L4

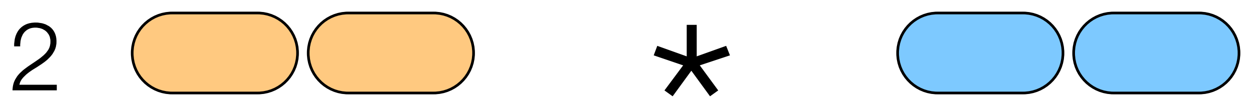
shelat

4102

sep 5 2013

$$T(n) = 3T(n/2) + 8O(n)$$

$$O(n^{\log_2(3)}) \quad O(n^{1.589})$$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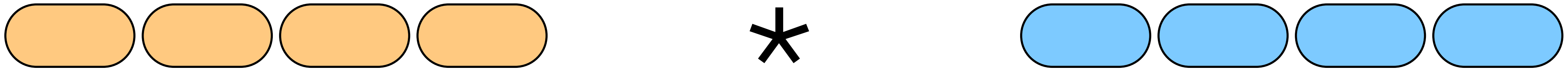

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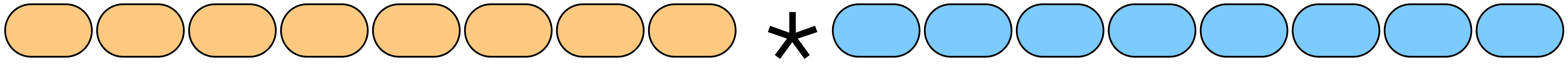

Digits	# operations	$14n^{\log_2 3} - 16n$
2	10	10
4	62	62
8	250	250
16	878	878
32	2890	2890
64	9182	9182
128	28570	28570

$$T(n) = 3T(n/2) + 8O(n)$$ (guess +chk)

goal: Prove $T(n) = O(n^{\log_2 3})$. More specifically,

Prove $T(n) = n^{\log_2 3} - 16n$

Base case: $T(n) < n^{\log_2 3} - 16n$ for small values of n

Spse $T(n) < n^{\log_2 3} - 16n$ holds for $n < n_0$.

Consider: $T(n_0+1) = 3T\left(\frac{n_0+1}{2}\right) + 8(n_0+1)$

(this argument is $< n_0$, so

$< 3\left[\left(\frac{n_0+1}{2}\right)^{\log_2 3} - 16\left(\frac{n_0+1}{2}\right)\right] + 8(n_0+$

$< (n_0+1)^{\log_2 3} - 16(n_0+1) - 8(n_0+1) +$

$$T(n) = 3T(n/2) + 8O(n)$$ (guess +chk)

Assume that this term is 8n

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Lets prove that $T(n) \leq 14n^{\log_2 3} - 16n$

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Lets prove that $T(n) \leq 14n^{\log_2 3} - 16n$

By inspection, indeed, $T(n) \leq 14n^{\log_2 3} - 16n$ when n<1024.

$$T(n) = 3T(n/2) + 8O(n)$$ (guess +chk)

Assume that this term is 8n

Lets prove that $T(n) \leq 14n^{\log_2 3} - 16n$

By inspection, indeed, $T(n) \leq 14n^{\log_2 3} - 16n$ when n<1024.

A1: Lets assume that $T(n) \leq 14n^{\log_2 3} - 16n$ when $n < n_0$

$$T(n) = 3T(n/2) + 8O(n)$$ (guess +chk)

Assume that this term is 8n

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A1: Lets assume that $T(n) \leq 14n^{\log_2 3} - 16n$ when $n < n_0$

Consider the case of $T(n_0 + 1)$

$$T(n) = 3T(n/2) + 8O(n)$$ (guess +chk)

Assume that this term is 8n

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Consider the case of $T(n_0 + 1)$

$$T(n_0 + 1) = 3T((n_0 + 1)/2) + 8(n_0 + 1)$$ By definition

$$T(n) = 3T(n/2) + 8O(n)$$ (guess +chk)

Assume that this term is 8n

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But since $(n_0 + 1)/2 < n_0$ and A1, it follows that

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But since $(n_0 + 1)/2 < n_0$ and A1, it follows that

$$T(n_0 + 1) < 3\left[14\left(\frac{n_0 + 1}{2}\right)^{\log_2 3} - 16\left(\frac{n_0 + 1}{2}\right)\right] + 8(n_0 + 1)$$

$$T(n_0+1) < 3\left[14\left(\frac{n_0+1}{2}\right)^{\log_2 3} - 16\left(\frac{n_0+1}{2}\right)\right] + 8(n_0+1)$$

$$T(n_0+1) < 3\left[14\left(\frac{n_0+1}{2}\right)^{\log_2 3} - 16\left(\frac{n_0+1}{2}\right)\right] + 8(n_0+1)$$

$$< 14(n_0+1)^{\log_2 3} - 24(n_0+1) + 8(n_0+1)$$

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This expression matches our Assumption A1.

A1: Lets assume that $T(n) \leq 14n^{\log_2 3} - 16n$ when $n < n_0$

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This expression matches our Assumption A1.

A1: Lets assume that $T(n) \leq 14n^{\log_2 3} - 16n$ when $n < n_0$

This establishes that $T(n) = O(n^{\log_2 3})$

$$T(n_0+1) < 3\left[14\left(\frac{n_0+1}{2}\right)^{\log_2 3} - 16\left(\frac{n_0+1}{2}\right)\right] + 8(n_0+1)$$

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This expression matches our Assumption A1.

A1: Lets assume that $T(n) \leq 14n^{\log_2 3} - 16n$ when $n < n_0$

Thus, we can conclude the proof via induction.

This establishes that $T(n) = O(n^{\log_2 3})$

Induction summary

1 $T(n) \leq 14n^{\log_2 3} - 16n$ IS TRUE for one case.

2 $T(n) \leq 14n^{\log_2 3} - 16n$ Suppose TRUE for $n < n_0$

3 Showed that 1,2 imply that

$$T(n_0+1) \leq 14(n_0+1)^{\log_2 3} - 16(n_0+1)$$

4 Lather, Rinse, Repeat! (Induction)

What happens if
we skip the -16n?

$$T(n) = 3T(n/2) + 8O(n)$$ (guess +chk)

Assume that this term is 8n

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$$T(n_0+1) < 3\left[14\left(\frac{n_0+1}{2}\right)^{\log_2 3}\right] + 8(n_0+1)$$

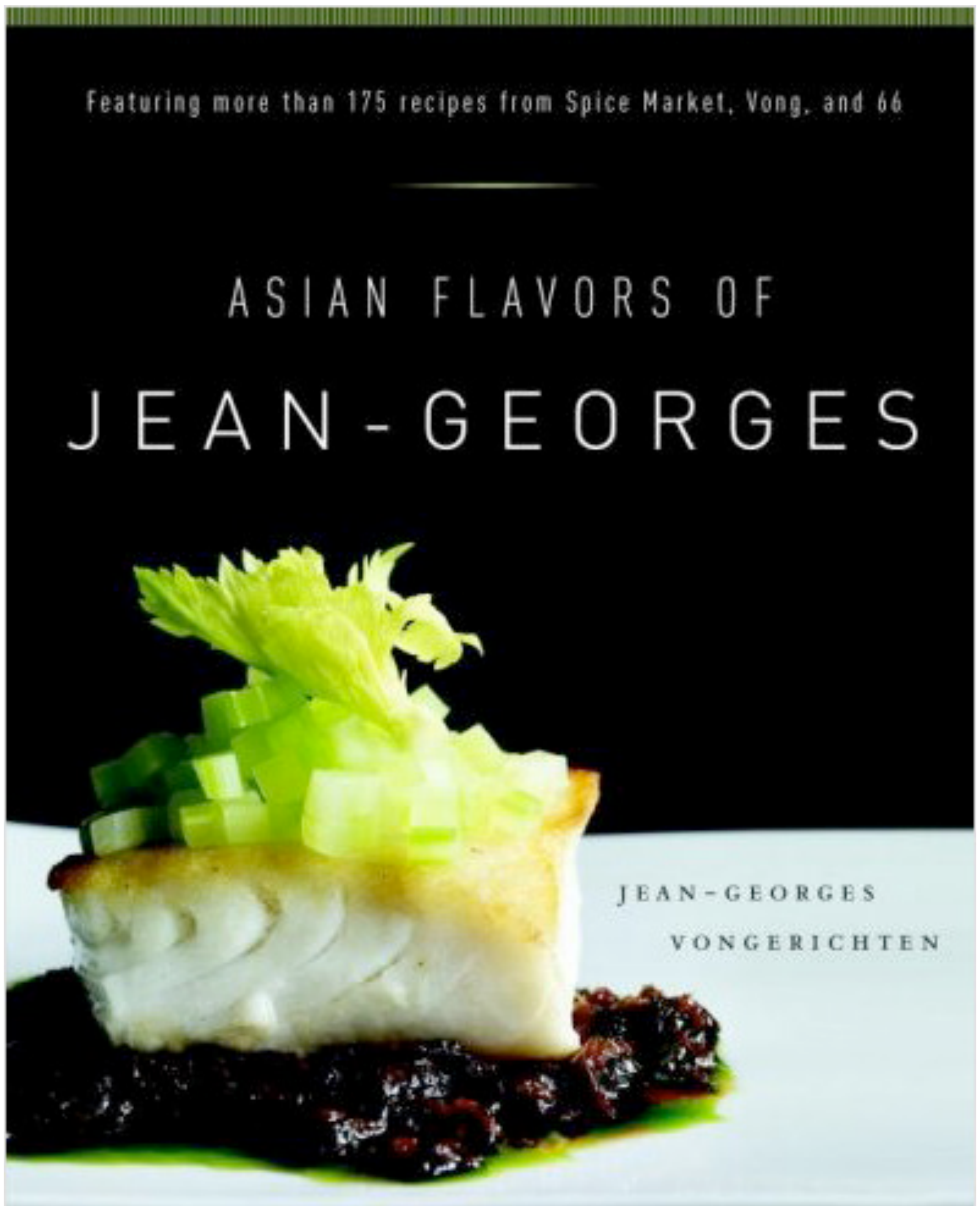
$$< 14(n_0+1)^{\log_2 3} + 8(n_0+1)$$

This expression DOES NOT matches our Assumption A1.
So the induction STOPS!

A1: Lets assume that $T(n) \leq 14n^{\log_2 3} - 16n$ when $n < n_0$

$$T(n) = 8T(n/2) + \Theta(n^2)$$ (guess +chk)

cookbook


Featuring more than 175 recipes from Spice Market, Vong, and 66
ASIAN FLAVORS OF
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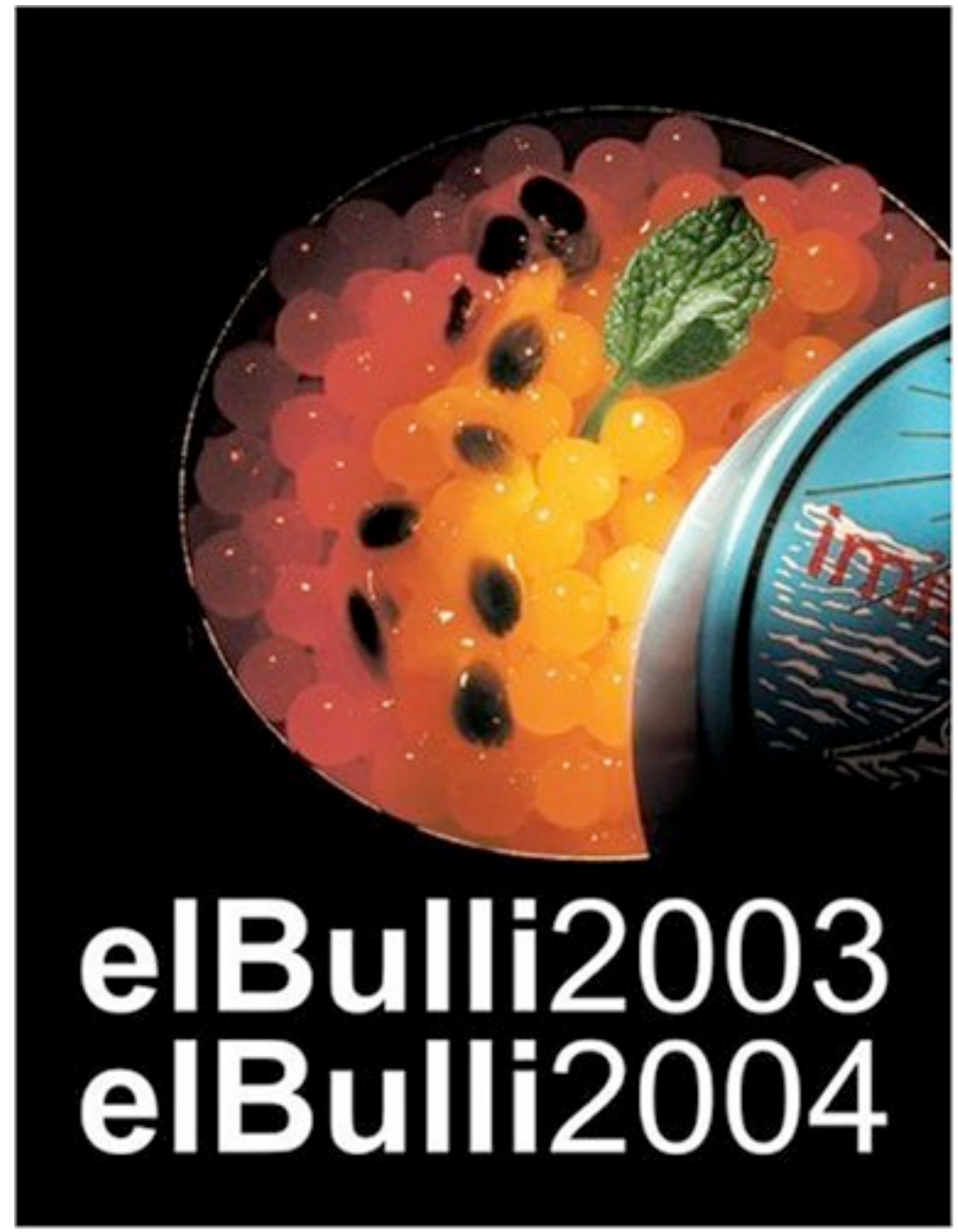
JEAN-GEORGES
VONGERICHTEN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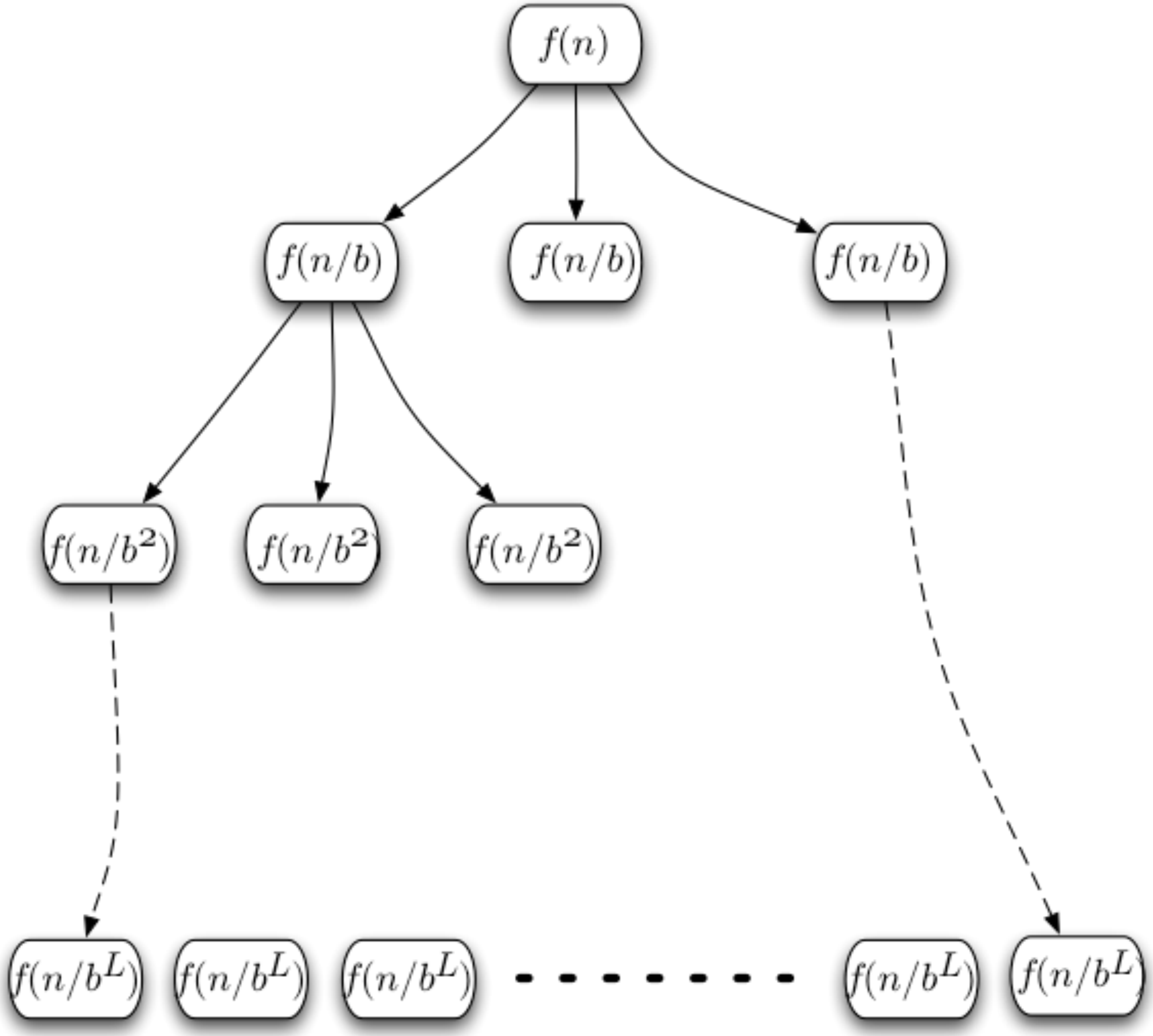
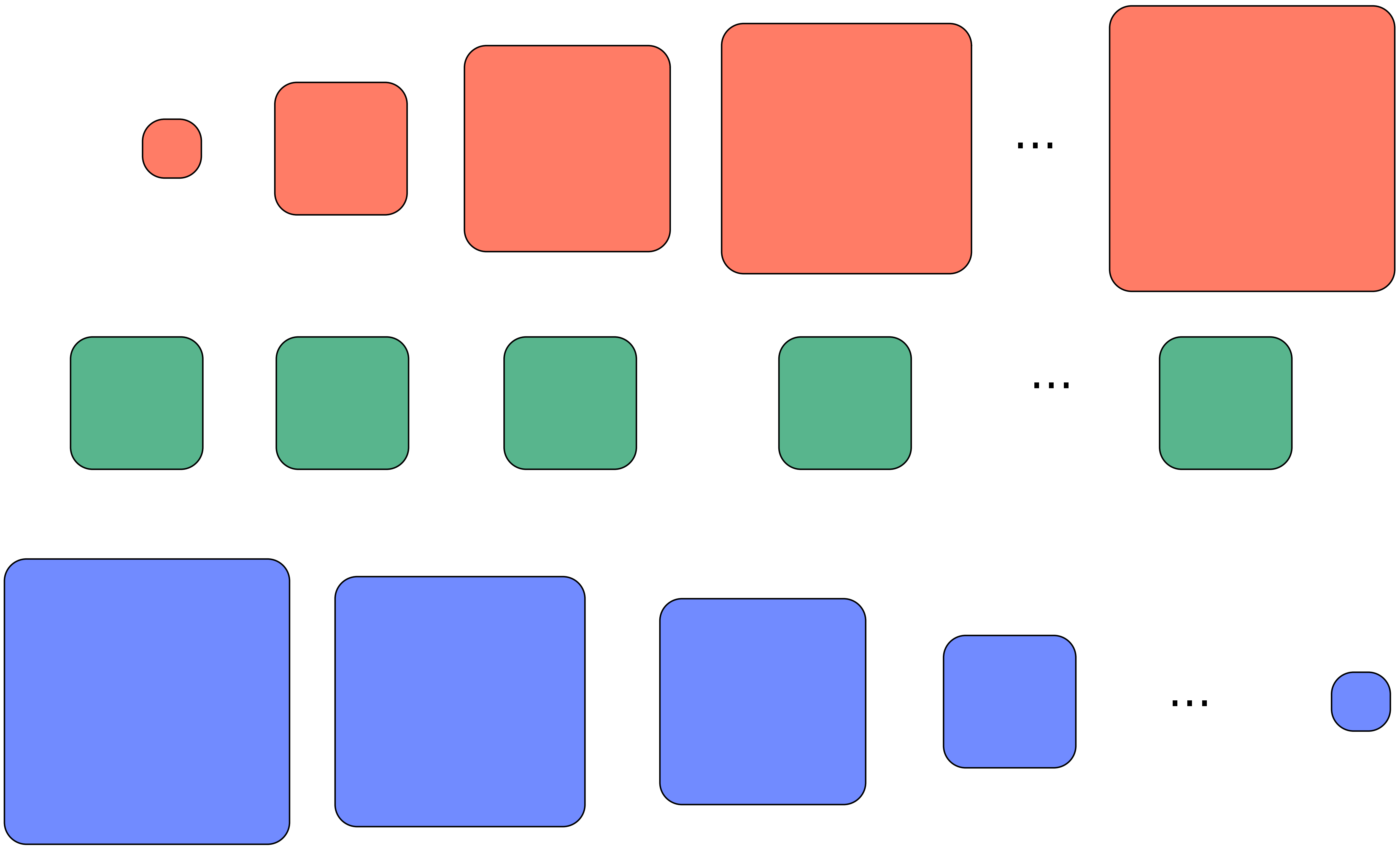
$$T(n) = aT(n/b) + f(n)$$

$T(n) = aT(n/b) + f(n)$



$$T(n) = f(n) + af\left(\frac{n}{b}\right) + a^2 f\left(\frac{n}{b^2}\right) + a^3 f\left(\frac{n}{b^3}\right) + \cdots + a^L f\left(\frac{n}{b^L}\right)$$

$$T(n) = f(n) + af\left(\frac{n}{b}\right) + a^2 f\left(\frac{n}{b^2}\right) + a^3 f\left(\frac{n}{b^3}\right) + \cdots + a^L f\left(\frac{n}{b^L}\right)$$



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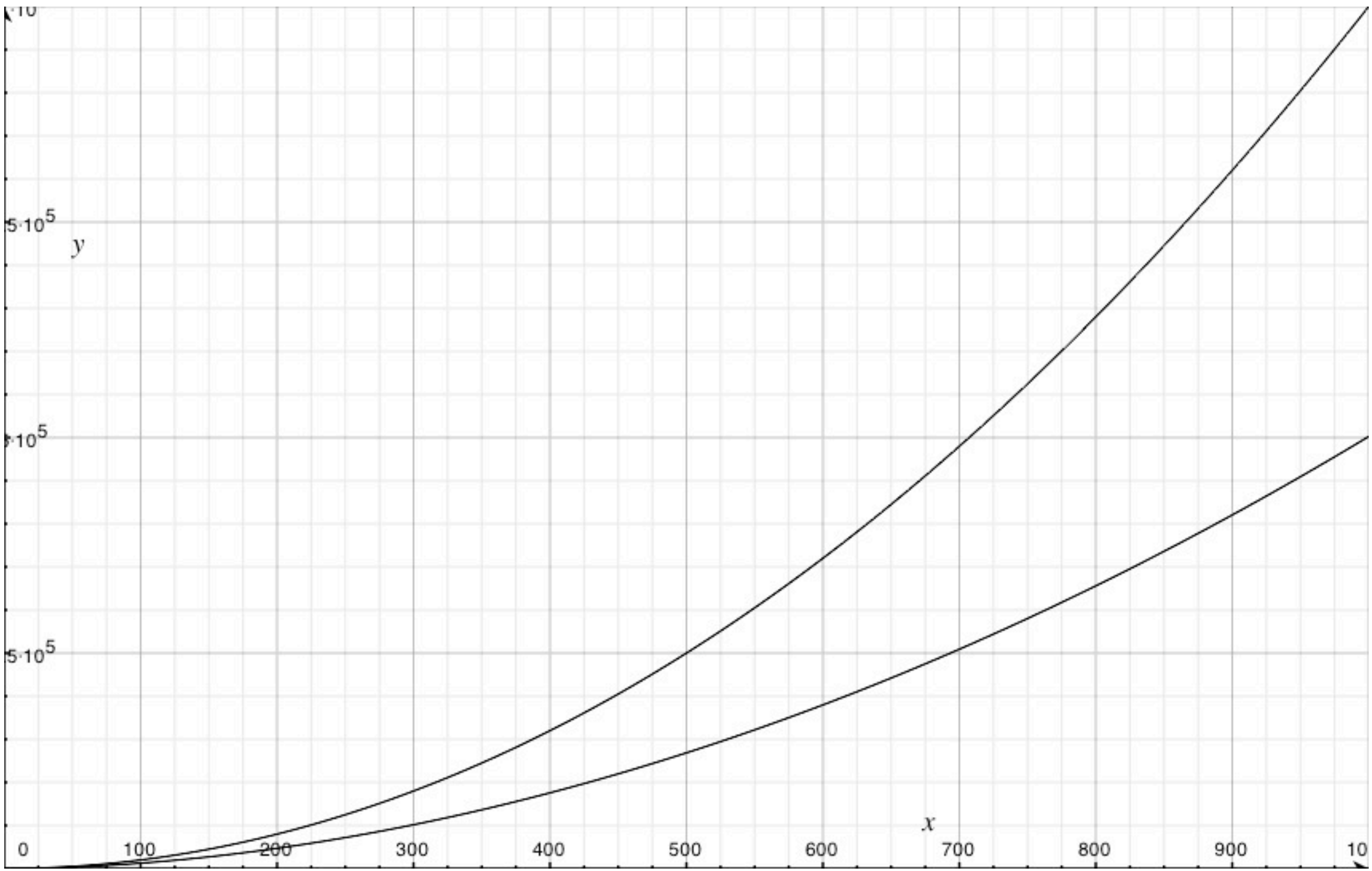
case 1: $f(n) = O(n^{\log_b a - \epsilon})$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$$T(n) = f(n) + af\left(\frac{n}{b}\right) + a^2 f\left(\frac{n}{b^2}\right) + a^3 f\left(\frac{n}{b^3}\right) + \cdots + a^L f\left(\frac{n}{b^L}\right)$$

case 1: $$f(n) = O(n^{\log_b a - \epsilon})$$

example:

a b f(n)

$$T(n) = 4T(n/2) + n$$

GOAL:

f(n) = O(n^{log_2(4)}), so case 1 applies here.

Thus, T(n) = Theta(n^{log_2(4)}) = Theta(n^2)

$$T(n) = f(n) + af\left(\frac{n}{b}\right) + a^2 f\left(\frac{n}{b^2}\right) + a^3 f\left(\frac{n}{b^3}\right) + \cdots + a^L f\left(\frac{n}{b^L}\right)$$

case 1: $f(n) = O(n^{\log_b a - \epsilon})$

L is the # of recursive levels,
and L=log_b(n)

Aim is to show that T(n) = Theta(n^{log_b a}). Towards this goal, we expand the eqtn above: f(n) < cn^{log_b(a) - eps} by the definition of big-Oh

```
T(n) = cn^{log_b(a)-eps} + a*c(n/b)^{log_b(a)-eps} + a^2*c(n/b^2)^{log_b(a)-eps} + … +
             a^(L-1)*c(n/b^(L-1))^{log_b(a)-eps} + a^L*c(n/b^L)^{log_b(a)-eps}
                                              this last term^^^ simplifies to c*a^L
```

```
obs I: a^L = a^{log_b(n)} = b^({log_b(a)}*{log_b(n)}) = (b^{log_b(n)})^{log_b(a)}
                                                       = n^{log_b(a)}

obs II: a^i*c(n/b^i)^{log_b(a)-eps} =  c*n^{log_b(a)-eps} (a^i / (b^i)^{log_b(a)-eps} )
                                    =  c*n^{log_b(a)-eps} (a^i / (a^i b^{-eps*i} )
                                    =  c*n^{log_b(a)-eps}  b^{eps*i} )
```

$$1 + b + b^2 + \cdots + b^{L-1} =$$

$$T(n) = f(n) + af\left(\frac{n}{b}\right) + a^2 f\left(\frac{n}{b^2}\right) + a^3 f\left(\frac{n}{b^3}\right) + \cdots + a^L f\left(\frac{n}{b^L}\right)$$

L=log_b(n)

case 1 (cont):

$$T(n) \leq cn^{\log_b a - \epsilon}\left[1 + b^{\epsilon} + b^{2\epsilon} + \cdots + b^{\epsilon(L-1)}\right] + n^{\log_b a}$$

>=

b^{L*eps} -1
b^eps -1

n^epsilon - 1
b^eps - 1

T(n) < c'' * n^{log_b(a)}

T(n) = O(n^{\log_b(a)}

ALso need to show that T(n) = Omega(n^{\log_b(a)})

$$T(n) = f(n) + af\left(\frac{n}{b}\right) + a^2 f\left(\frac{n}{b^2}\right) + a^3 f\left(\frac{n}{b^3}\right) + \cdots + a^L f\left(\frac{n}{b^L}\right)$$

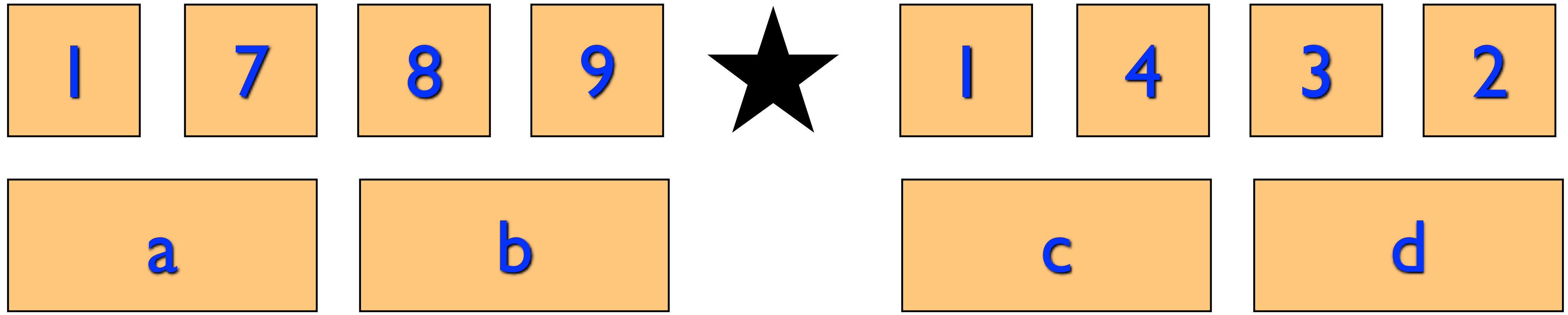
case 2: $f \in \Theta(n^{\lg_b a})$

then T(n) = Theta (n^{log_b a} log(n))

case 3: $f \in \Omega(n^{\lg_b a+\epsilon})$ and...

then T(n) = Theta(f(n))

example 2: $T(n) = 8T(n/2) + \Theta(n^2)$



1
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4
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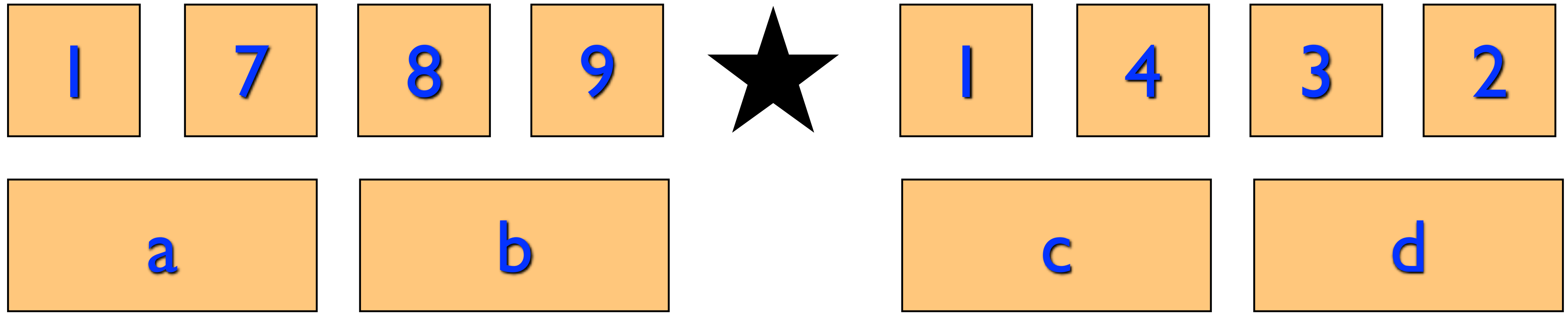
a
b
c
d



$$T(n) = 4T(n/2) + 3O(n)$$

$$T(n) = 2T(n/2) + n^3$$

$$T(n) = \mathbf{7}T(n/2) + O(n^2)$$

example:

$$T(n) = T\left(\frac{14}{17}n\right) + 24$$

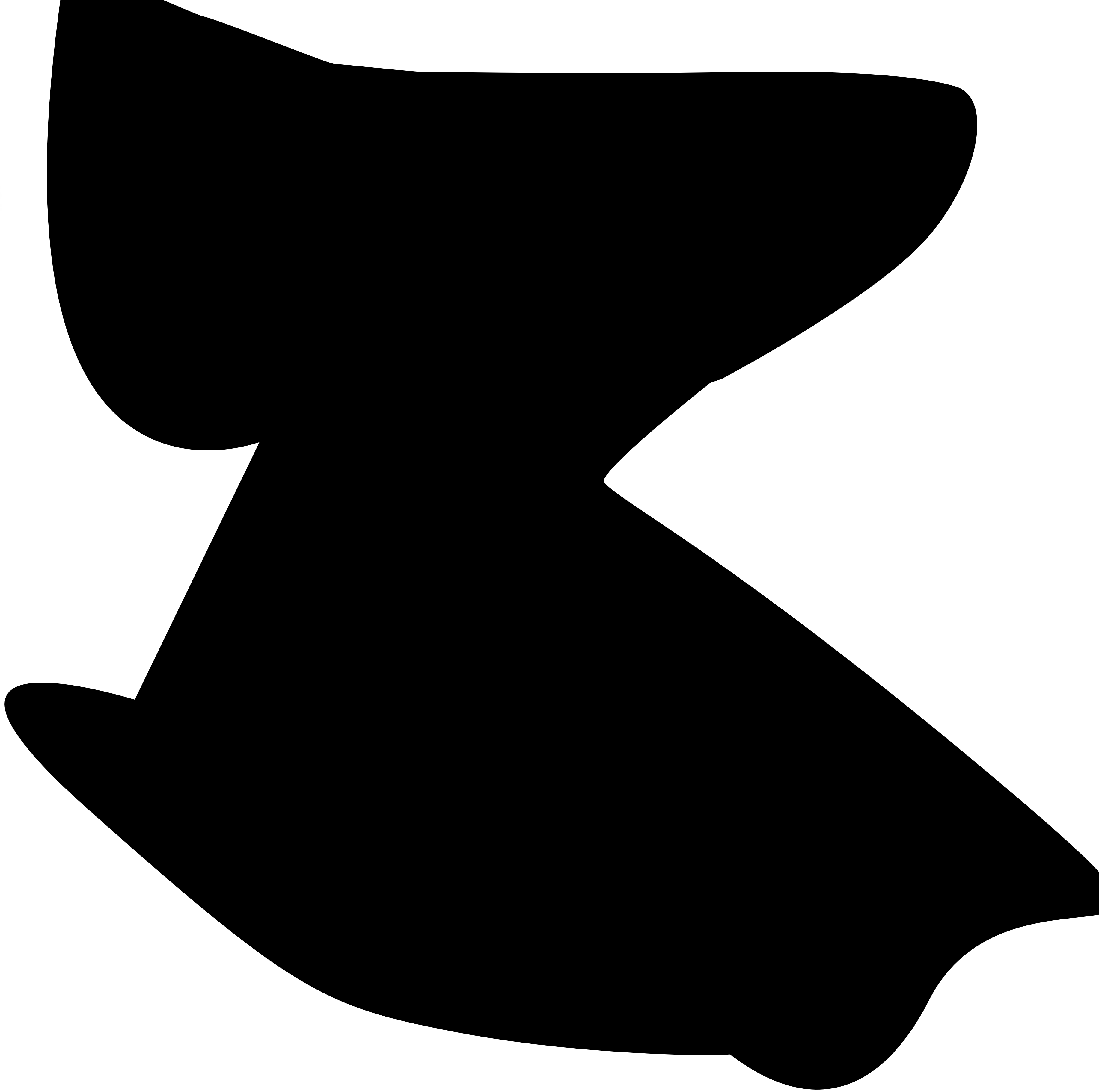

8
13
FIFA

$$T(n) = 2T\left(\sqrt{n}\right) + \lg n$$

```
T(2^m) = 2T( sqrt(2^m) ) + lg(2^m)
         = 2T( 2^{m/2} ) + c*m     --- c is some constant to hangle the "natural" lg
So in other words, 2^m = n

S(m)   = T(2^m)
S(m/2) = T(2^{m/2})

S(m) = 2S(m/2) + c*m

S(m) = Theta(m * log m)

If 2^m = n, then m = log(n).

Thus, T(n) = T(2^m) = Theta(m * log m) = Theta( log(n)log(log(n)) )
```

divide

& conquer



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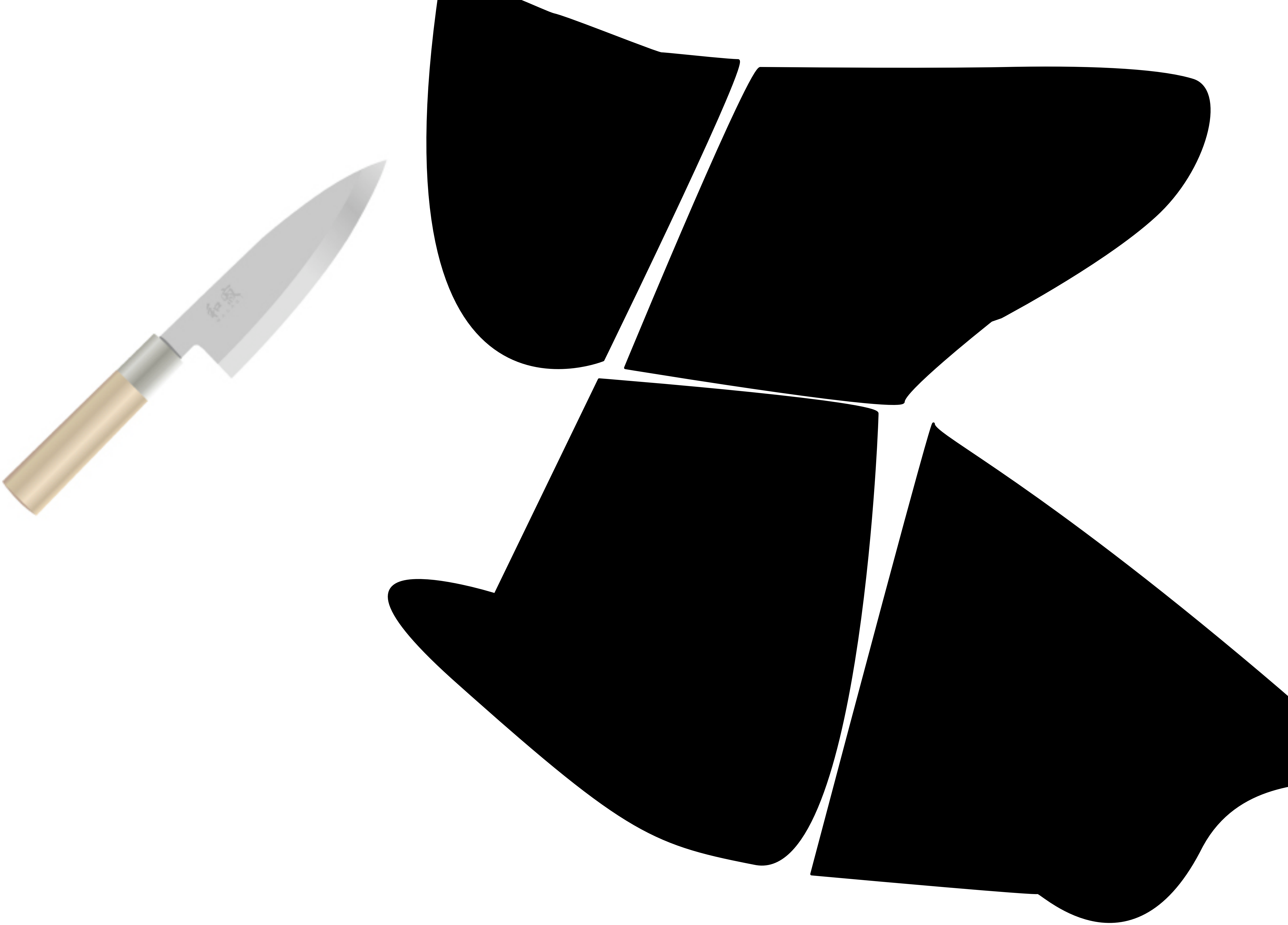

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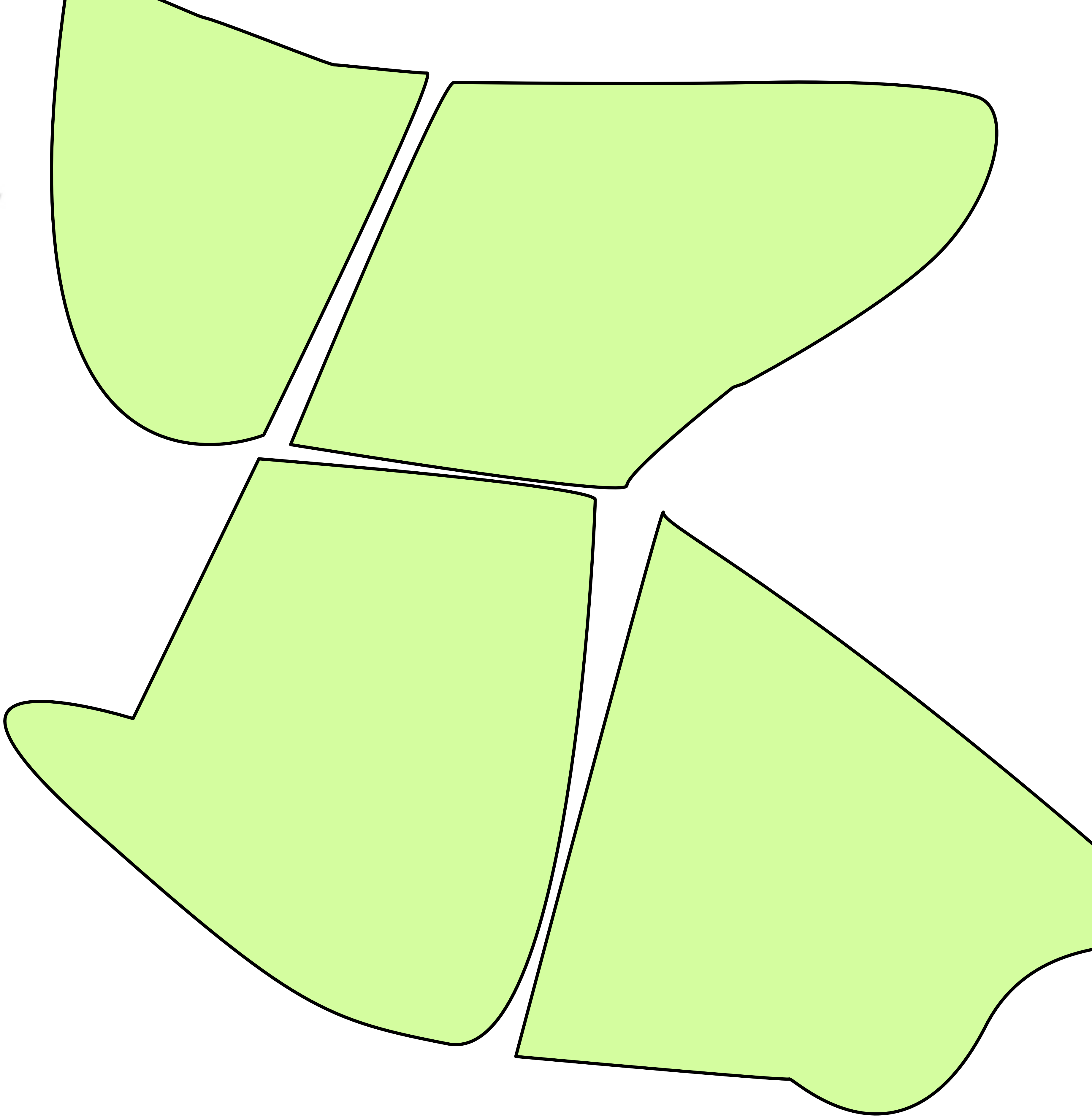

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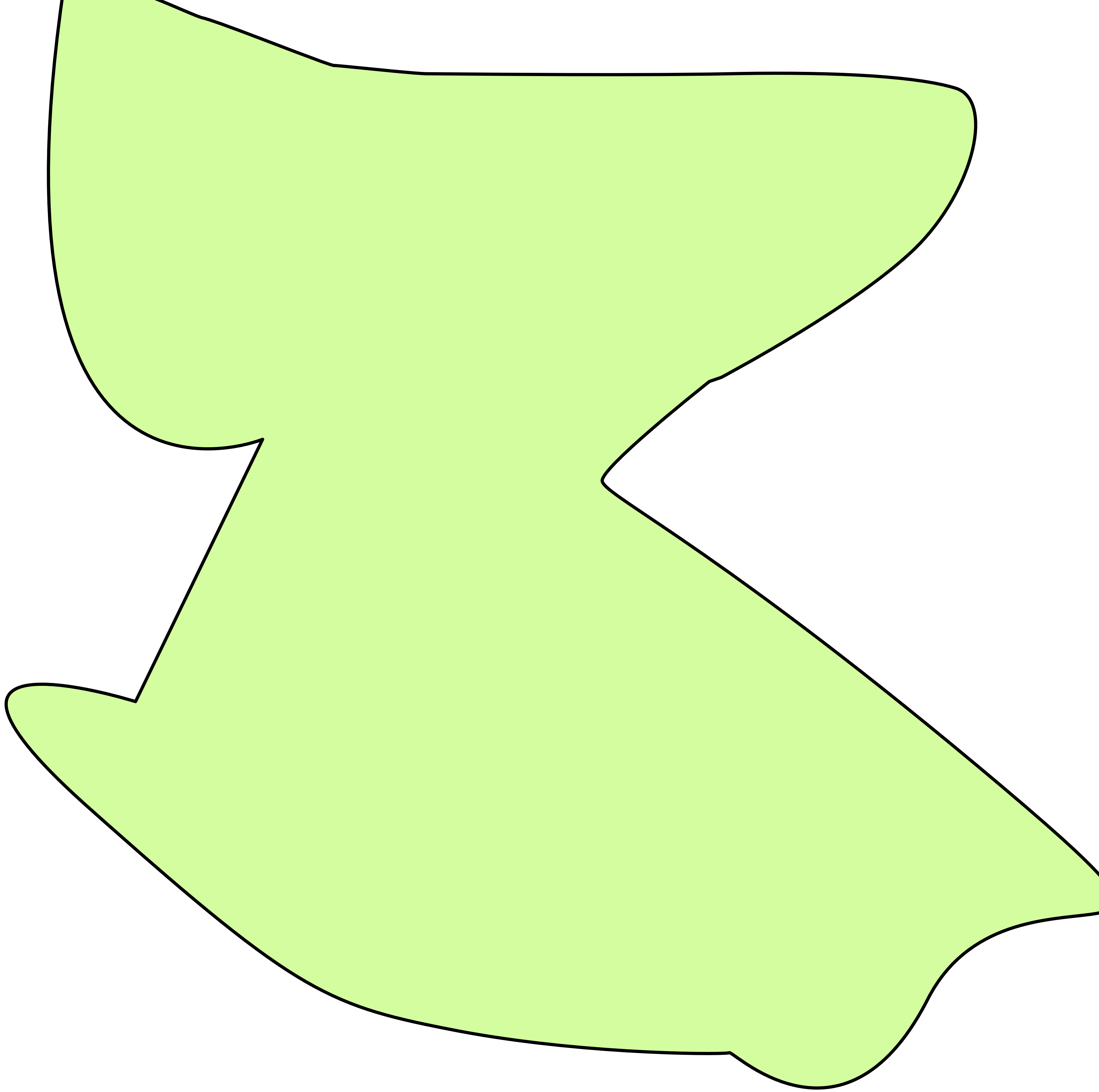


http://www.kitchenknifedrawer.com/files/1696205/uploaded/K6615D.jpg

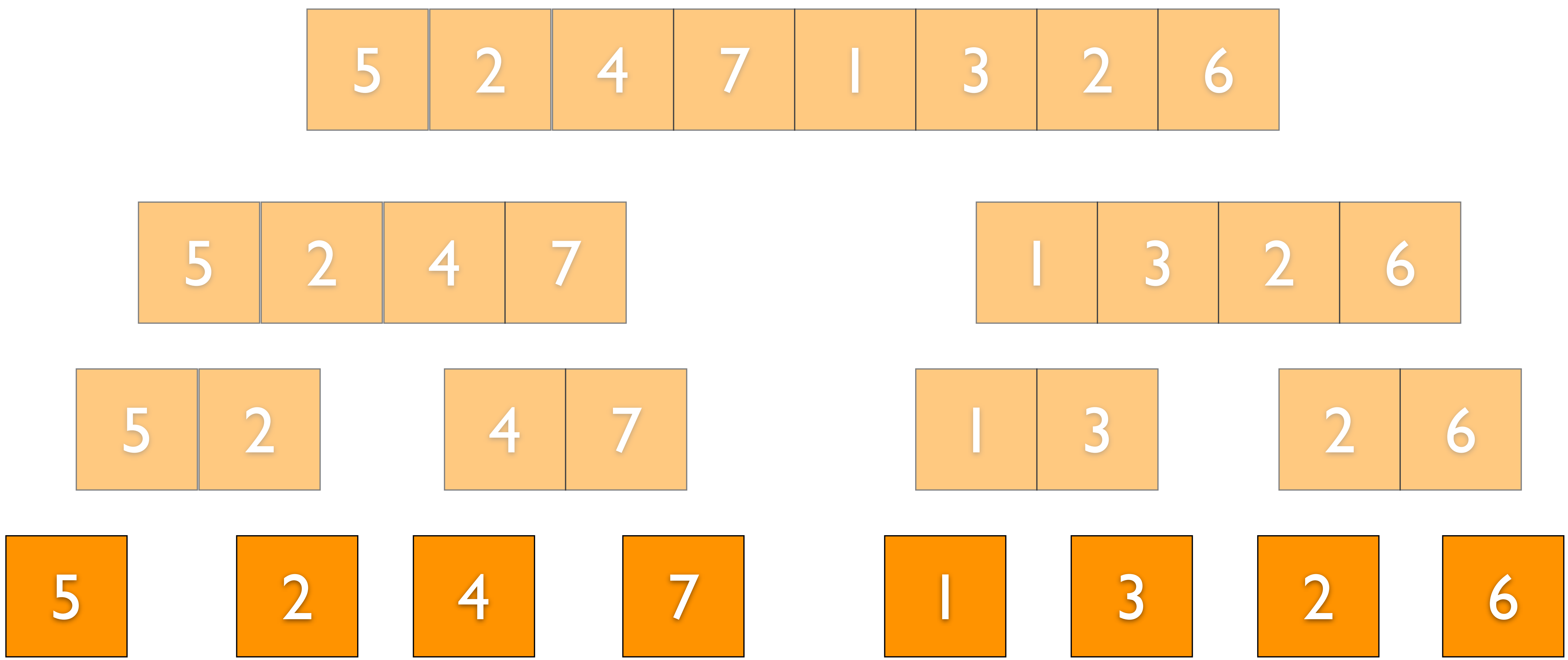
examples

Merge

merge-sort (A, p, r)
  if $p < r$
    $q \leftarrow \lfloor (p + r)/2 \rfloor$
    merge-sort (A, p, q)
    merge-sort $(A, q + 1, r)$
    merge(A, p, q, r)

MERGE$(A[1..n], m)$:
  $i \leftarrow 1$; $j \leftarrow m + 1$
  for $k \leftarrow 1$ to n
    if $j > n$
      $B[k] \leftarrow A[i]$; $i \leftarrow i + 1$
    else if $i > m$
      $B[k] \leftarrow A[j]$; $j \leftarrow j + 1$
    else if $A[i] < A[j]$
      $B[k] \leftarrow A[i]$; $i \leftarrow i + 1$
    else
      $B[k] \leftarrow A[j]$; $j \leftarrow j + 1$
  for $k \leftarrow 1$ to n
    $A[k] \leftarrow B[k]$

jeff erickson



merge-sort (A, p, r)

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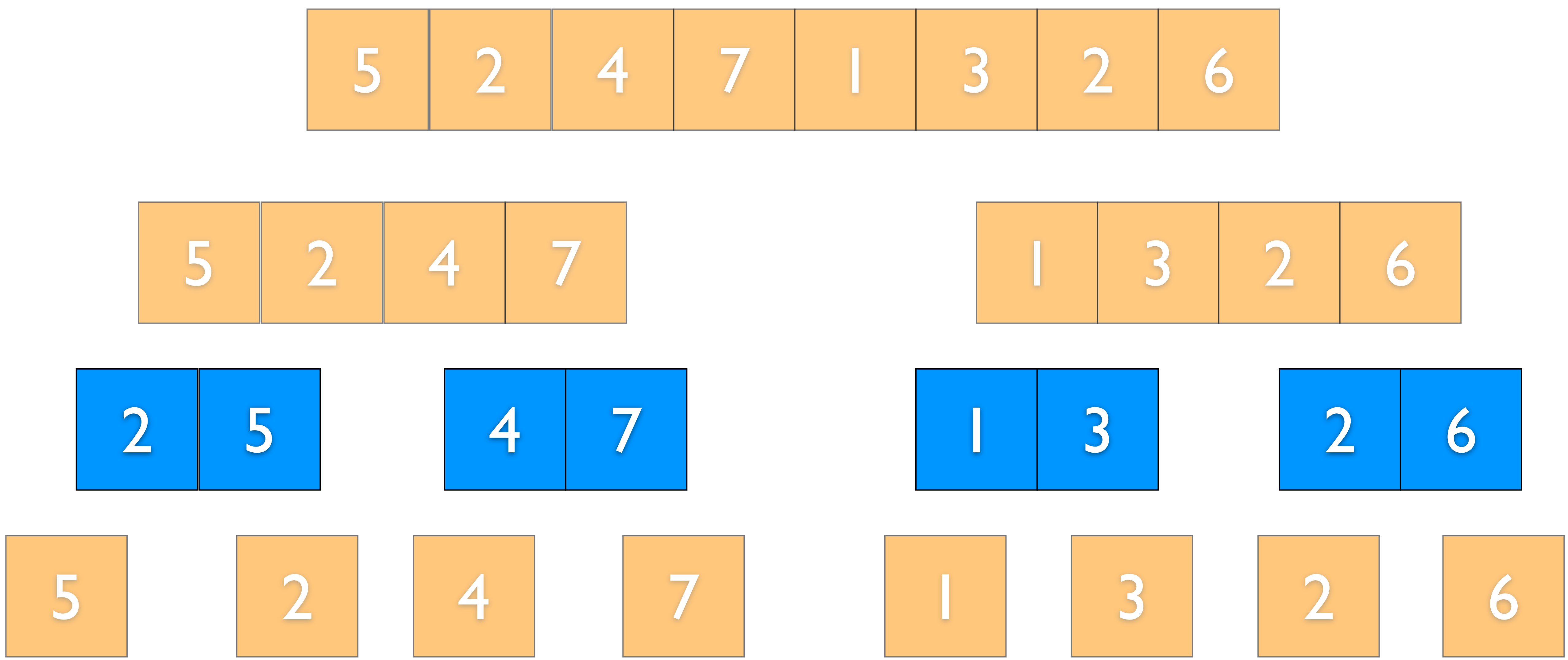
    merge-sort $(A, q+1, r)$

    merge(A, p, q, r)



merge-sort (A, p, r)

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merge-sort (A, p, q)

merge-sort $(A, q + 1, r)$

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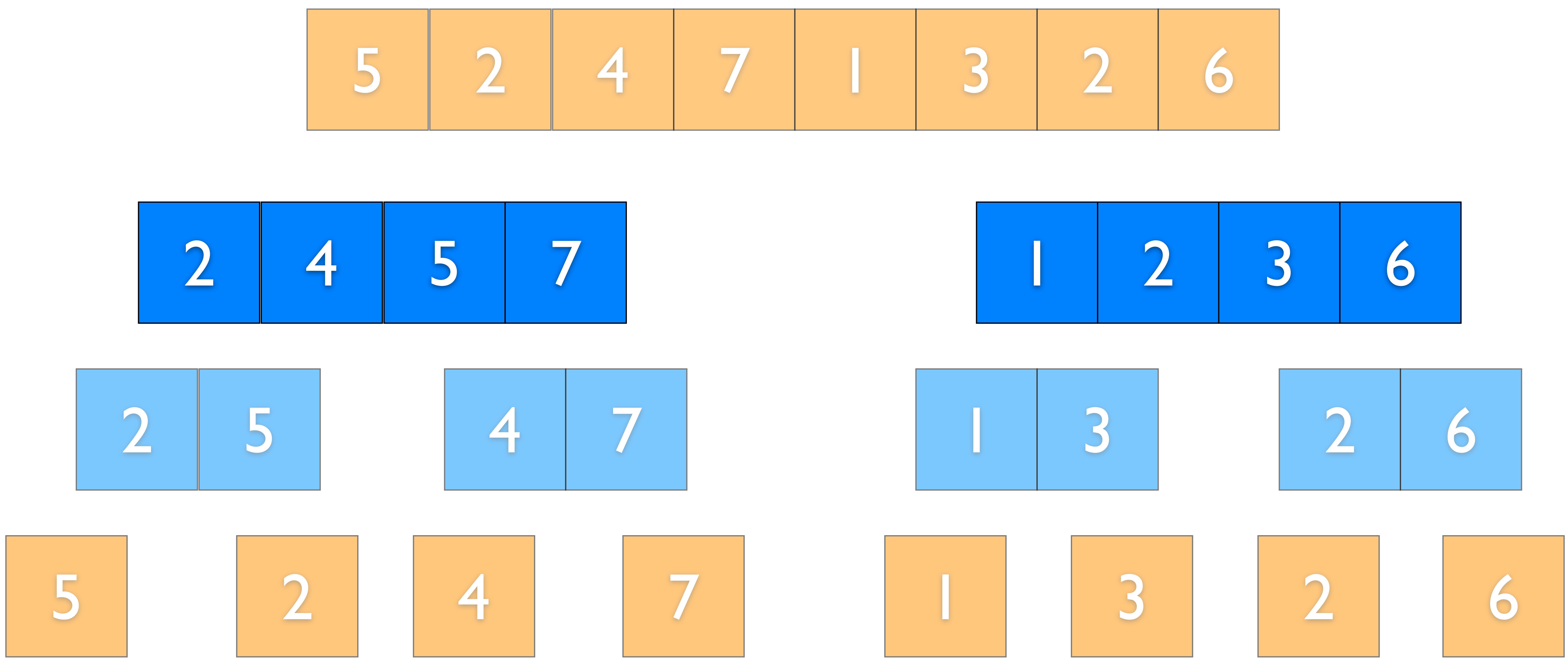

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if $p < r$

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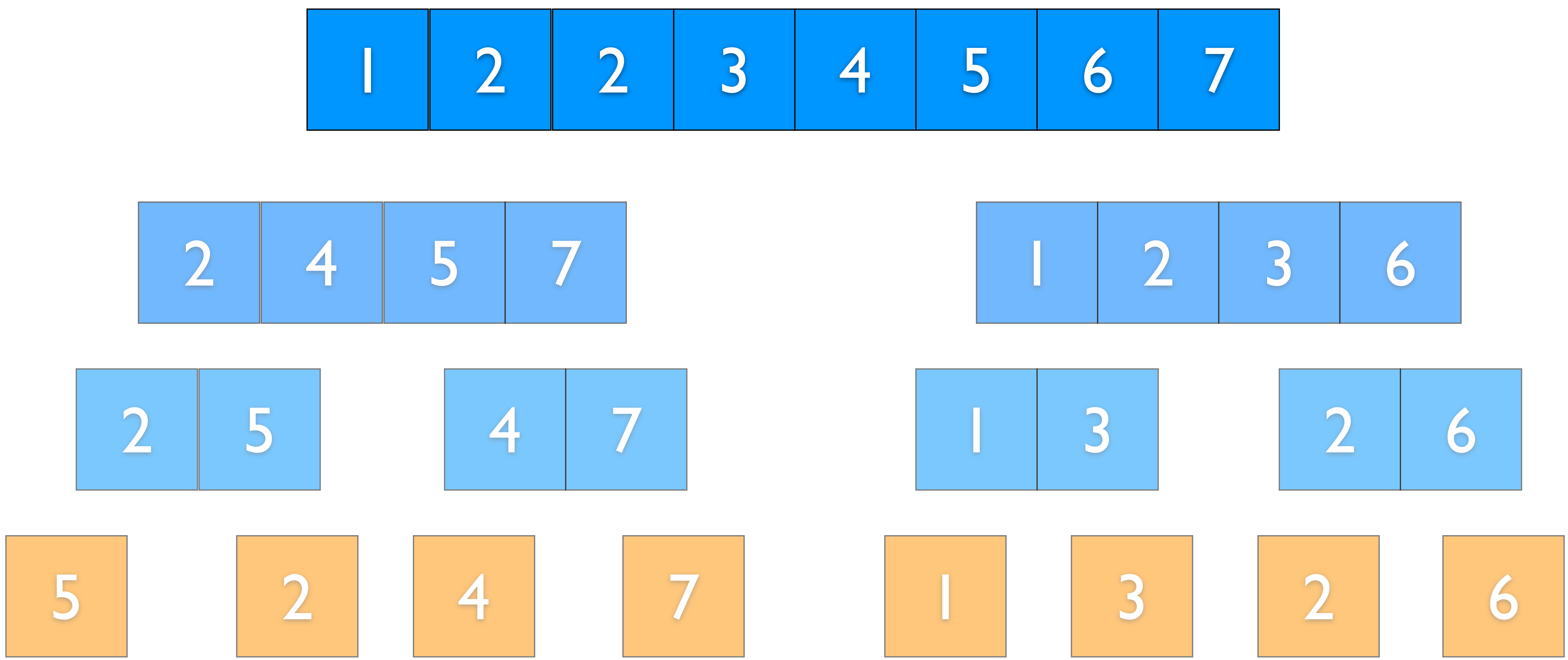

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        merge-sort (A, p, q)
        merge-sort $(A, q+1, r)$
        merge(A, p, q, r)

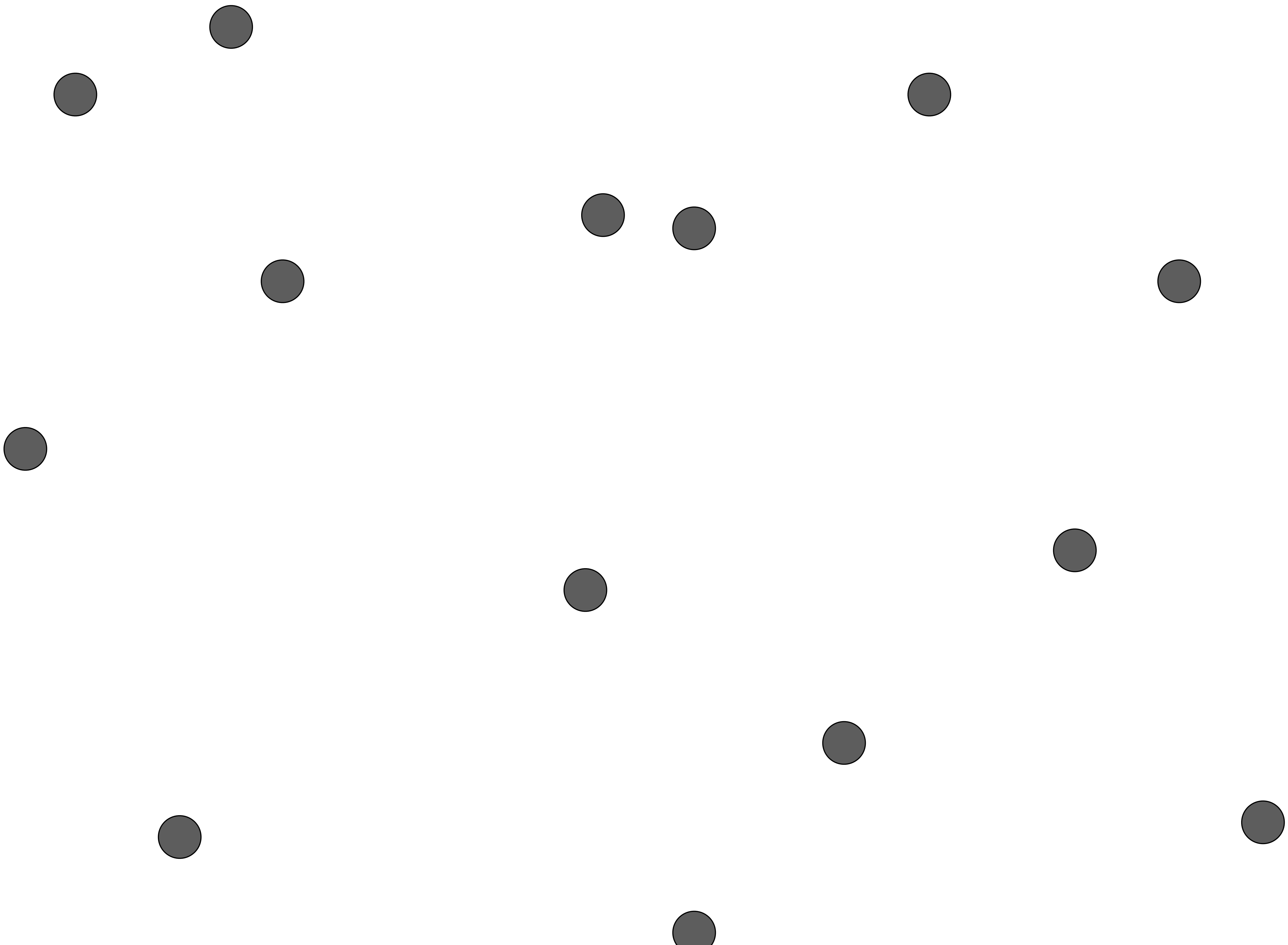
$$T(n) = 2T(n/2) + O(n)$$

$$= \Theta(n \log n)$$

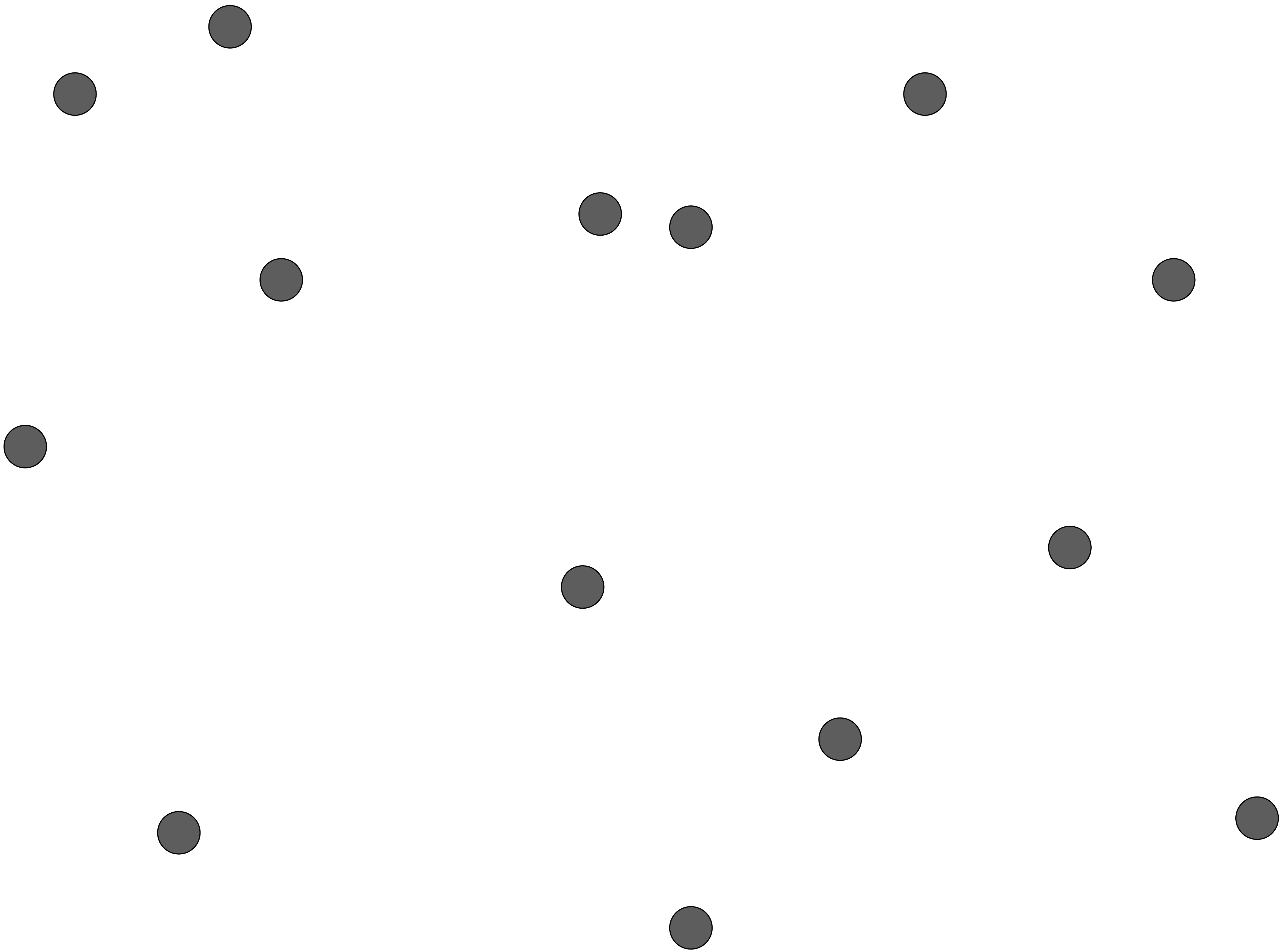
closest

of points


n points in D dimensions

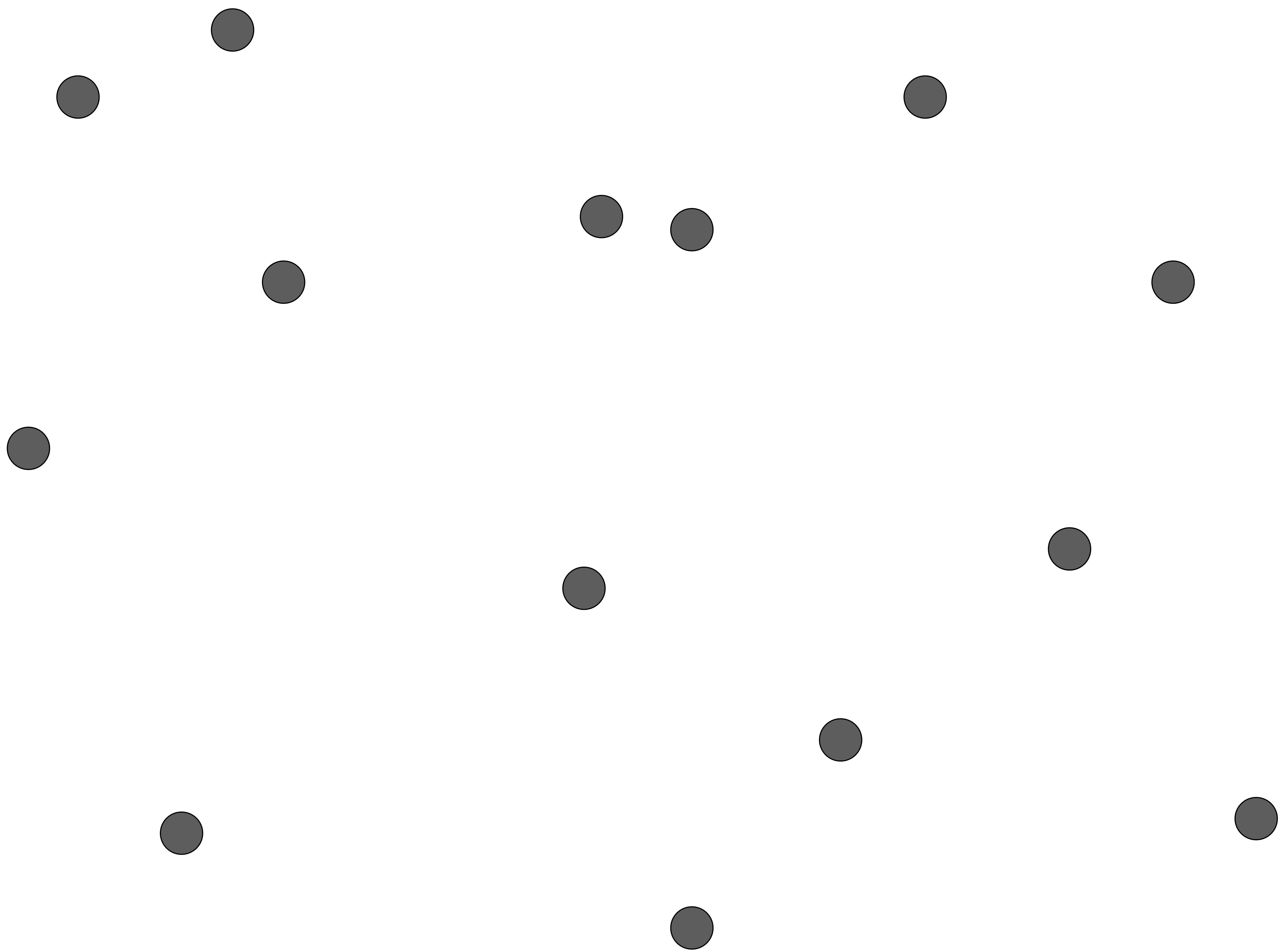


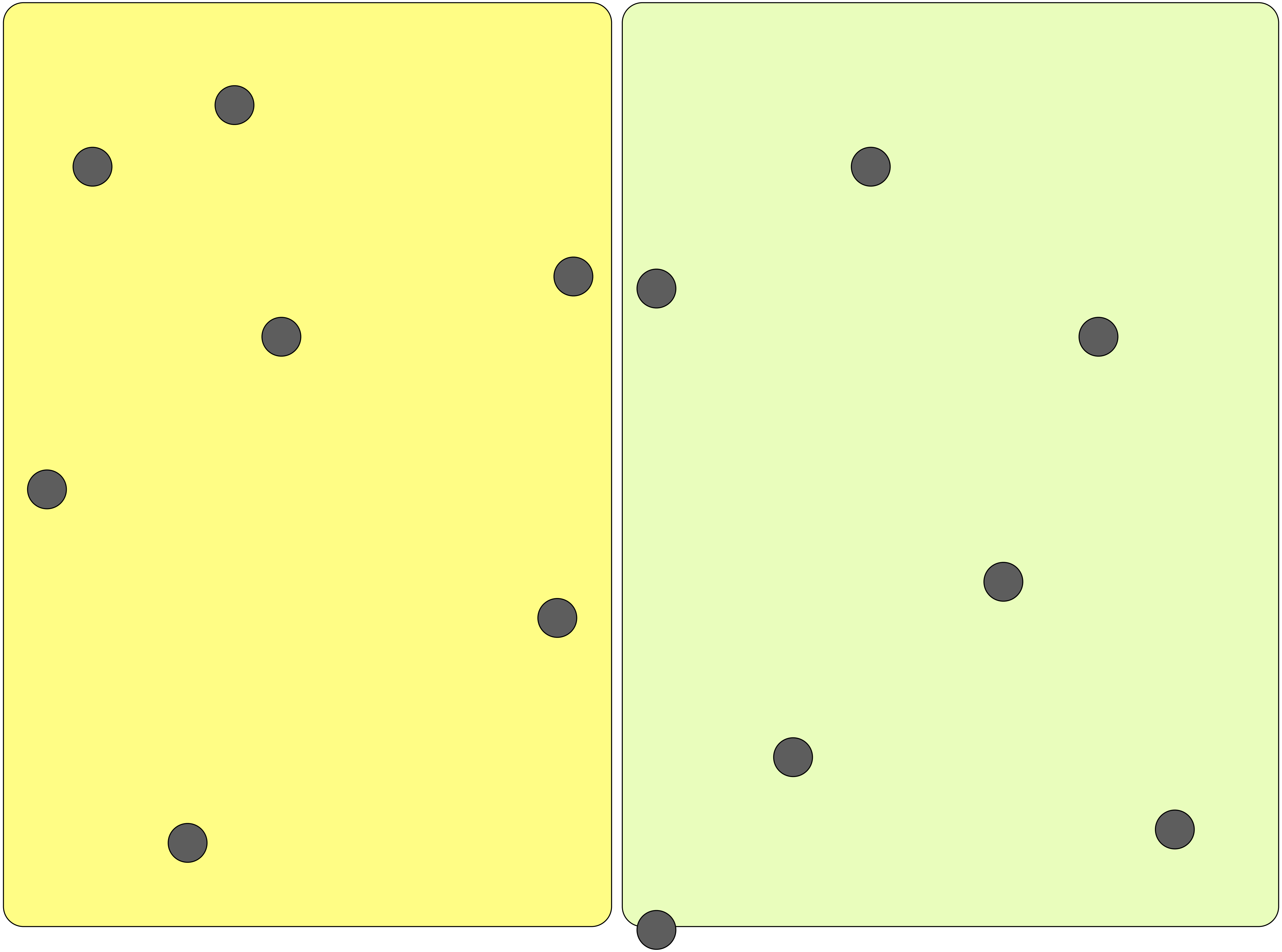
simple solution: brute force:

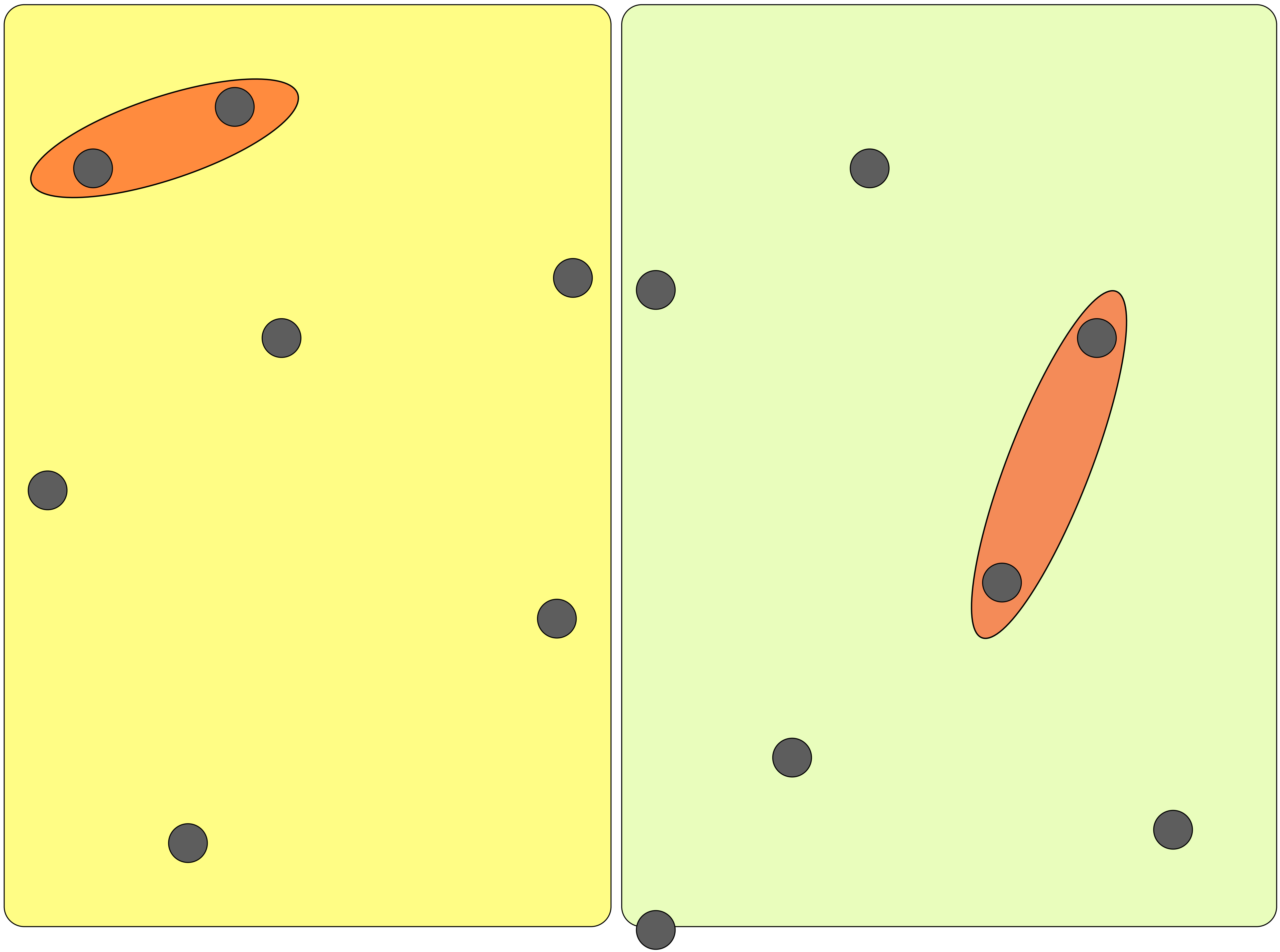


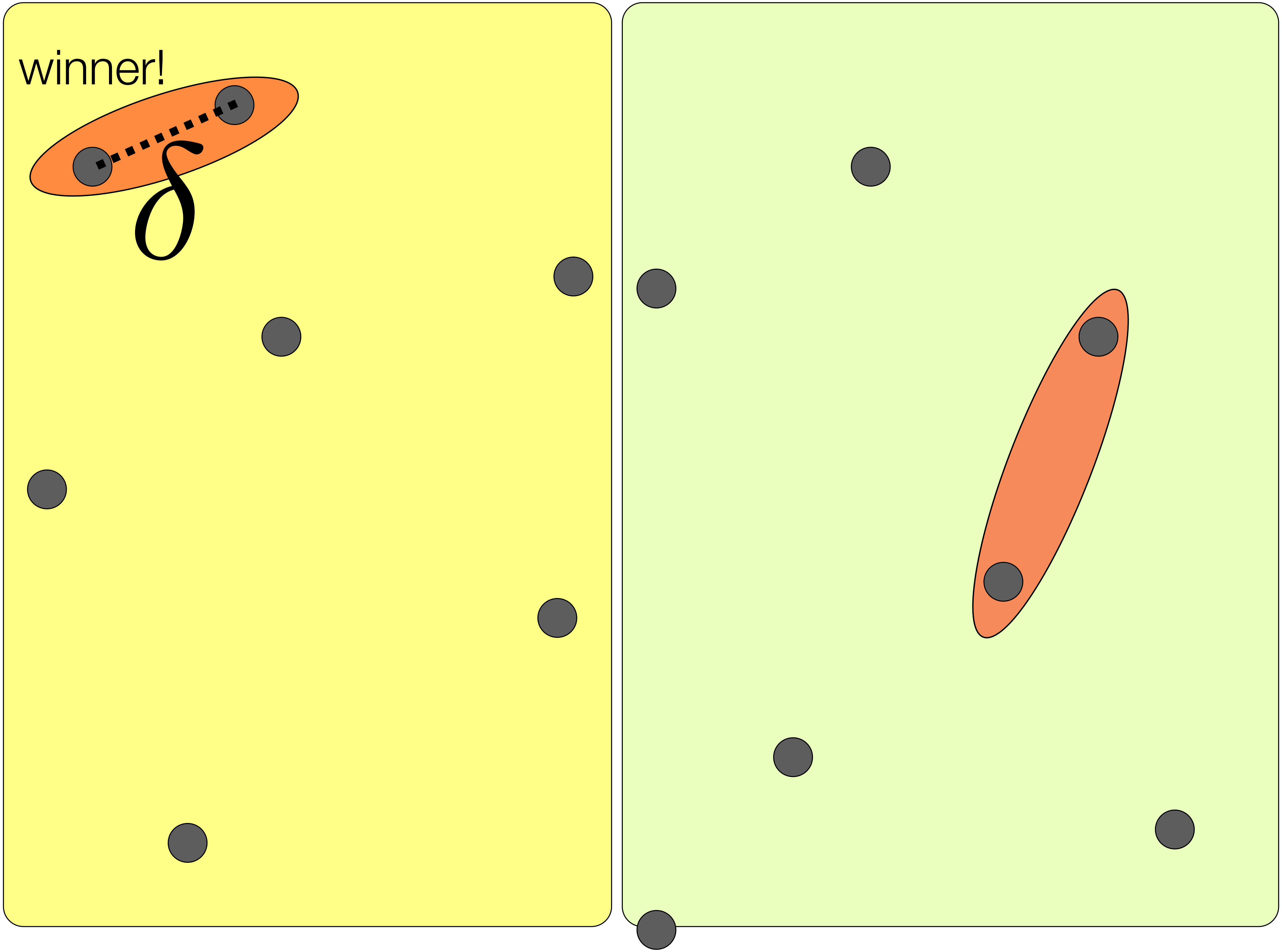
solve the large problem by

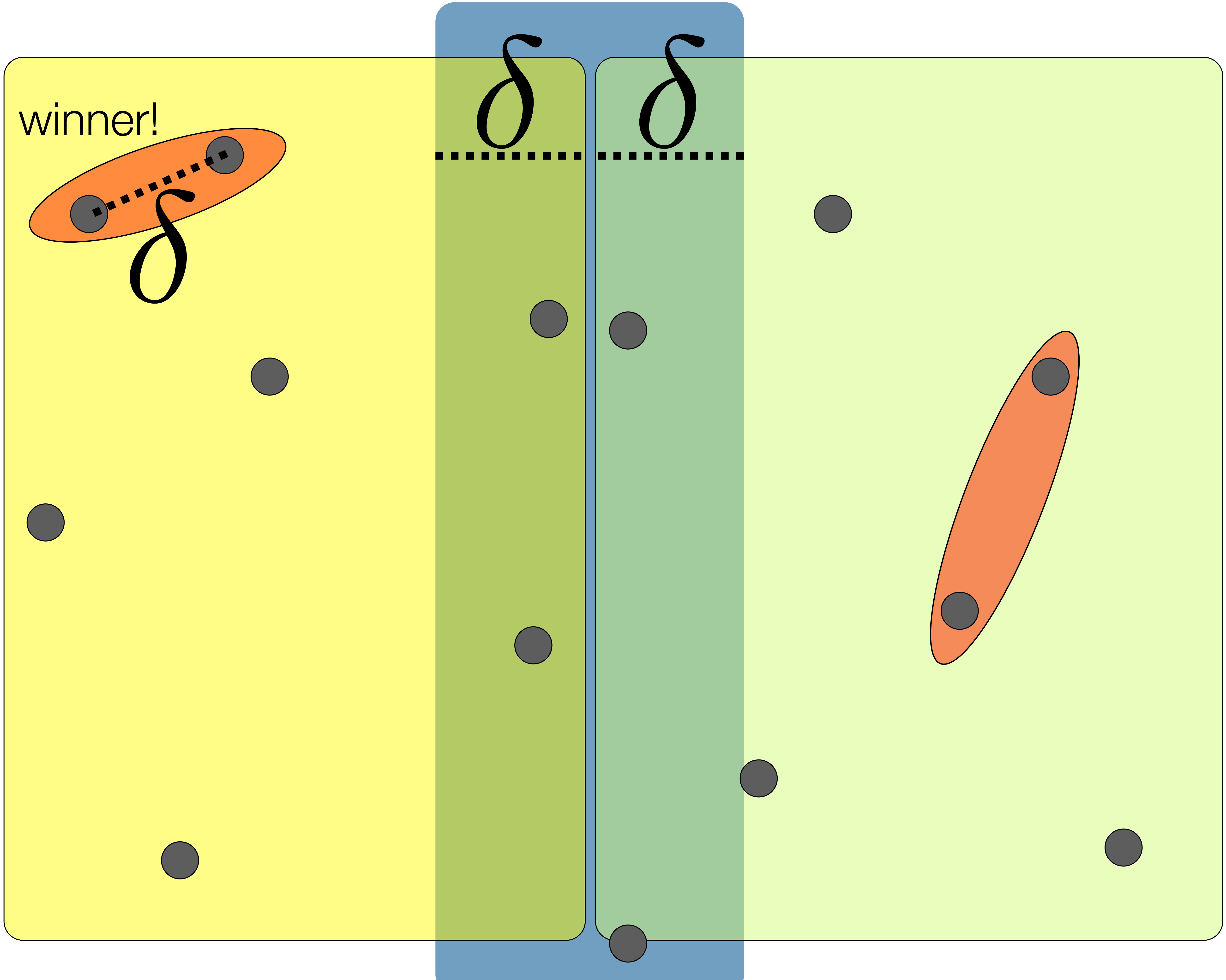
solving smaller problems
and combining solutions

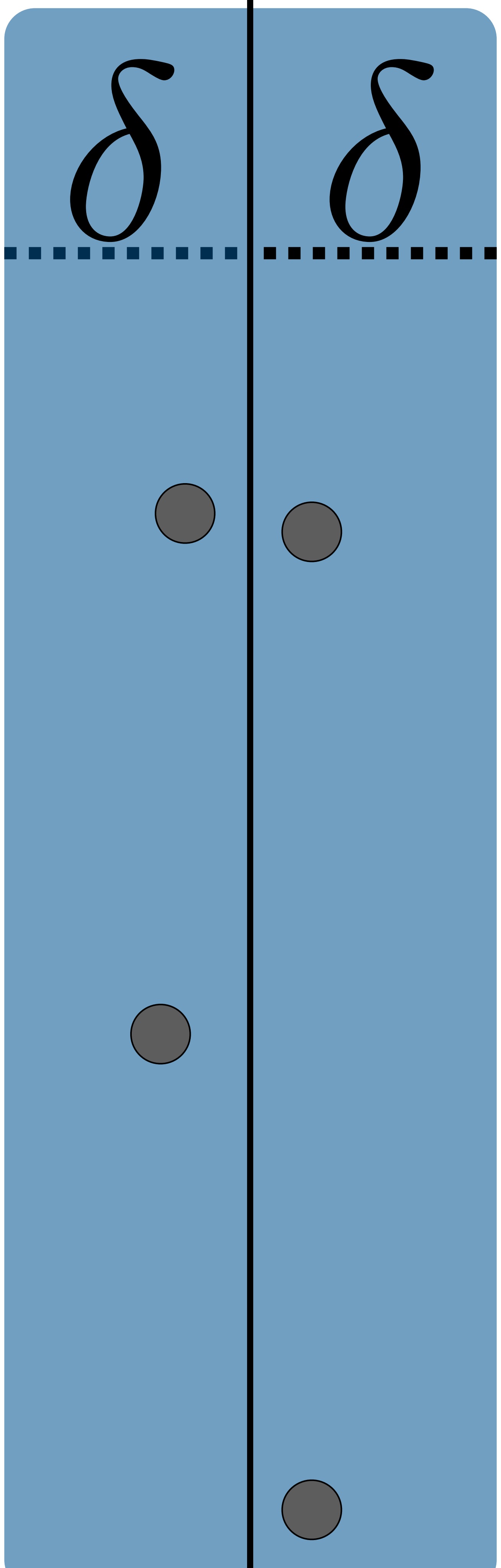


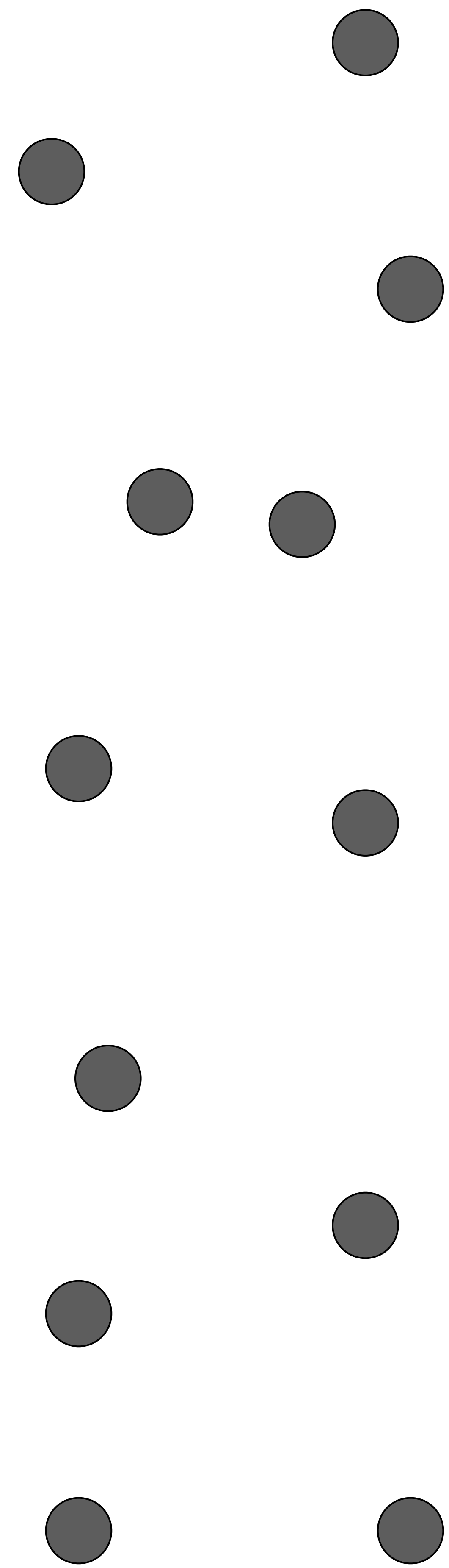


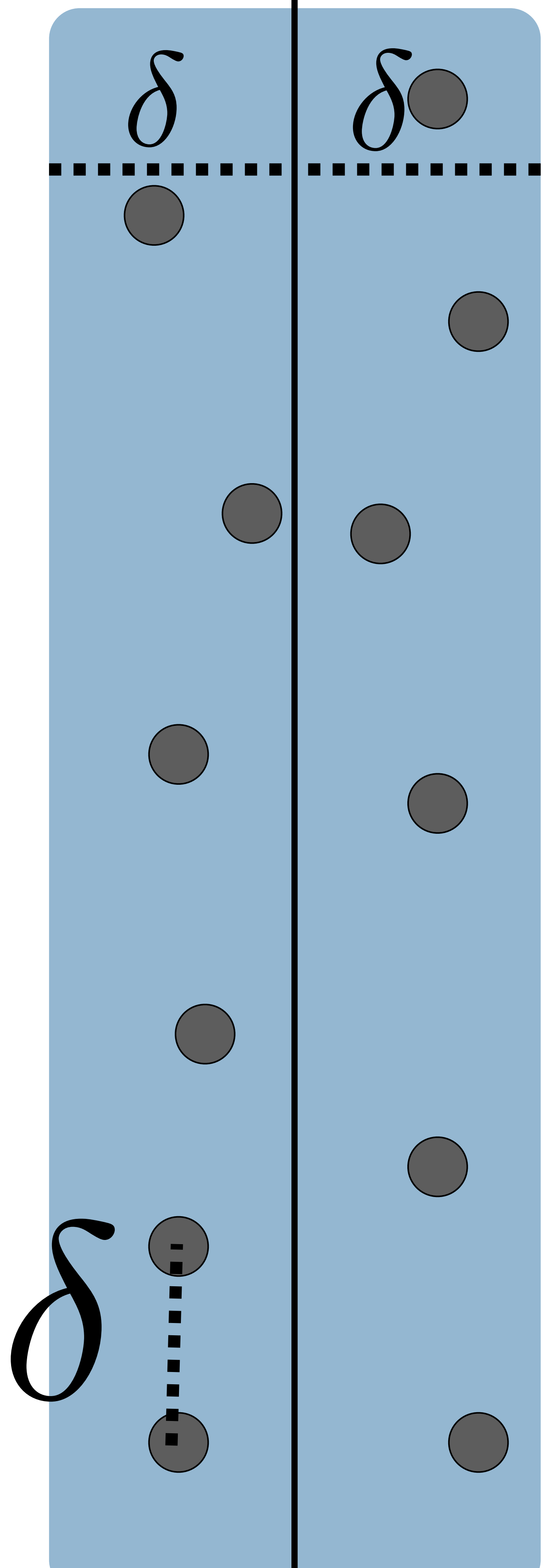


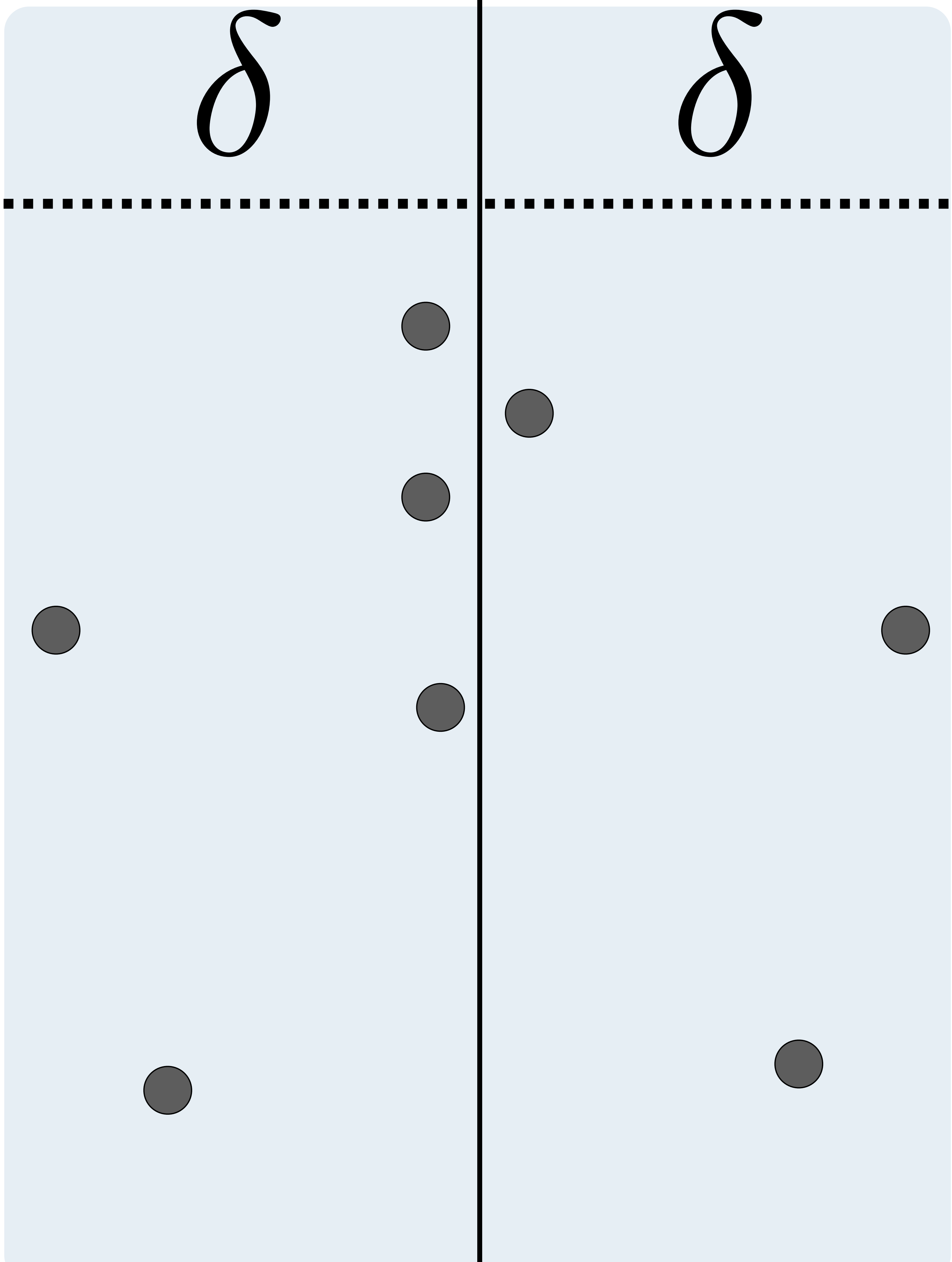

winner!
δ

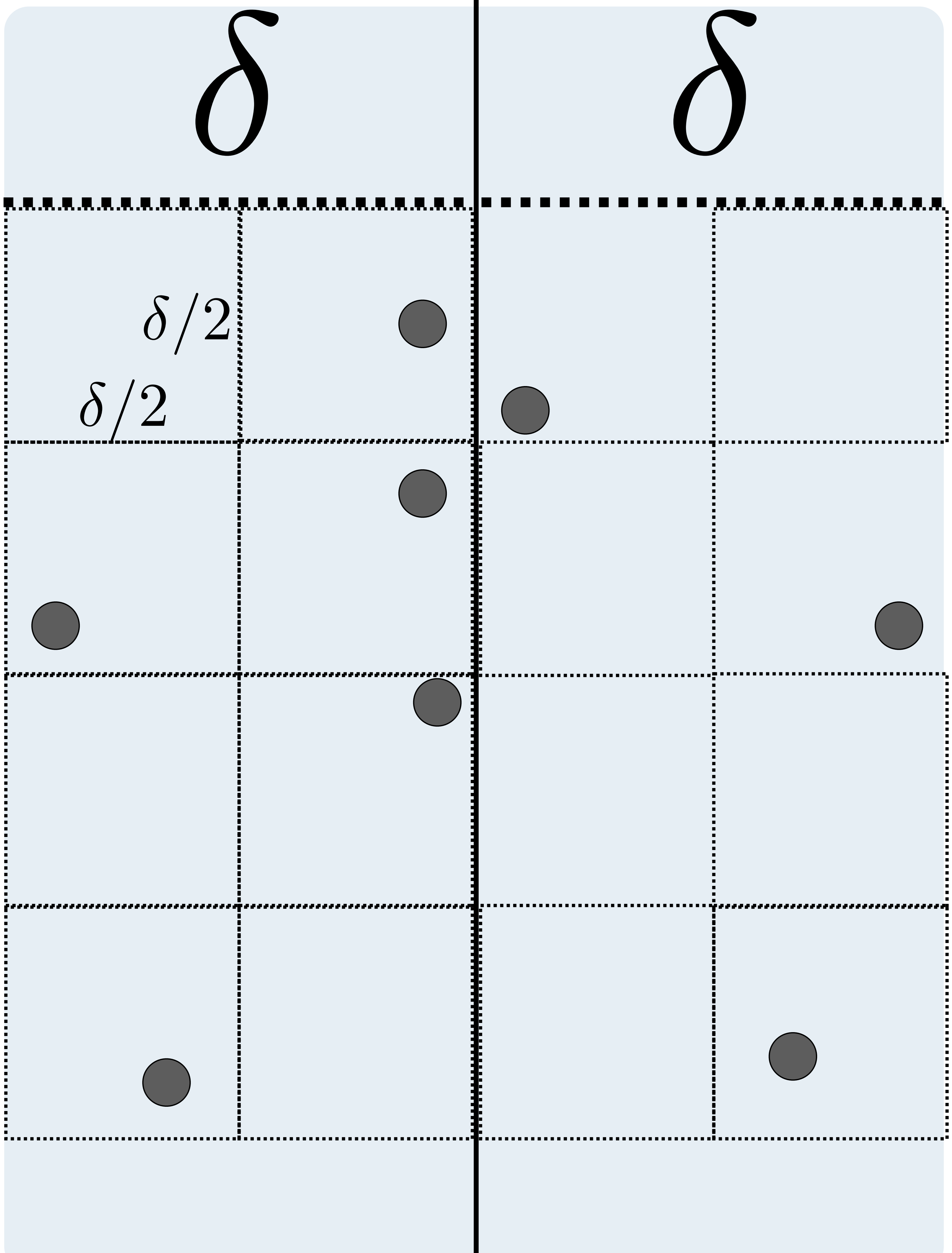

winner!
δ
δ
δ

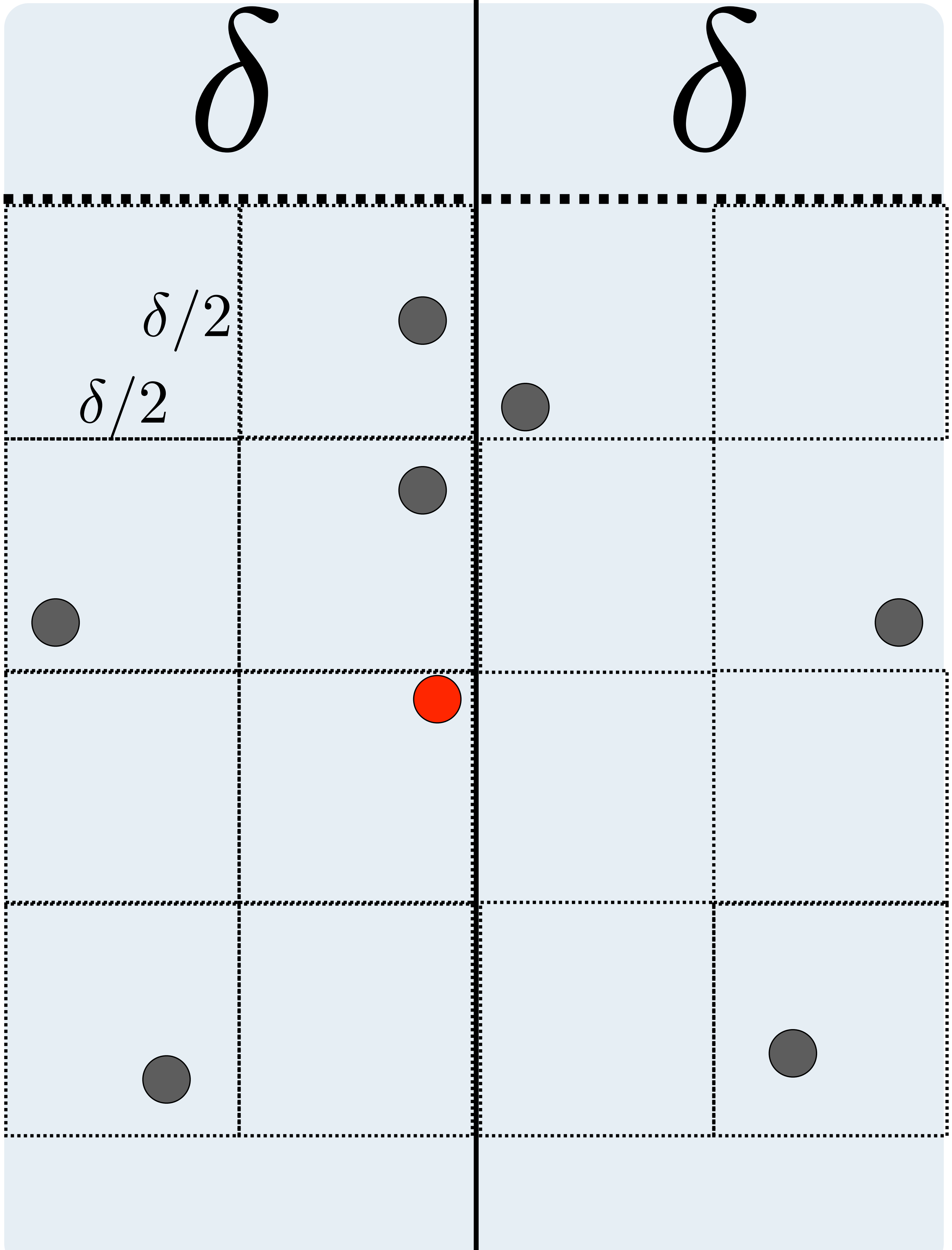

δ
δ

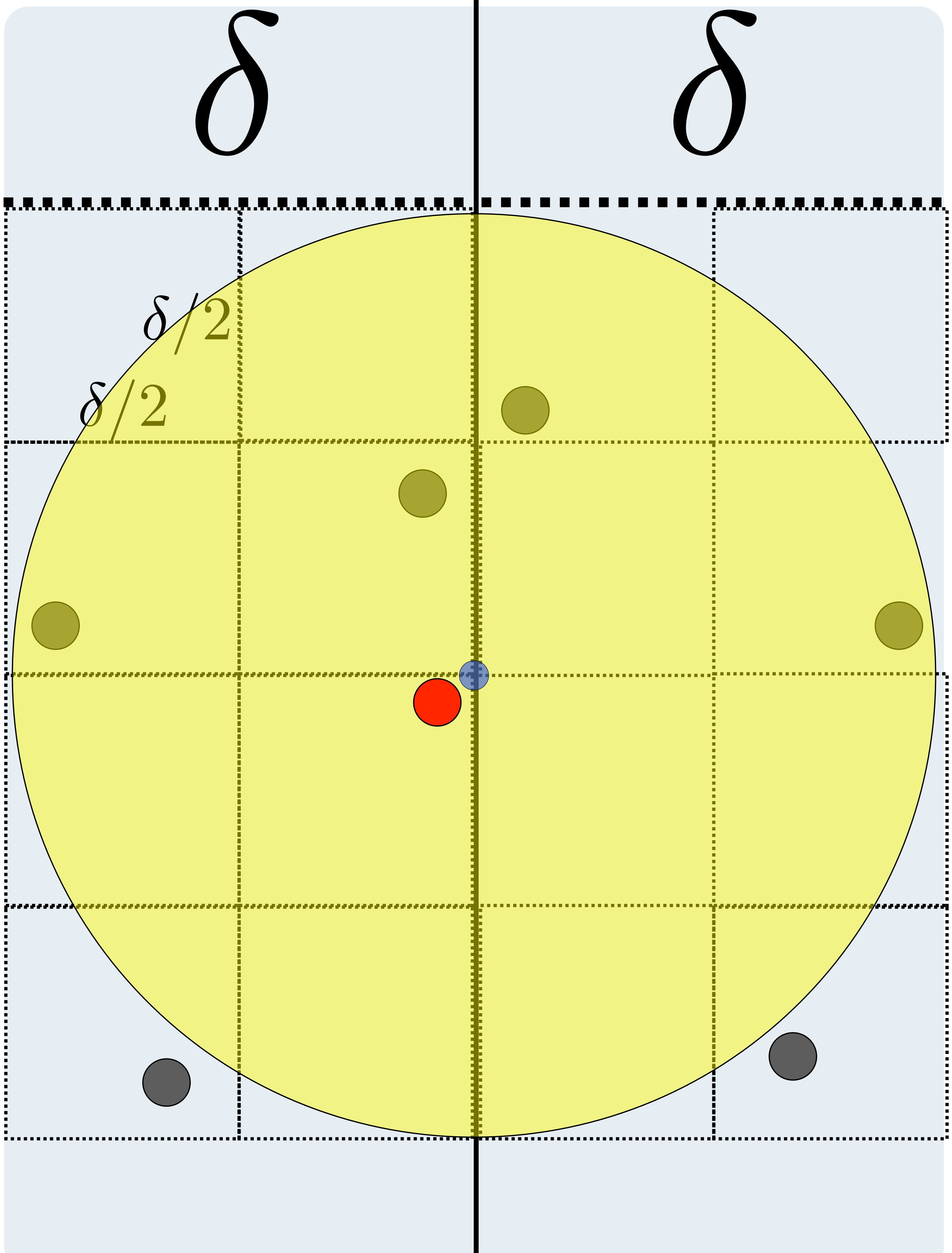



δ
δ
δ

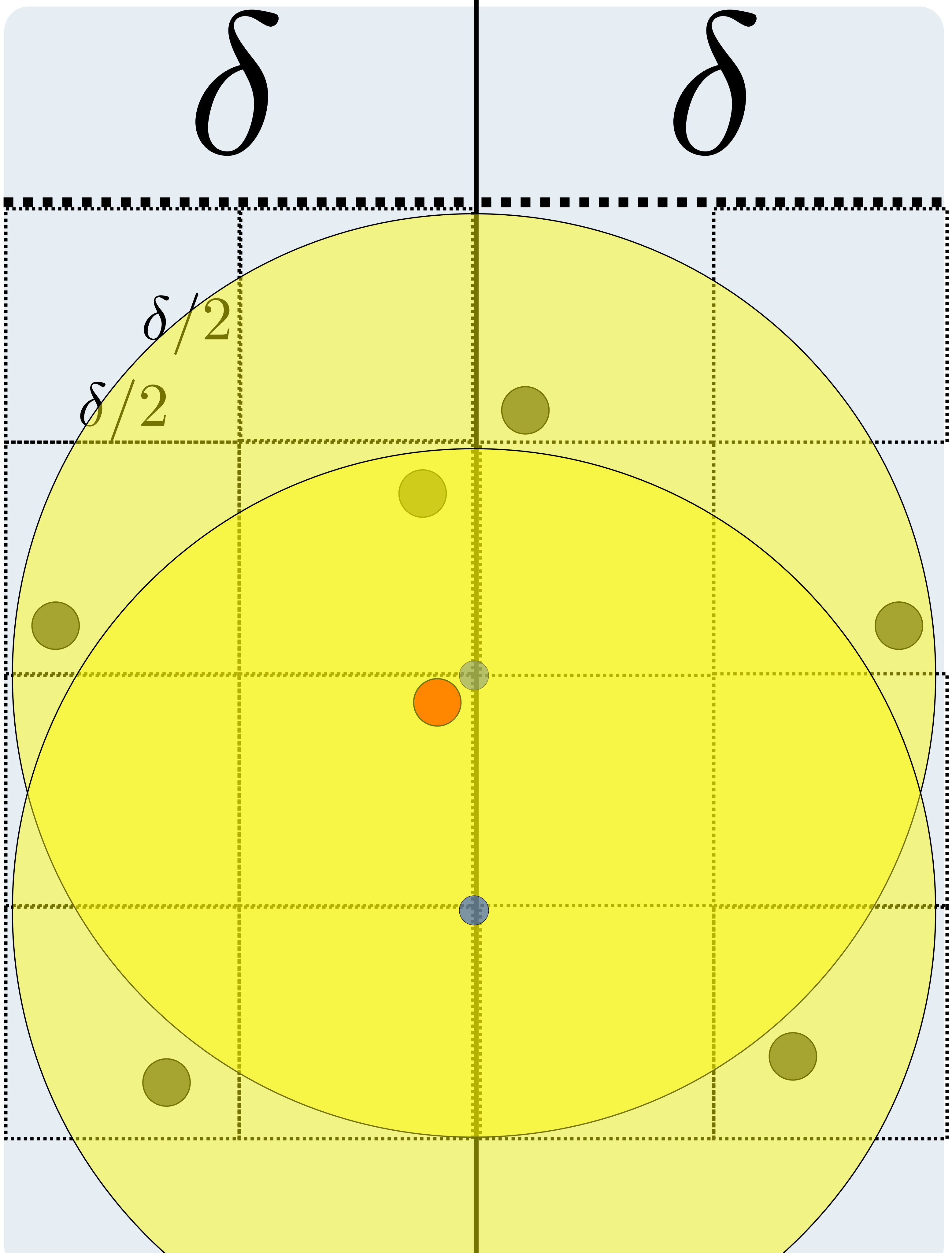

δ
δ


δ
δ
δ/2
δ/2

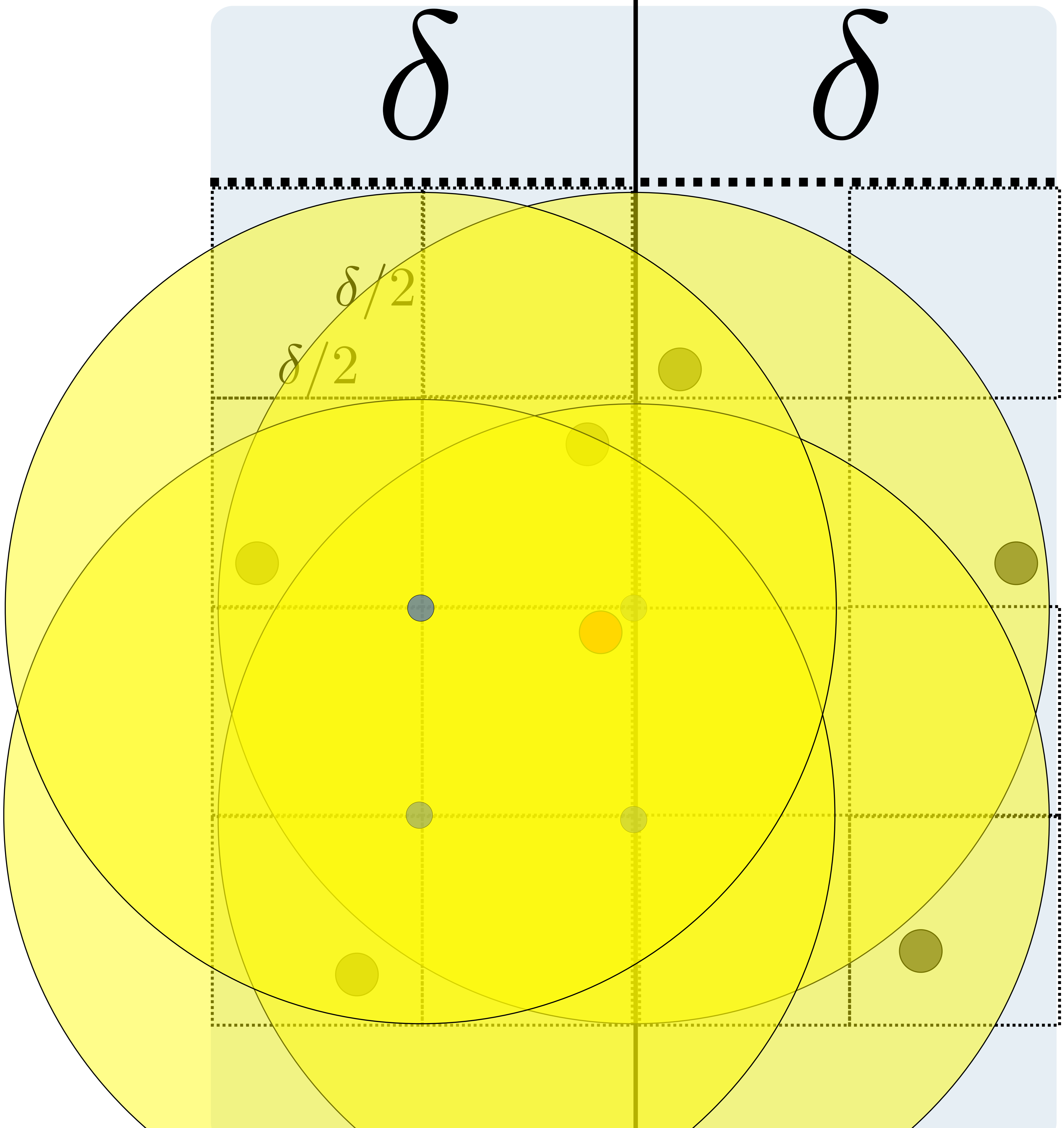

δ
δ
$\delta/2$
$\delta/2$

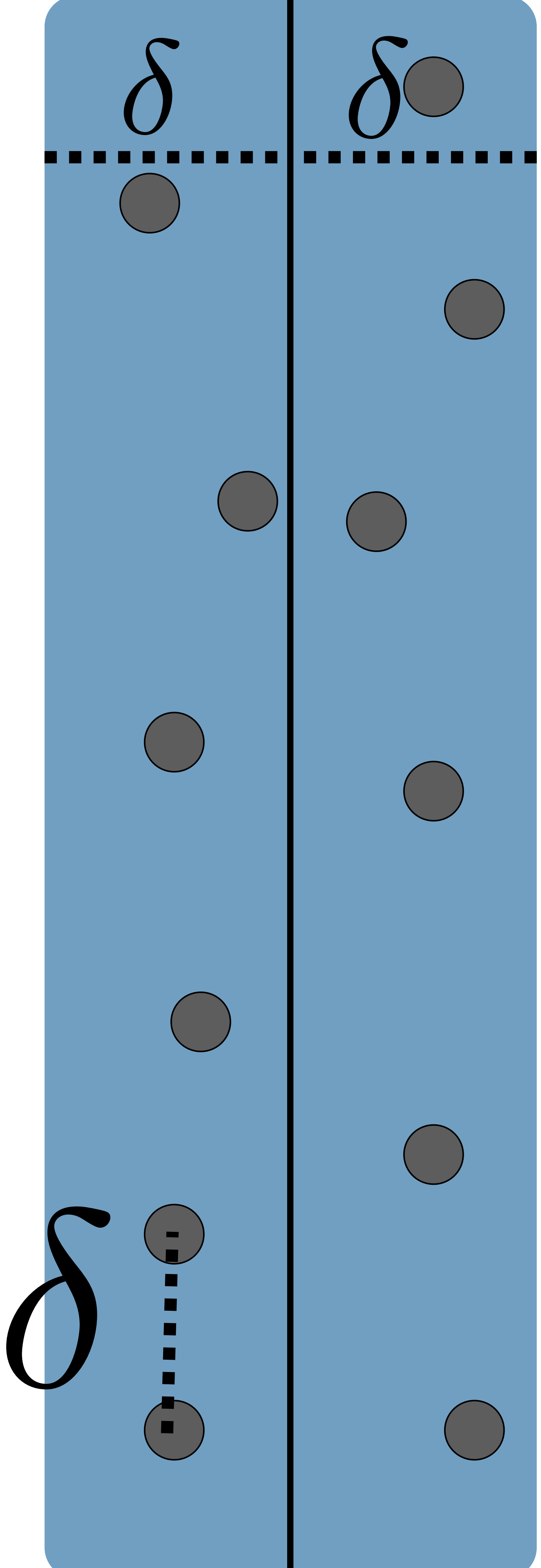


δ
δ
δ/2
δ/2



δ
δ
$\delta/2$
$\delta/2$


δ
δ
δ/2
δ/2



(make data structures, only once)

closest pair:

base case of <5 points

solve left half, right half

let δ be min from left/right

add points δ from middle to set S

assign points to boxes of side $\delta/2$

for each point in S,

compare w/10 neighbor boxes
find minimum in this list

return closest pair

$$T(n) =$$

$$T(n) = 2T(n/2) + \Theta(n) = \Theta(n \log n)$$


Matrix

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \bigstar \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \star \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 5+14 & 6+16 \\ 15+28 & 18+32 \end{bmatrix}$$

$$= \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

$$\begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \vdots & & & \\ a_{n,1} & a_{n,2} & \cdots & a_{n,n} \end{bmatrix} \begin{bmatrix} b_{1,1} & b_{1,2} & \cdots & b_{1,n} \\ b_{2,1} & b_{2,2} & \cdots & b_{2,n} \\ \vdots & & & \\ b_{n,1} & b_{n,2} & \cdots & b_{n,n} \end{bmatrix} = \begin{bmatrix} c_{1,1} & c_{1,2} & \cdots & c_{1,n} \\ c_{2,1} & c_{2,2} & \cdots & c_{2,n} \\ \vdots & & & \\ c_{n,1} & c_{n,2} & \cdots & c_{n,n} \end{bmatrix}$$

$$\begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \vdots & & & \\ a_{n,1} & a_{n,2} & \cdots & a_{n,n} \end{bmatrix} \begin{bmatrix} b_{1,1} & b_{1,2} & \cdots & b_{1,n} \\ b_{2,1} & b_{2,2} & \cdots & b_{2,n} \\ \vdots & & & \\ b_{n,1} & b_{n,2} & \cdots & b_{n,n} \end{bmatrix} = \begin{bmatrix} c_{1,1} & c_{1,2} & \cdots & c_{1,n} \\ c_{2,1} & c_{2,2} & \cdots & c_{2,n} \\ \vdots & & & \\ c_{n,1} & c_{n,2} & \cdots & c_{n,n} \end{bmatrix}$$

$$c_{i,j} = \sum_{k=0}^{n} a_{i,k} \cdot b_{k,j}$$

$$
\begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \vdots & & & \\ a_{n,1} & a_{n,2} & \cdots & a_{n,n} \end{bmatrix} \begin{bmatrix} b_{1,1} & b_{1,2} & \cdots & b_{1,n} \\ b_{2,1} & b_{2,2} & \cdots & b_{2,n} \\ \vdots & & & \\ b_{n,1} & b_{n,2} & \cdots & b_{n,n} \end{bmatrix} = \begin{bmatrix} c_{1,1} & c_{1,2} & \cdots & c_{1,n} \\ c_{2,1} & c_{2,2} & \cdots & c_{2,n} \\ \vdots & & & \\ c_{n,1} & c_{n,2} & \cdots & c_{n,n} \end{bmatrix}
$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} E & F \\ G & H \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} E & F \\ G & H \end{bmatrix}$$

$$= \begin{bmatrix} AE + BG & AF + BH \\ CE + DG & CF + DH \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} E & F \\ G & H \end{bmatrix}$$

$$= \begin{bmatrix} AE + BG & AF + BH \\ CE + DG & CF + DH \end{bmatrix}$$

$$T(n) = 8T(n/2) + \Theta(n^2) \quad \Theta(n^3)$$

$$= \begin{bmatrix} AE + BG & AF + BH \\ CE + DG & CF + DH \end{bmatrix}$$

[strassen]

$$P_1 = A(F - H)$$

$$P_2 = (A + B)H$$

$$P_3 = (C + D)E$$

$$P_4 = D(G - E)$$

$$P_5 = (A + D)(E + H)$$

$$P_6 = (B - D)(G + H)$$

$$P_7 = (A - C)(E + F)$$

$$= \begin{bmatrix} AE + BG & AF + BH \\ CE + DG & CF + DH \end{bmatrix}$$

[strassen]

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$$= \begin{bmatrix} AE + BG & AF + BH \\ CE + DG & CF + DH \end{bmatrix}$$

$$R = P_5 + P_4 - P_2 + P_6 \qquad S = P_1 + P_2$$

$$T = P_3 + P_4 \qquad U = P_5 + P_1 - P_3 - P_7$$

[strassen]

$$P_1 = A(F - H)$$

$$P_2 = (A + B)H$$

$$P_3 = (C + D)E$$

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$$= \begin{bmatrix} AE + BG & AF + BH \\ CE + DG & CF + DH \end{bmatrix}$$

$R = P_5 + P_4 - P_2 + P_6$

$S = P_1 + P_2$

$T = P_3 + P_4$

$U = P_5 + P_1 - P_3 - P_7$

[strassen]

$P_1 = A(F - H)$

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$P_4 = D(G - E)$

$P_5 = (A + D)(E + H)$

$P_6 = (B - D)(G + H)$

$P_7 = (A - C)(E + F)$

$$M(n) = 7M(n/2) + 18n^2$$

$$= \Theta(n^{\log_2 7})$$

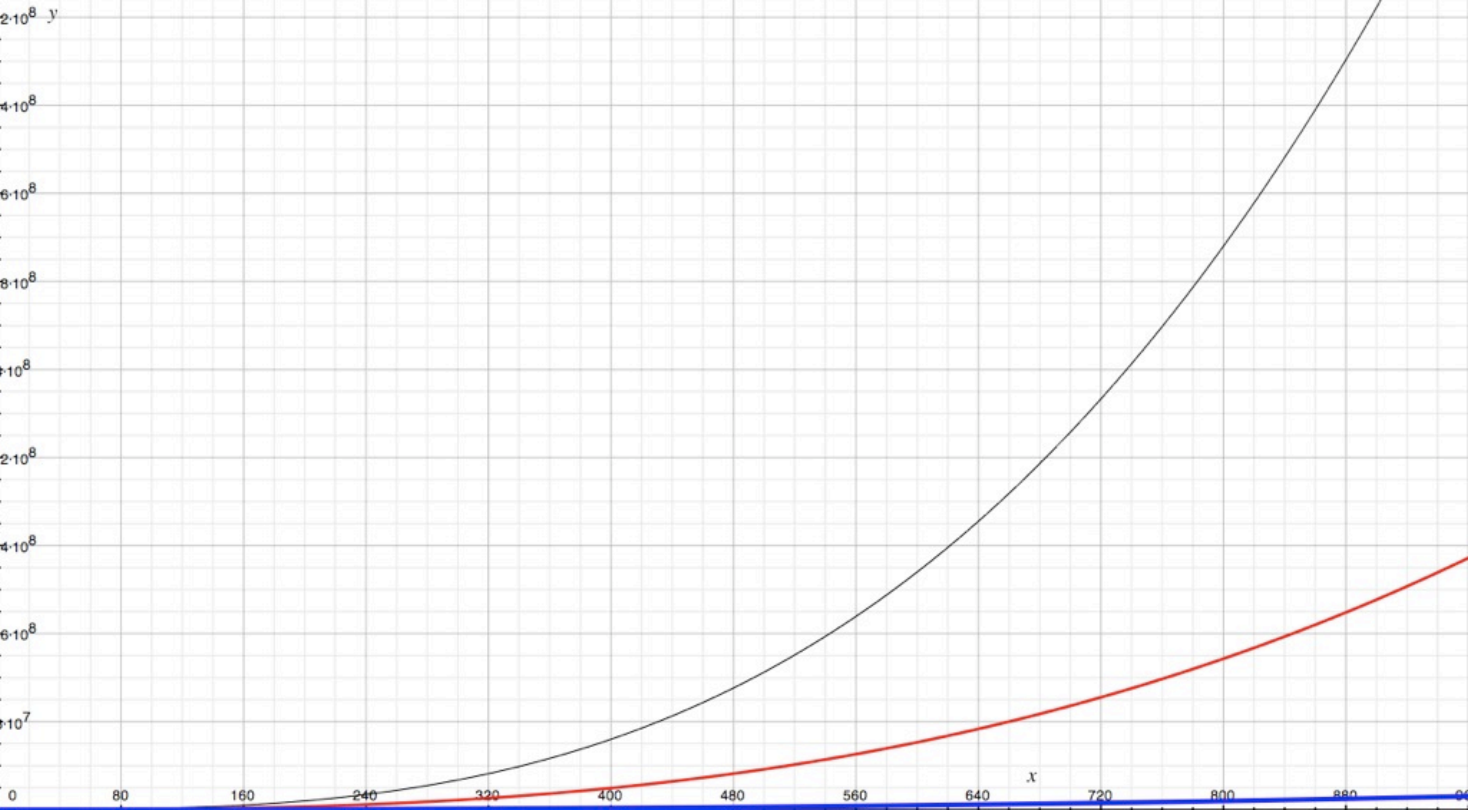
taking this idea further

3x3 matricies

1978 victor pan method

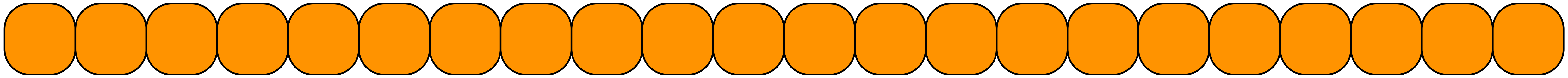
70x70 matrix using 143640 mults

what is the recurrence:

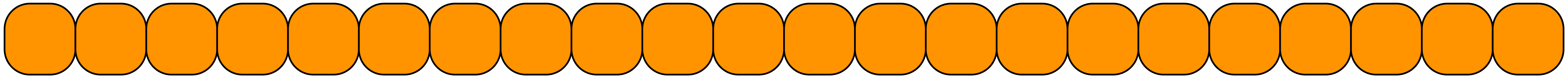


y
x
0
80
160
240
320
400
480
560
640
720
800
880

MEDIAN



problem: given a list of n elements, find the element of rank $n/2$. (half are larger, half are smaller)



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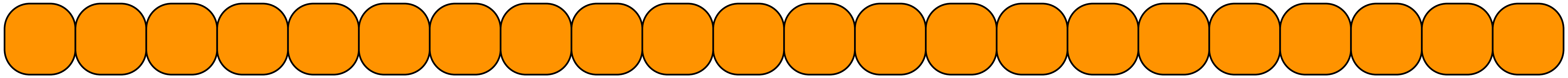
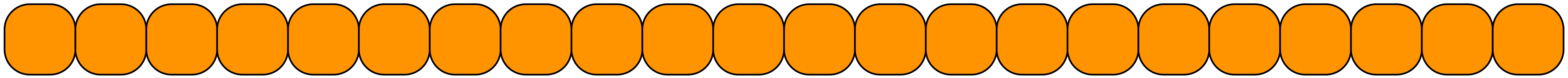


problem: given a list of n elements, find the element of rank $n/2$. (half are larger, half are smaller)

can generalize to i

first solution: sort and pluck.

$$O(n \log n)$$



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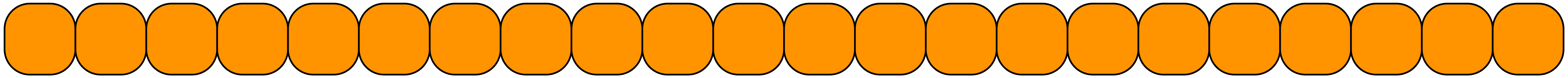
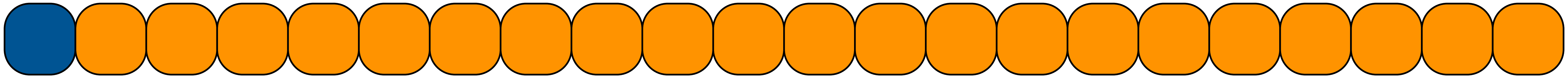


problem: given a list of n elements, find the element of rank i.

key insight:

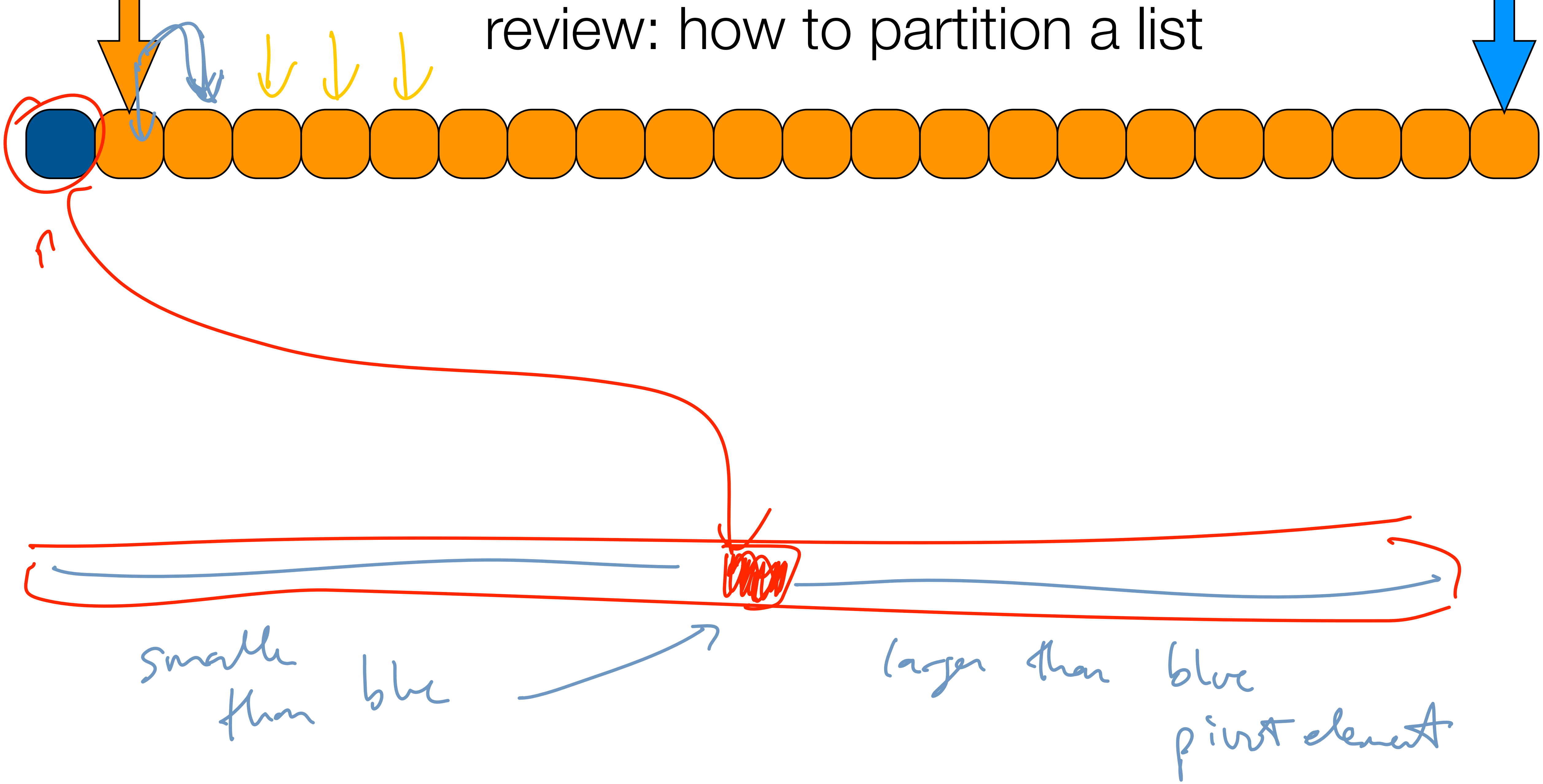
we do not have to “fully” sort.

semi sort can suffice.



pick first element
partition list about this one
see where we stand

review: how to partition a list



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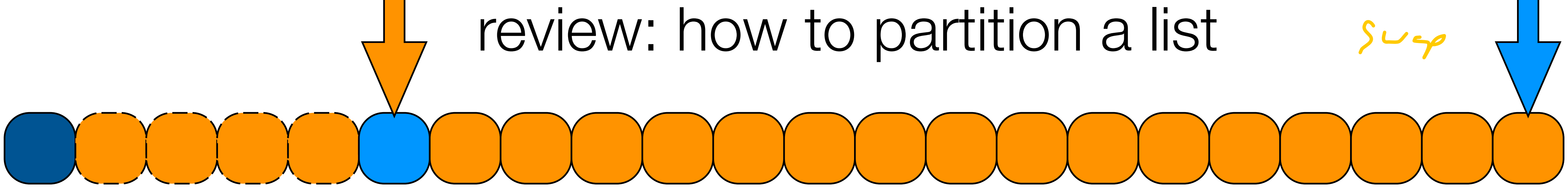

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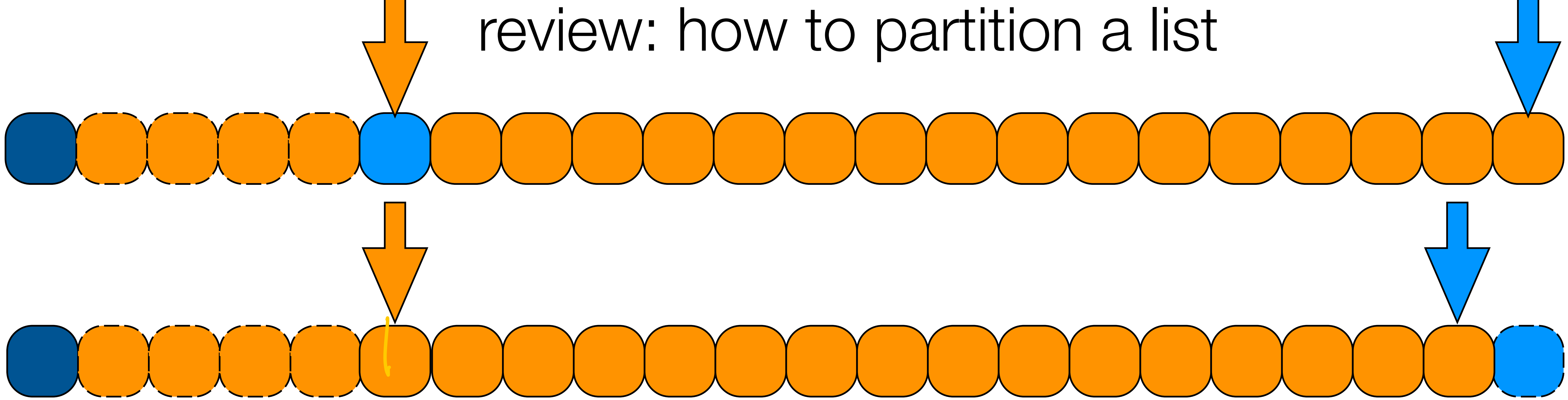
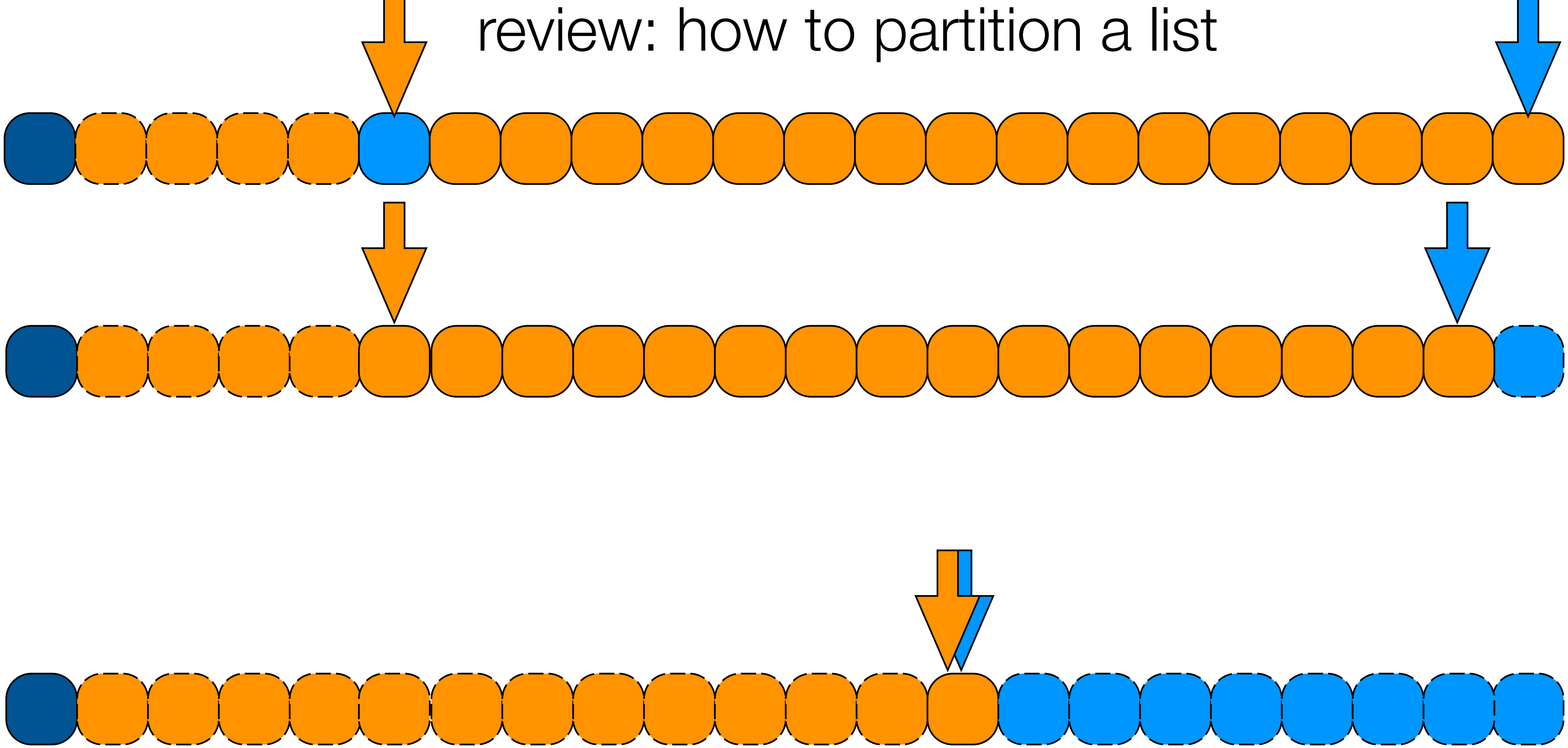
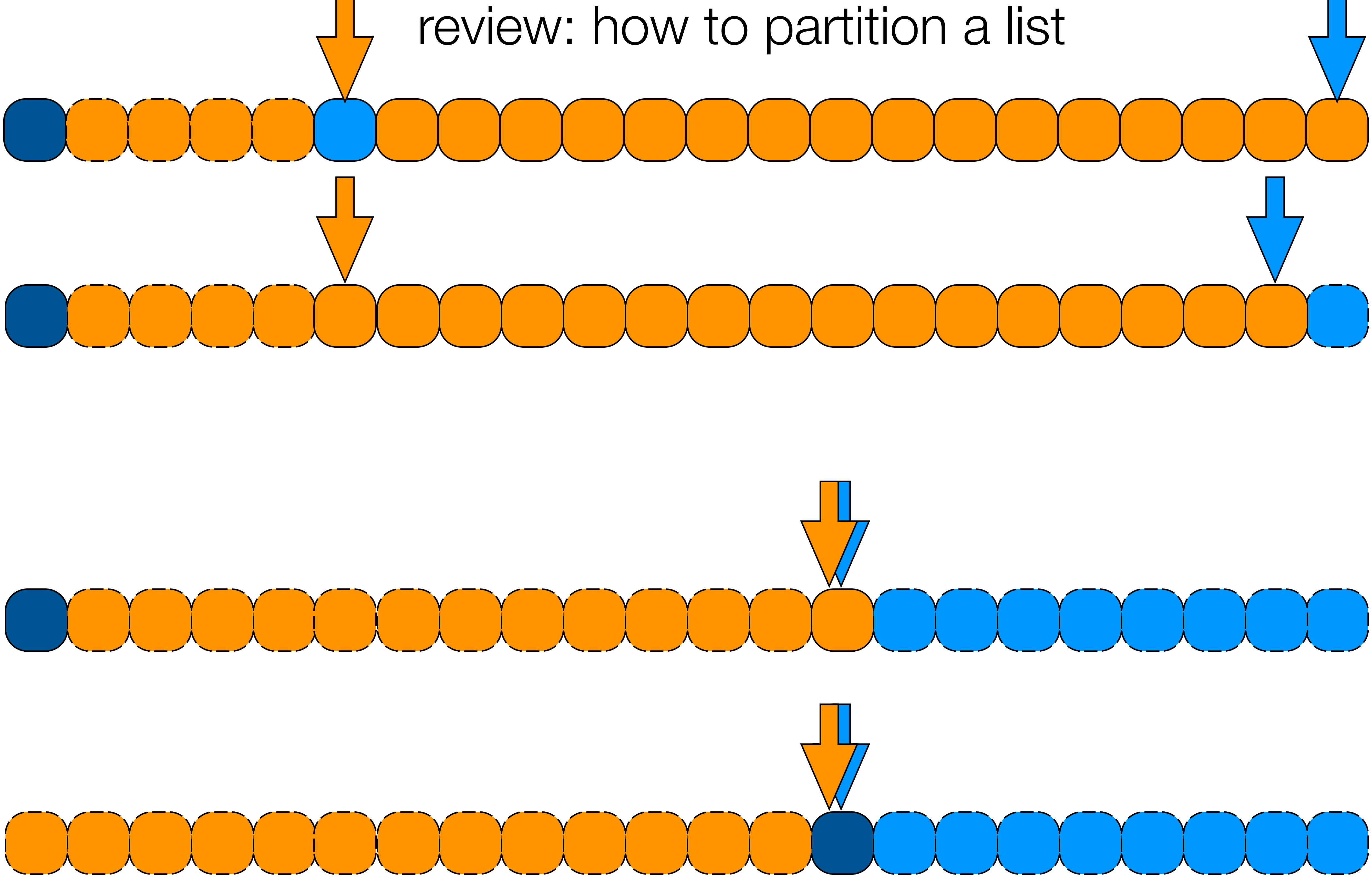


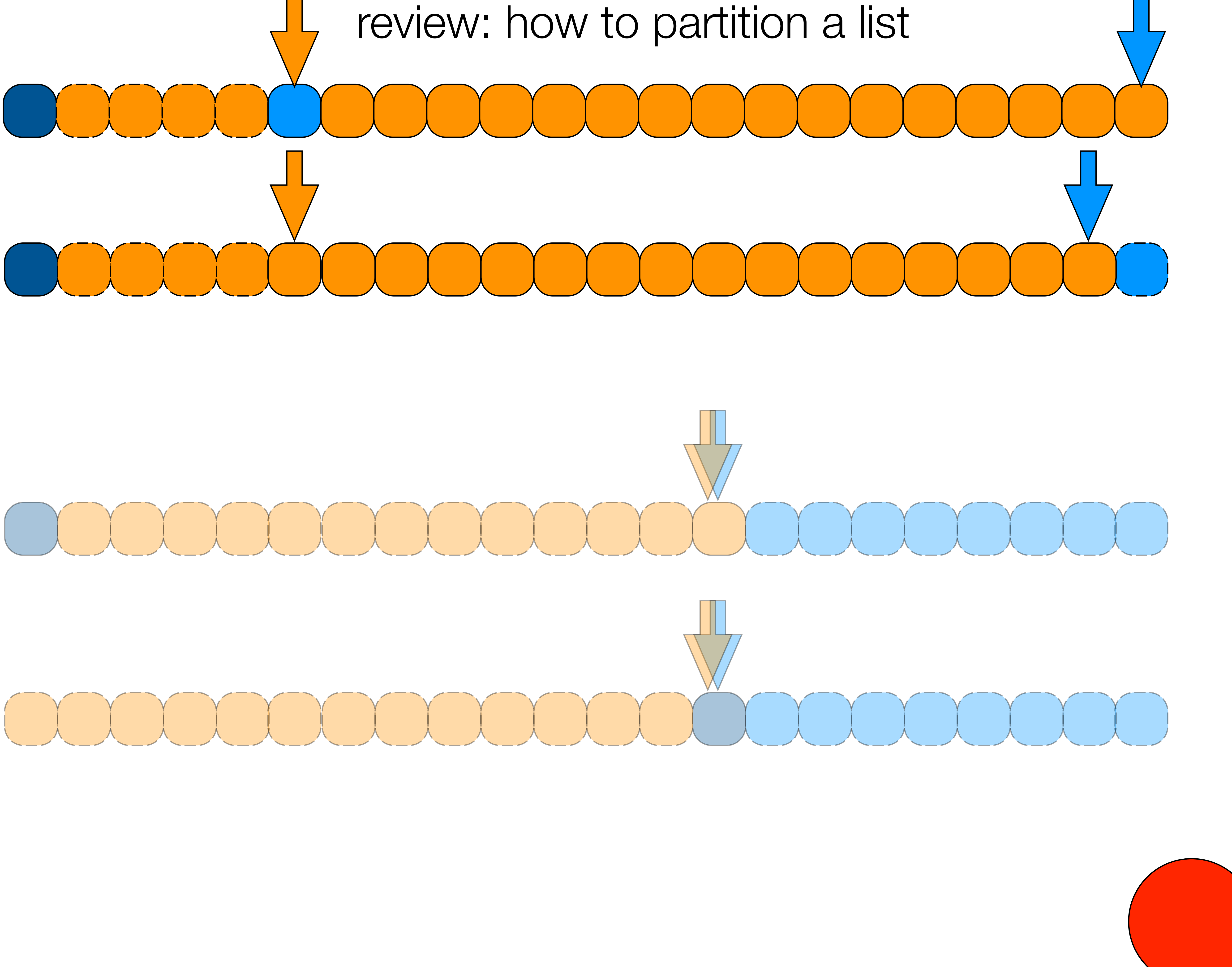
review: how to partition a list

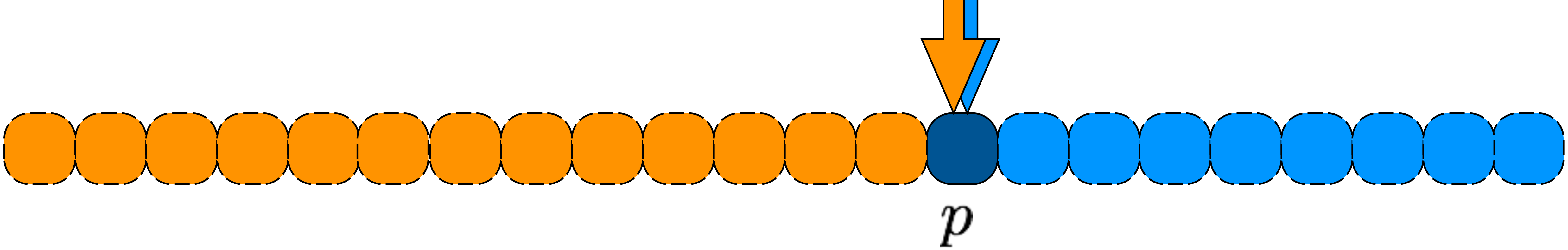


review: how to partition a list



review: how to partition a list





pick first element
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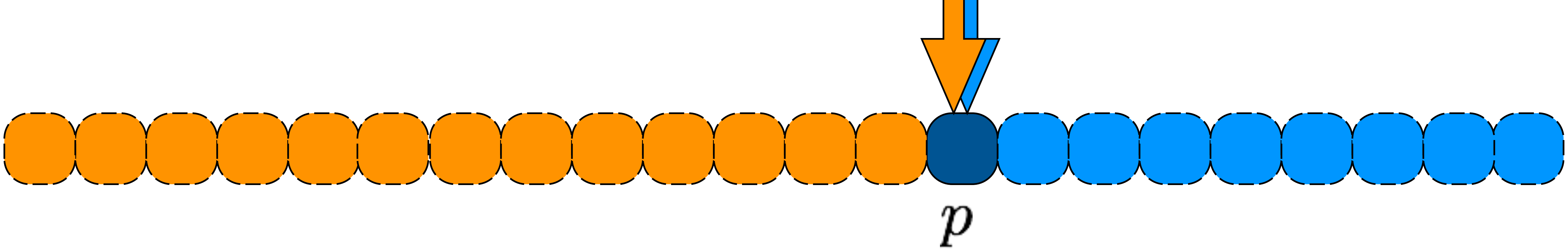

select $(i, A[1, \ldots, n])$

pick first element
partition list about this one
if pivot is position i, return pivot
else if pivot is in position $> i$ select $(i, A[1, \ldots, p-1])$
else select $((i-p-1), A[p+1, \ldots, n])$

select $(i, A[1, \ldots, n])$

pick first element

partition list about this one

if pivot is position i, return pivot

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if pivot is position i, return pivot

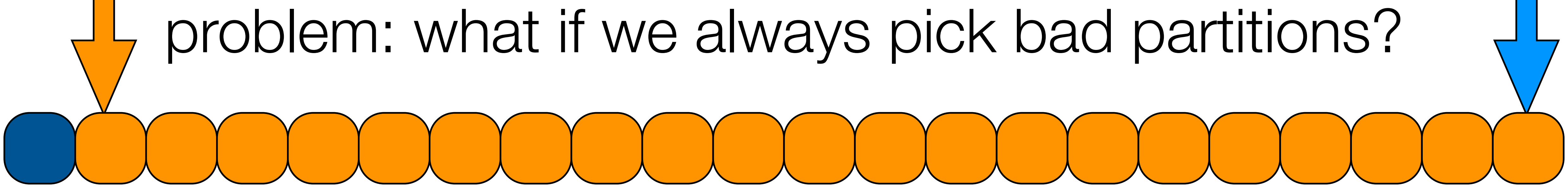
else if pivot is in position $> i$ select $(i, A[1, \ldots, p-1])$

else select $((i-p-1), A[p+1, \ldots, n])$

$$T(n) = T(n/2) + O(n)$$

$$\Theta(n)$$

problem: what if we always pick bad partitions?



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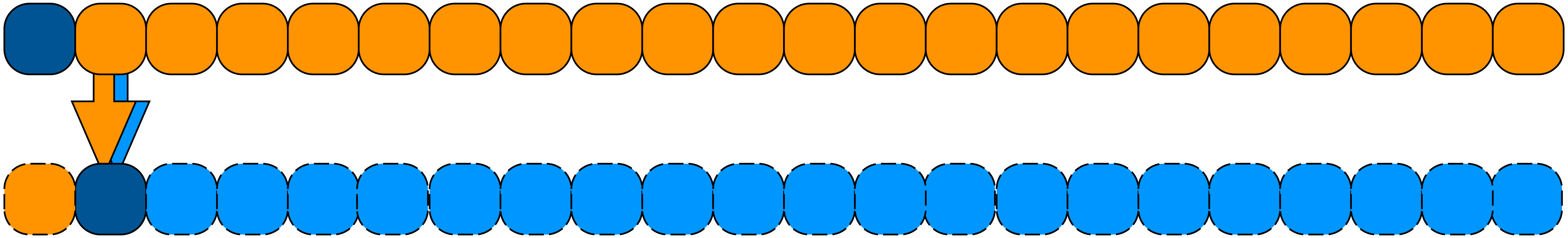

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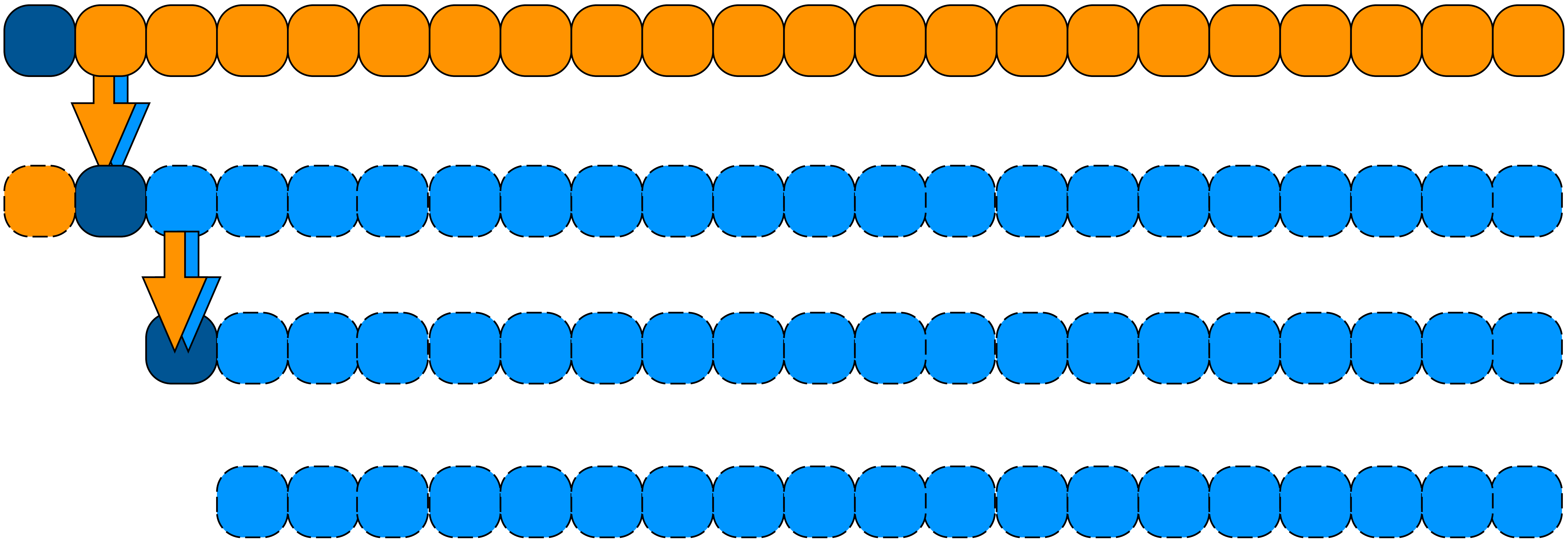
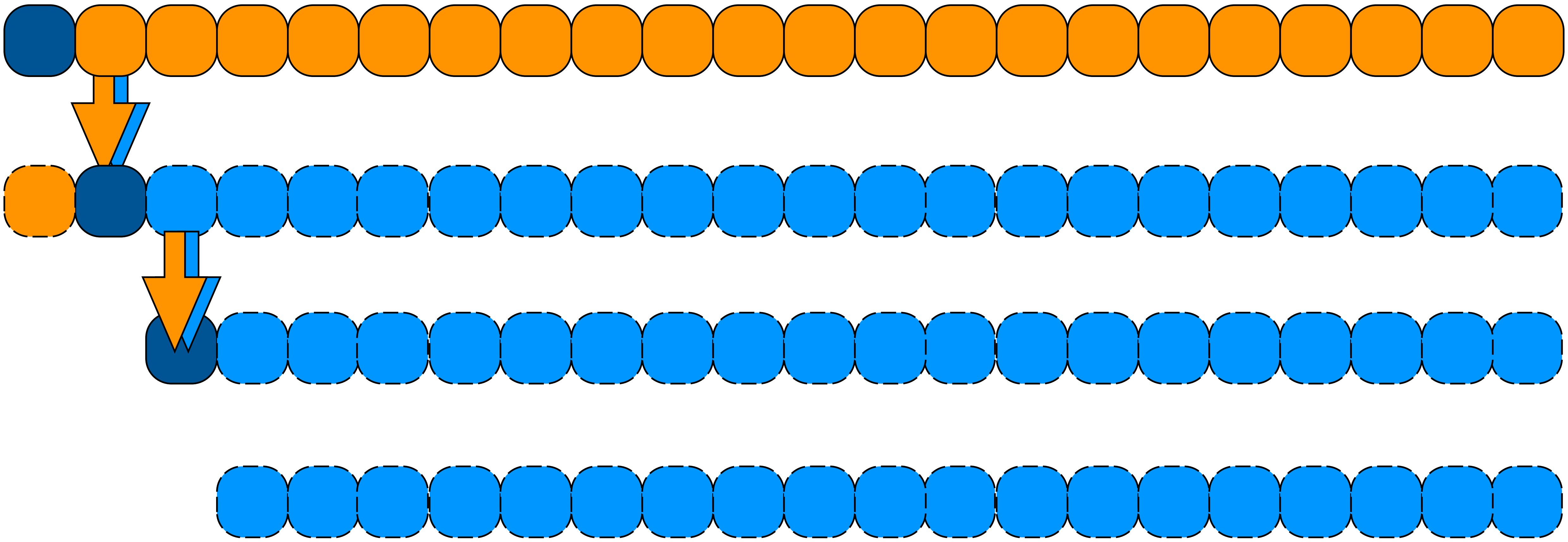




problem: what if we always pick bad partitions?

select $(i, A[1, \ldots, n])$

pick first element

partition list about this one

if pivot is position i, return pivot

else if pivot is in position $> i$

select $(i, A[1, \ldots, p-1])$

else

select $((i-p-1), A[p+1, \ldots, n])$

Needed

a good partition element

partition $(A[1, \ldots, n])$

Needed:

a good partition element

partition $(A[1, \ldots, n])$

produce an element where
30% smaller, 30% larger

DDDDD

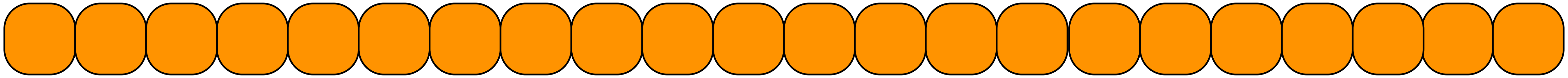
solution: bootstrap

image: gucci

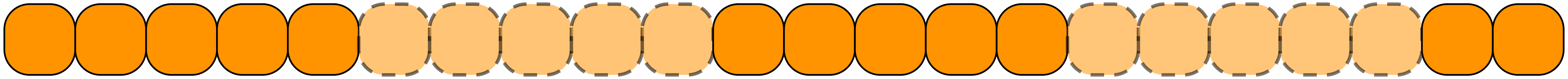
image: mark nason

image: d&g

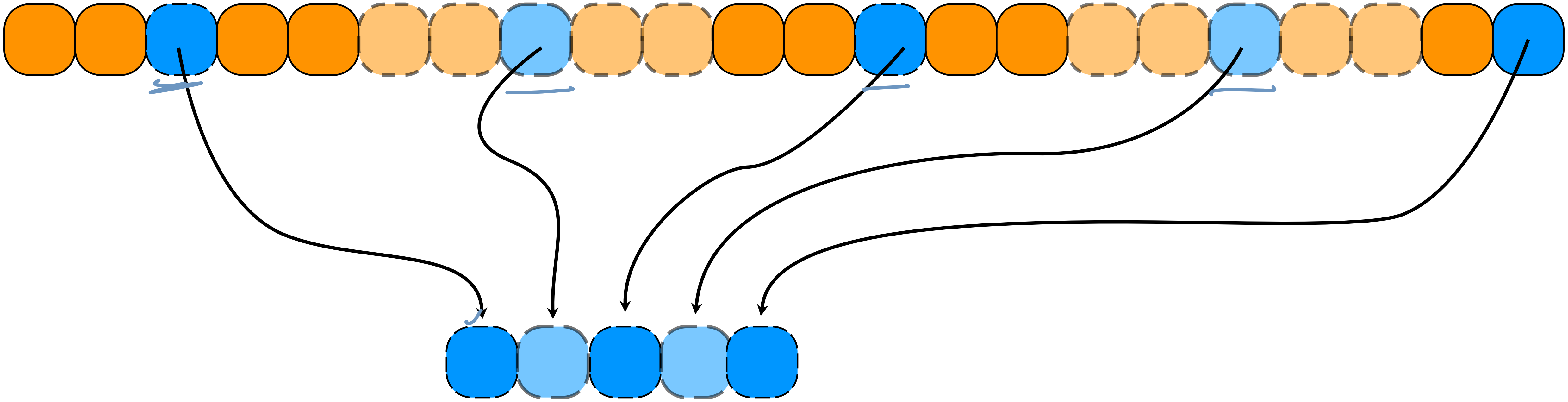
partition $(A[1, \ldots, n])$

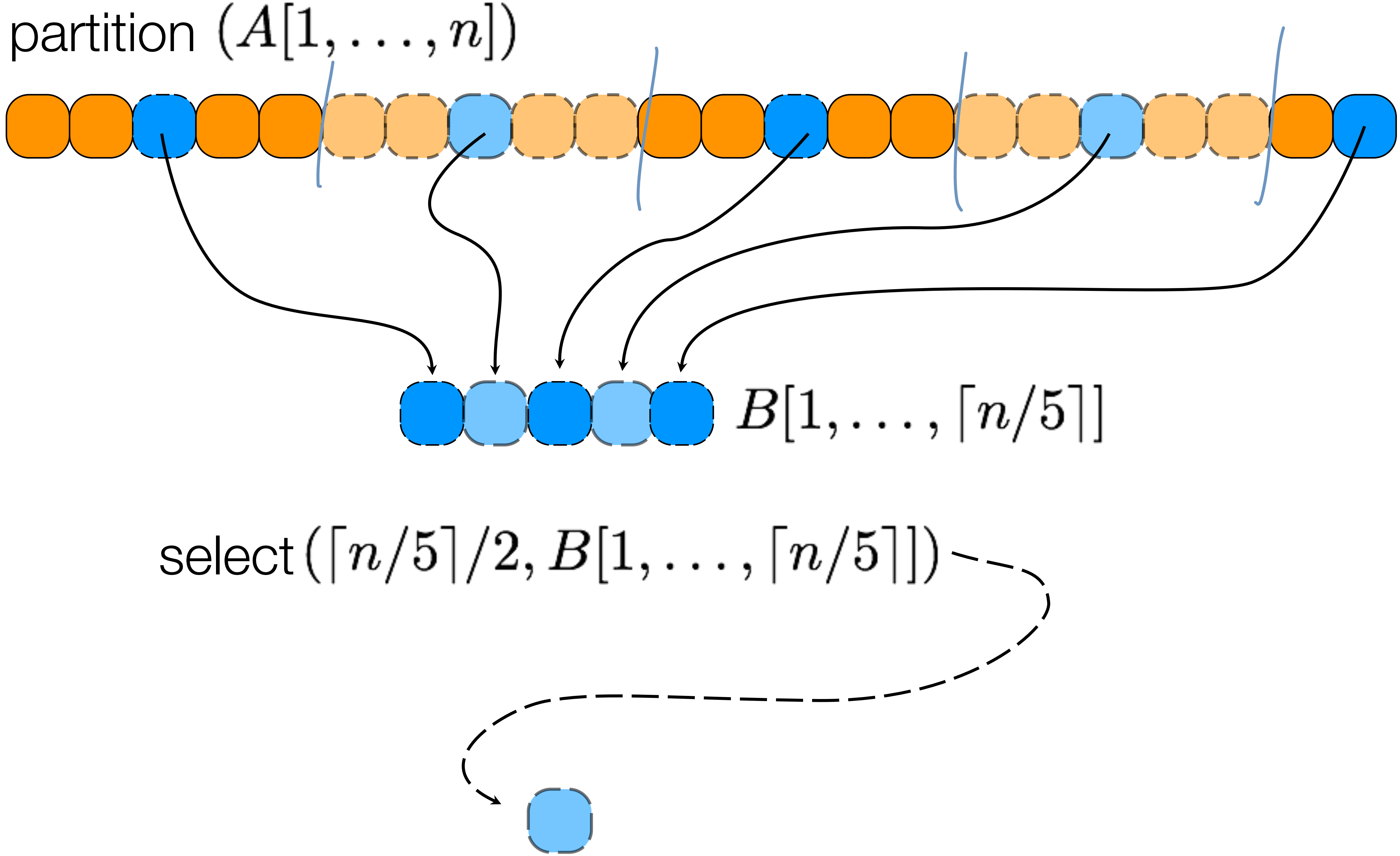


partition $(A[1,\ldots,n])$

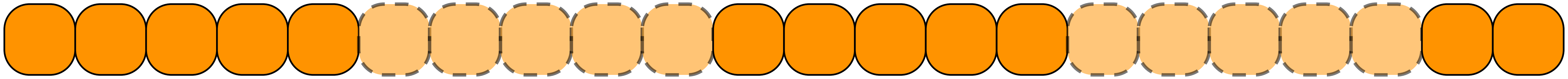


partition $(A[1, \ldots, n])$




partition $(A[1, \ldots, n])$
$B[1, \ldots, \lceil n/5 \rceil]$
select $(\lceil n/5 \rceil / 2, B[1, \ldots, \lceil n/5 \rceil])$

partition $(A[1,...,n])$



divide list into groups of 5 elements

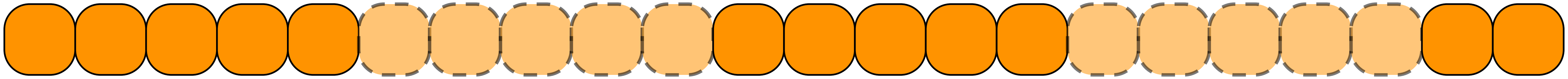
find median of each small list

gather all medians

call select(...) on this sublist to find median

return the result

partition $(A[1,\ldots,n])$



divide list into groups of 5 elements

find median of each small list

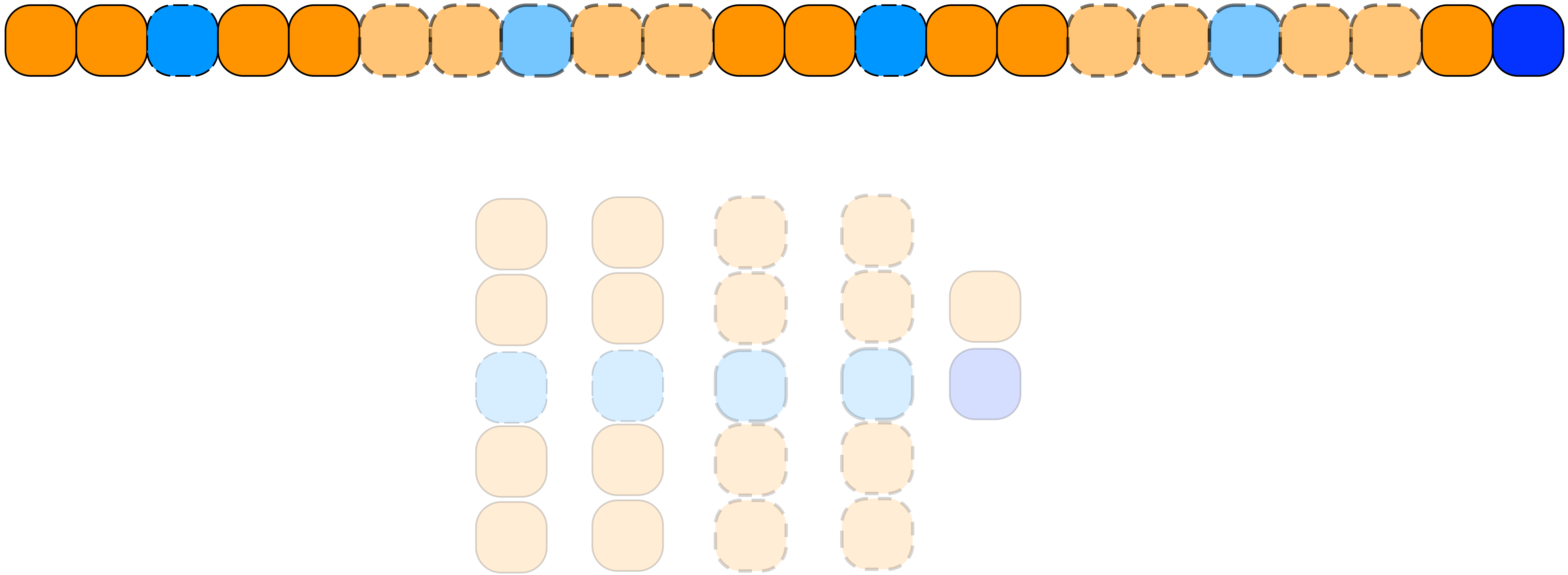
gather all medians

call select(...) on this sublist to find median

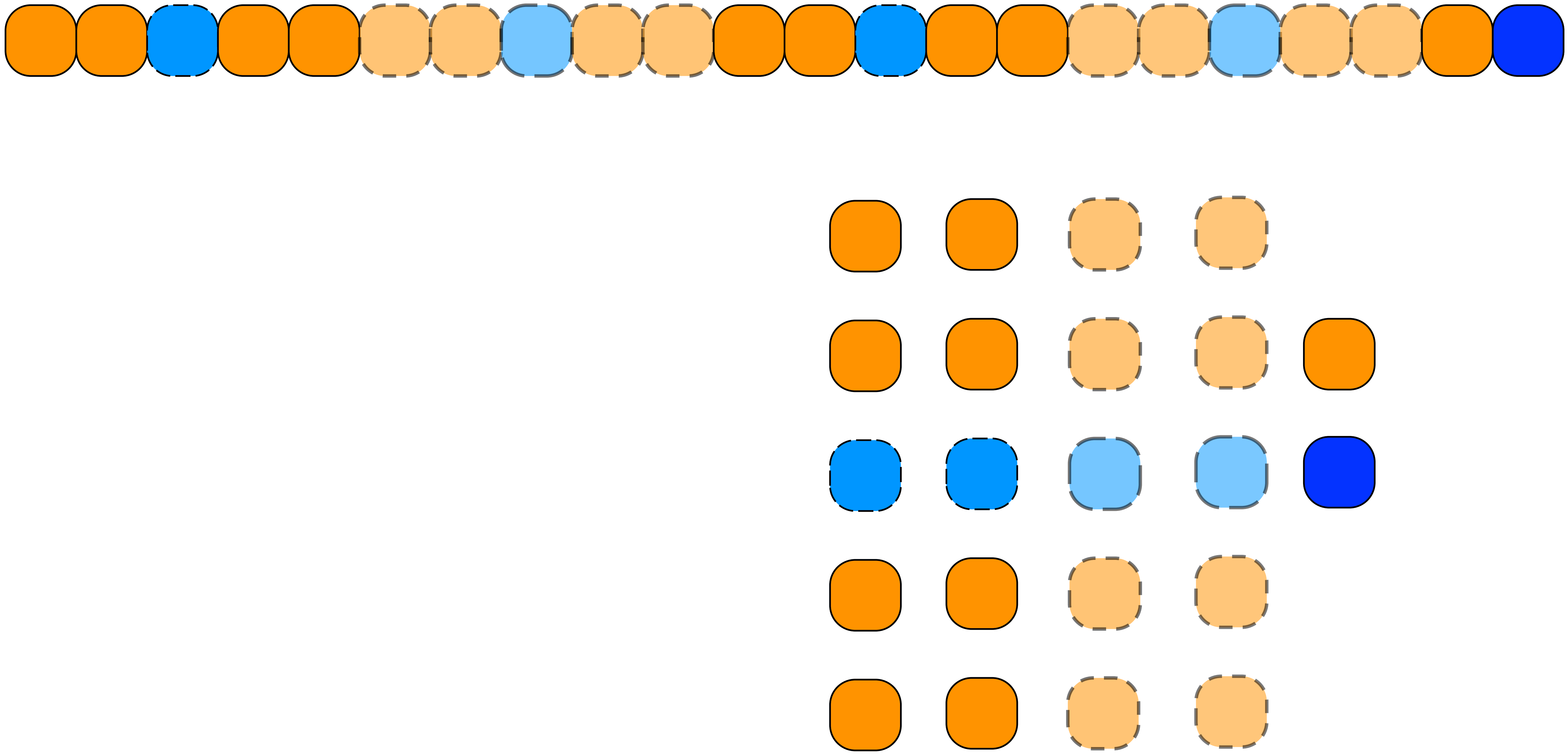
return the result

$$P(n) = S(\lceil n/5 \rceil) + O(n)$$

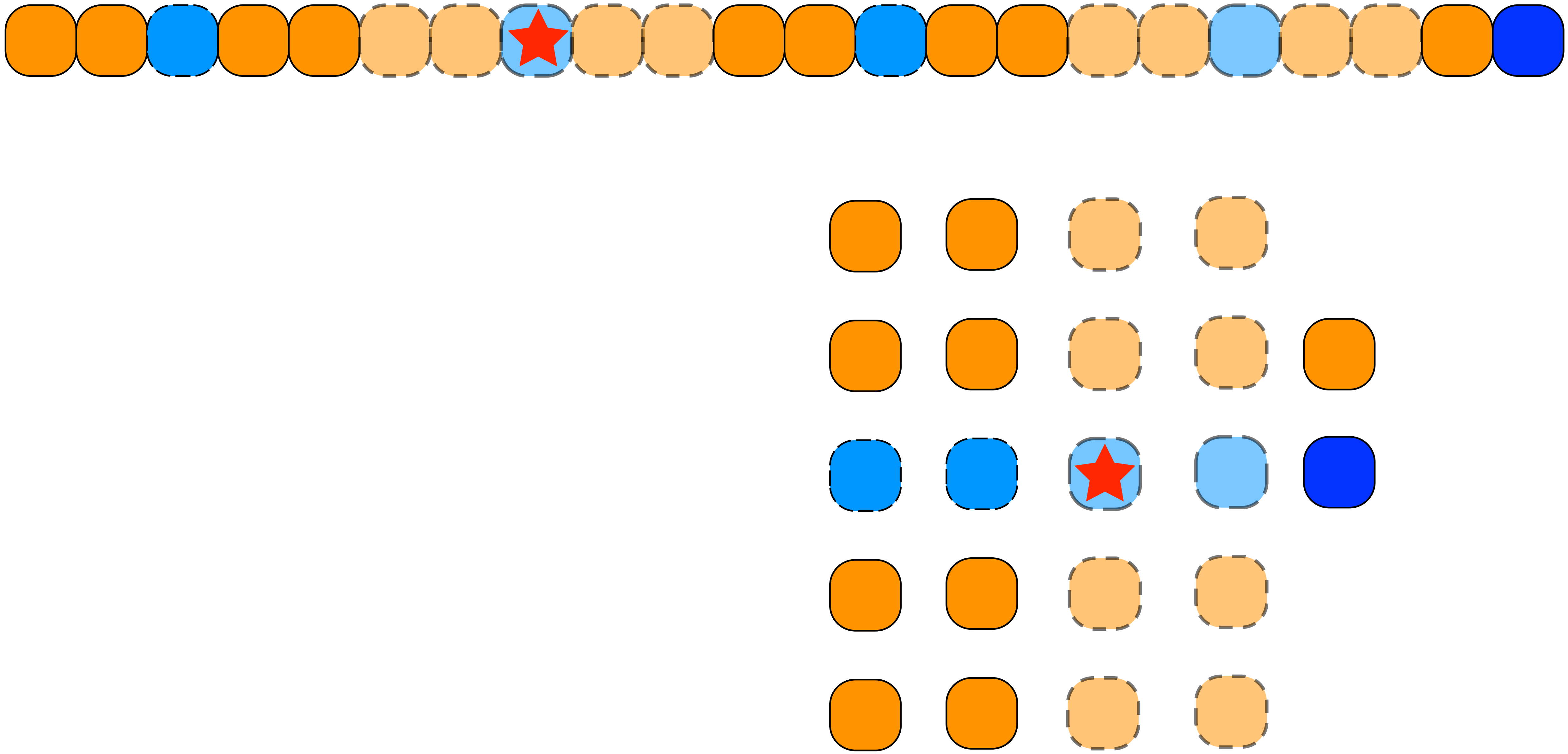
a nice property of our partition



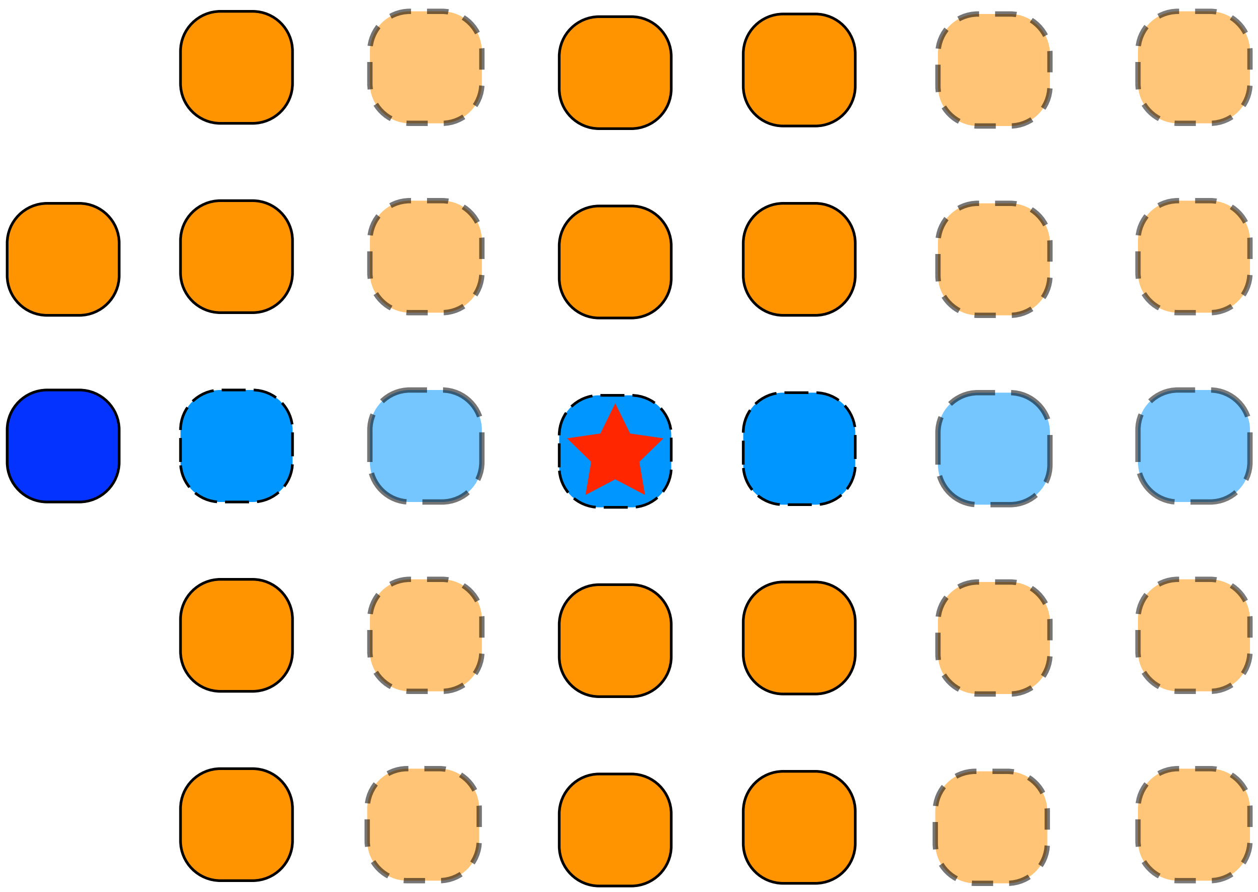
a nice property of our partition



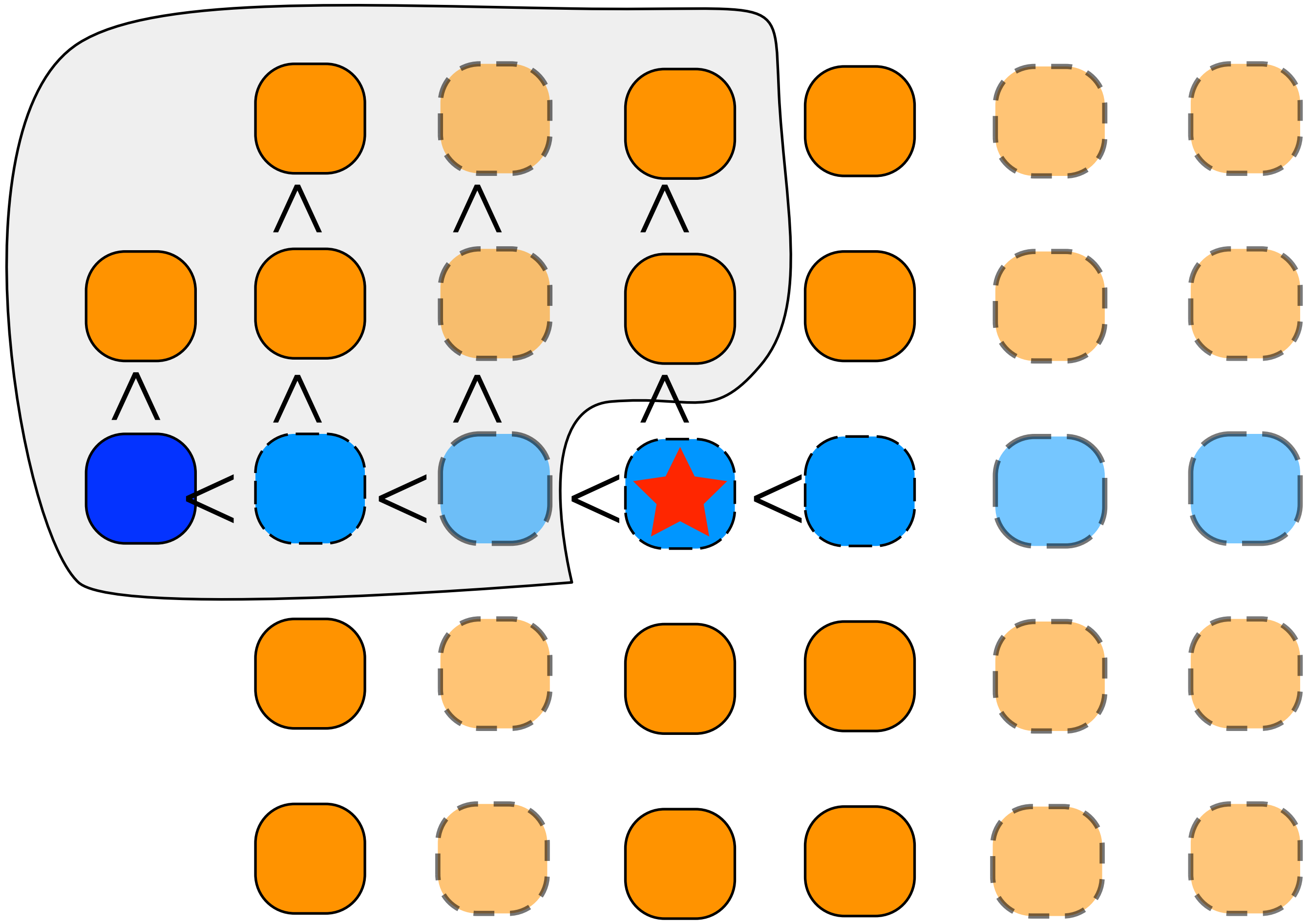
a nice property of our partition



SWITCH TO A BIGGER EXAMPLE



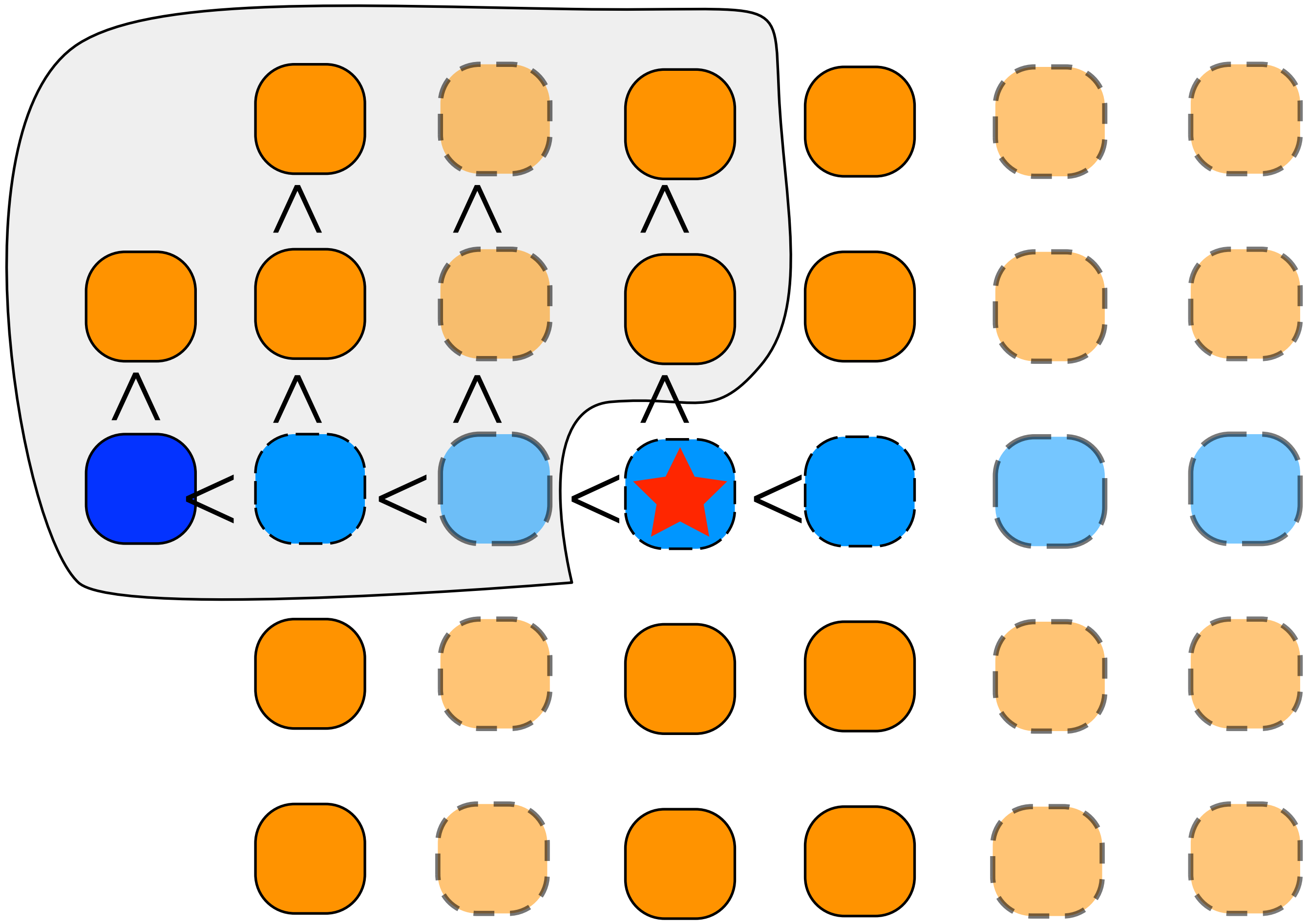
a nice property of our partition



a nice property of our partition

$$3\left(\left\lceil \frac{1}{2}\lceil n/5 \rceil \right\rceil - 2\right)$$

$$\geq \frac{3n}{10} - 6$$



a nice property of our partition

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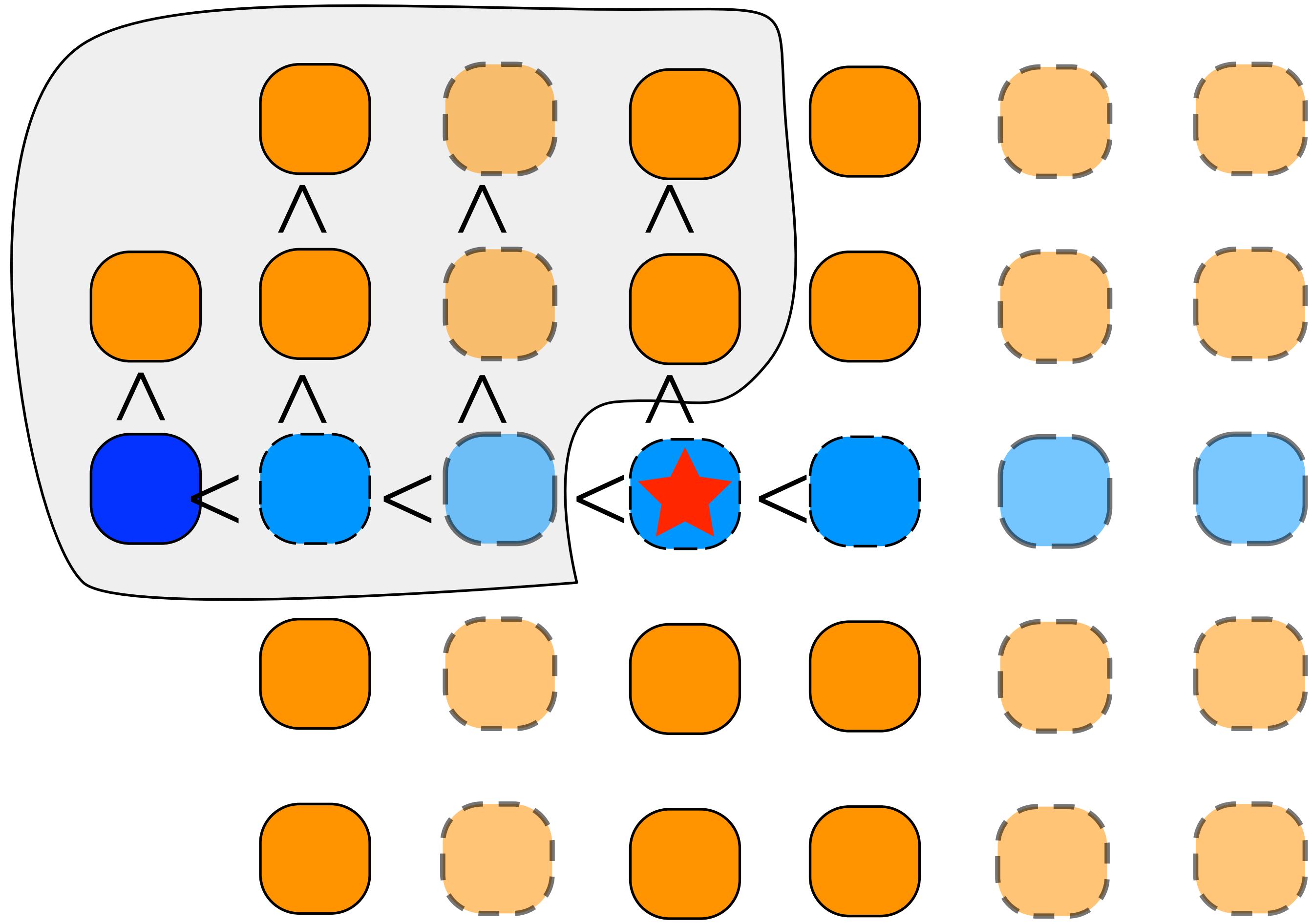


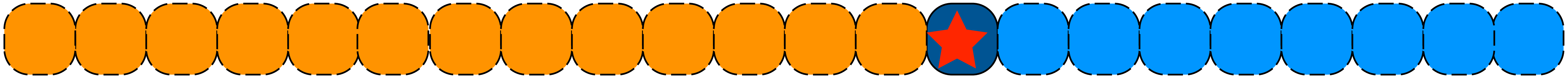
this implies there are

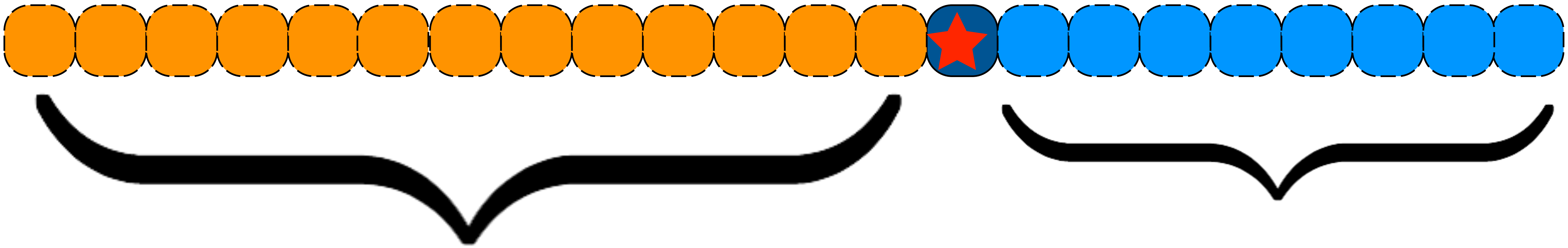
at most $\frac{7n}{10} + 6$ numbers

larger than 

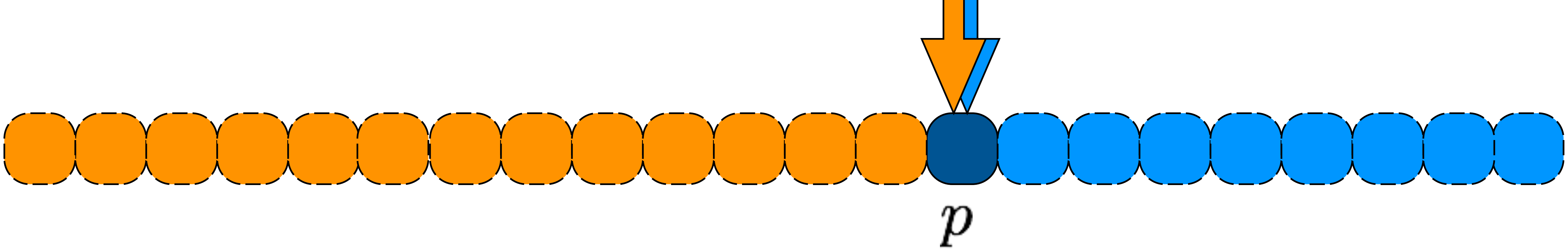
/smaller

a nice property of our partition





$$\leq \frac{7n}{10} + 6 \qquad\qquad \leq \frac{7n}{10} + 6$$



select $(i, A[1, \ldots, n])$

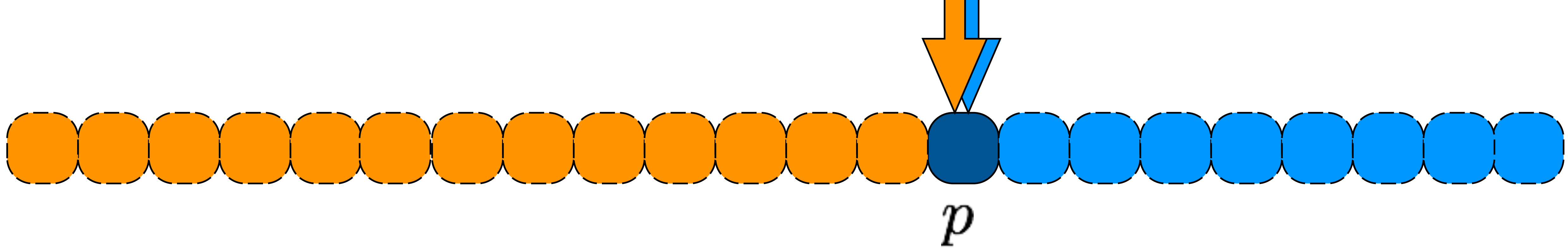
pick first element

partition list about this one

if pivot is position i, return pivot

else if pivot is in position $> i$ select $(i, A[1, \ldots, p-1])$

else select $((i-p-1), A[p+1, \ldots, n])$



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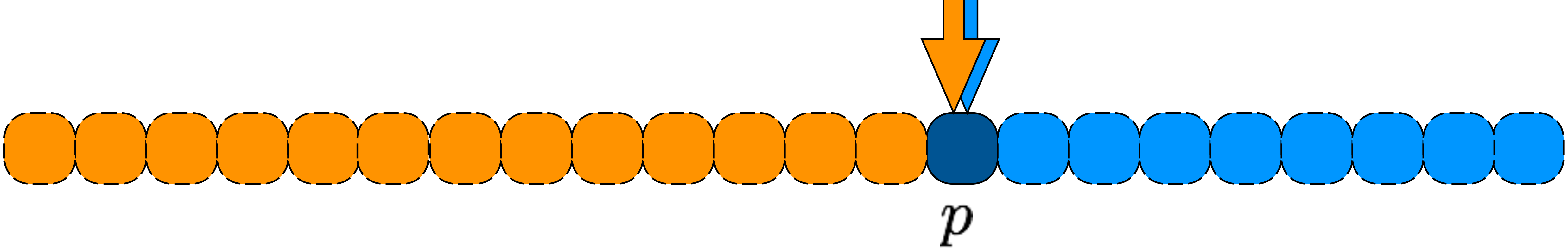
$$S(n) = S(\lceil n/5 \rceil) + O(n) + S(7n/10 + 6)$$



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$$\Theta(n)$$