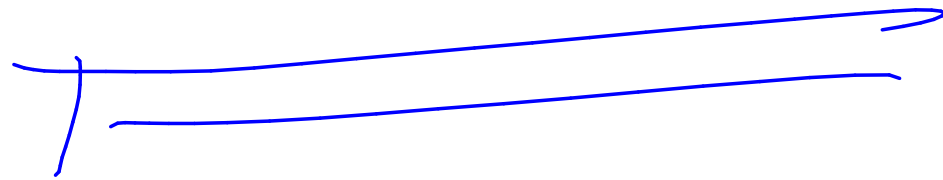
$$\text{Dec}(\overset{\text{AMZ}}{\text{SK}}, \underset{=}{c}) \rightarrow m$$

MOD-EXP

1000 bits !!

$$(a, x, n) \rightarrow a^x \bmod n$$

a mod N



a^{12}

a^n

MOD-EXP

$$(a, x, n) \rightarrow a^x \bmod n$$

Algorithm 2: ModularExponentiation(a, x, n)

Input: $a, x \in [1, n]$

1 $r \leftarrow 1$

2 **while** $x > 0$ **do**

3 **if** x is odd **then**

4 $r \leftarrow r \cdot a \bmod n$

5 $x \leftarrow \lfloor x/2 \rfloor$

6 $a \leftarrow a^2 \bmod n$

7 **Return** r

$$a^x \rightarrow \left(a^{x/2} \right)^2$$

MOD-EXP

$$(a, x, n) \rightarrow a^x \bmod n$$
$$a^x \bmod n = \prod_{i=0}^{\ell} x_i a^{2^i} \bmod n$$

Algorithm 2: ModularExponentiation(a, x, n)

Input: $a, x \in [1, n]$

```
1  $r \leftarrow 1$ 
2 while  $x > 0$  do
3   if  $x$  is odd then
4      $r \leftarrow r \cdot a \bmod n$ 
5    $x \leftarrow \lfloor x/2 \rfloor$ 
6    $a \leftarrow a^2 \bmod n$ 
7 Return  $r$ 
```

EUCLID

greatest common divisor of

$$\text{gcd}(35 \text{ and } 14) = 7$$

$5 \textcircled{7}$ $2 \textcircled{7}$

EUCLID AND THE GCD

1237918278937

142104160622754

what is the GCD?

Algorithm 1: ExtendedEuclid(a, b)

of bits in $a = n$ **Input:** (a, b) s.t $a > b \geq 0$ **Output:** (x, y) s.t. $ax + by = \text{gcd}(a, b)$ 1 **if** $a \bmod b = 0$ **then**2 | Return $(0, 1)$ 3 **else**4 | $(x, y) \leftarrow \text{ExtendedEuclid}(b, a \bmod b)$ 5 | Return $(y, x - y(\lfloor a/b \rfloor))$

 $\Theta(\log a)$ **GIVEN (A,B):****FINDS (X,Y) S.T. AX + BY = GCD(A,B)**

35 and 14

$$\gcd(35, 14) = 7$$

$$(1)\underline{35} + (\underline{14})(-2) = 7$$

13 and 73

$$\gcd(13, 73) = 1$$

$$(13)(-28) + (73) \cdot 5 = 1$$

$$\begin{array}{r} 365 \\ -364 \\ \hline 1 \end{array}$$

CRYPTOGRAPHY

32964031794323944819653393490459747322286350
31500646399521148595996590847768392238771217
69252874938669758963521262177684757622917354
10764395167469005450386721087598087995167019
51260209070780169584330401159403323161691626
51931932385937935848982371478700671595968131
07098610562722922433990122345442992245859824
74364293651925019779584845838833700838150940
56504167483874319231730153624474523841938831
33113697736378643670286581890300666191500953
329742364829

LARGE PRIME NUMBER

```
import java.io.*;
import java.math.*;
import java.util.*;

public class pr {
    public static void main(String args[]) {

        BigInteger prime = new BigInteger(1500, 80, new Random());
        System.out.println("prime is " + prime);
    }
}
```

2^{-80}

RABIN-MILLER

$$L_N = \{\alpha \in \mathbb{Z}_N \mid \alpha^{N-1} = 1 \text{ and if } \alpha^{u2^{j+1}} = 1 \text{ then } \alpha^{u2^j} = 1\}$$

RABIN-MILLER

$$L_N = \{\alpha \in \mathbb{Z}_N \mid \alpha^{N-1} = 1 \text{ and if } \alpha^{u2^{j+1}} = 1 \text{ then } \alpha^{u2^j} = 1\}$$

Algorithm 3: Miller-Rabin Primality Test

- 1 Handle base case $N = 2$
- 2 **for** t times **do**
- 3 Pick a random $\alpha \in \mathbb{Z}_N$
- 4 **if** $\alpha \notin L_N$ **then** Output "composite"
- 5 Output "prime"

$t = 0 \rightarrow$

$$\alpha^{N-1} = 1$$

RABIN-MILLER

$$L_N = \{\alpha \in \mathbb{Z}_N \mid \alpha^{N-1} = 1 \text{ and if } \alpha^{u2^{j+1}} = 1 \text{ then } \alpha^{u2^j} = 1\}$$

Algorithm 3: Miller-Rabin Primality Test

- 1 Handle base case $N = 2$
 - 2 **for** t times **do**
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 - 5 Output “prime”
-

Theorem 38.1. *If N is composite, then the Miller-Rabin test outputs “composite” with probability $1 - 2^{-t}$. If N is prime, then the test outputs “prime.”*

EULER TOTIENT

$\phi(n)$: # of ^{positive} integers that are smaller & relatively prime to n

$$\phi(7) = \underbrace{1\ 2\ 3\ 4\ 5\ 6}_7$$

$$\phi(p) = p - 1$$

$$\gcd(7, 1) = 1$$

EULER TOTIENT

$\Phi(n)$

$$\phi(p) = p-1$$

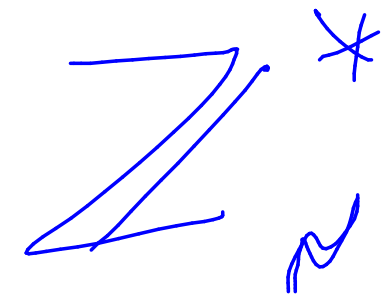
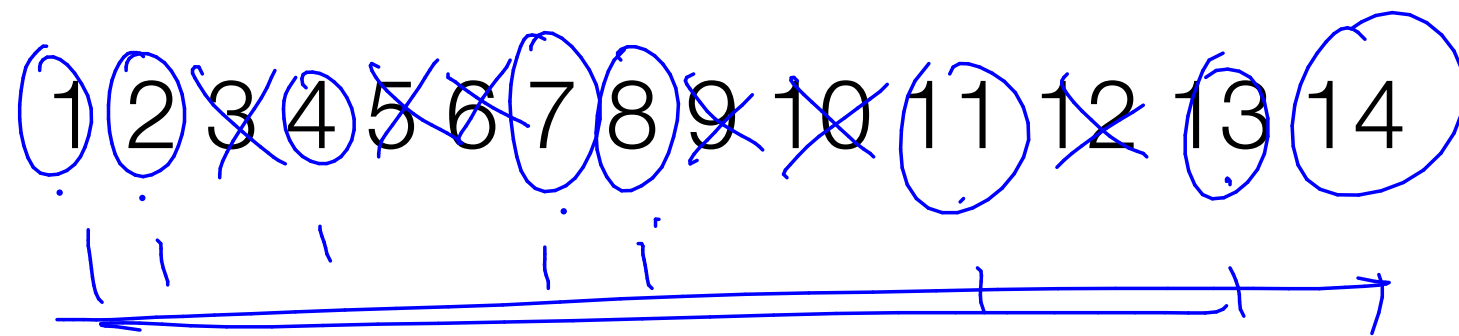
prime

$$\phi(n) = \phi(p) \cdot \dots \cdot \phi(q)$$

$$\phi(15) = 8 = 2 \cdot 4 = \phi(3) \cdot \phi(5)$$

15

↪ 3·5



EULER TOTIENT

$$|\mathbb{Z}_n^\star| = \Phi(n)$$

prime

$$\Phi(p) = p - 1$$

product
of 2 primes

$$\Phi(n) = (p - 1)(q - 1)$$

EULER THEOREM

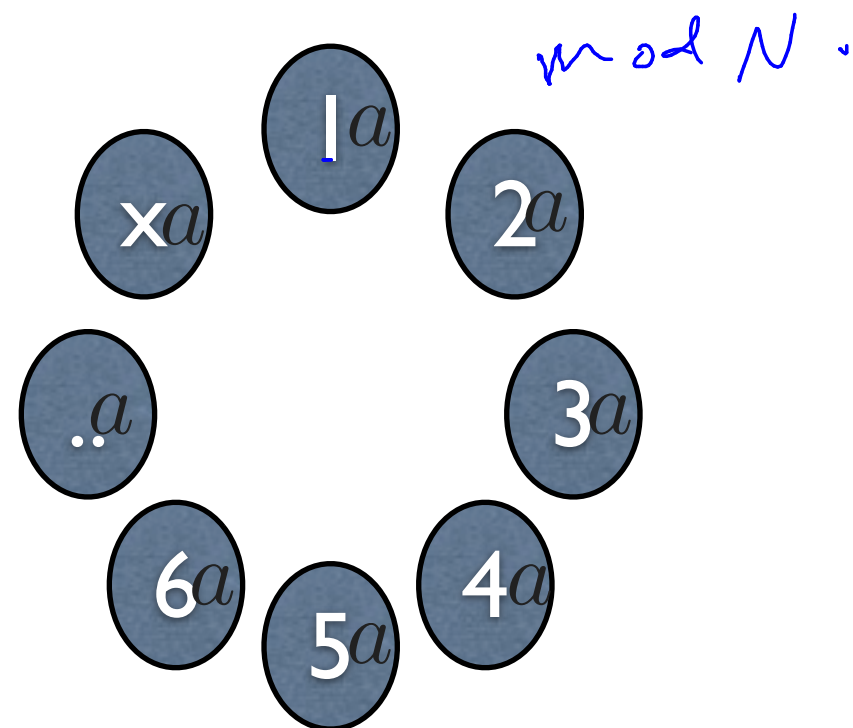
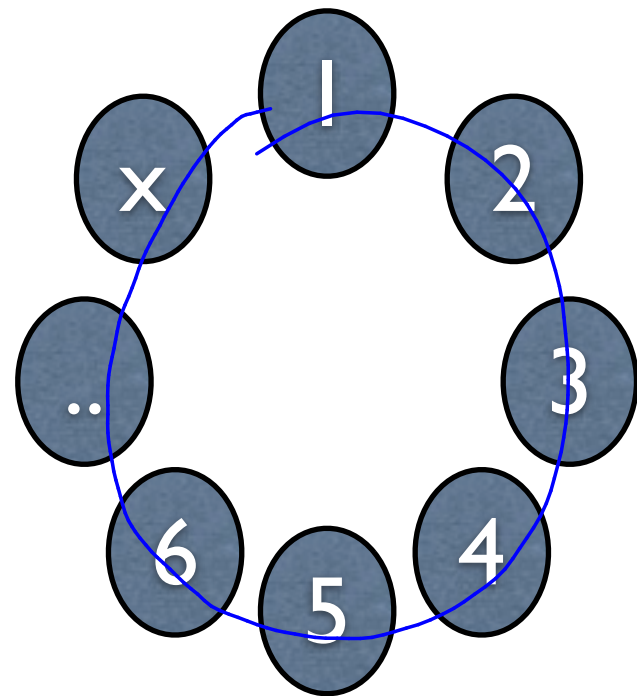
$$\text{if } \underline{\gcd(a, n)} = 1 \quad \underline{a^{\Phi(n)}} = \underline{1 \pmod n}$$

(why RSA works)

EULER THEOREM

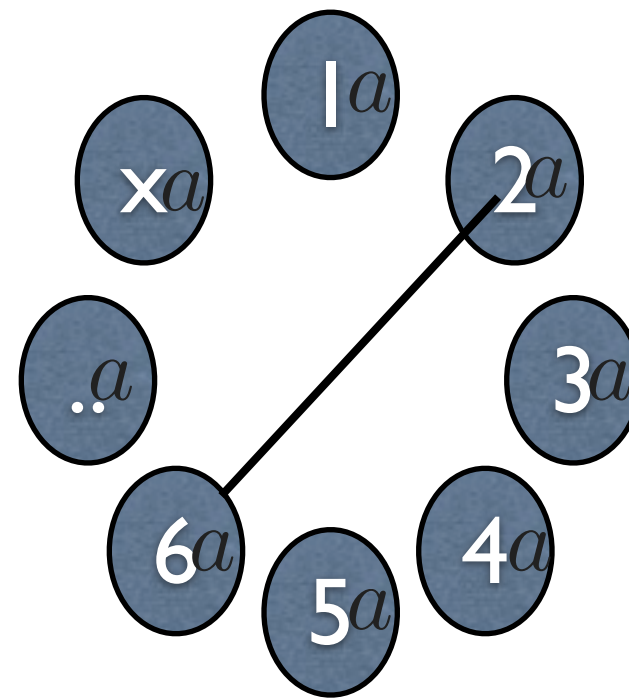
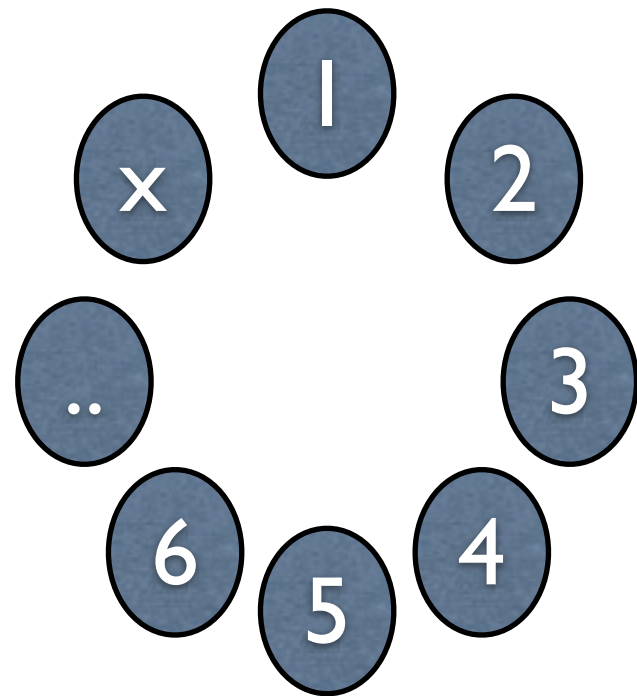
$\phi(N)$

$$\forall a \in \mathbb{Z}_N^*, a^{\phi(N)} = 1 \pmod N$$



EULER THEOREM

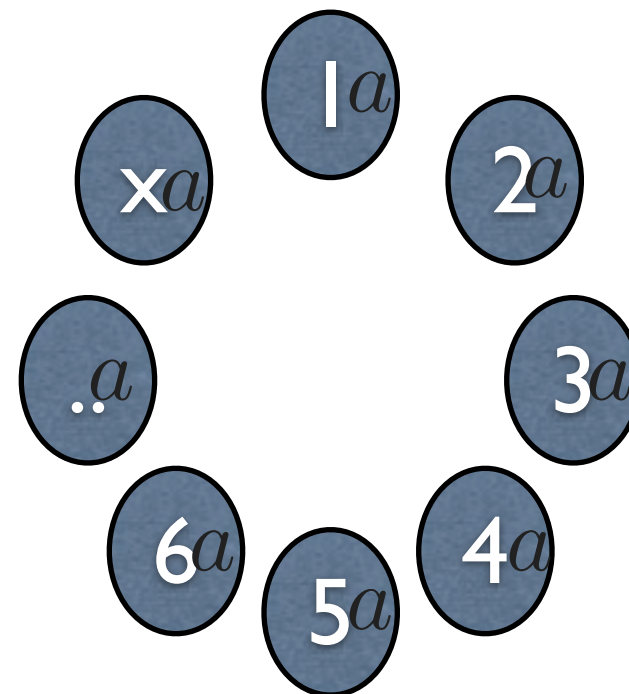
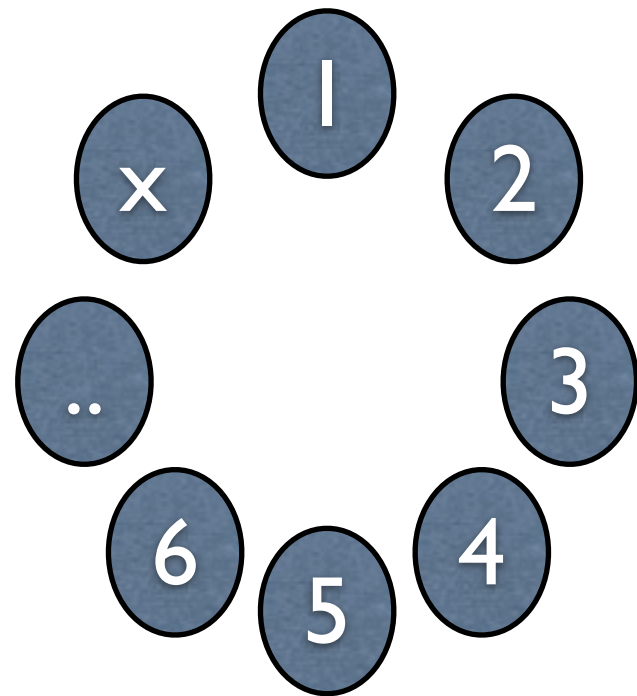
$$\forall a \in \mathbb{Z}_N^*, a^{\Phi(N)} = 1 \pmod N$$



argue: all are distinct
suppose two are equal.
multiply by a^{-1}
this implies $2=6$!

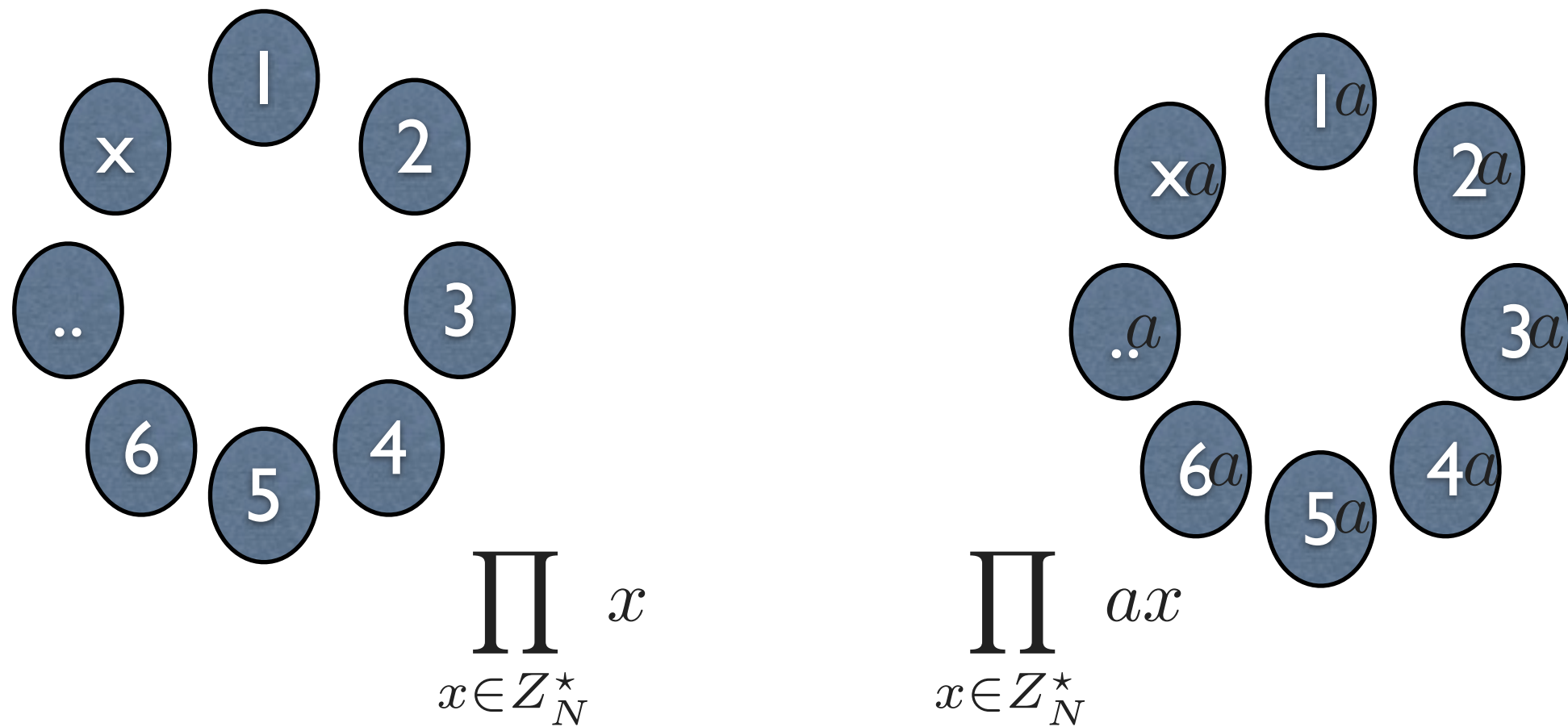
EULER THEOREM

$$\forall a \in \mathbb{Z}_N^*, a^{\Phi(N)} = 1 \pmod N$$



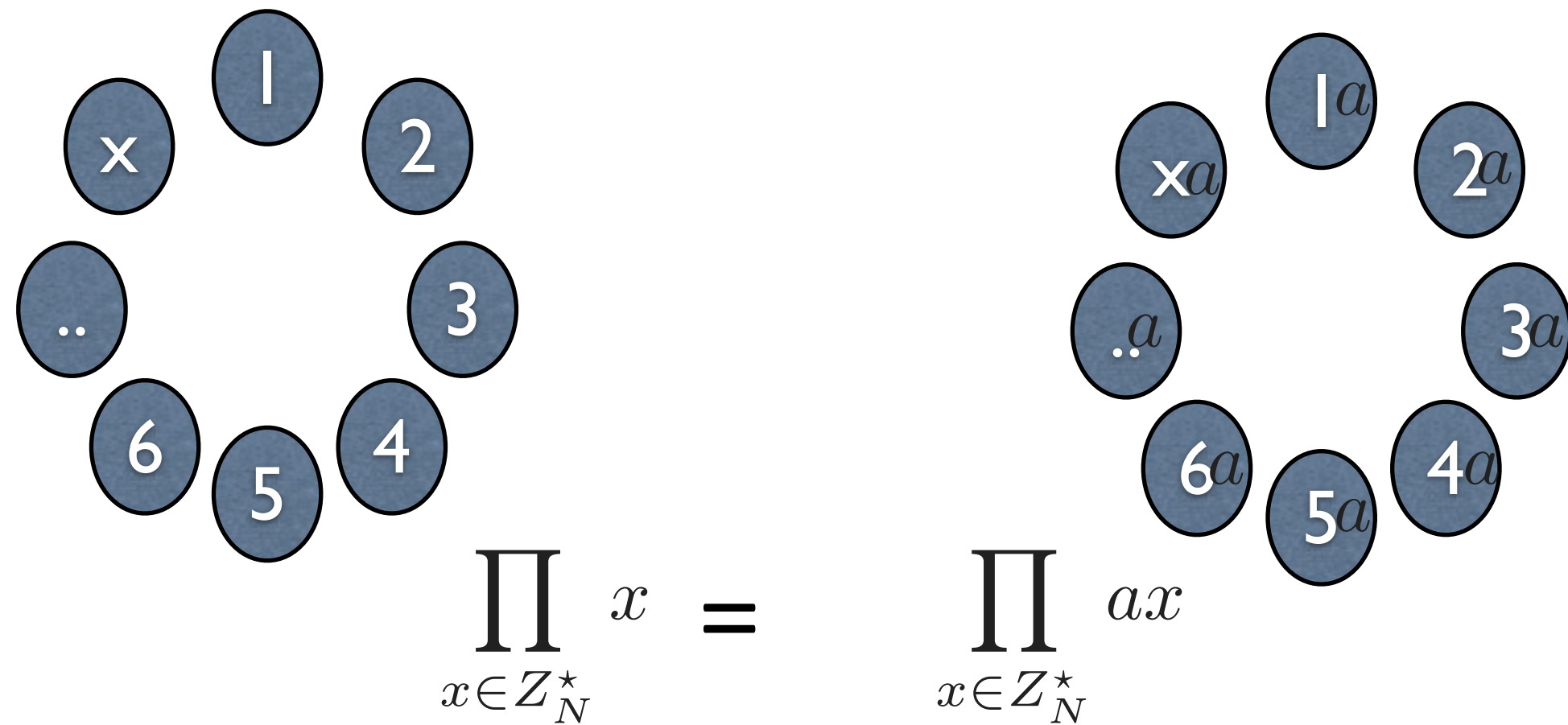
EULER THEOREM

$$\forall a \in \mathbb{Z}_N^*, a^{\Phi(N)} = 1 \pmod N$$



EULER THEOREM

$$\forall a \in \mathbb{Z}_N^*, a^{\Phi(N)} = 1 \pmod{N}$$



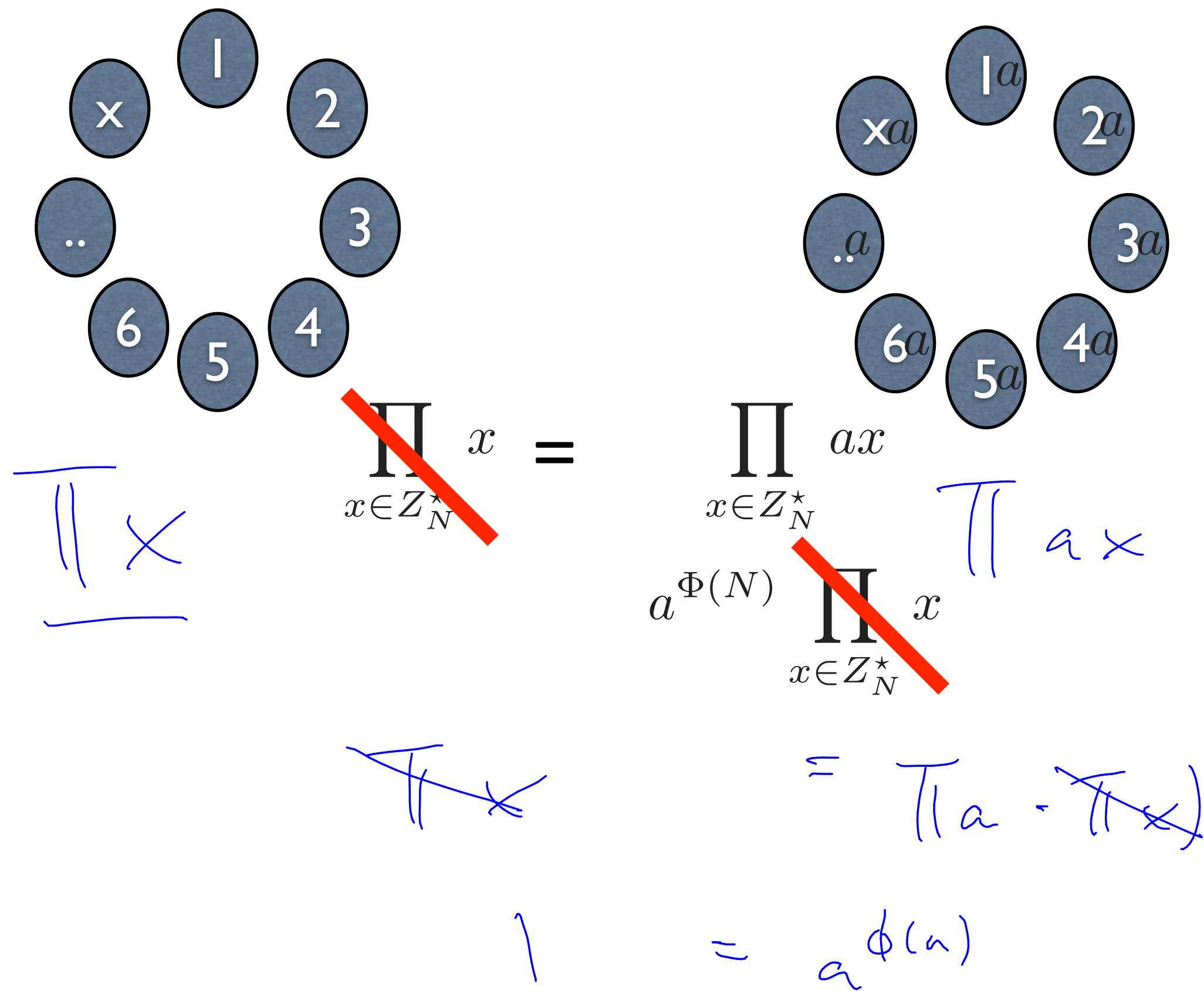
EULER THEOREM

$$\forall a \in \mathbb{Z}_N^*, a^{\Phi(N)} = 1 \pmod N$$

$$\prod_{x \in \mathbb{Z}_N^*} x = \prod_{x \in \mathbb{Z}_N^*} ax$$
$$a^{\Phi(N)} \prod_{x \in \mathbb{Z}_N^*} x$$

EULER THEOREM

$$\forall a \in \mathbb{Z}_N^*, a^{\Phi(N)} = 1 \pmod N$$



TEXTBOOK RSA

GEN(1^n)

① pick 2 primes p, q ~ 1000 bit #s

② $N = p \cdot q$

③ $\phi(N) = (p-1)(q-1)$

④ pick e s.t.

$\gcd(e, \phi(N)) = 1$

$e = 17, \underline{\underline{65537}}$

⑤ use euclid to compute d s.t.

$e \cdot d = 1 \pmod{\phi(N)}$

$e \cdot d + k \cdot \phi(N) = 1$

TEXTBOOK RSA

GEN(1^n)

$$N = pq \quad \Phi(N) = (p-1)(q-1)$$

e is a number such that $\gcd(e, \Phi(N)) = 1$

d is such that $e \cdot d = 1 \pmod{\Phi(N)}$

$$pk = (N, e)$$

$$sk = (N, d)$$

ENC_{pk}(m):

$$m^e \pmod N$$

Dec_{sk}(c) = $c^d \pmod N$


$$\text{Dec}(\text{Enc}(m)) =$$

$$\text{Dec}(m^e) = m^{ed} \pmod N$$

$$= m^{k \cdot \Phi(N) + 1} \pmod N$$

$$= (m^{\Phi(N)})^k \cdot m \pmod N$$
$$= 1^k \cdot m \pmod N$$

$N = 949$ $E = 11$ $D = 707$



TEXTBOOK RSA

GEN($\mathbf{1}^n$)

$$N \leftarrow pq, p, q \in \Pi_n, e \in \mathbb{Z}_{\phi(n)}^*$$

$$pk \leftarrow (N, e)$$

$$sk \leftarrow (N, d)$$

ENC_{pk}(\mathbf{m})

$$c \leftarrow m^e \bmod N$$

DEC_{sk}(\mathbf{c})

$$m \leftarrow c^d \bmod N$$

JUNE 1942

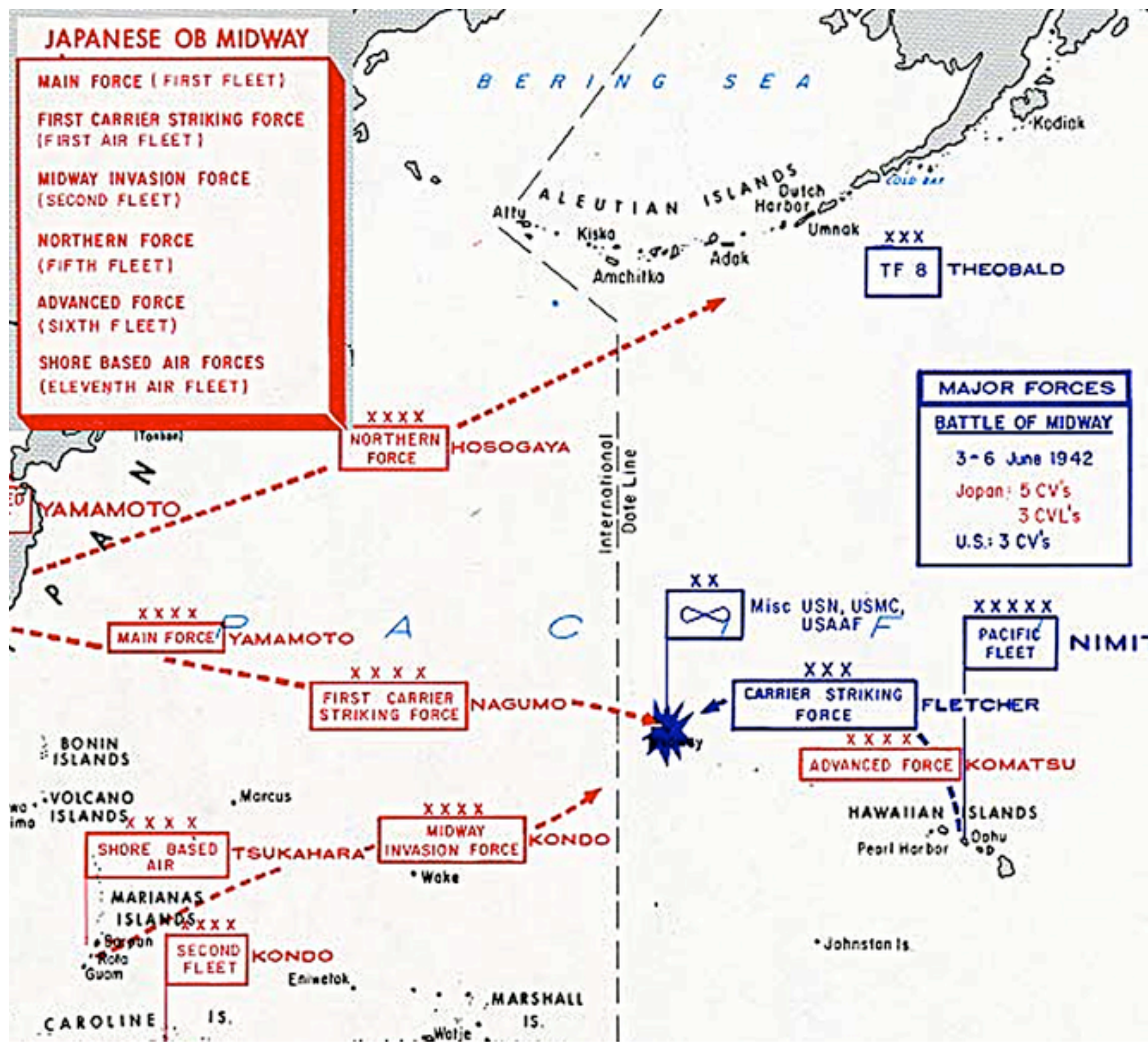
JN-25B

CMDR EDWARD T LAYTON

(FLEET INTELLIGENCE OFFICER)

LT CMDR JOSEPH ROCHEFORT

(COMBAT INTELLIGENCE UNIT)



secure encryption
schemes **need** to use
randomness!


The word "randomness!" is underlined with two red lines. The top line is slightly above the bottom line, and both are centered under the word.

PKCS1.5

m^e

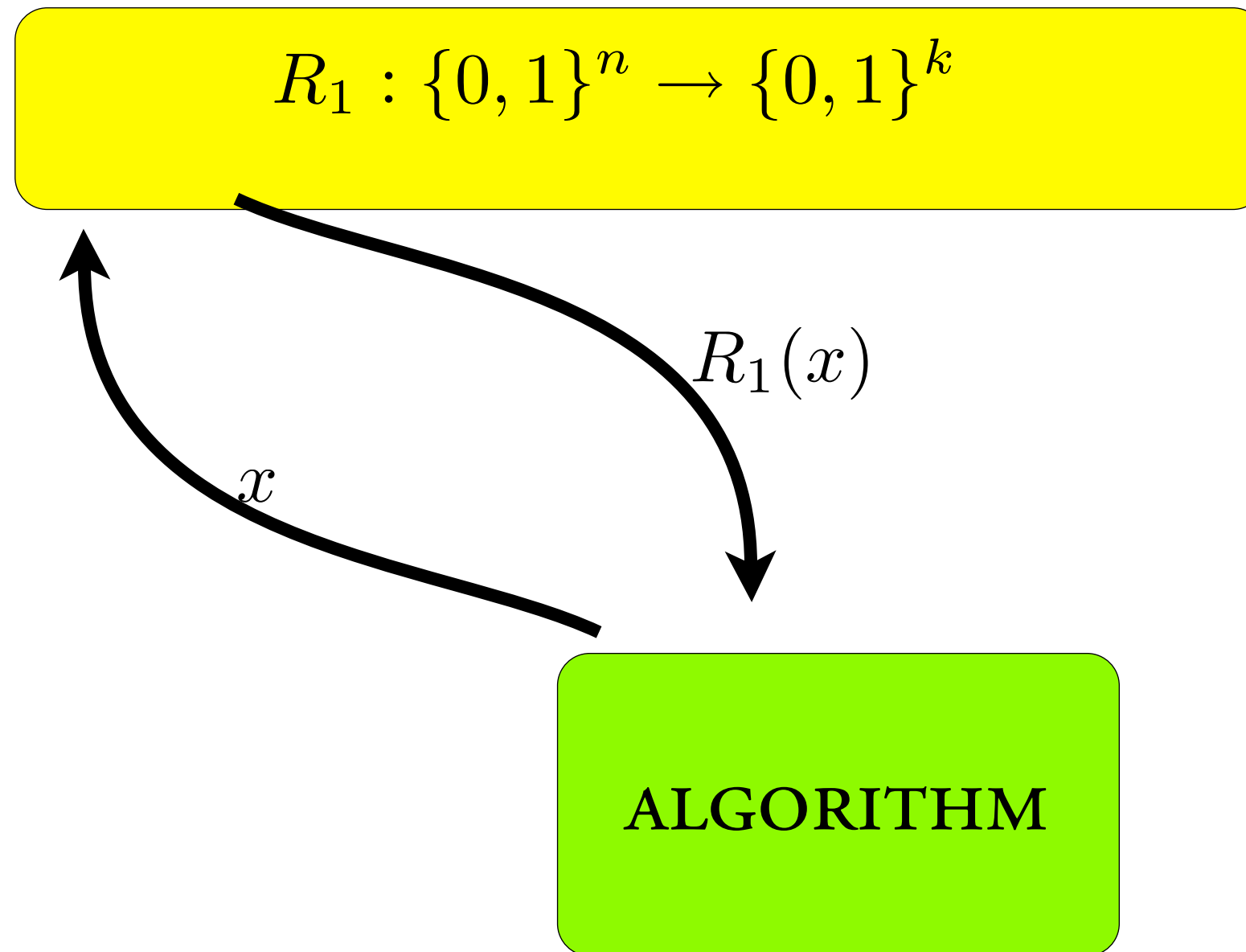
$\text{ENC}_{pk}(m)$

PICK r AS A RANDOM STRING WITH NO **0s** (TYPICALLY 8 BYTES)

$$c \leftarrow (0||2||r||0||m)^e \bmod N$$


CCA2 ATTACK AGAINST THIS SCHEME

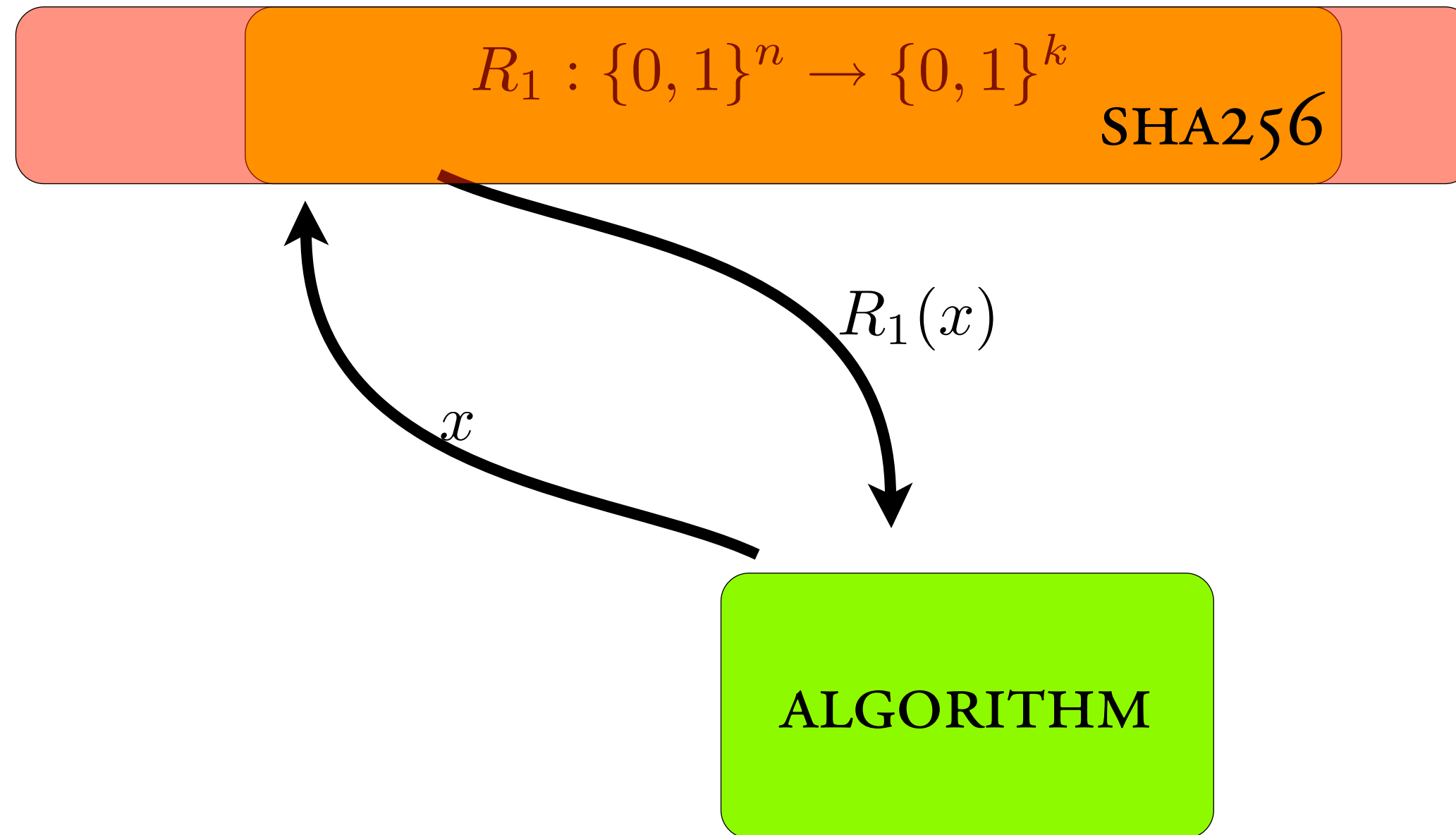
RANDOM ORACLE MODEL



PUBLIC FUNCTION. NOT KEYED.

ANYONE CAN EVALUATE, OUTPUT IS UNPREDICTABLE.

RANDOM ORACLE MODEL



HEURISTIC SECURITY ONLY

CANNOT BE ALWAYS BE SECURELY INSTANTIATED

OAEP+

GEN($\mathbf{1}^n$)

$f, f^{-1} \leftarrow \text{TRAPDOOR OWP}()$

ENC_{pk}(\mathbf{m})

$r \leftarrow U_n$

$s \leftarrow R_1(r) \oplus m \parallel R_2(r||m)$

$t \leftarrow R_3(s) \oplus r$

$c \leftarrow f(s||t)$

$R_1 : \{0, 1\}^{k_0} \rightarrow \{0, 1\}^n$

$R_2 : \{0, 1\}^{n+k_0} \rightarrow \{0, 1\}^{k_1}$

$R_3 : \{0, 1\}^{n+k_1} \rightarrow \{0, 1\}^{k_0}$

DEC_{sk}(\mathbf{C})

$(s = (s_1, s_2), t) \leftarrow f^{-1}(c)$

$r \leftarrow R_3(s) \oplus t$

$m \leftarrow R_1(r) \oplus s_1$

$R_2(r||m) \stackrel{?}{=} s_2$ OUTPUT \mathbf{m} ELSE FAIL

Theme

“SMALL PROBLEMS ARE EASY TO SOLVE.”

“SOLVE BIG PROBLEMS BY MAKING THEM
INTO SMALLER ONES.”

INTRO

MIDTERM

FINAL

LI5 OCT 17

DEC 5-7

DYNAMIC

GRAPH

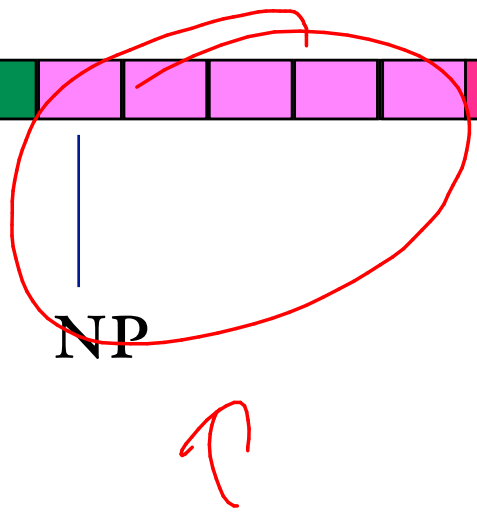
RAND



DIV & CONQ

GREEDY

NP



TOPICS

D&C	DP	GREEDY	GRAPH	RAND	NPC	OTHER
MULT	LOG	SCHED	MST	MATCH	RED	GPU
QUICK	CHAIN	HUFF	BFS	FINGER	IND	DISSENT
CLOSE	TYPESET	ESPRESSO	DIJKSTRA	STRING	VC	PQ
MEDIAN	GERRY	CACHING	BELL-FORD	ENC	3COL	
FFT	ZAP		ALLSHORT		SUBSET	
MATMUL	POSTER		MAXFLOW		SET	
MASTERS	TUG		BIPARTITE			
BUS			EDGE-DISJ			
NIFTY			BASEBALL			
			ASSIGNMENT			
			STABLE			

Hand-drawn red annotations on the table:

- A large red bracket encloses the entire GRAPH column.
- A red box encloses the entire NPC column.
- Red circles are drawn around the following terms: MST, DIJKSTRA, BIPARTITE, STABLE, GPU, and DISSENT.

first goal: create an
amazing learning
experience

second goal: *instill*

my enthusiasm for this

area

third goal: *enjoy* every
second of this semester