L6

4102

feb 9 2016

shelat

divide&conquer

closest points

matrixmult

median

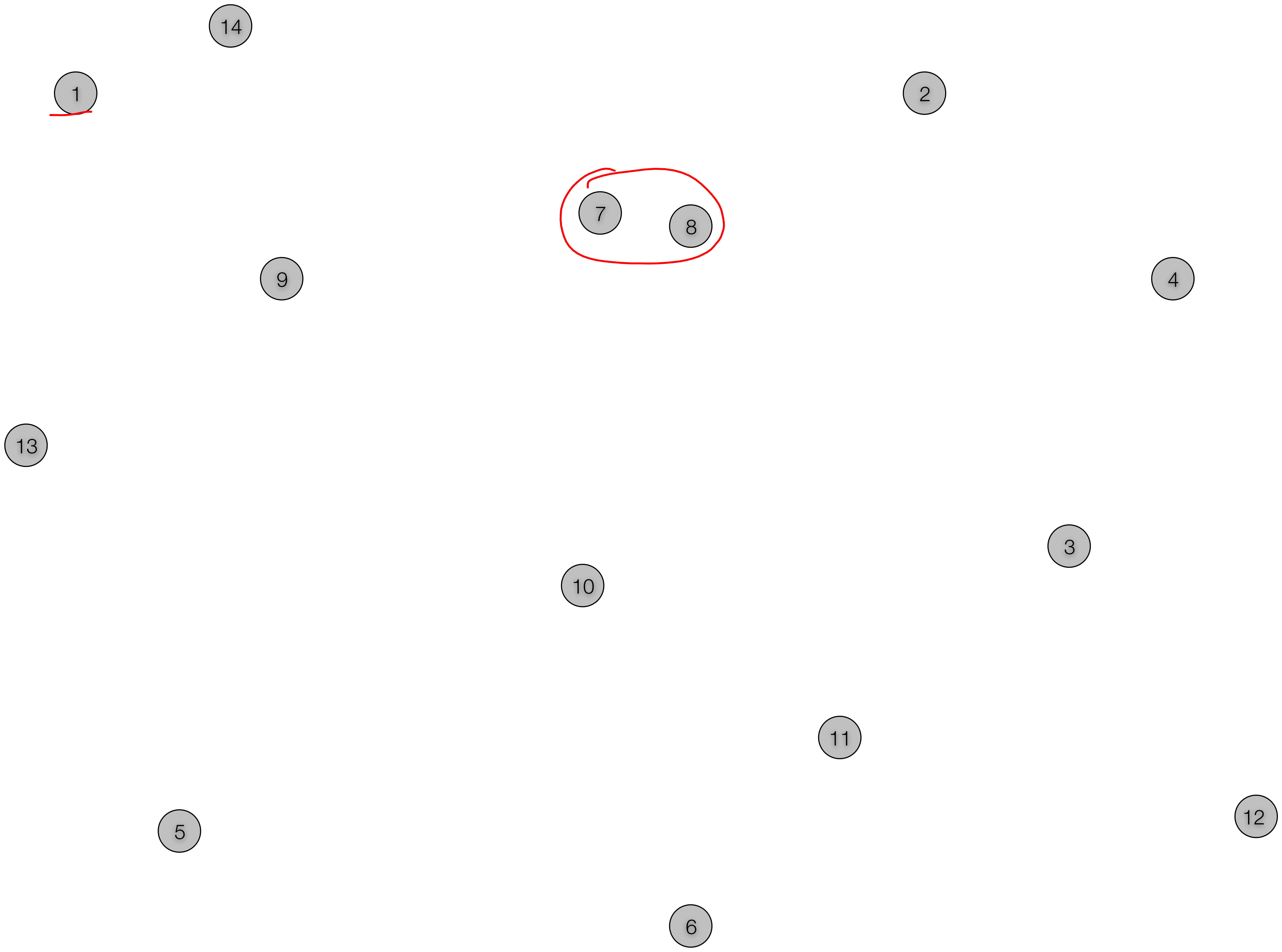
7.5	7.8	9.5	3.7	26.1

closest pair

of points

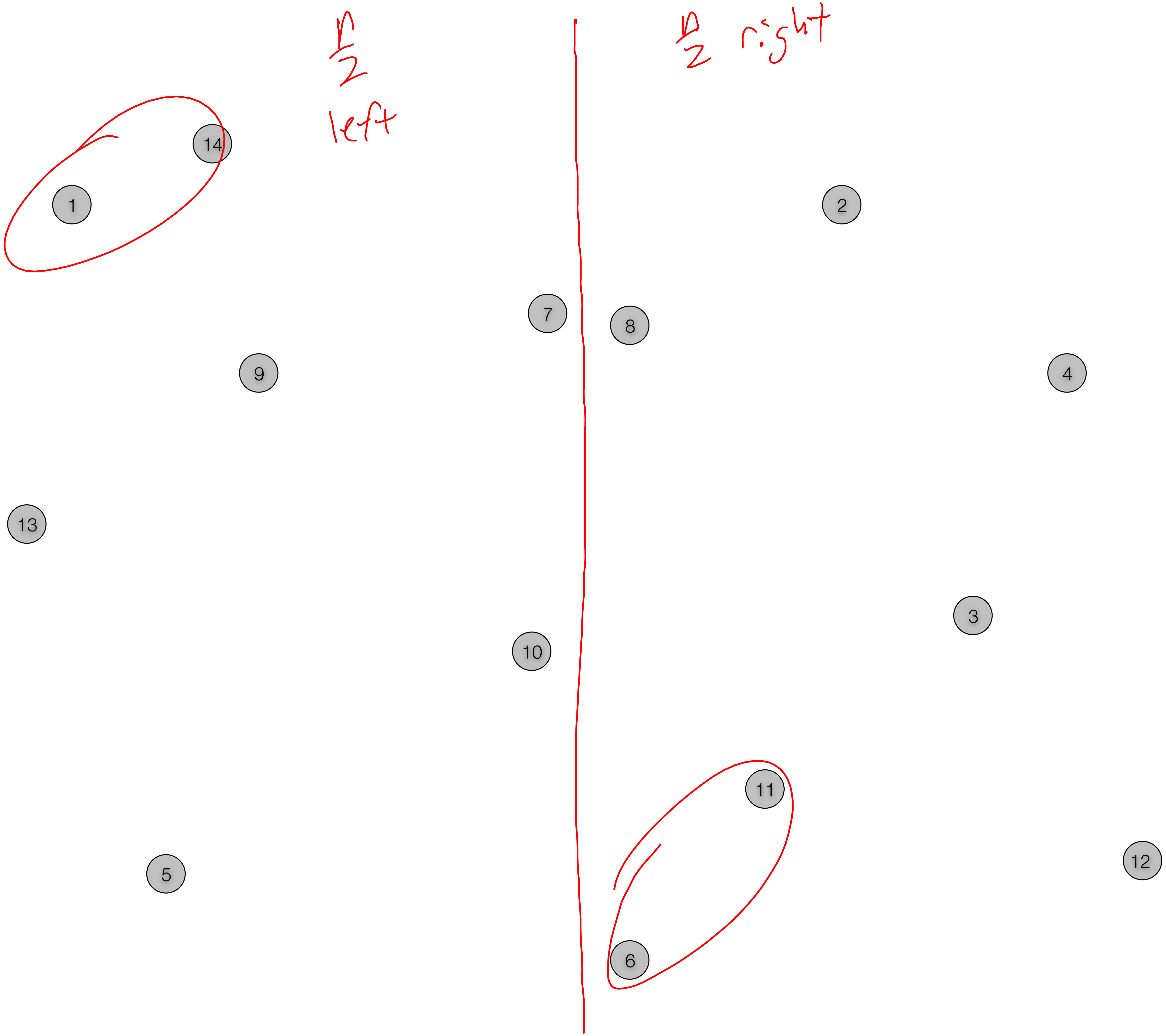
simple brute force approach takes

n points in 2D.

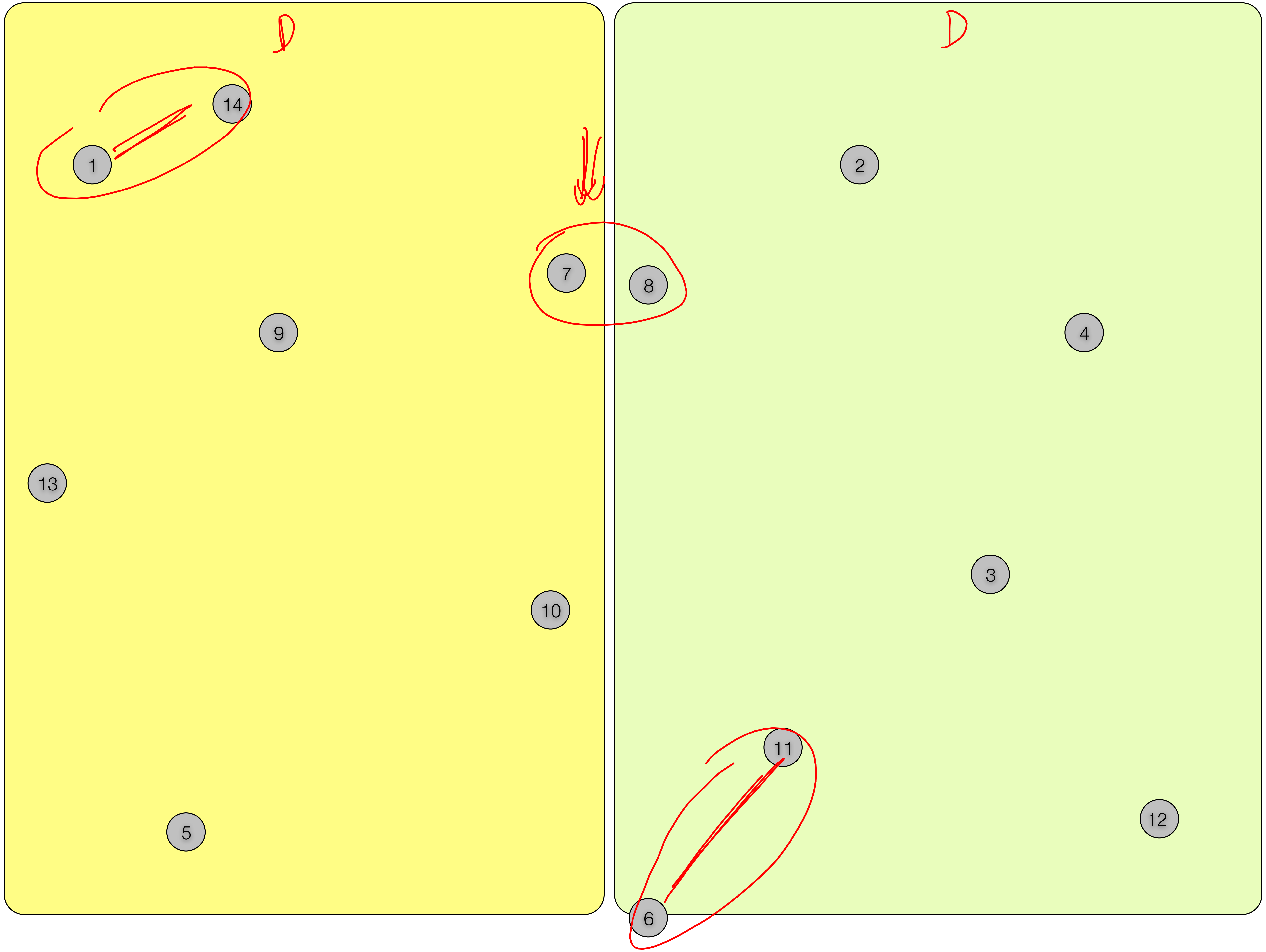


solve the large problem by

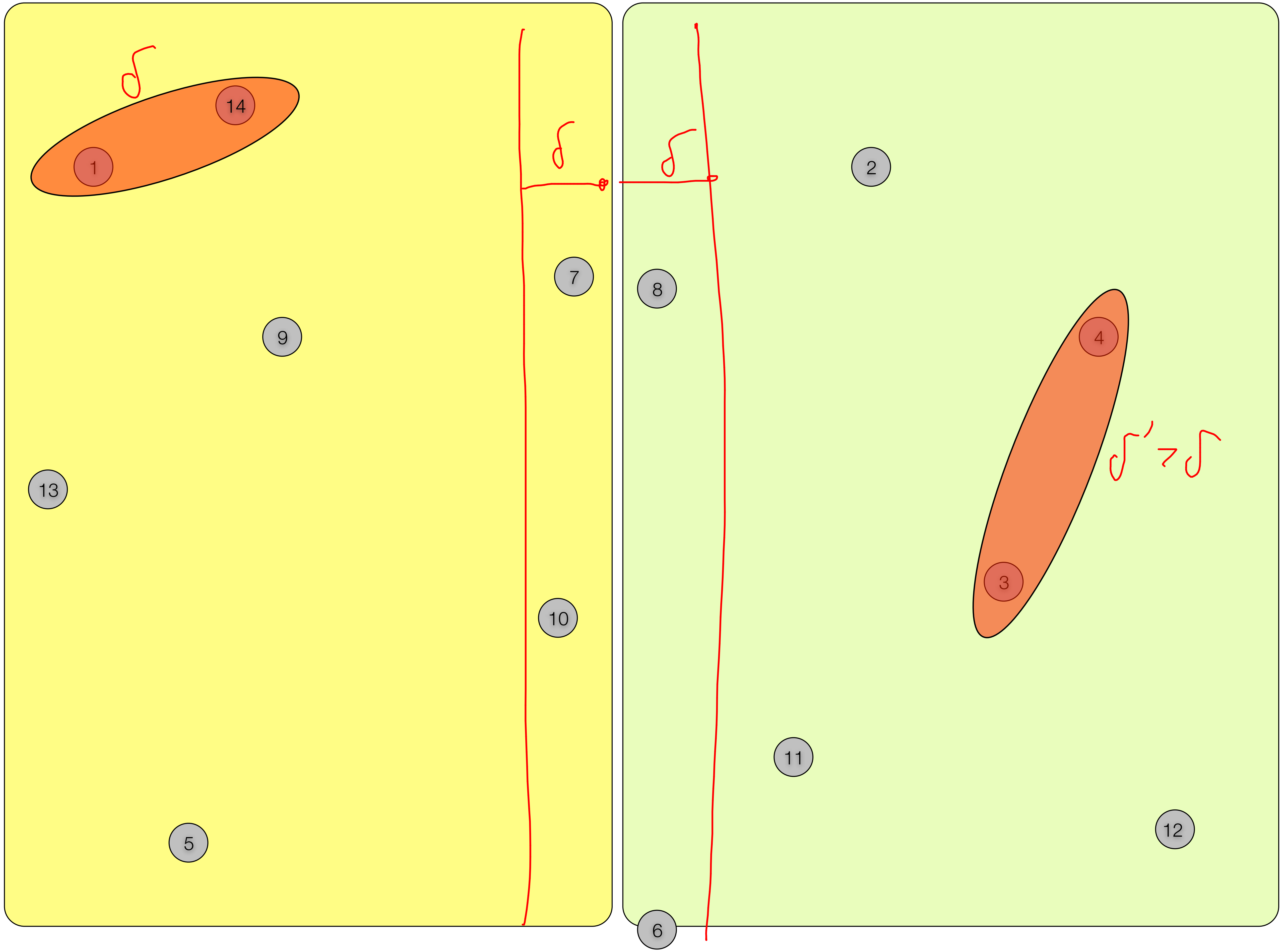
solving smaller problems
and combining solutions


n/2
left
n/2 right
14
1
2
7
8
9
4
13
3
10
11
12
5
6

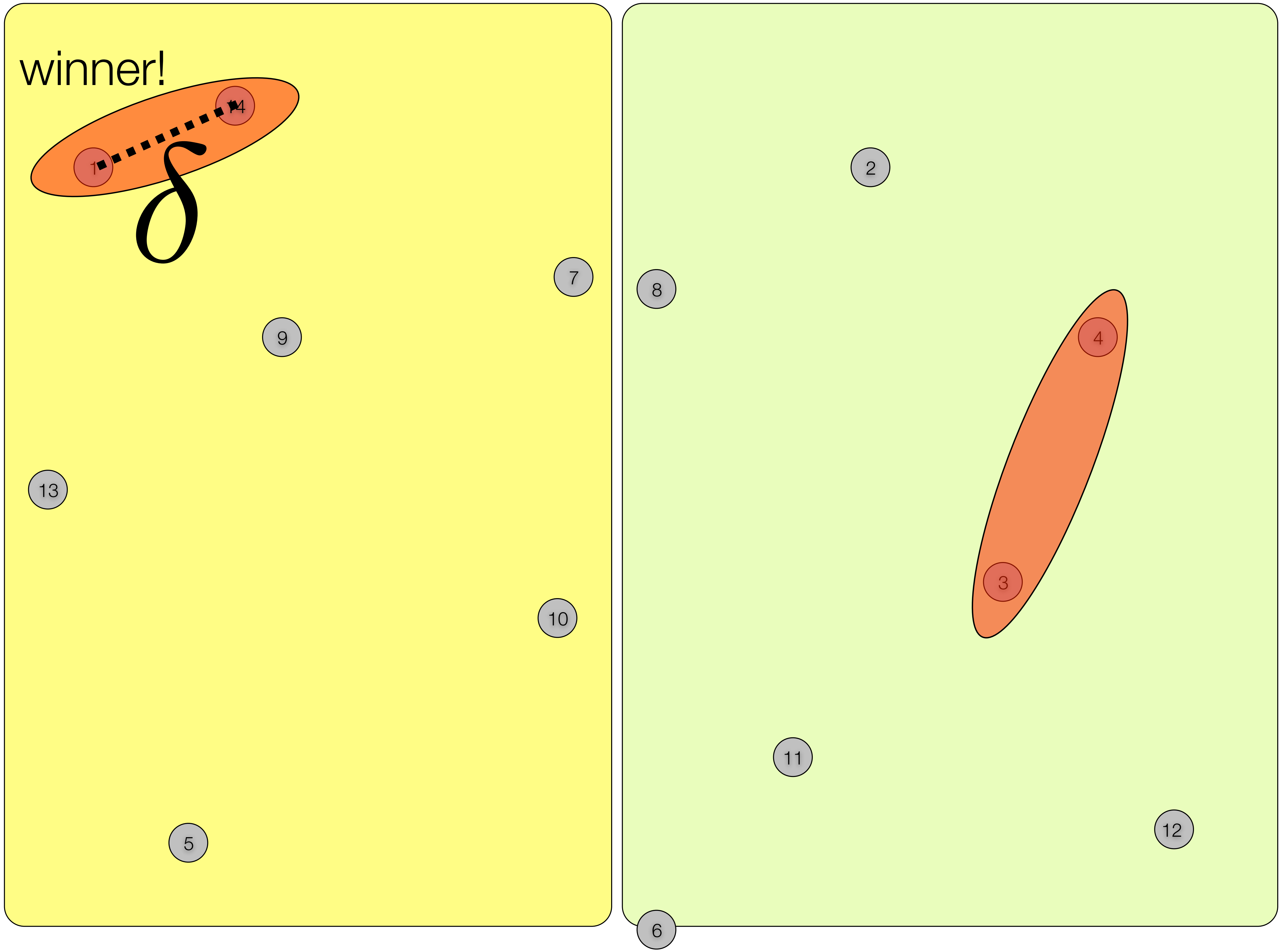
Divide & Conquer



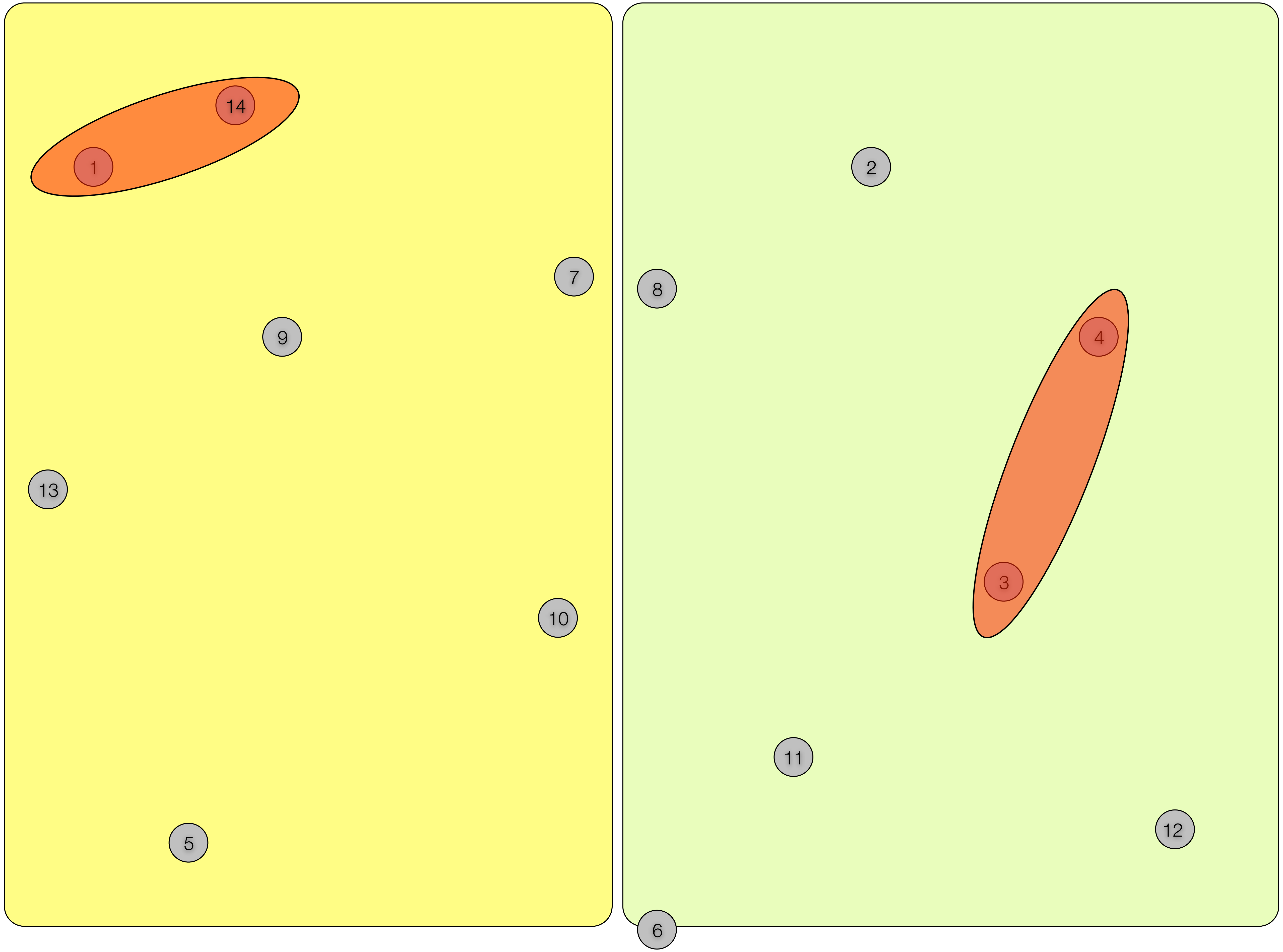
Divide & Conquer



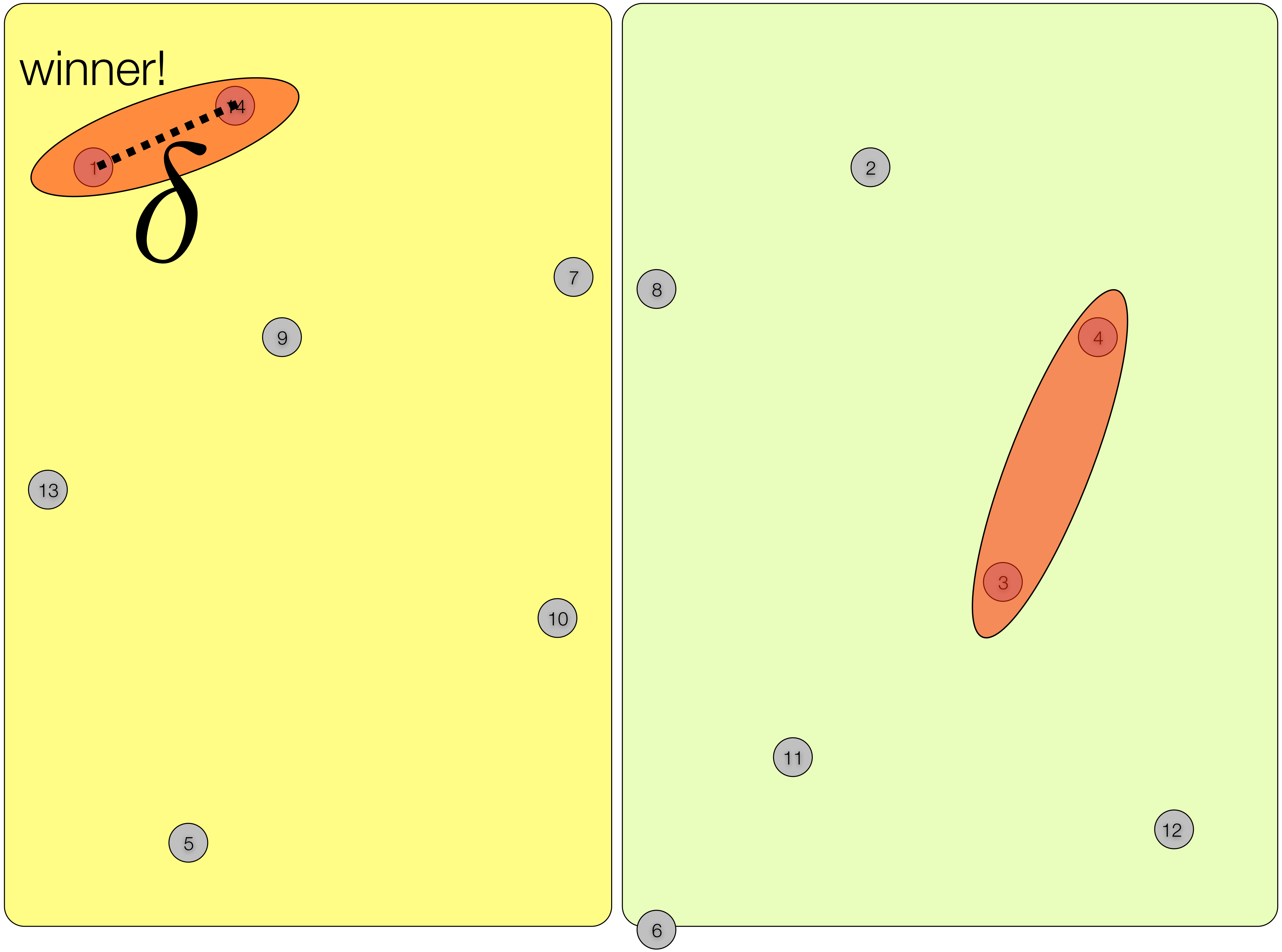
Divide & Conquer

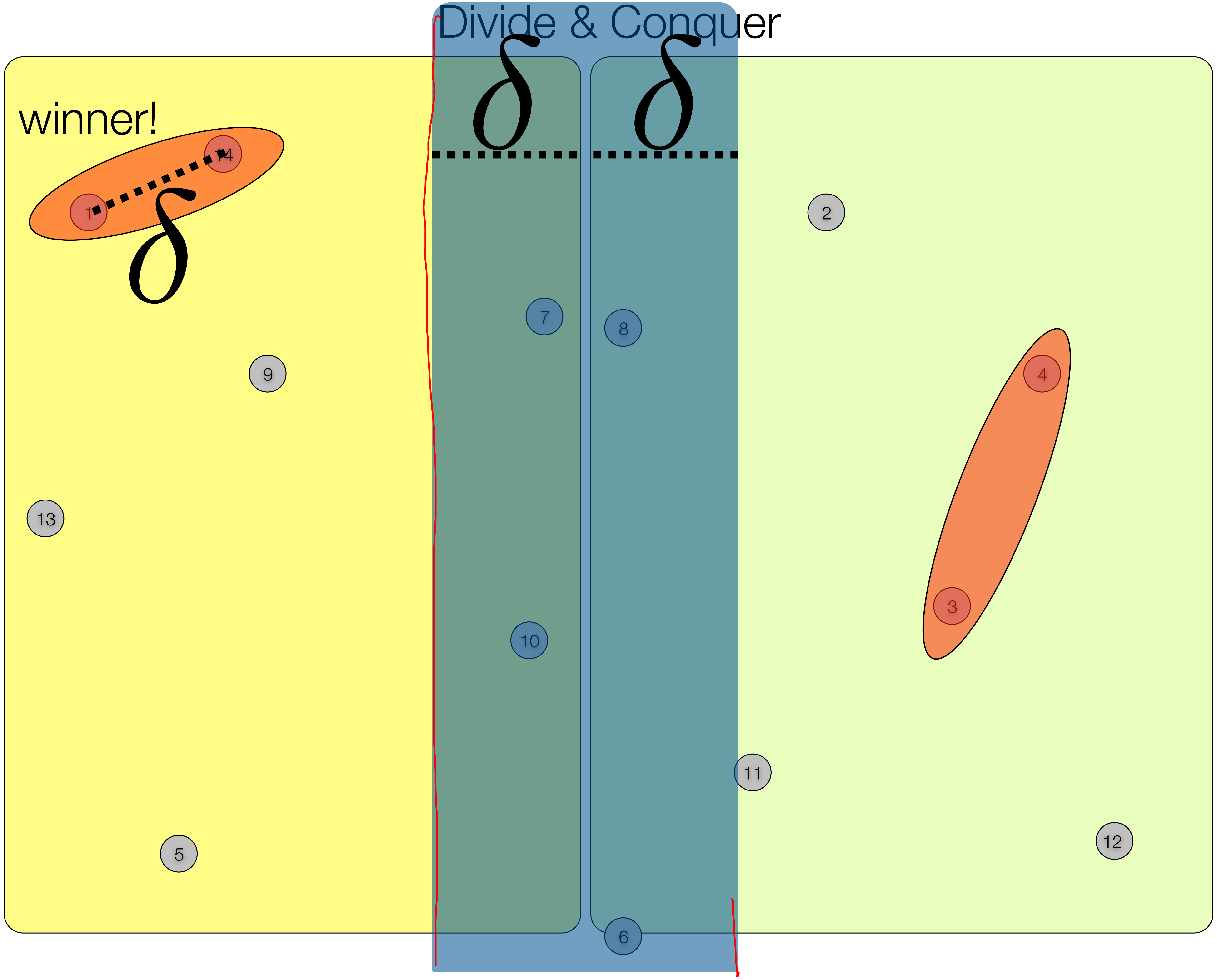


Divide & Conquer

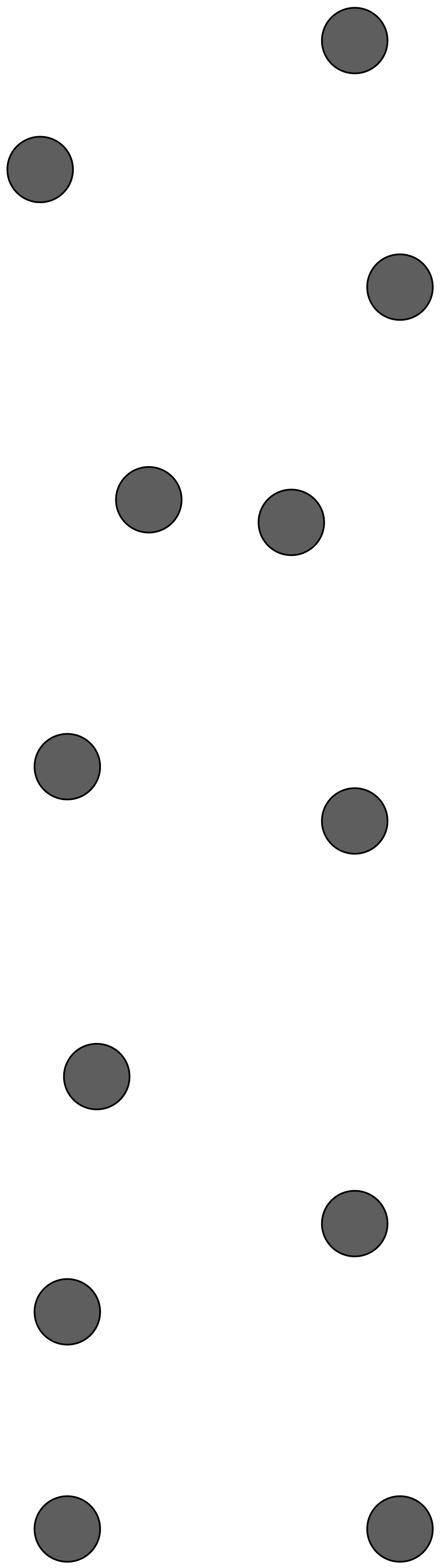


Divide & Conquer

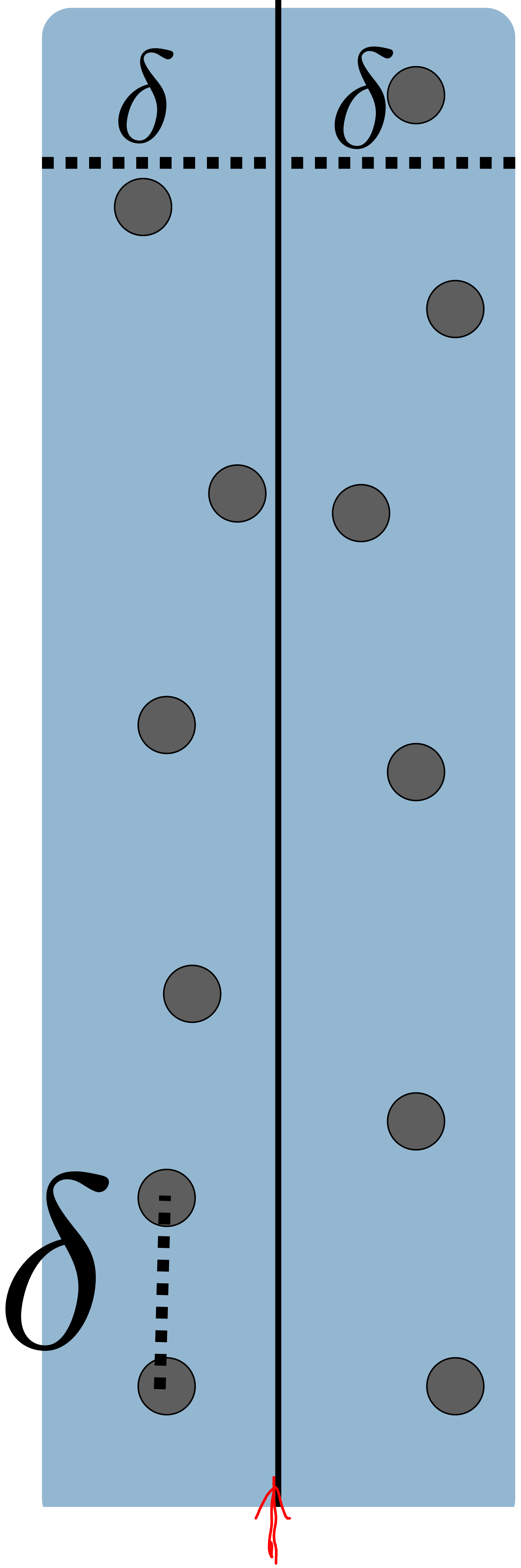


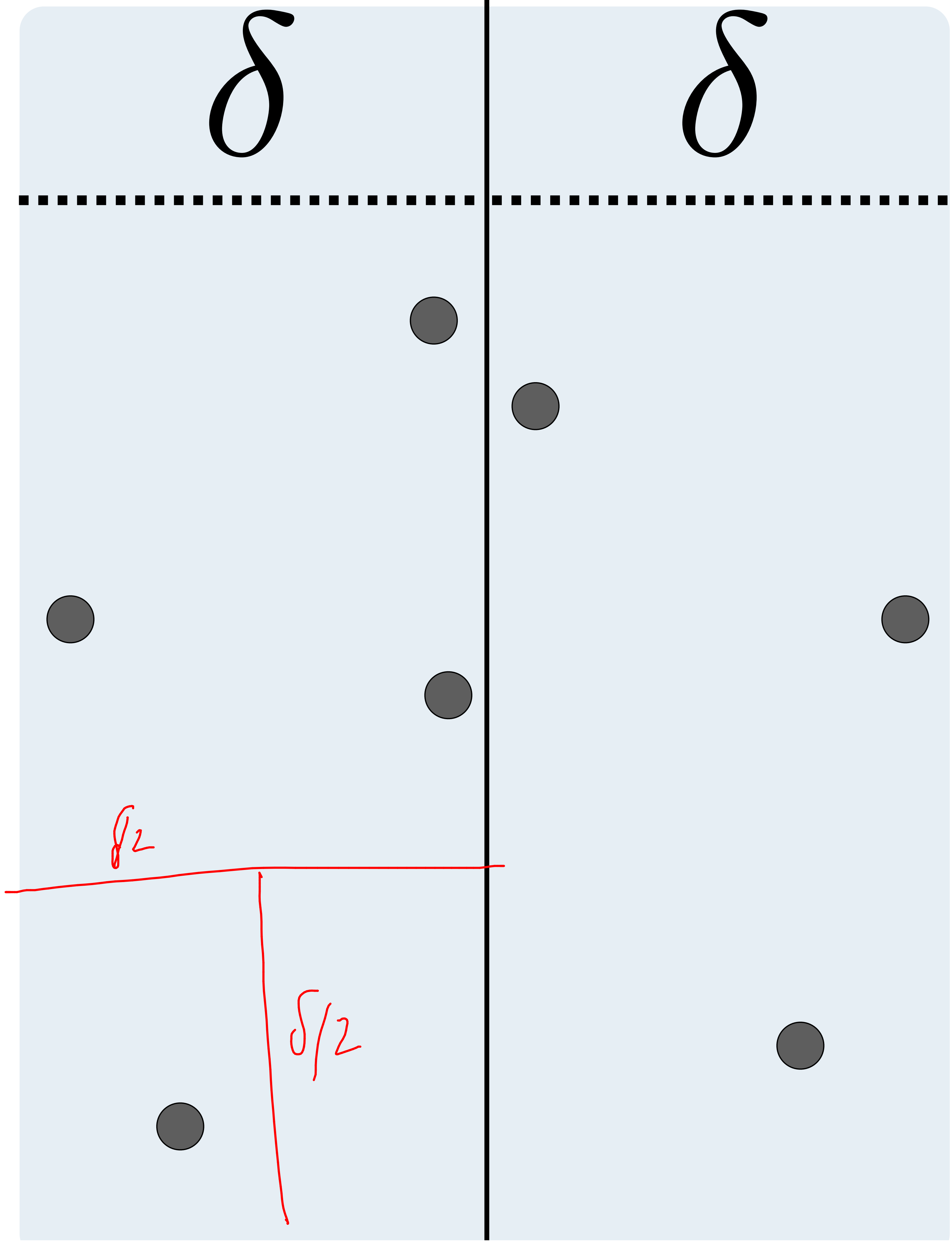


Divide & Conquer
winner!
δ
δ
δ
1
14
2
7
8
9
4
13
3
10
11
12
5
6



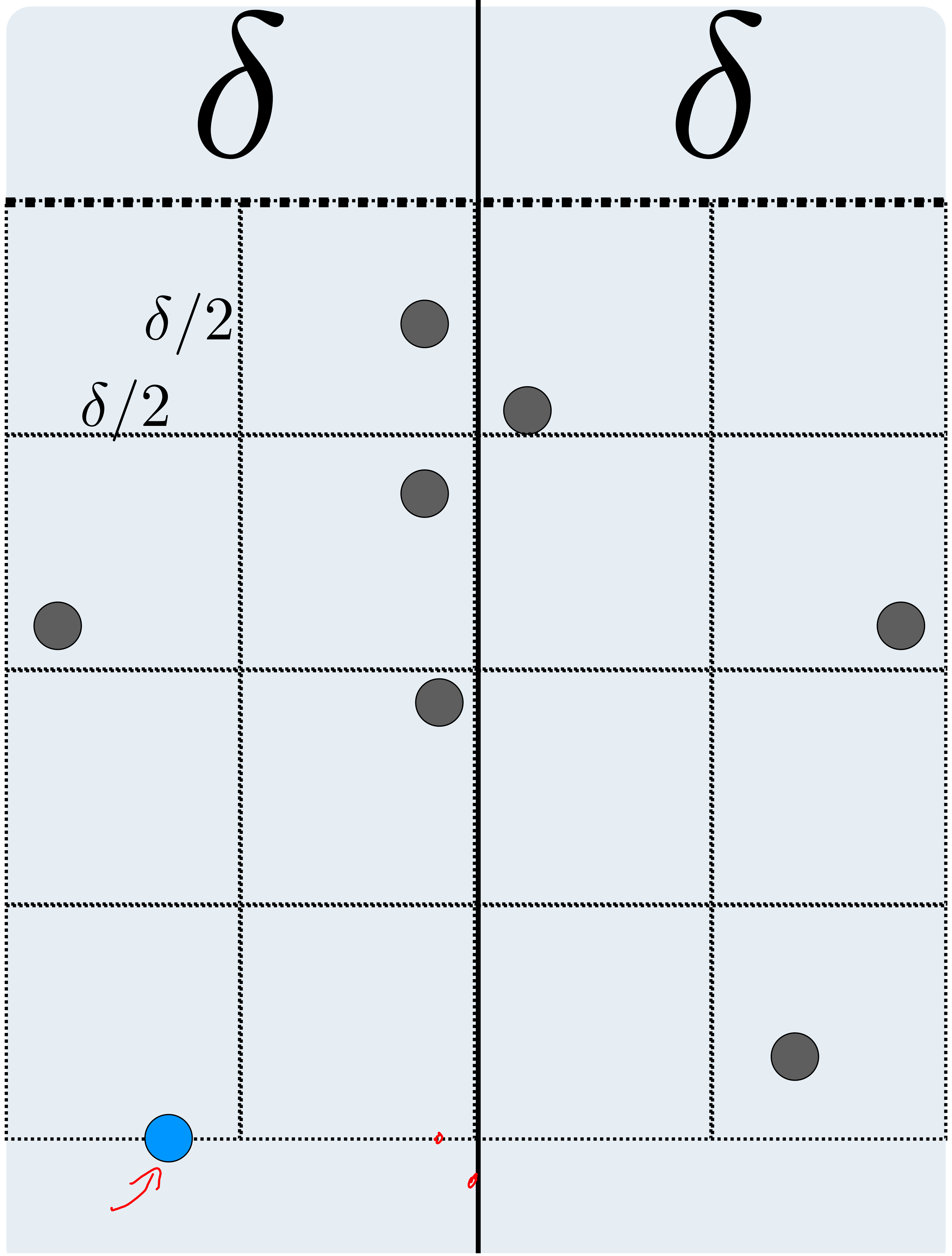
Mohawk can contain all of the input points.

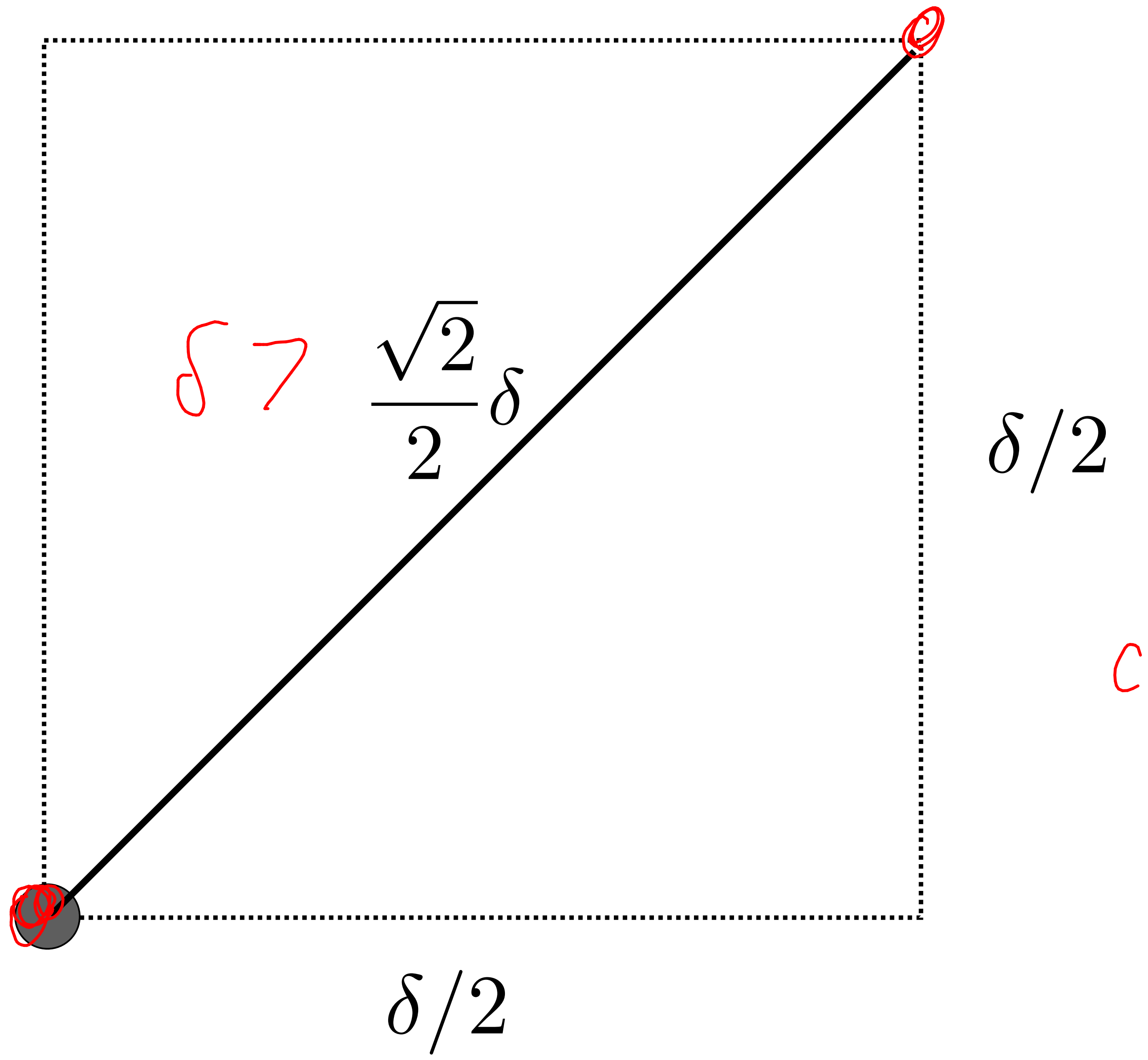

δ
δ
δ



δ
δ
δ/2
δ/2

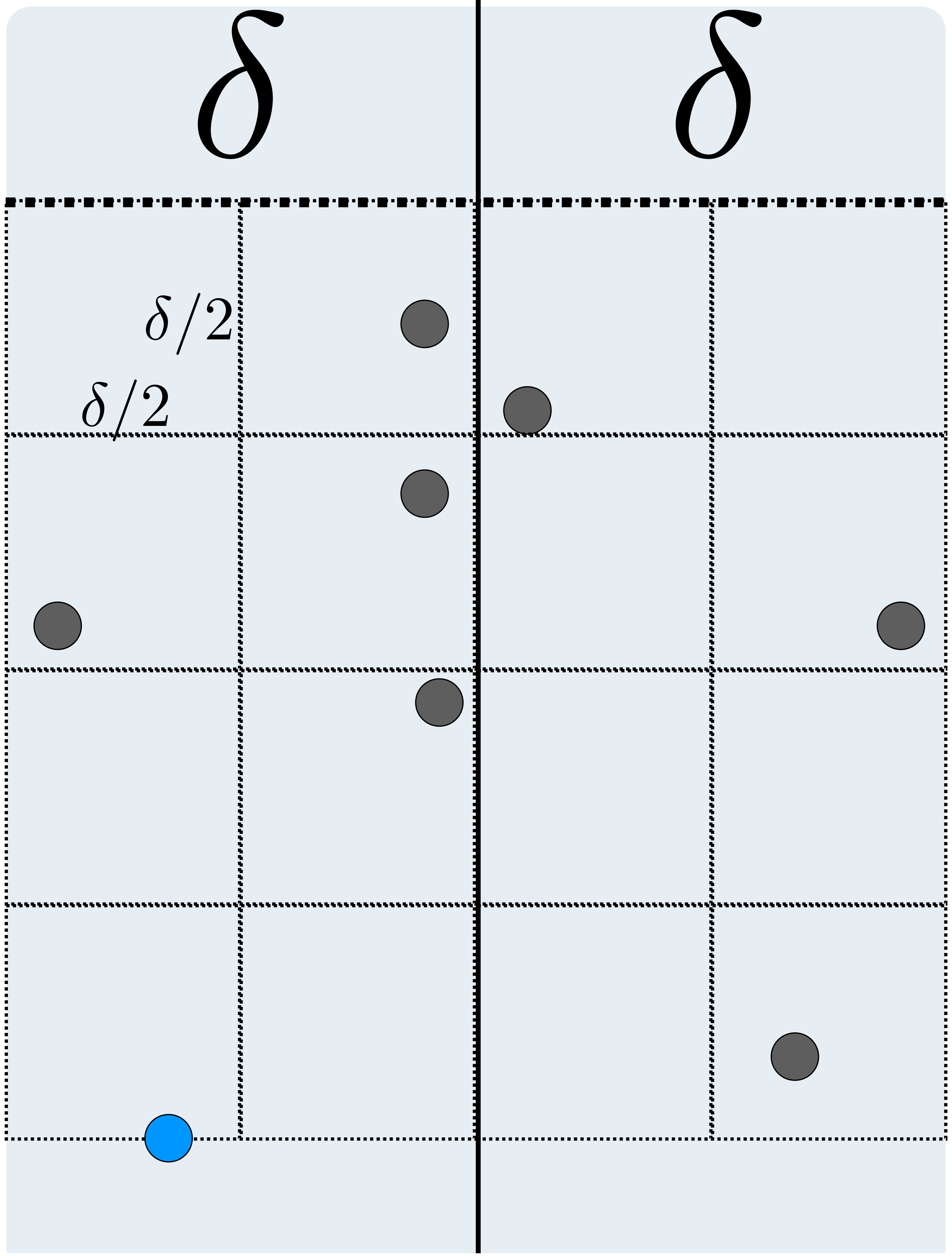
Imagine there is a grid of cubbies starting at the lowest Y point



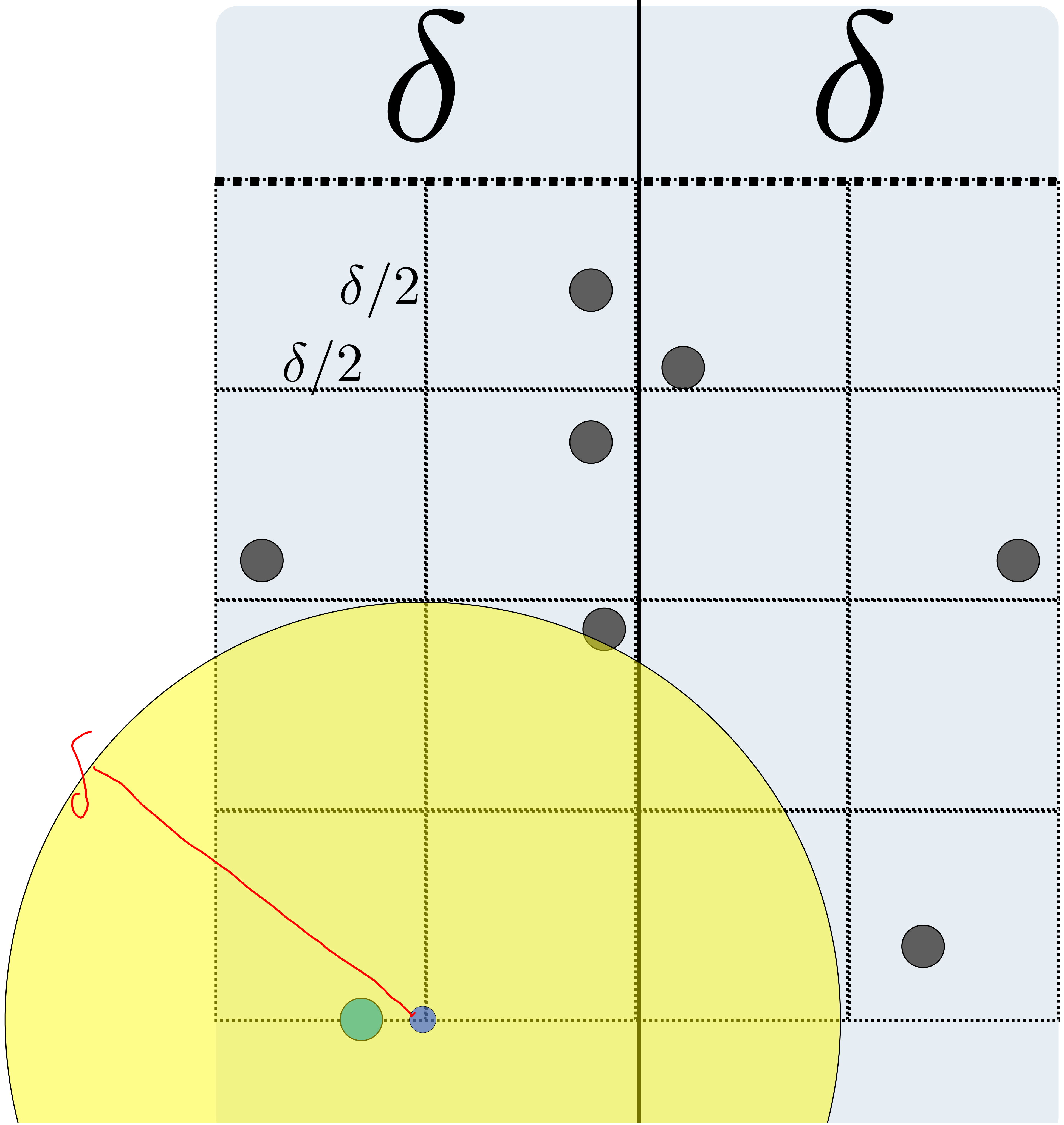


② insight.

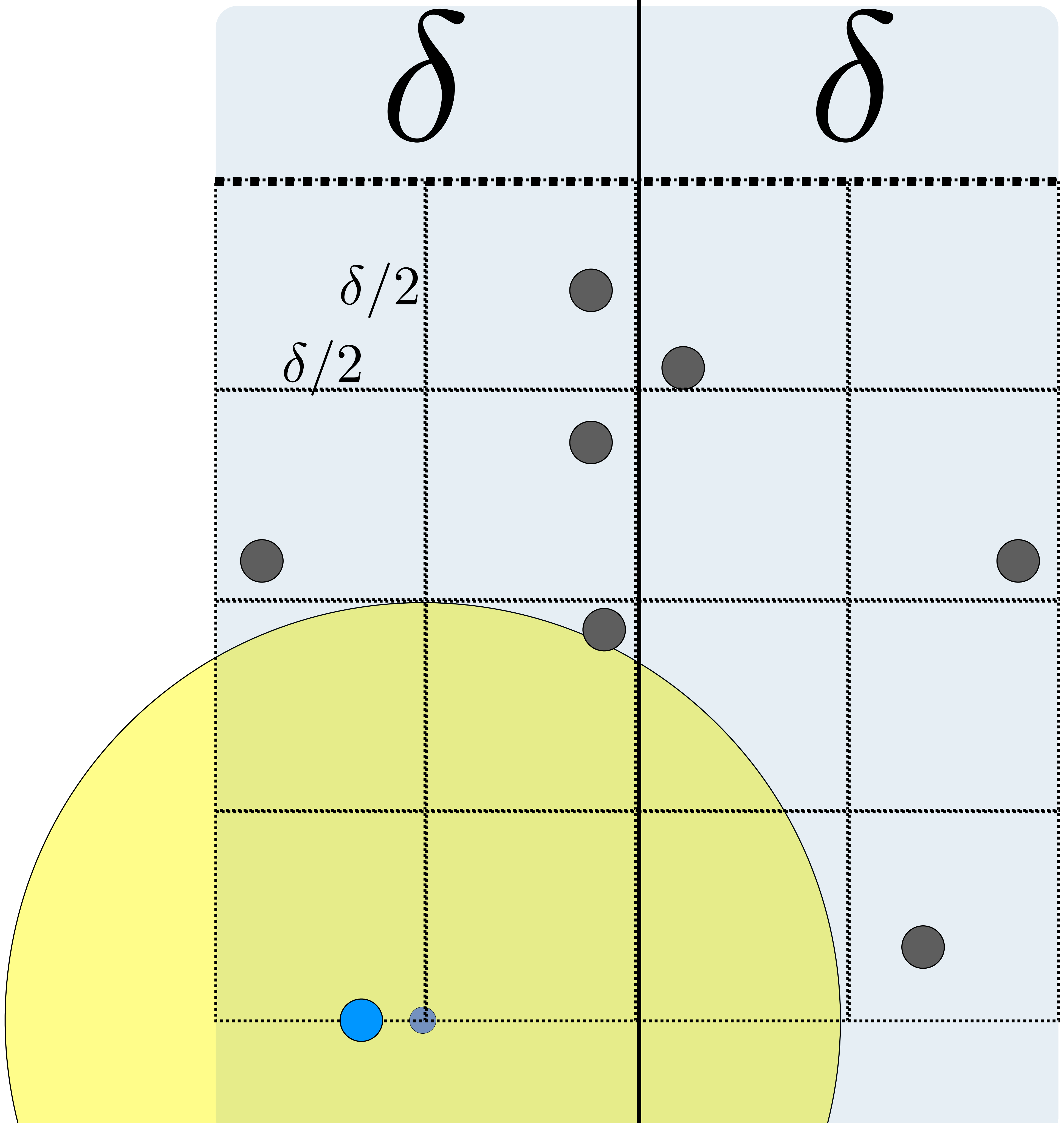
cubbies have ≤ 1 point.



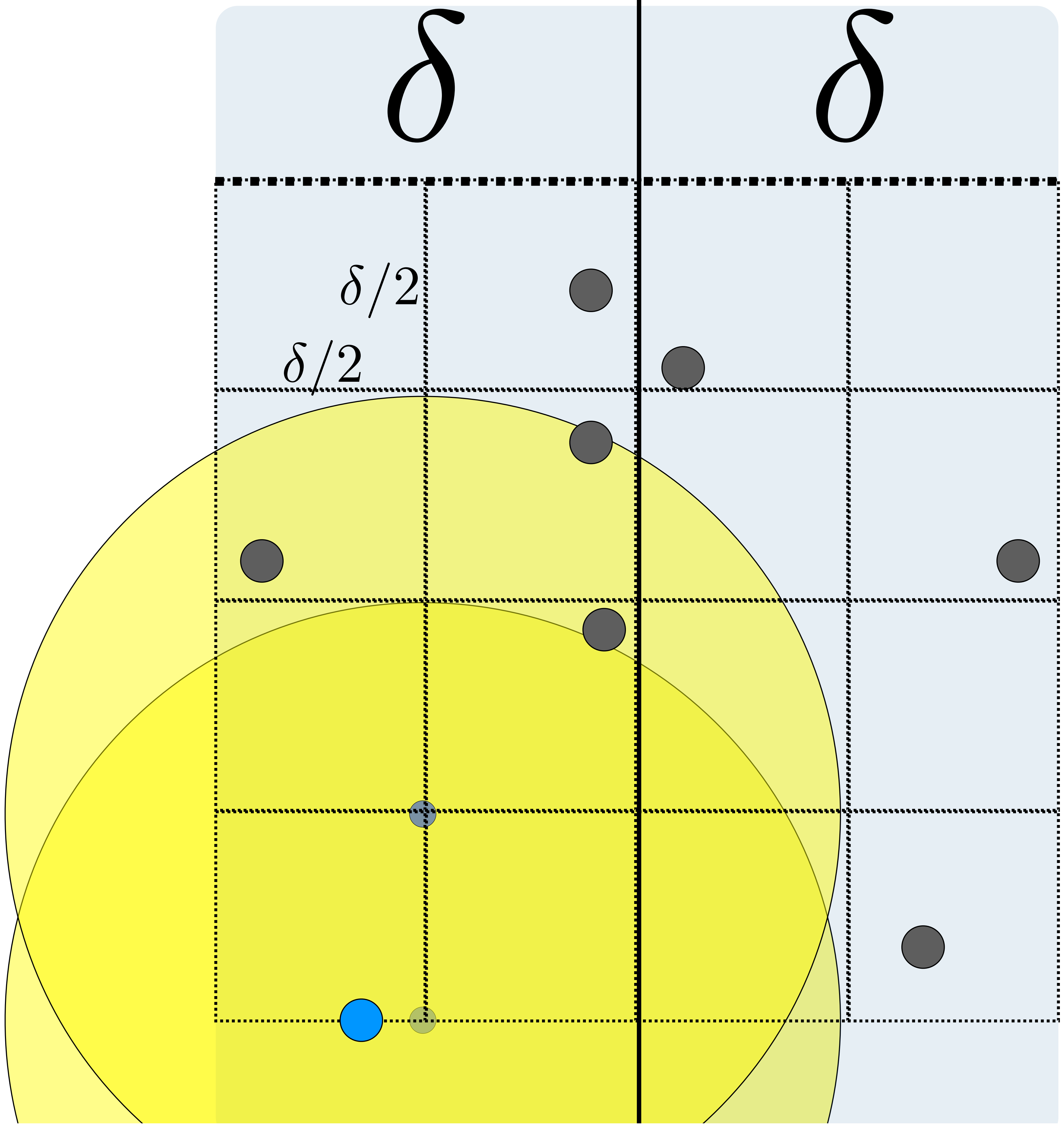
FACT: At most 1 point in each cubby



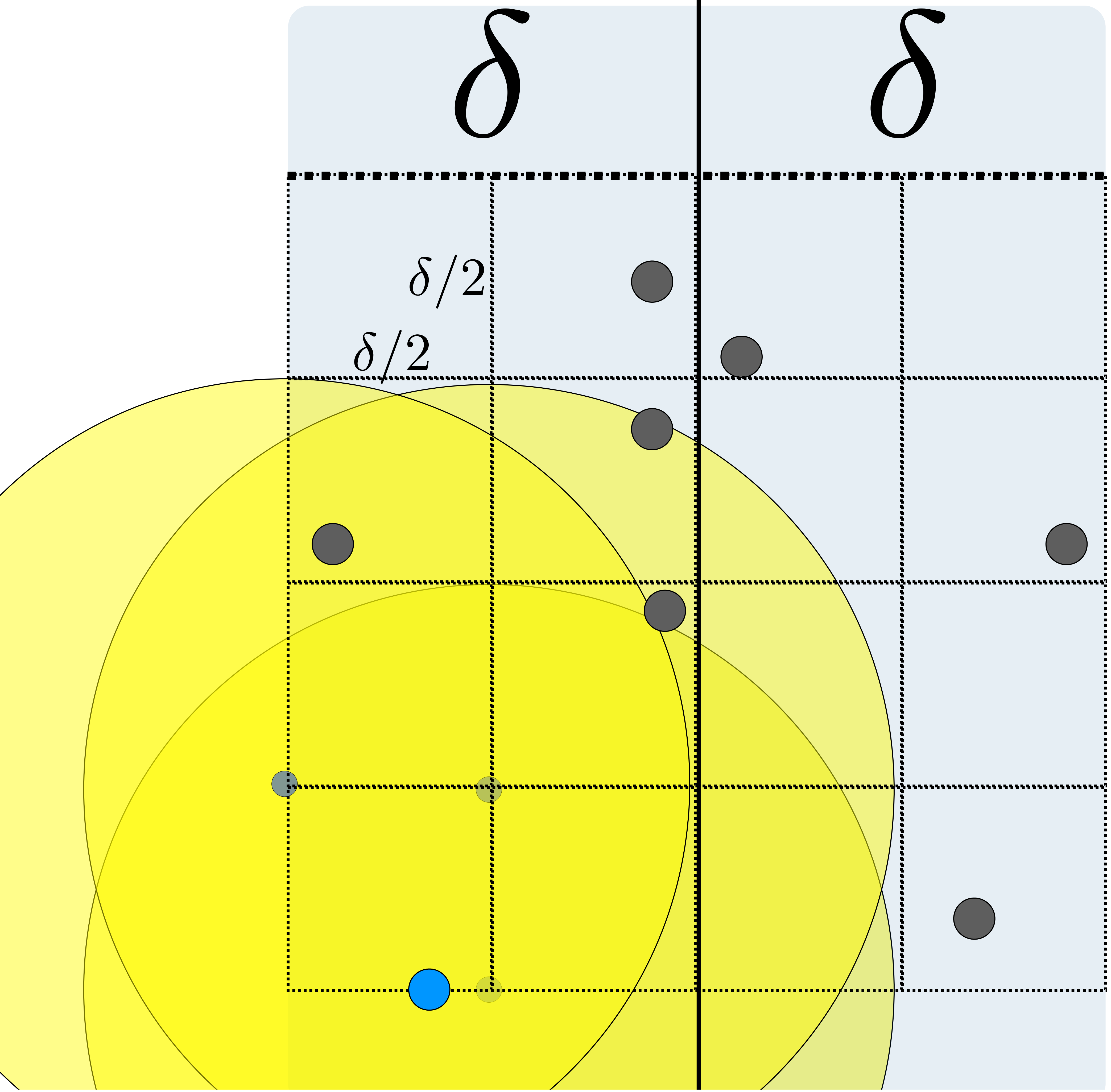
FACT: <=1 point per cubby



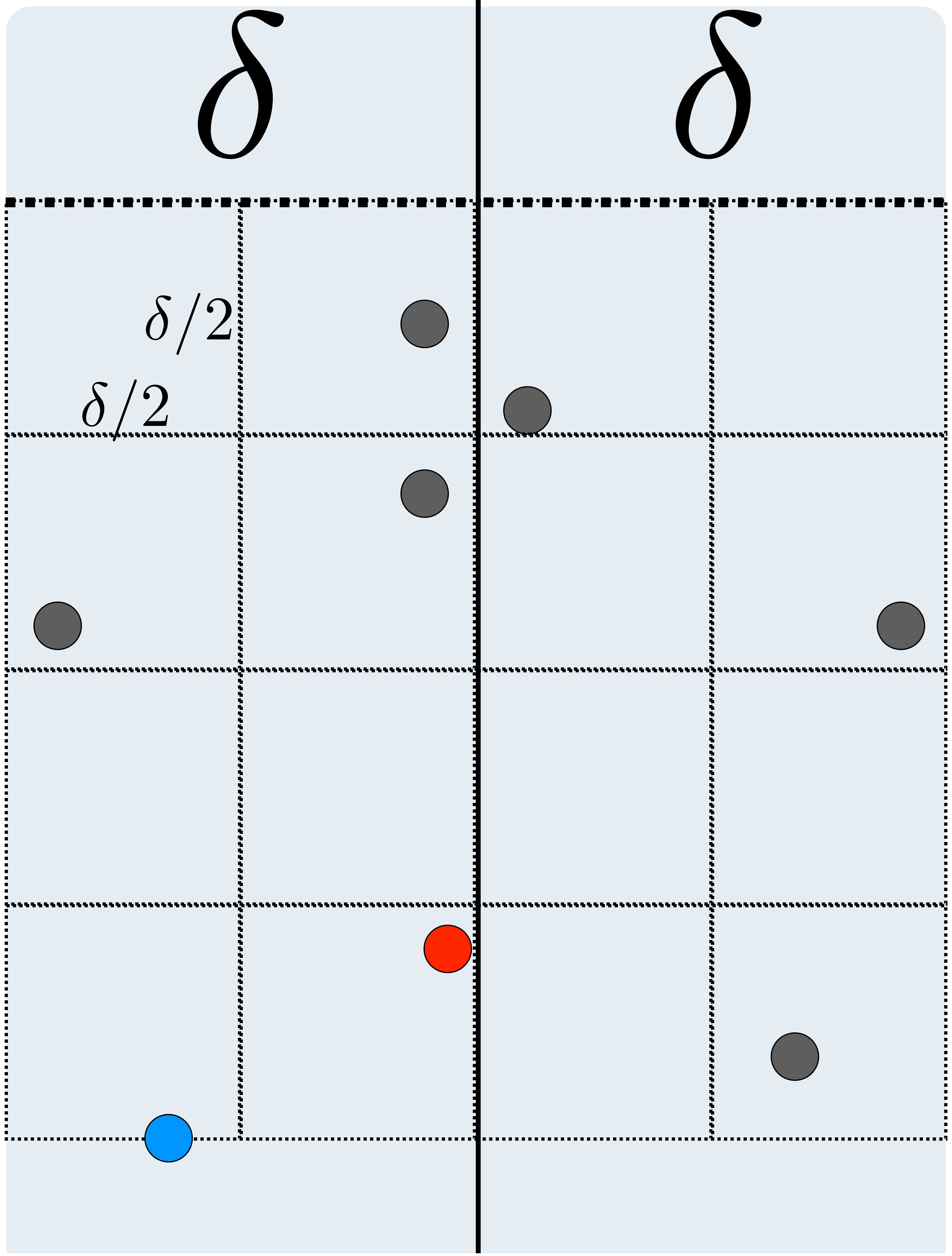
FACT: <=1 point per cubby

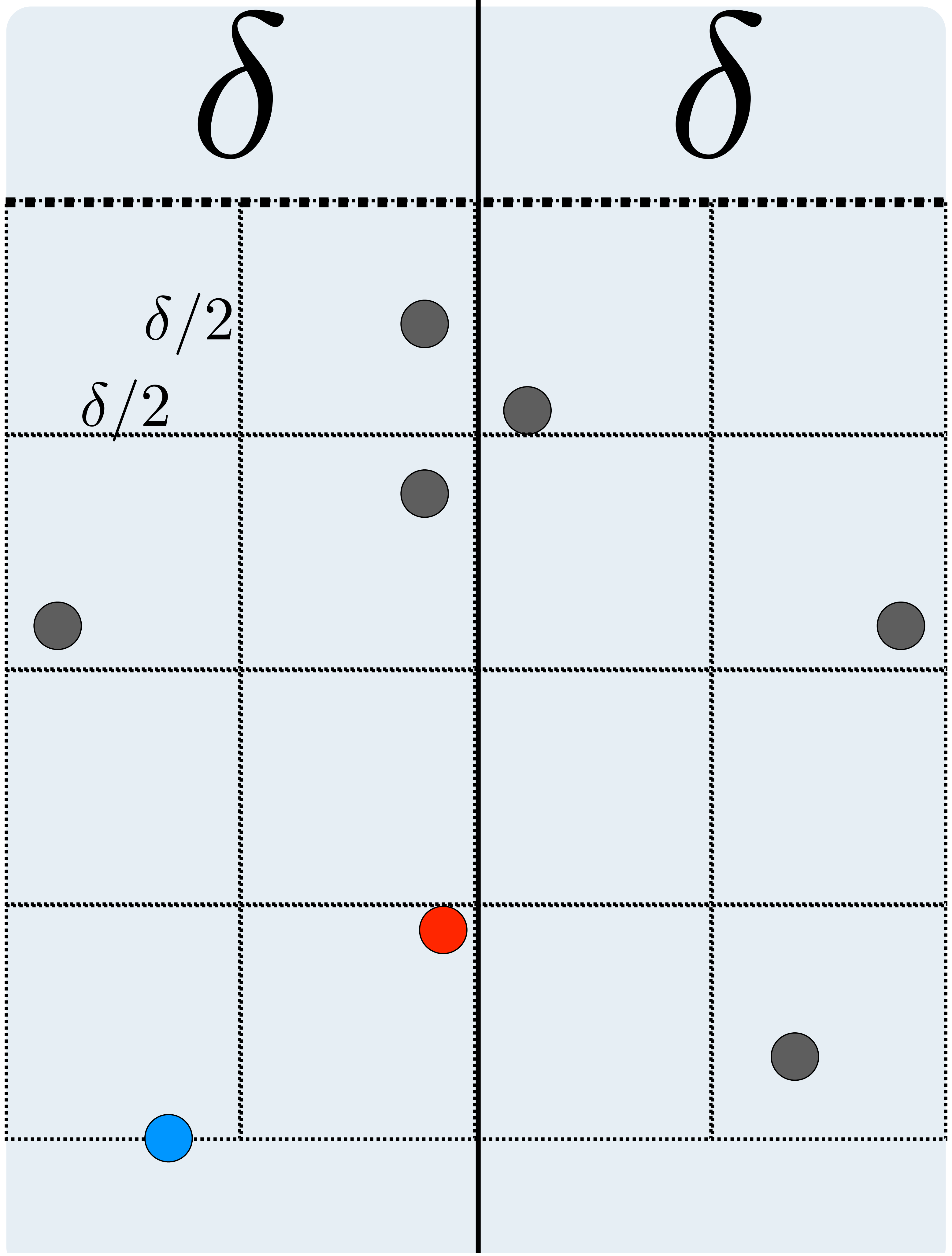


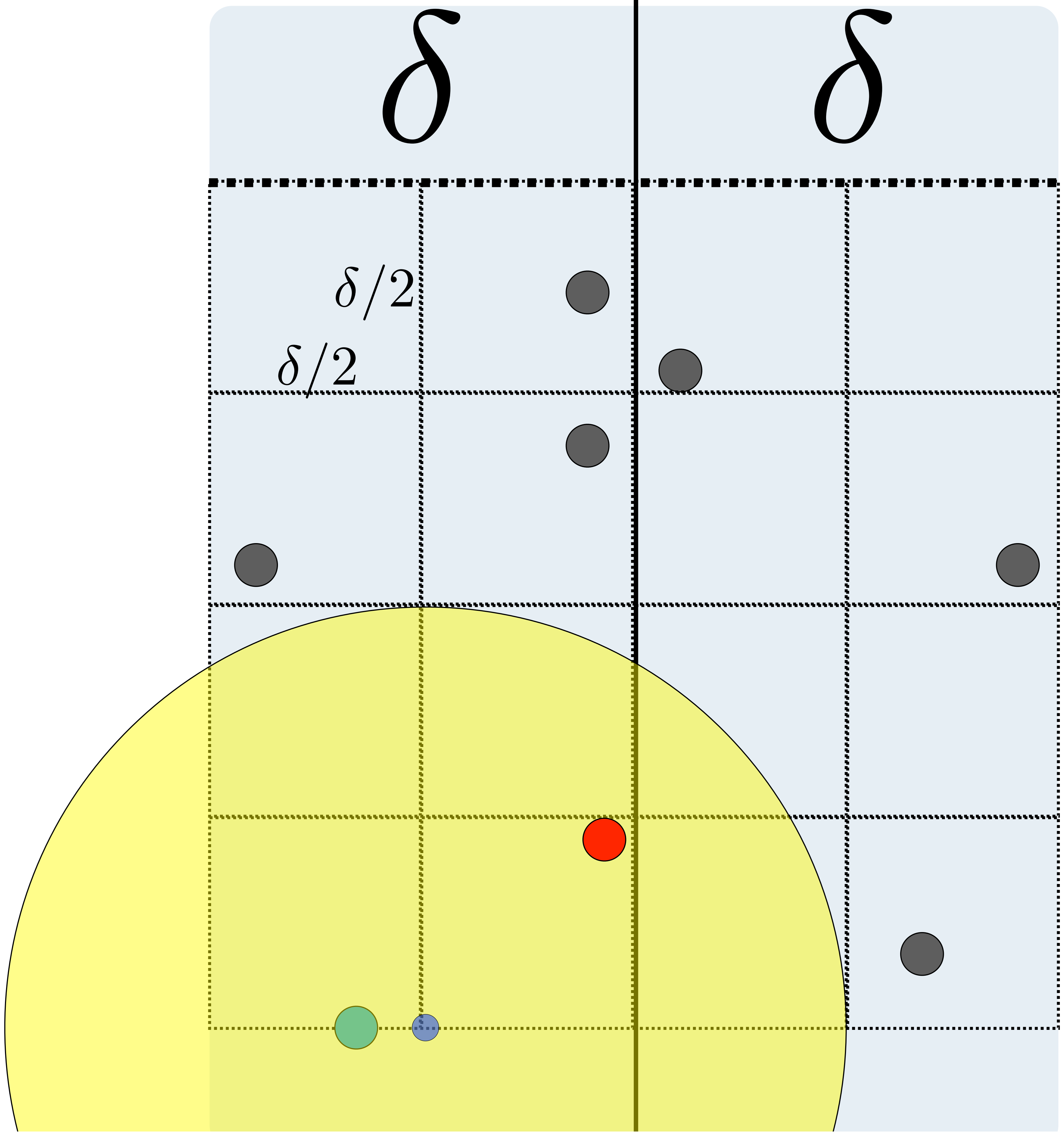
FACT: <=1 point per cubby



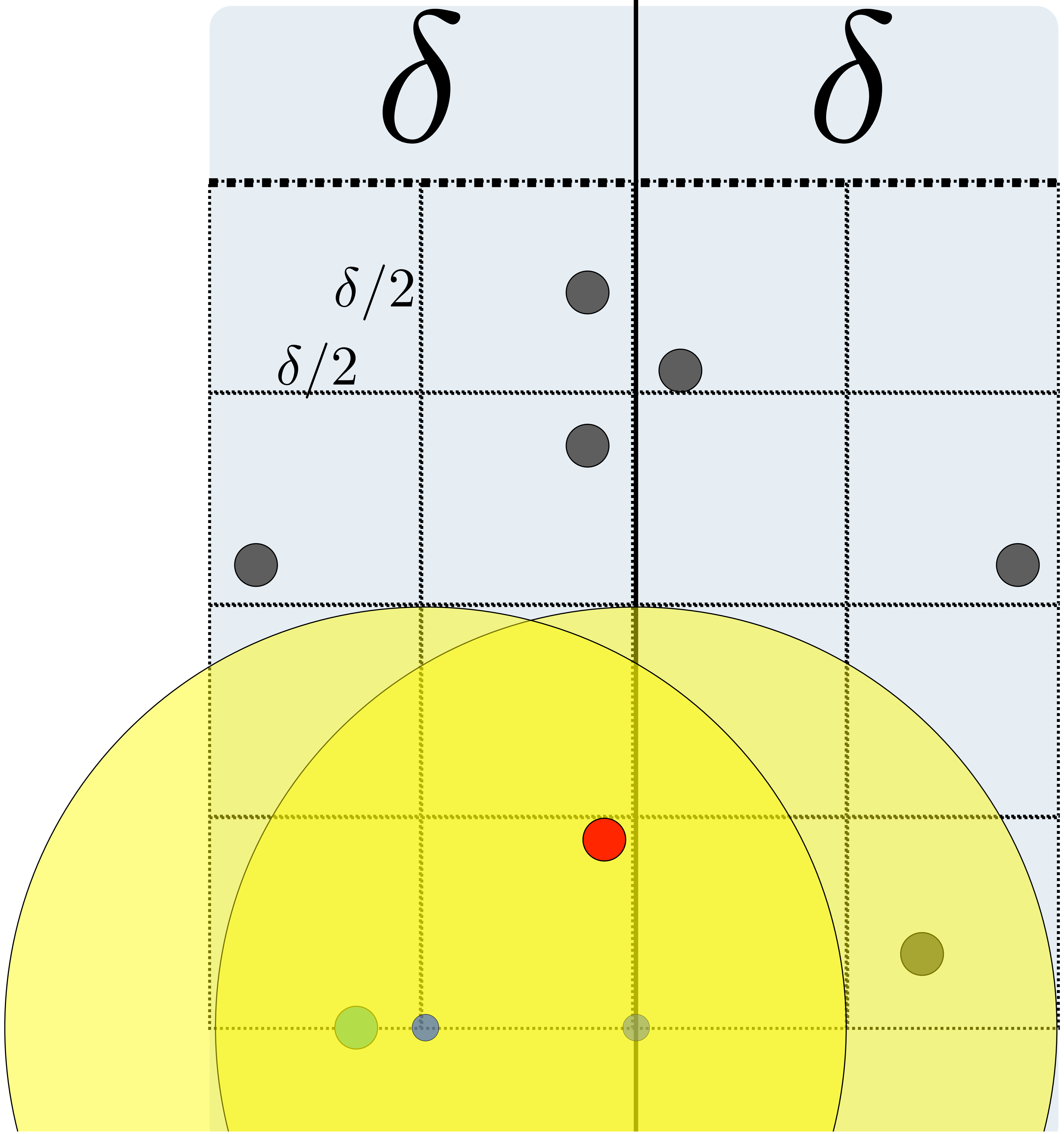
FACT: <=1 point per cubby


δ
δ
$\delta/2$
$\delta/2$

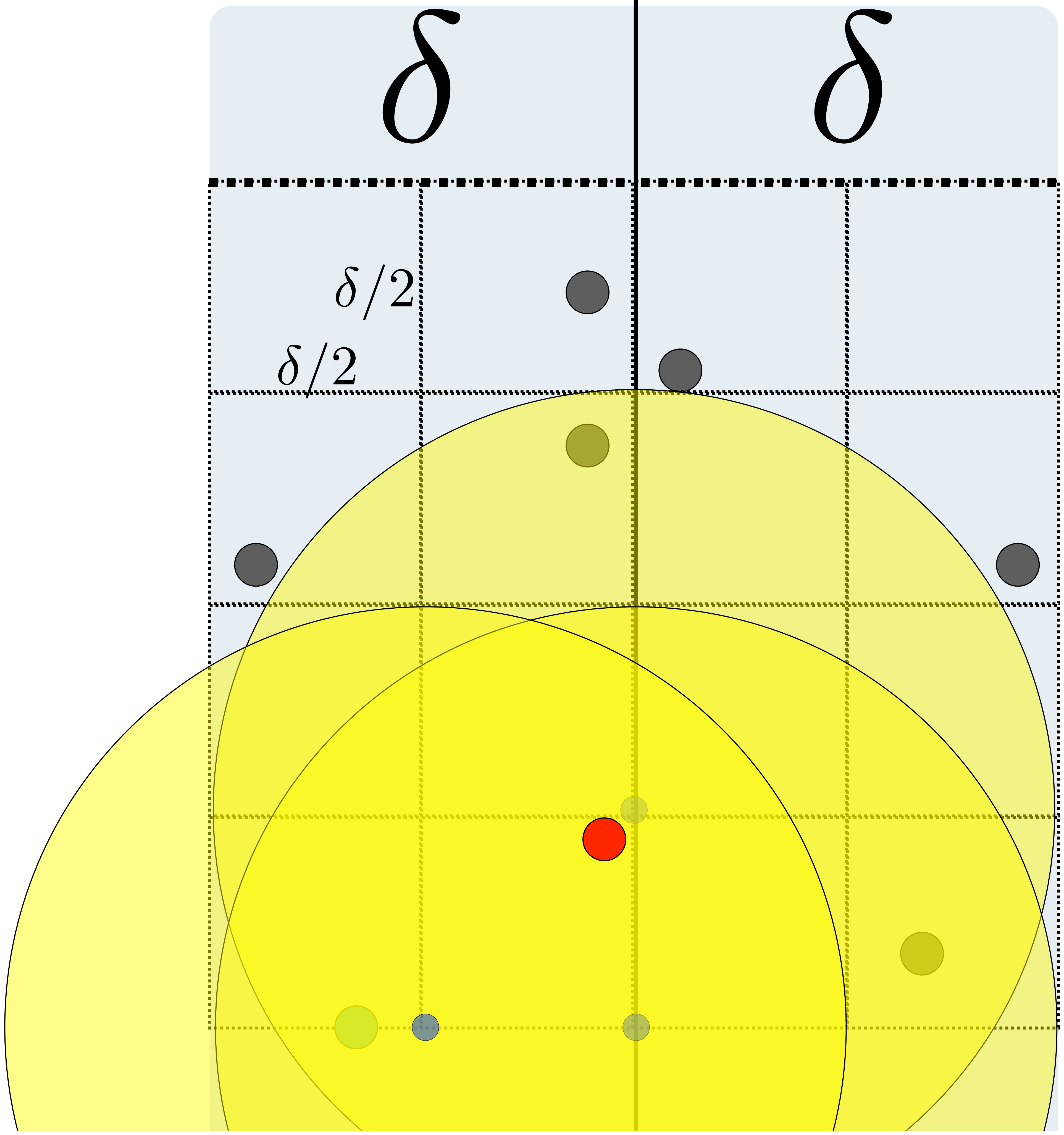

δ
δ
$\delta/2$
$\delta/2$



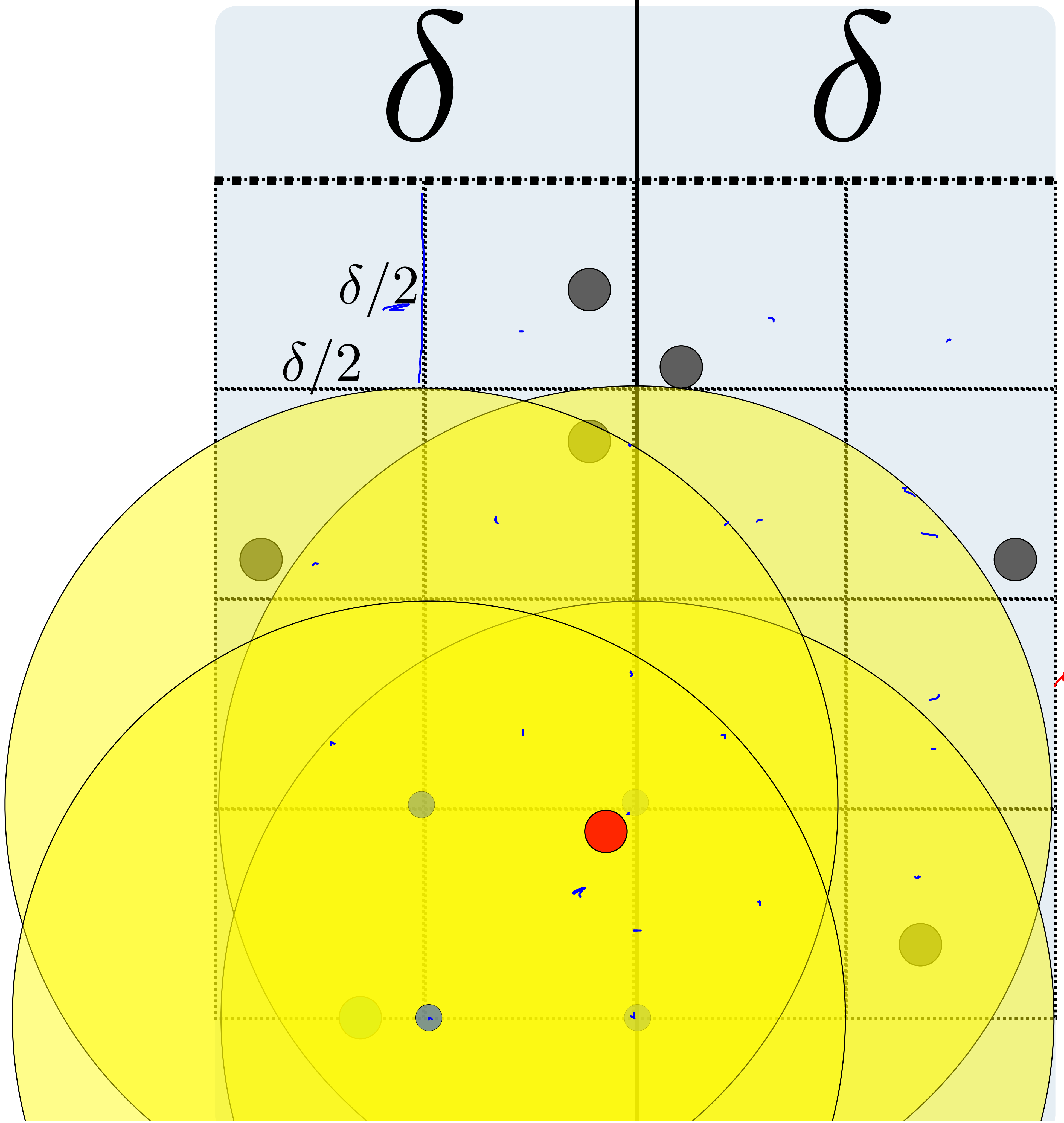
δ
δ
$\delta/2$
$\delta/2$



δ
δ
$\delta/2$
$\delta/2$



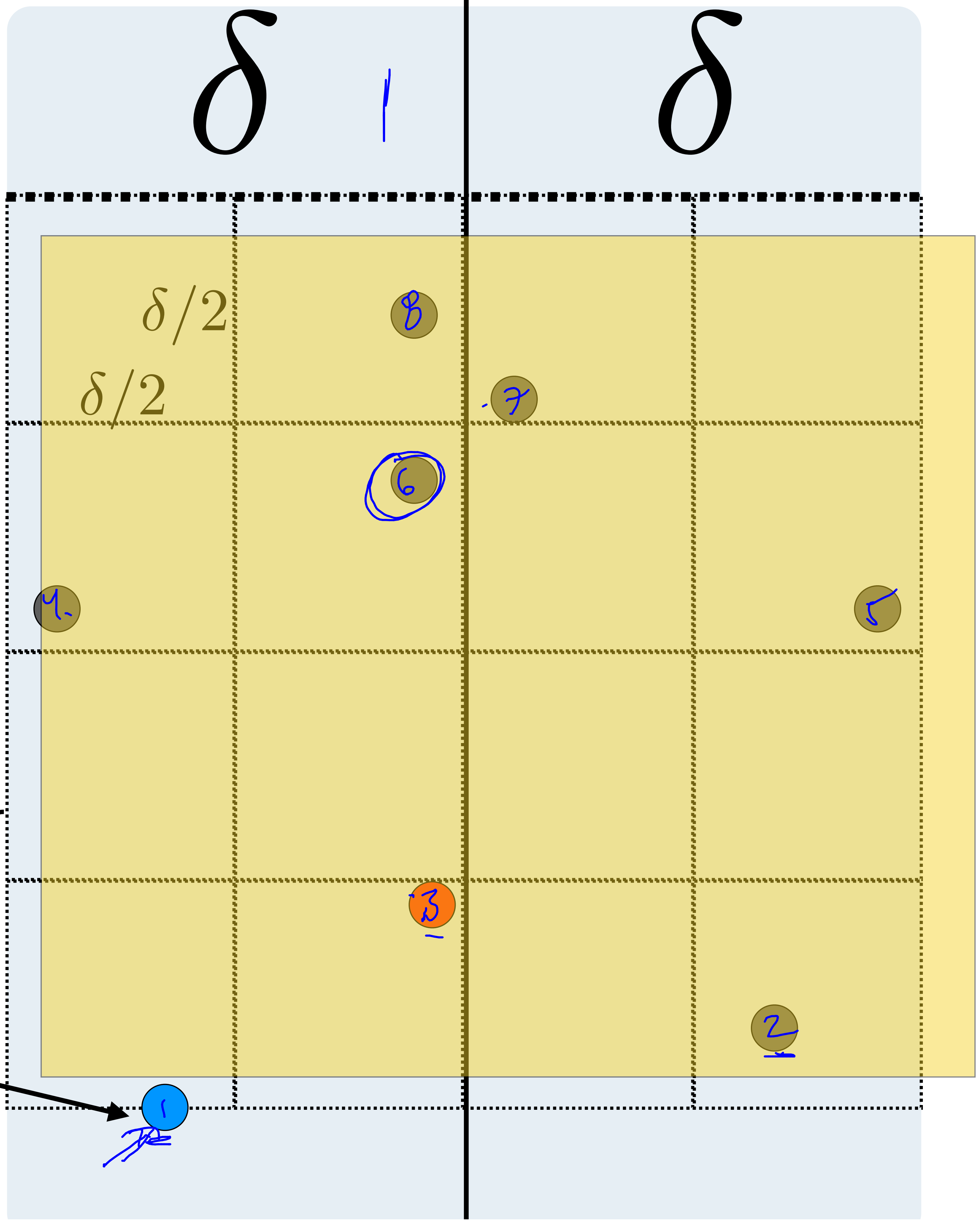
δ
δ
δ/2
δ/2



count the number of cubbies that juliet can reside in.

$\leq 15.$

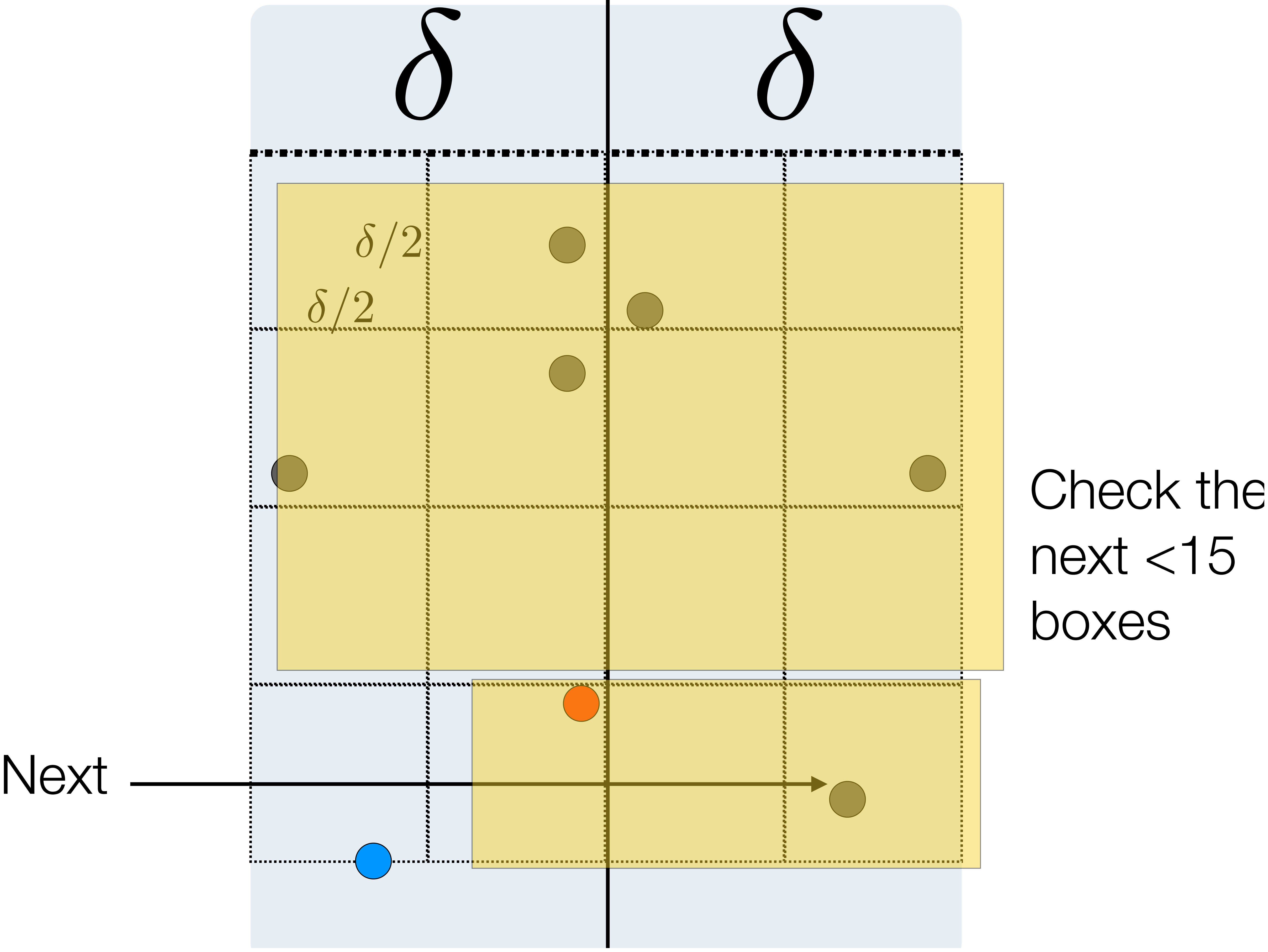
≤ 7 buckets

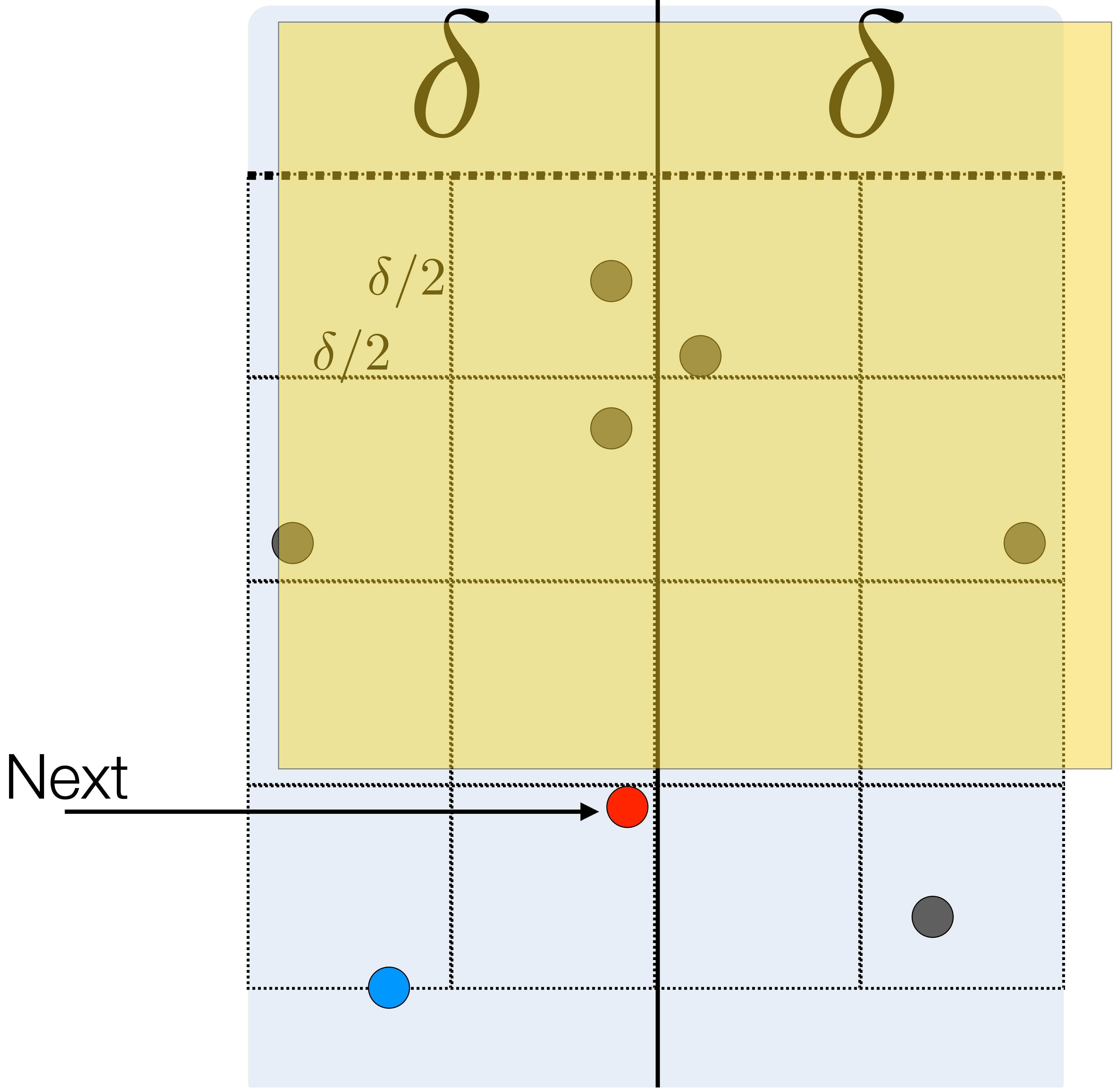


Check the next 15 boxes

Visit its by y-order

Start


δ
δ
$\delta/2$
$\delta/2$
Check the next <15 boxes
Next



Check the next <15 boxes

Closest(P) → points in 2D.

Base case: if $|P| \le 2$, brute force.

Let q be the mid-point along x coordinates.

$L, R =$ split points into left & right halves according to q

$\delta_L = \text{closest}(L)$
$\delta_R = \text{closest}(R)$
$\Longrightarrow$ let $\delta = \min(\delta_L, \delta_R)$

$M = \text{Mohawk}(q, \delta)$ // set of points that are w/in δ of q_x

for all points r in M (sorted according to y-coordinate)

check next 15 points (in y-order) for a j.

$\delta = \min(\delta, d(r, j))$

Return δ.

Closest(P)

Running Analysis

need

Base Case: If <8 points, brute force.

1. Let q be the "middle-element" of points → $\Theta(n)$

2. Divide P into Left, Right according to q → $\Theta(n)$

3. delta,r,j = MIN(Closest(Left) , Closest(Right)) → $2T(\frac{n}{2})$

$\frac{n}{2}$ $\frac{n}{2}$

4. Mohawk = { Scan P, add pts that are delta from q.x } $\Theta(n)$

5. For each point x in Mohawk (in y-order):
   Compute distance to its next 15 neighbors
   Update delta,r,j if any pair (x,y) is < delta

$\Theta(n)$

6. Return (delta,r,j)

$\Rightarrow T(n) = 2T(\frac{n}{2}) + \Theta(n)$ by Master's

$\Rightarrow T(n) = \Theta(n \log n)$

Closest(P)

Base Case: If <8 points, brute force.

1. Let q be the “middle-element” of points

2. Divide P into Left, Right according to q

3. delta,r,j = MIN(Closest(Left) , Closest(Right))

4. Mohawk = { Scan P, add pts that are delta from q.x }

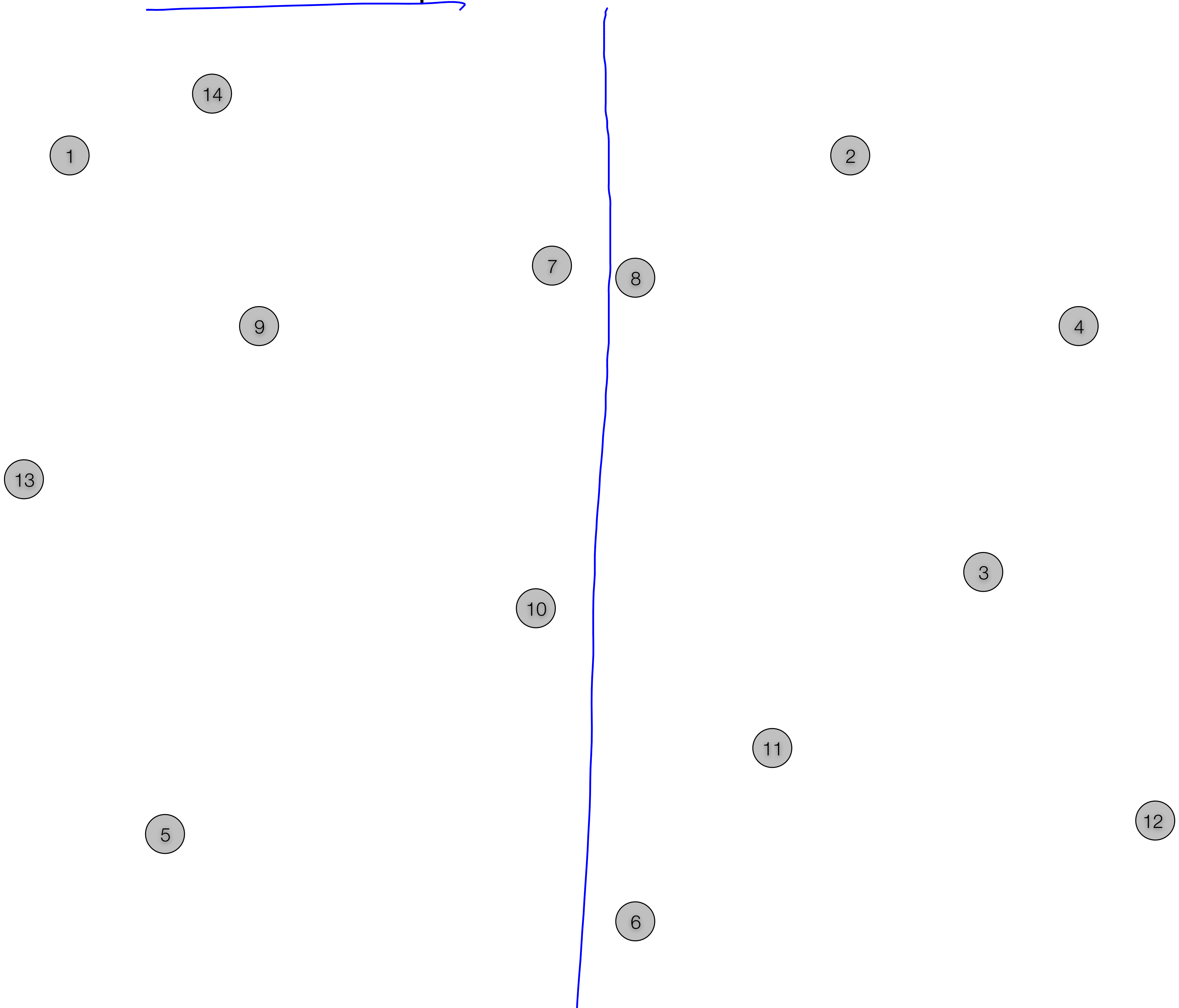
5. For each point x in Mohawk (in y-order):

   Compute distance to its next 15 neighbors
   Update delta,r,j if any pair (x,y) is < delta

6. Return (delta,r,j)

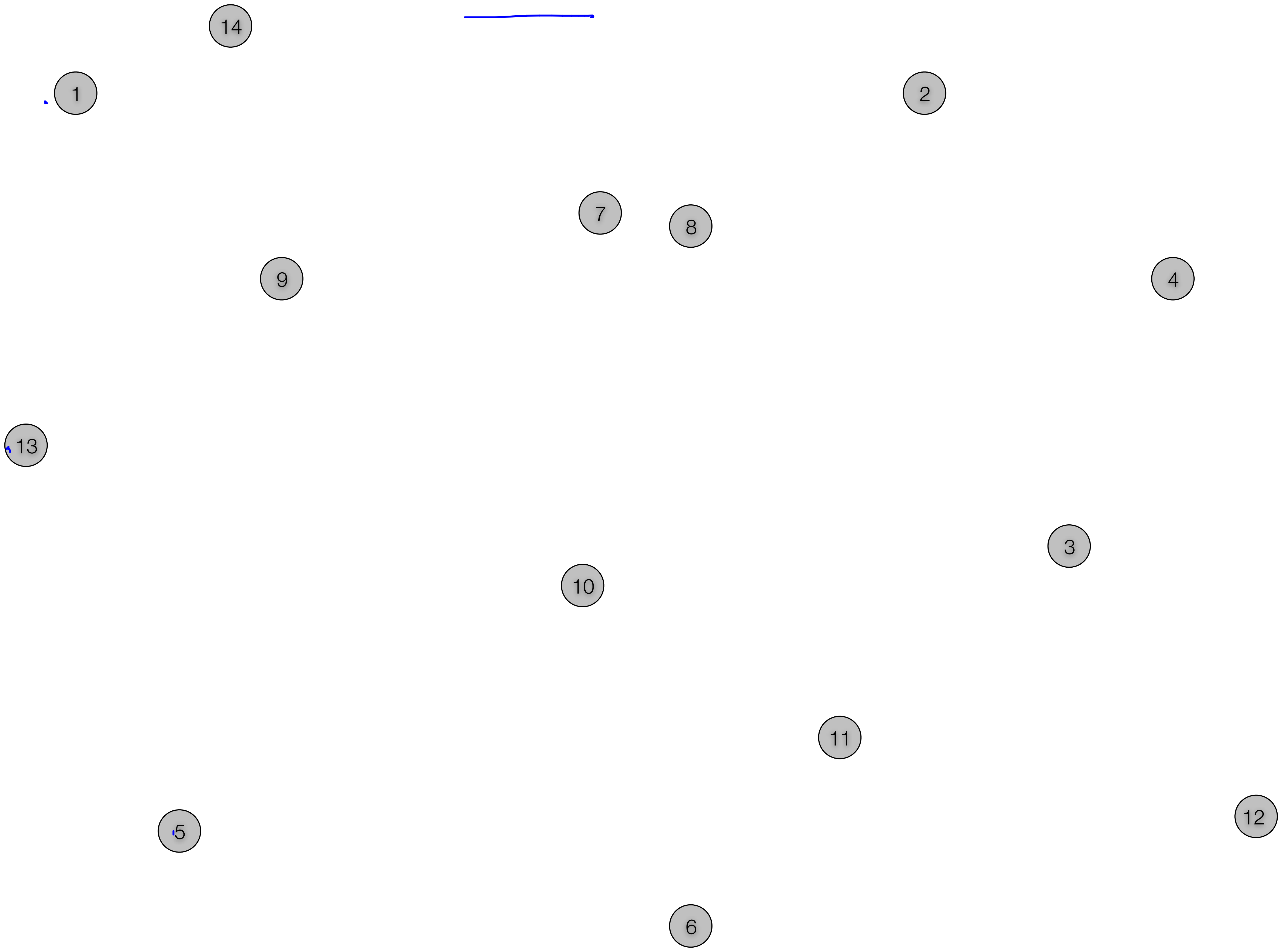
Can be reduced to 7!

Details: How to do step 1?



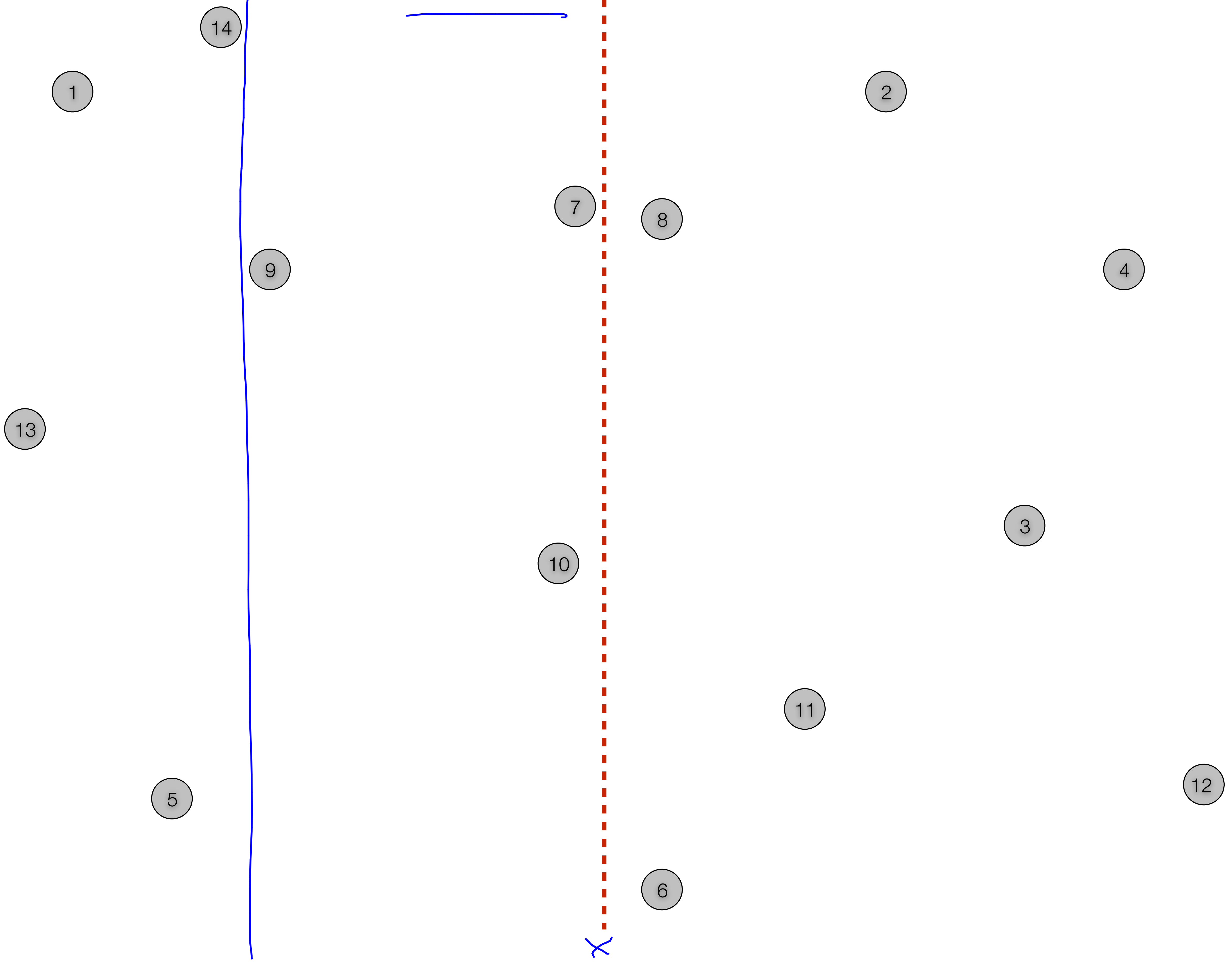
sorted in X: 13 1 5 14 9 10 7 9 8 11 2 3 4 12

sorted in Y: 6 5 12 11 10 3 13 4 9 8 7 2 1 14



sorted in X: 13 1 5 14 9 10 7 9 8 11 2 3 4 12

sorted in Y: 6 5 12 11 10 3 13 4 9 8 7 2 1 14



ClosestPair(P)

Compute Sorted-in-X list SX → mergesort $\Theta(n\log n)$

Compute Sorted-in-Y list SY → $\Theta(n\log n)$

Closest(P,SX,SY) → $\Theta(n\log n)$

Overall solution is still

$\Theta(n\log n)$

Closest(P,SX,SY)

Let q be the middle-element of SX $\Theta(1)$

Divide P into Left, Right according to q → $\Theta(n)$

LX – left points sorted
LY – left point sorted by y.

delta,r,j = MIN(Closest(Left, LX, LY) Closest(Right, RX, RY))

how to compute

Mohawk = { Scan SY, add pts that are delta from q.x }

For each point x in Mohawk (in order):

Compute distance to its next 15 neighbors
Update delta,r,j if any pair (x,y) is < delta

Return (delta,r,j)

Closest(P,SX,SY)

Let q be the middle-element of SX

Divide P into Left, Right according to q

delta,r,j = MIN(Closest(Left, LX, LY) Closest(Right, RX, RY))

Mohawk = { Scan SY, add pts that are delta from q.x }

For each point x in Mohawk (in order):

Compute distance to its next 15 neighbors

Update delta,r,j if any pair (x,y) is < delta

Return (delta,r,j)

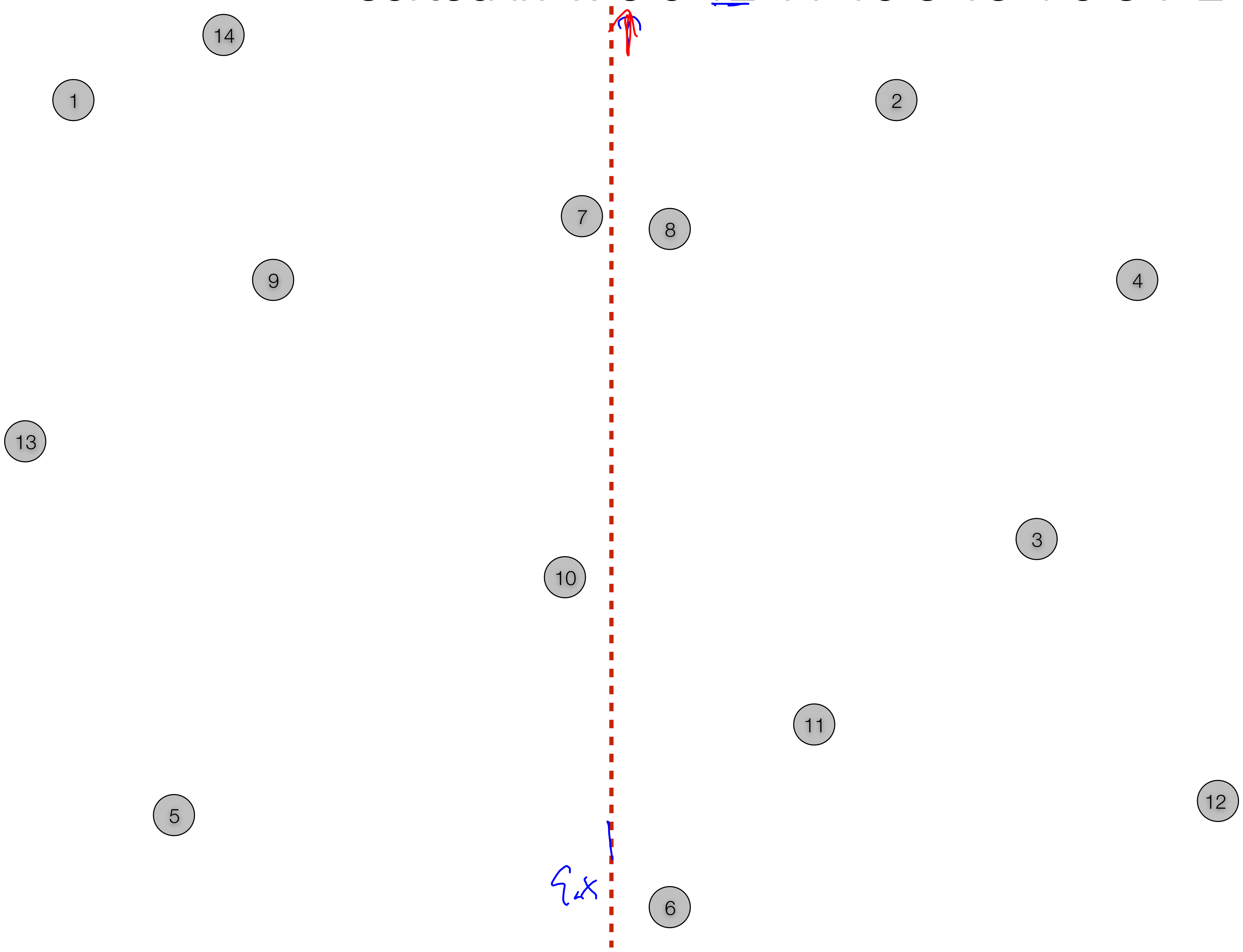
Can be reduced to 7!

Lx Ly

sorted in X: 13 1 5 14 9 10 7 6 8 11 2 3 4 12

Rx Ry

sorted in Y: 6 5 12 11 10 3 13 4 9 8 7 2 1 14



LY	RY
5	6
10	12
13	11
9	3
7	4
1	8
14	2

Closest(P,SX,SY)

Let q be the middle-element of SX

Divide P into Left, Right according to q. Scan to get LY, RY.

(SY, q)

delta,r,j = MIN(Closest(Left, LX, LY)   Closest(Right, RX, RY))

Mohawk = { Scan SY, add pts that are delta from q.x }

For each point x in Mohawk (in order):

Compute distance to its next 15 neighbors

Update delta,r,j if any pair (x,y) is < delta

Return (delta,r,j)

Closest(P,SX,SY)

Let q be the middle-element of SX

Divide P into Left, Right according to q. Scan to get LY, RY.

delta,r,j = MIN(Closest(Left, LX, LY) Closest(Right, RX, RY))

Mohawk = { Scan SY, add pts that are delta from q.x }

For each point x in Mohawk (in order):

Compute distance to its next 15 neighbors

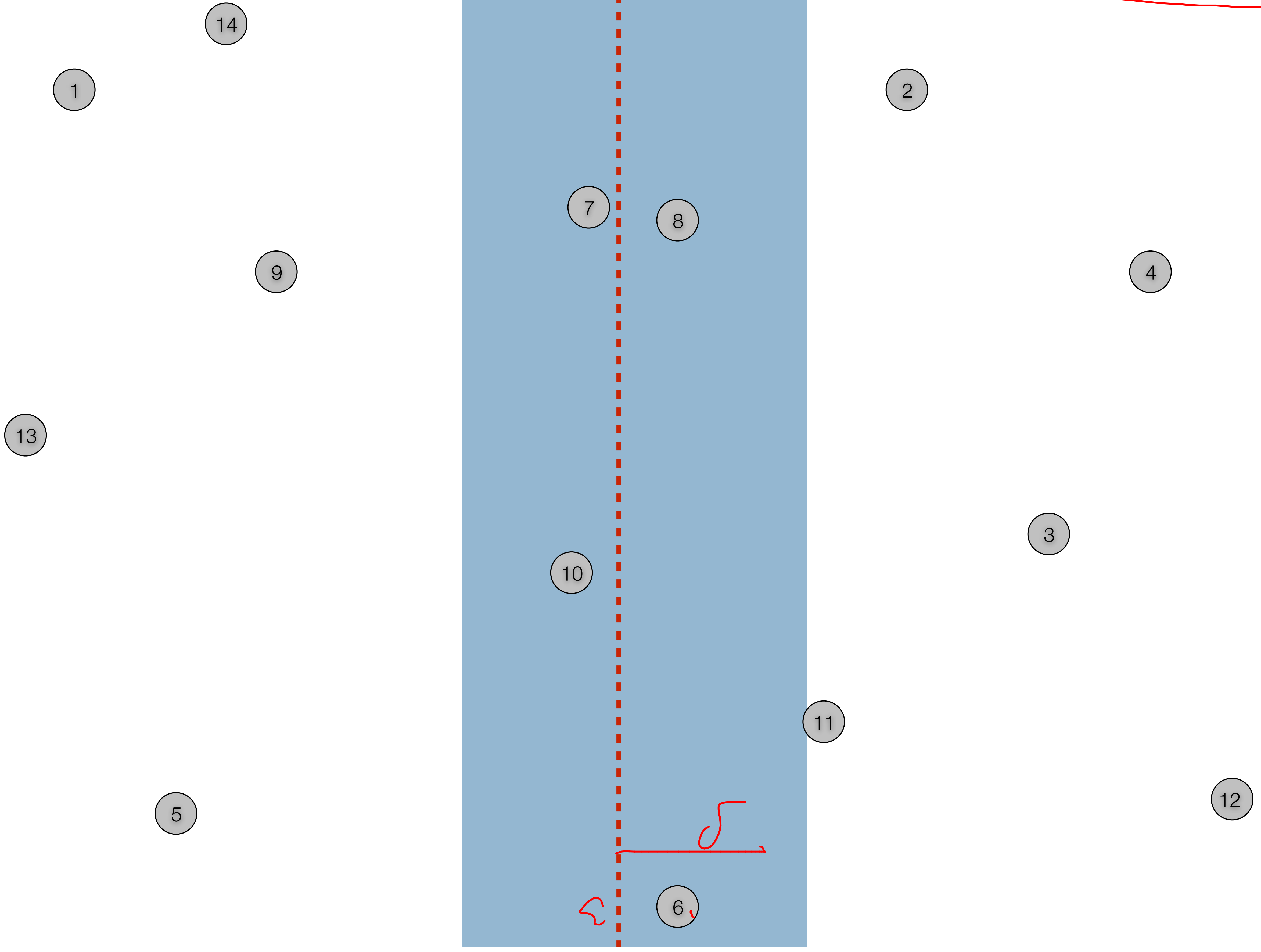
Update delta,r,j if any pair (x,y) is < delta

Return (delta,r,j)

Can be reduced to 7!

sorted in X: 13 1 5 14 9 10 7 9 8 11 2 3 4 12

sorted in Y: 6 5 12 11 10 3 13 4 9 8 7 2 1 14



Closest(P,SX,SY)

Let q be the middle-element of SX

Divide P into Left, Right according to q. Scan to get LY, RY.

delta,r,j = MIN(Closest(Left, LX, LY) Closest(Right, RX, RY))

Mohawk = { Scan SY, add pts that are delta from q.x }

For each point x in Mohawk (in order):

Compute distance to its next 15 neighbors
Update delta,r,j if any pair (x,y) is < delta

Return (delta,r,j)

Closest(P,SX,SY)

Let q be the middle-element of SX

Divide P into Left, Right according to q. Scan to get LY, RY.

delta,r,j = MIN(Closest(Left, LX, LY) Closest(Right, RX, RY))

Mohawk = { Scan SY, add pts that are delta from q.x }

For each point x in Mohawk (in order):

Compute distance to its next 15 neighbors
Update delta,r,j if any pair (x,y) is < delta

Return (delta,r,j)

Can be reduced to 7!

Running time for Closest pair algorithm

$$T(n) =$$

$$T(n) = 2T(n/2) + \Theta(n) = \Theta(n \log n)$$

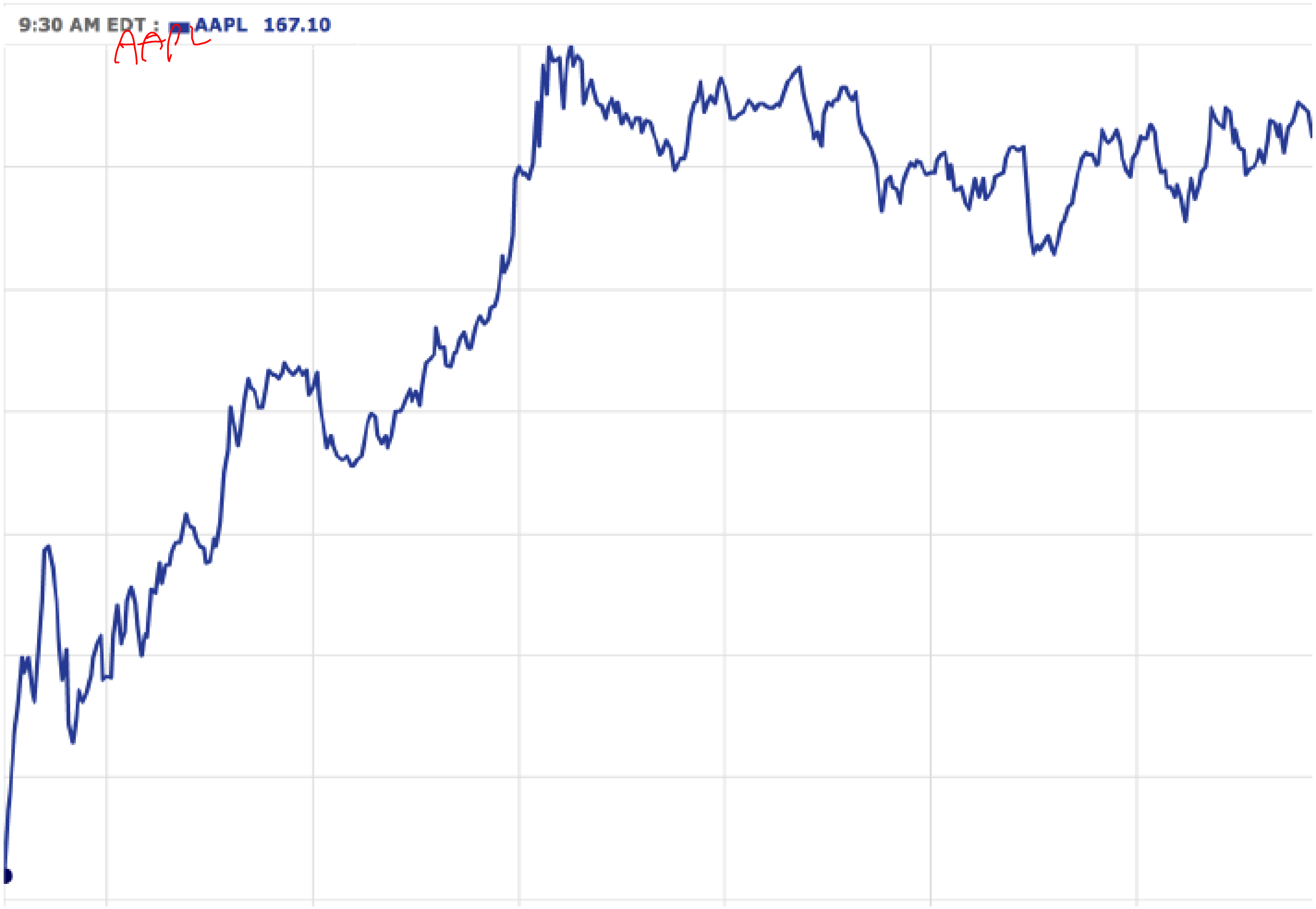
@author Robert Sedgewick
@author Kevin Wayne

http://algs4.cs.princeton.edu/99hull/ClosestPair.java.html

```java
public ClosestPair(Point2D[] points) {
    int N = points.length;
    if (N <= 1) return;

    // sort by x-coordinate (breaking ties by y-coordinate)
    Point2D[] pointsByX = new Point2D[N];
    for (int i = 0; i < N; i++)
        pointsByX[i] = points[i];
    Arrays.sort(pointsByX, Point2D.X_ORDER);

    // check for coincident points
    for (int i = 0; i < N-1; i++) {
        if (pointsByX[i].equals(pointsByX[i+1])) {
            bestDistance = 0.0;
            best1 = pointsByX[i];
            best2 = pointsByX[i+1];
            return;
        }
    }

    // sort by y-coordinate (but not yet sorted)
    Point2D[] pointsByY = new Point2D[N];
    for (int i = 0; i < N; i++)
        pointsByY[i] = pointsByX[i];

    // auxiliary array
    Point2D[] aux = new Point2D[N];

    closest(pointsByX, pointsByY, aux, 0, N-1);
}
```

```java
// find closest pair of points in pointsByX[lo..hi]
// precondition:  pointsByX[lo..hi] and pointsByY[lo..hi] are the same sequence of points, sorted by x,y-coord
private double closest(Point2D[] pointsByX, Point2D[] pointsByY, Point2D[] aux, int lo, int hi) {
    if (hi <= lo) return Double.POSITIVE_INFINITY;

    int mid = lo + (hi - lo) / 2;
    Point2D median = pointsByX[mid];

    // compute closest pair with both endpoints in left subarray or both in right subarray
    double delta1 = closest(pointsByX, pointsByY, aux, lo, mid);
    double delta2 = closest(pointsByX, pointsByY, aux, mid+1, hi);
    double delta = Math.min(delta1, delta2);

    // merge back so that pointsByY[lo..hi] are sorted by y-coordinate
    merge(pointsByY, aux, lo, mid, hi);

    // aux[0..M-1] = sequence of points closer than delta, sorted by y-coordinate
    int M = 0;
    for (int i = lo; i <= hi; i++) {
        if (Math.abs(pointsByY[i].x() - median.x()) < delta)
            aux[M++] = pointsByY[i];
    }

    // compare each point to its neighbors with y-coordinate closer than delta
    for (int i = 0; i < M; i++) {
        // a geometric packing argument shows that this loop iterates at most 7 times
        for (int j = i+1; (j < M) && (aux[j].y() - aux[i].y() < delta); j++) {
            double distance = aux[i].distanceTo(aux[j]);
            if (distance < delta) {
                delta = distance;
                if (distance < bestDistance) {
                    bestDistance = delta;
                    best1 = aux[i];
                    best2 = aux[j];
                    // StdOut.println("better distance = " + delta + " from " + best1 + " to " + best2);
                }
            }
        }
    }
    return delta;
}
```

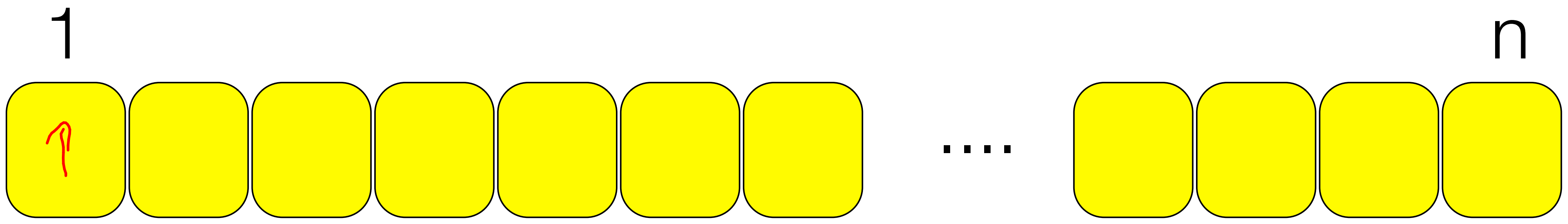
arbitrage


9:30 AM EDT : AAPL 167.10
AAPL

Open 27.46
Close 27.07
Low 26.65
High 27.69
Vol 33.79K
% Chg -75.68%
BPT 27.07
YAHOO! FINANCE


12:38 PM EDT : AIG 40.58
AIG
© 2008 Yahoo! Inc.
10:00 AM
11:00 AM
12:00 PM
1:00 PM
2:00 PM
3:00 PM
4:00

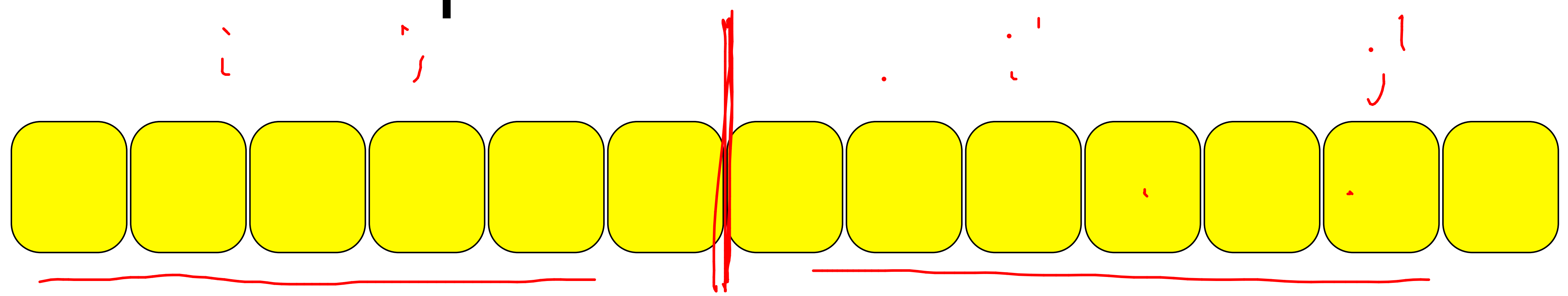
input: array of n numbers



goal: to find index i, j s.t. $i < j$ which maximize $A[j] - A[i]$

We want an $\Theta(n \log n)$ algorithm !!

first attempt



`arbit(A[1...n])` {

-handle base case. if $|A| \le 2$.

→ $(i,j) \leftarrow \text{Arbit}(A[1,\dots \frac{n}{2}])$

→ $(i',j') \leftarrow \text{Arbit}(A[\frac{n}{2}+1,\dots n])$

$r^* \leftarrow \min(A[1\dots \frac{n}{2}])$

$j^* \leftarrow \max(A[\frac{n}{2}+1, n])$

Return $\max\{(i,j)\ (i',j')\ (r^*,j^*)\}$

}

ℓ mean

$A[j]-A[i]$

$A[j']-A[i']$

$A[j]-A[r]$

first attempt

```
arbit(A[1...n])
    base case if |A|<=2
    lg = arbit(left(A))
    rg = arbit(right(A))
    minl = min(left(A))
    maxr = max(right(A))

    return max{maxr-minl,lg,rg}
```

$\to T(\frac{n}{2})$

$\to T(\frac{n}{2})$

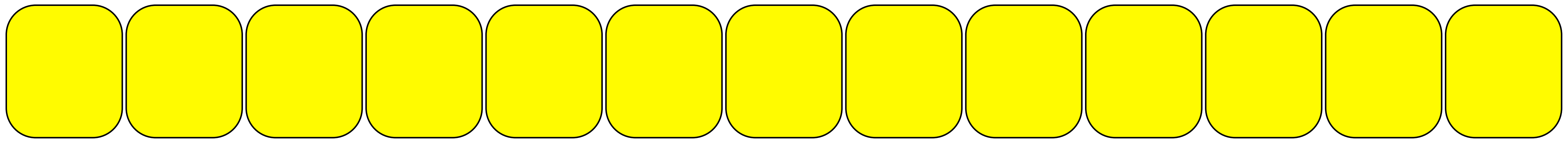
$\to \Theta(n)$

$\to \Theta(n)$

$\to$ too much

$T(n) = 2T(\frac{n}{2}) + \Theta(n) \Rightarrow \Theta(n \log n)$ solution.

first attempt: time $\Theta(n \log n)$



```
arbit(A[1...n])
    base case if |A|<=2
    lg = arbit(left(A))
    rg = arbit(right(A))
    minl = min(left(A))
    maxr = max(right(A))
    return max{maxr-minl,lg,rg}
```

better approach

better approach

Can we find a solution that has $T(n) = 2T(n/2) + O(1)$?

better approach

Can we find a solution that has $T(n) = 2T(n/2) + O(1)$?

```
minl = min(left(A))
maxr = max(right(A))

return max{maxr-minl,lg,rg}
```

$\Theta(n)$

second attempt

`arbit+(A[1...n])` returns (best trade, min, max)

min of the array A

max of A

`base case if |A|<=2` (_ , _ _ ,)

$T(n) = 2T(\frac{n}{2}) + \Theta(1)$

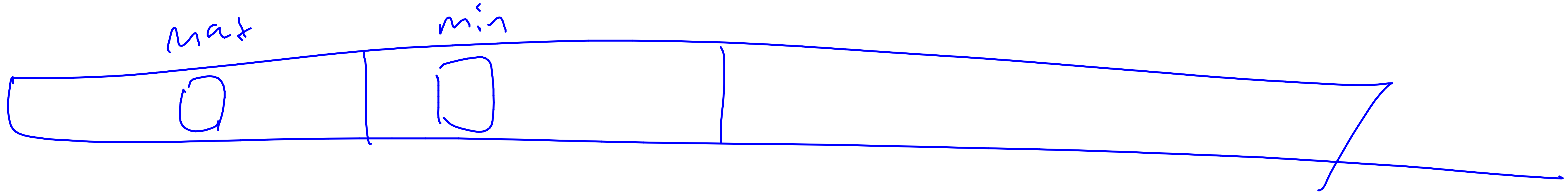
(lg, minl, maxl) ← Arbit (A[1.. n/2])

(rg, minr, maxr) ← Arbit (A[n/2+1.. n])

Return (max { lg, rg, maxr − minl }, $\Theta(1)$

min { minl, minr }

max { maxl, maxr })


max
0
min
0

second attempt

```
arbit+(A[1...n])
 base case if |A|<=2, …
 (lg,minl,maxl) = arbit(left(A))
 (rg,minr,maxr) = arbit(right(A))
   return max{maxr-minl,lg,rg},
          min{minl, minr},
          max{maxl, maxr}
```

Matrix

multiplication

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \bigstar \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \bigstar \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 5+14 & 6+16 \\ 15+28 & 18+32 \end{bmatrix}$$

$$= \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

$$\begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \vdots & & & \\ a_{n,1} & a_{n,2} & \cdots & a_{n,n} \end{bmatrix} \begin{bmatrix} b_{1,1} & b_{1,2} & \cdots & b_{1,n} \\ b_{2,1} & b_{2,2} & \cdots & b_{2,n} \\ \vdots & & & \\ b_{n,1} & b_{n,2} & \cdots & b_{n,n} \end{bmatrix} = \begin{bmatrix} c_{1,1} & c_{1,2} & \cdots & c_{1,n} \\ c_{2,1} & c_{2,2} & \cdots & c_{2,n} \\ \vdots & & & \\ c_{n,1} & c_{n,2} & \cdots & c_{n,n} \end{bmatrix}$$

$$\begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \vdots & & & \\ a_{n,1} & a_{n,2} & \cdots & a_{n,n} \end{bmatrix} \begin{bmatrix} b_{1,1} & b_{1,2} & \cdots & b_{1,n} \\ b_{2,1} & b_{2,2} & \cdots & b_{2,n} \\ \vdots & & & \\ b_{n,1} & b_{n,2} & \cdots & b_{n,n} \end{bmatrix} = \begin{bmatrix} c_{1,1} & c_{1,2} & \cdots & c_{1,n} \\ c_{2,1} & c_{2,2} & \cdots & c_{2,n} \\ \vdots & & & \\ c_{n,1} & c_{n,2} & \cdots & c_{n,n} \end{bmatrix}$$

$$c_{i,j} = \sum_{k=1}^{n} a_{i,k} \cdot b_{k,j}$$

$$\begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \vdots & & & \\ a_{n,1} & a_{n,2} & \cdots & a_{n,n} \end{bmatrix} \begin{bmatrix} b_{1,1} & b_{1,2} & \cdots & b_{1,n} \\ b_{2,1} & b_{2,2} & \cdots & b_{2,n} \\ \vdots & & & \\ b_{n,1} & b_{n,2} & \cdots & b_{n,n} \end{bmatrix} = \begin{bmatrix} c_{1,1} & c_{1,2} & \cdots & c_{1,n} \\ c_{2,1} & c_{2,2} & \cdots & c_{2,n} \\ \vdots & & & \\ c_{n,1} & c_{n,2} & \cdots & c_{n,n} \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} E & F \\ G & H \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} E & F \\ G & H \end{bmatrix}$$

$$= \begin{bmatrix} AE + BG & AF + BH \\ CE + DG & CF + DH \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} E & F \\ G & H \end{bmatrix} = \begin{bmatrix} AE + BG & AF + BH \\ CE + DG & CF + DH \end{bmatrix}$$

$$T(n) = 8T(n/2) + \Theta(n^2)$$

$$\Theta(n^3)$$

$$= \begin{bmatrix} AE + BG & AF + BH \\ CE + DG & CF + DH \end{bmatrix}$$

[Strassen]

$$P_1 = A(F - H)$$

$$P_2 = (A + B)H$$

$$P_3 = (C + D)E$$

$$P_4 = D(G - E)$$

$$P_5 = (A + D)(E + H)$$

$$P_6 = (B - D)(G + H)$$

$$P_7 = (A - C)(E + F)$$

$$= \begin{bmatrix} AE + BG & AF + BH \\ CE + DG & CF + DH \end{bmatrix}$$

$R = P_5 + P_4 - P_2 + P_6$

$S = P_1 + P_2$

$T = P_3 + P_4$

$U = P_5 + P_1 - P_3 - P_7$

[strassen]

$P_1 = A(F - H)$

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$$= \begin{bmatrix} AE+BG & AF+BH \\ CE+DG & CF+DH \end{bmatrix}$$

$$R = P_5 + P_4 - P_2 + P_6 \qquad S = P_1 + P_2$$

$$T = P_3 + P_4 \qquad U = P_5 + P_1 - P_3 - P_7$$

[strassen]

$$P_1 = A(F - H)$$

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$$P_4 = D(G - E)$$

$$P_5 = (A + D)(E + H)$$

$$P_6 = (B - D)(G + H)$$

$$P_7 = (A - C)(E + F)$$

$$M(n) = 7M(n/2) + 18n^2$$

$$= \Theta(n^{\log_2 7})$$

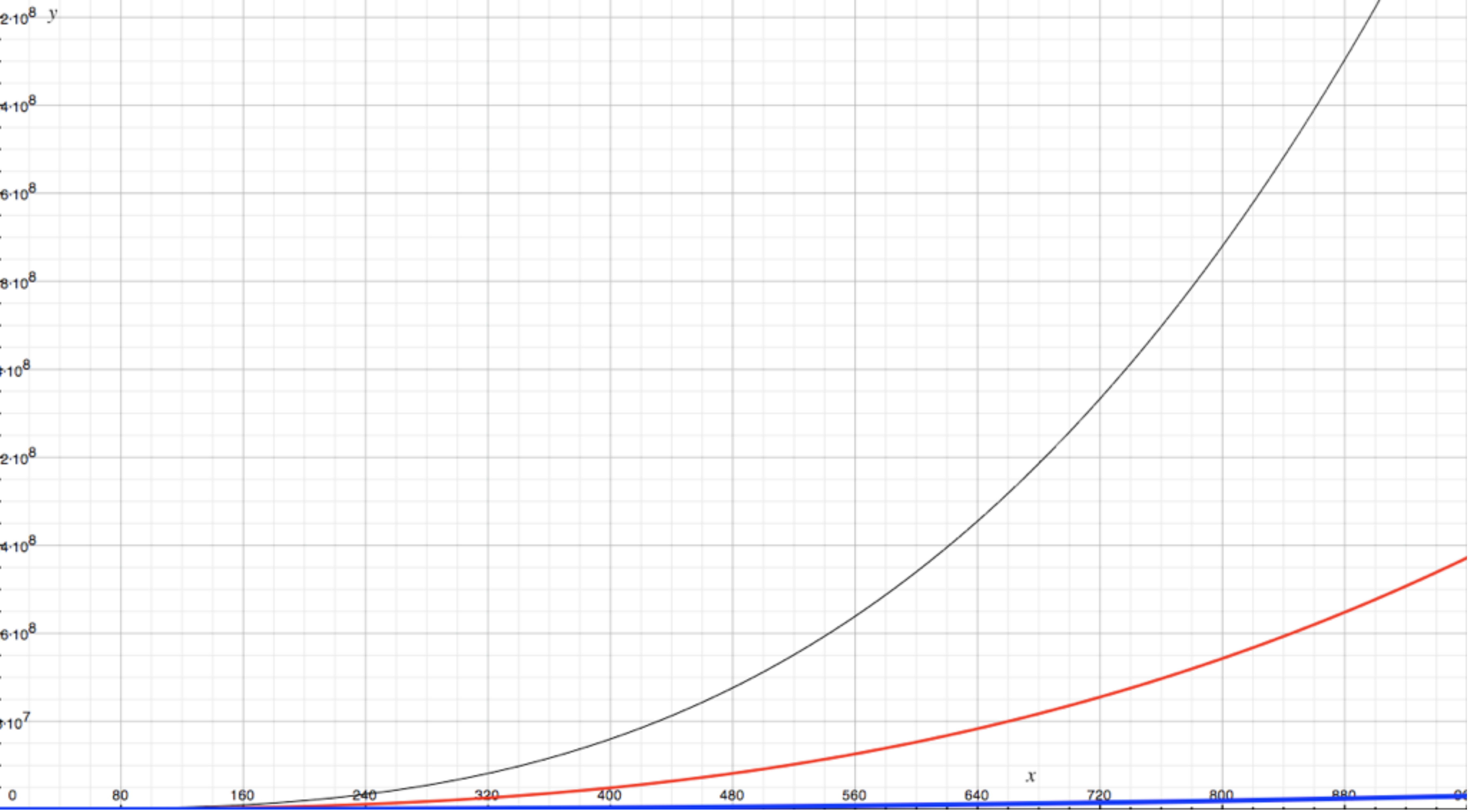
taking this idea further

3x3 matricies

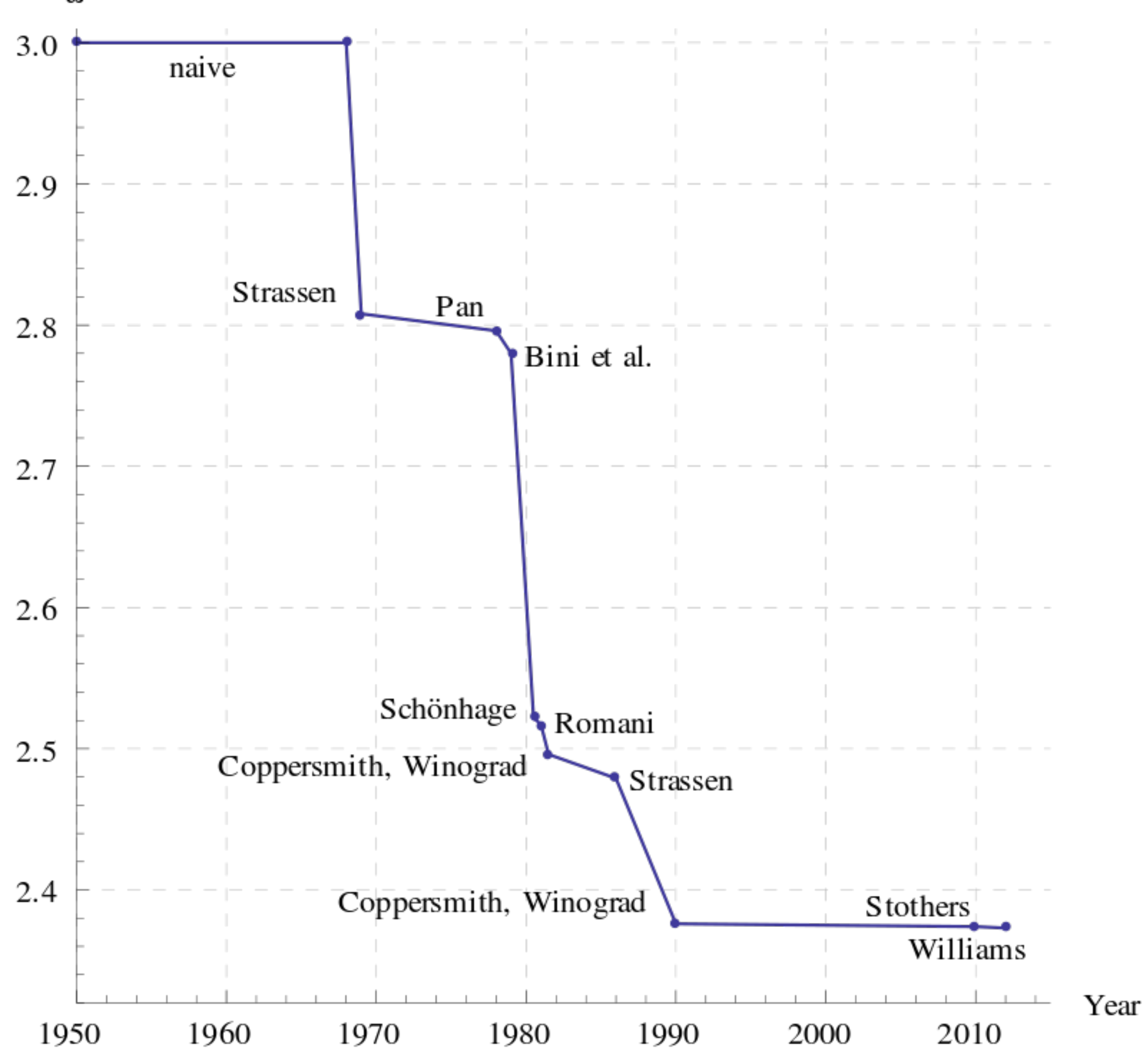
1978 victor pan method

70x70 matrix using 143640
mults

what is the recurrence:

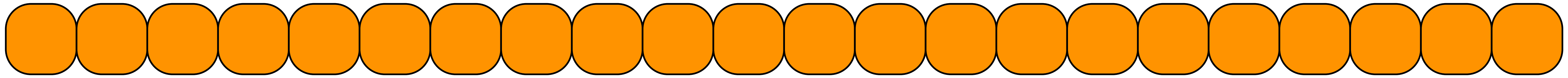


y
x
0
80
160
240
320
400
480
560
640
720
800
880

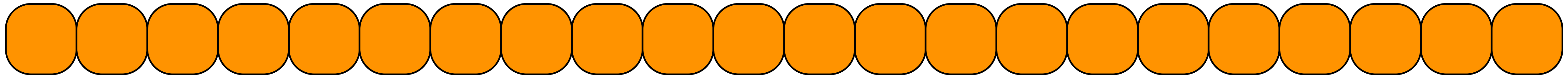


https://en.wikipedia.org/wiki/File:Bound_on_matrix_multiplication_omega_over_time.svg

MEDIAN



problem: given a list of **n** elements, find the element of rank **n**/2. (half are larger, half are smaller)

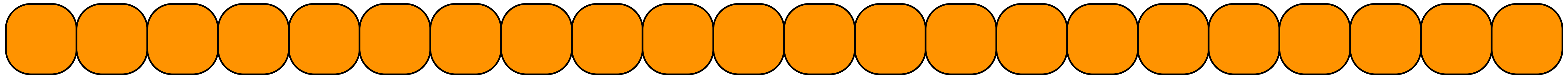


problem: given a list of **n** elements, find the element of rank n/2. (half are larger, half are smaller)

can generalize to **i**

first solution: sort and pluck.

$$O(n \log n)$$



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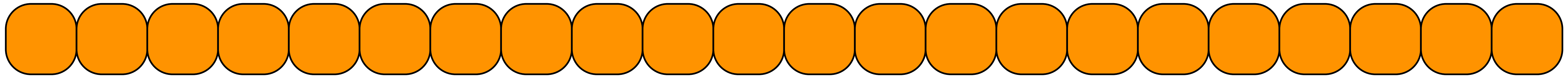
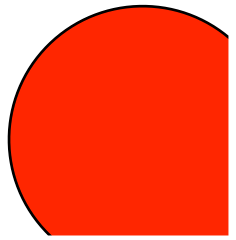


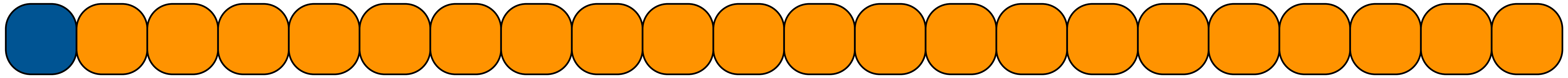
problem: given a list of **n** elements, find the element of rank **i**.

key insight:

we do not have to “fully” sort.

semi sort can suffice.





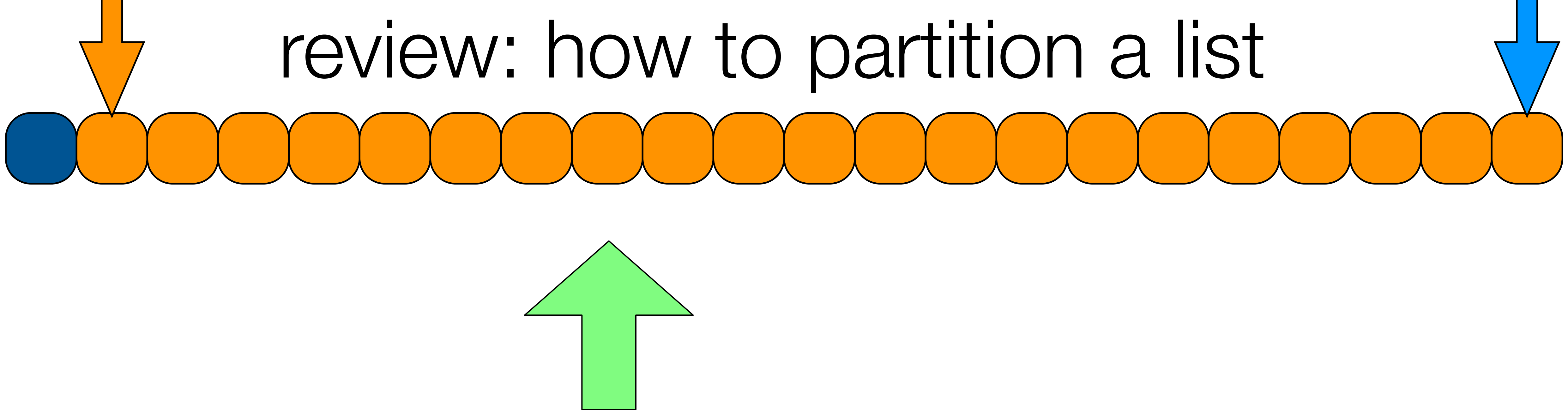
pick first element

partition list about this one

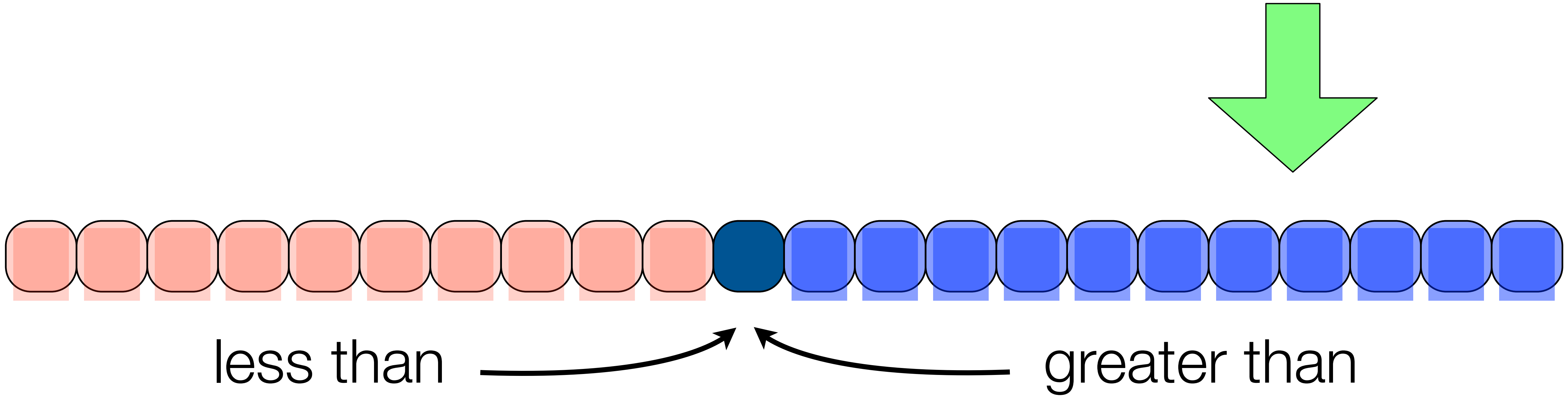
see where we stand

review: how to partition a list

review: how to partition a list



GOAL: start with THIS LIST and END with THAT LIST



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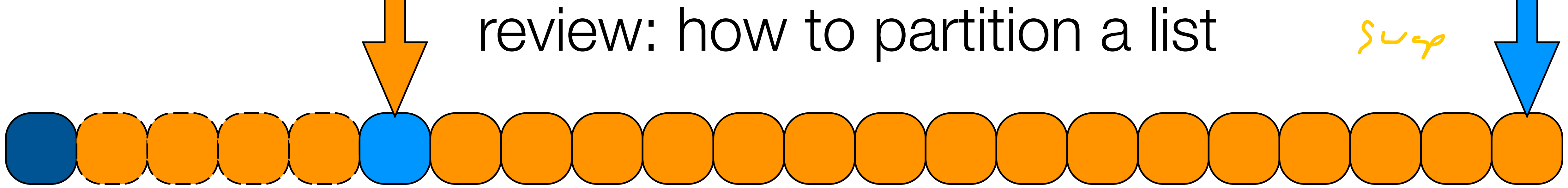

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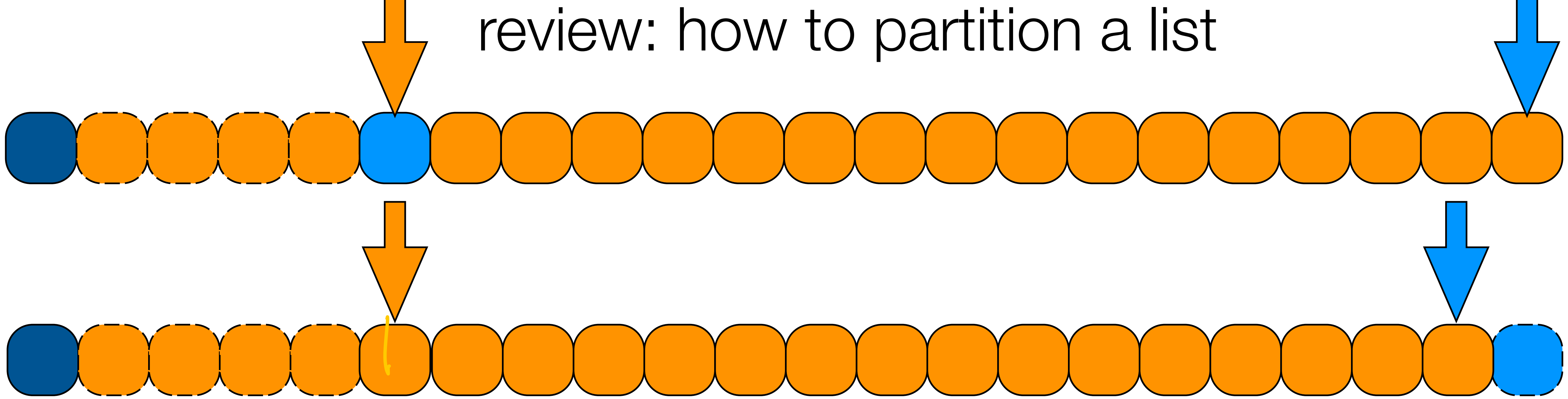
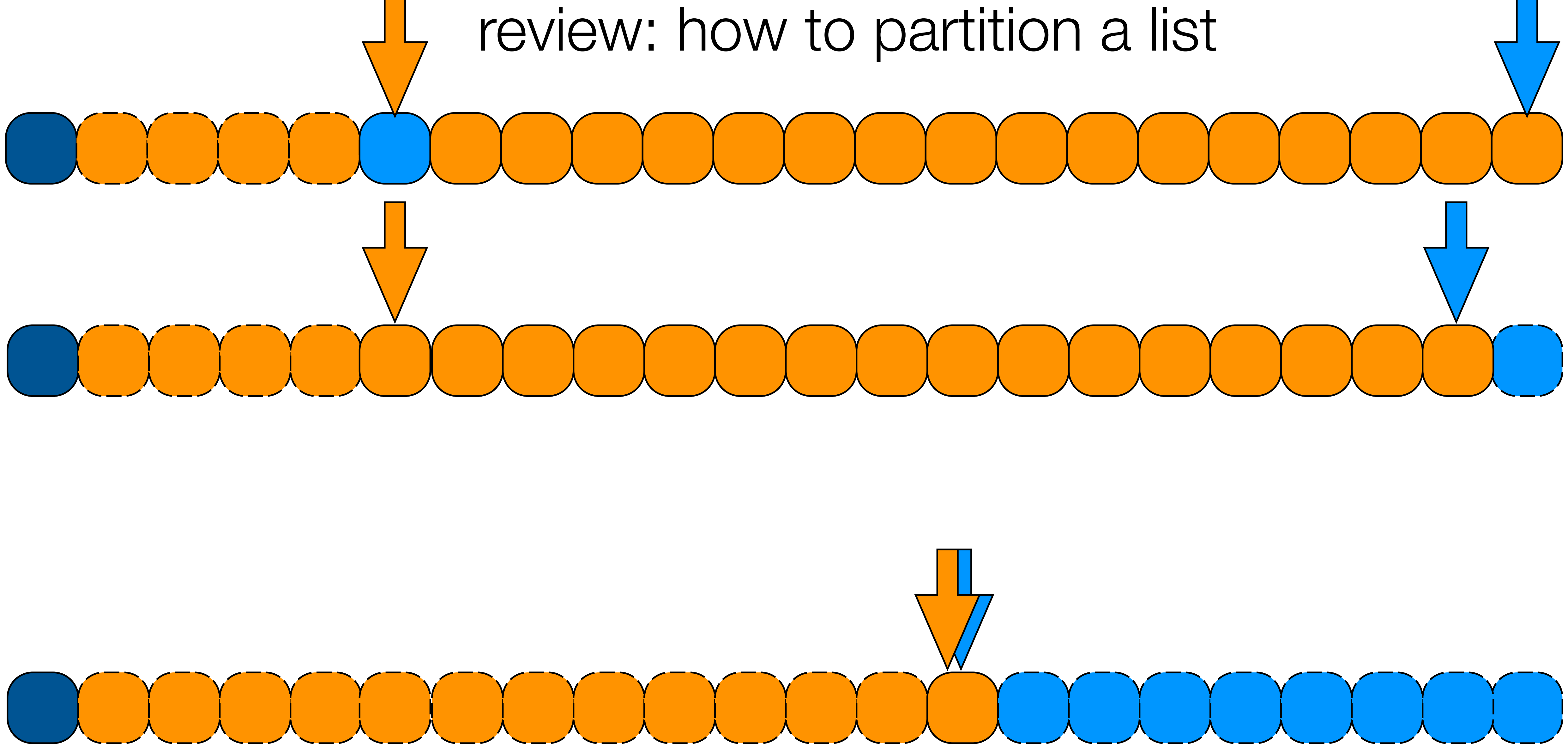
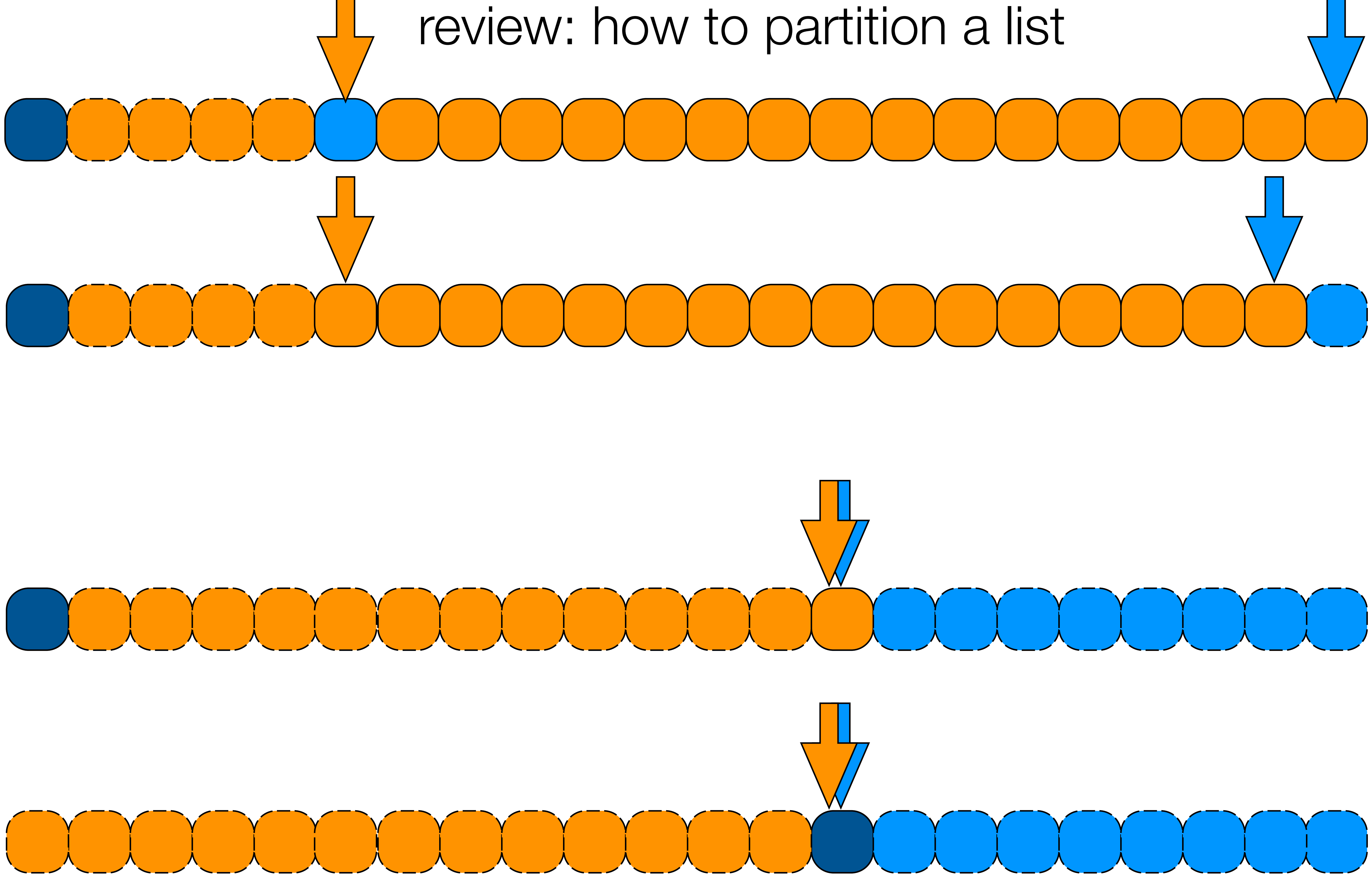


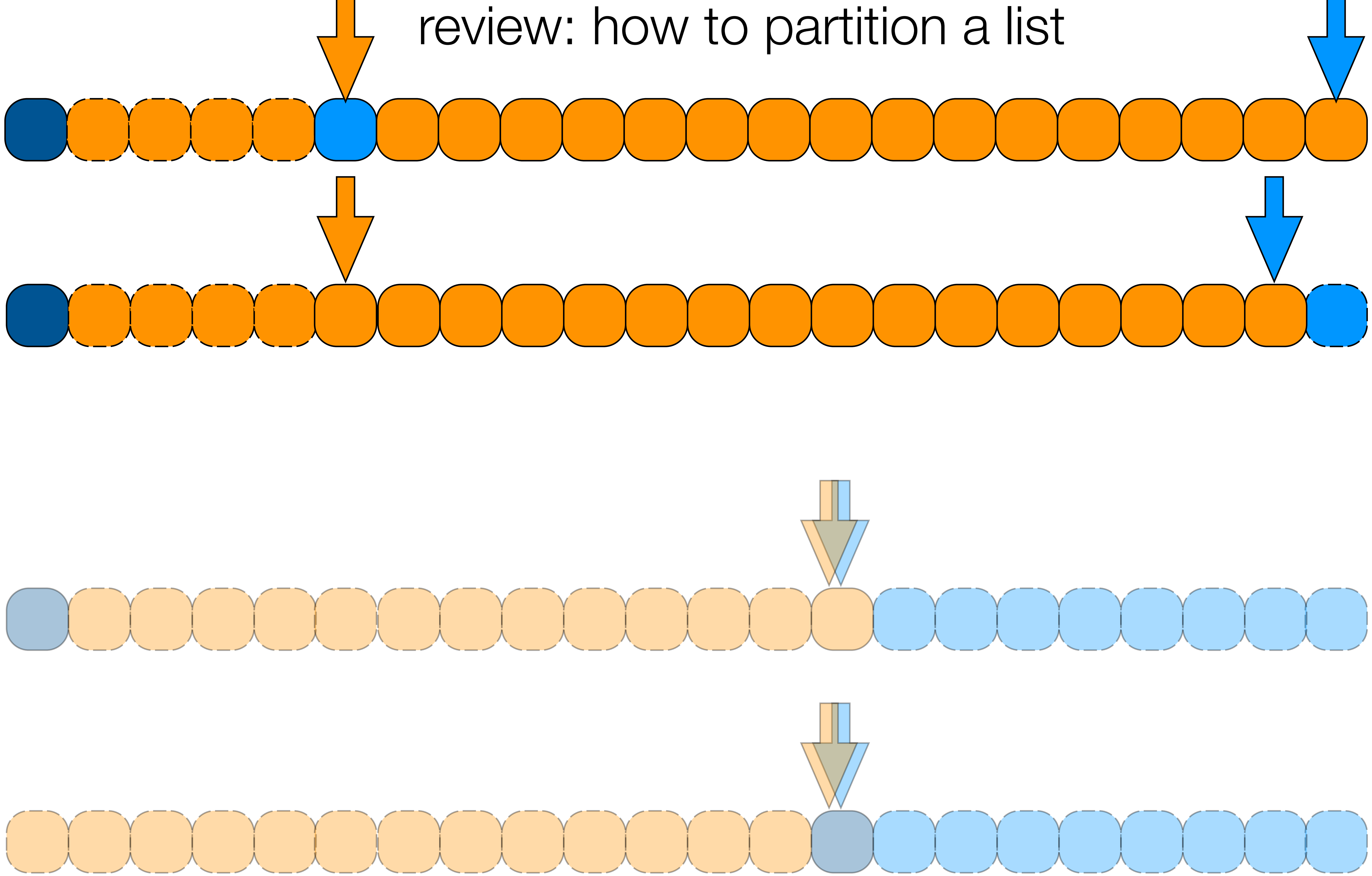
review: how to partition a list



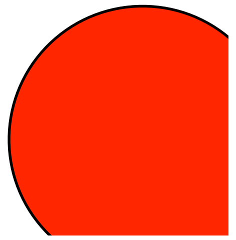
review: how to partition a list

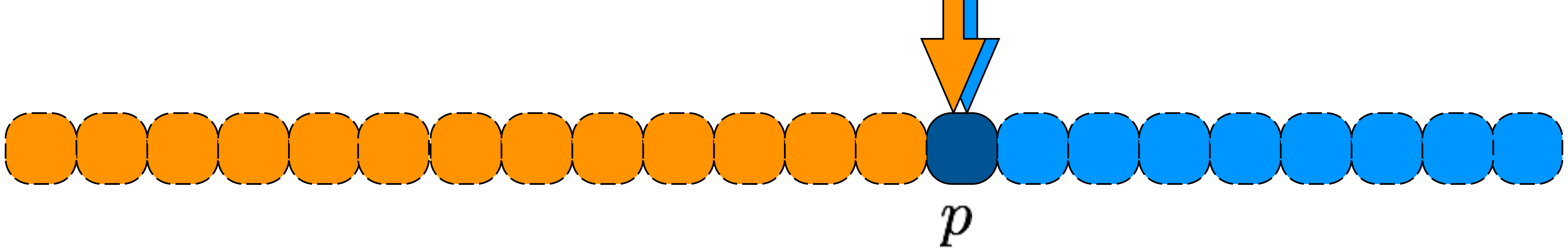


review: how to partition a list

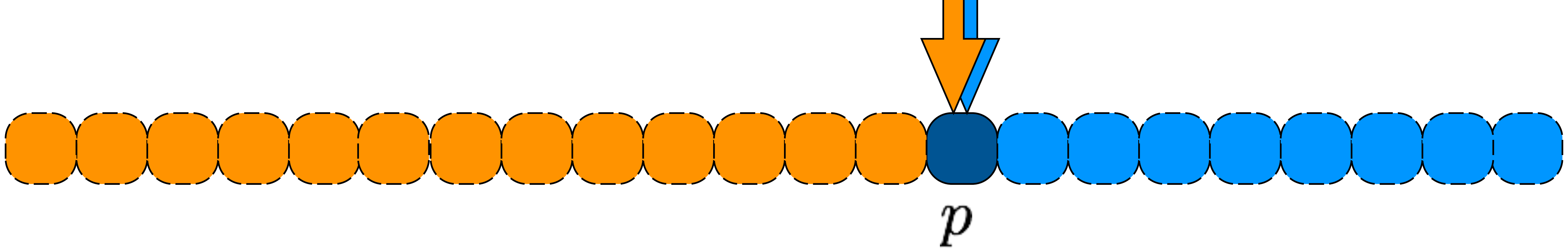


partitioning a list about an element takes linear time.





select $(i, A[1, \ldots, n])$



select $(i, A[1, \ldots, n])$

handle base case.

partition list about first element

if pivot is position **i**, return pivot

else if pivot is in position > **i** select $(i, A[1, \ldots, p-1])$

else select $((i-p-1), A[p+1, \ldots, n])$

select $(i, A[1, \ldots, n])$

Assume our partition always splits list into two eql parts

handle base case.

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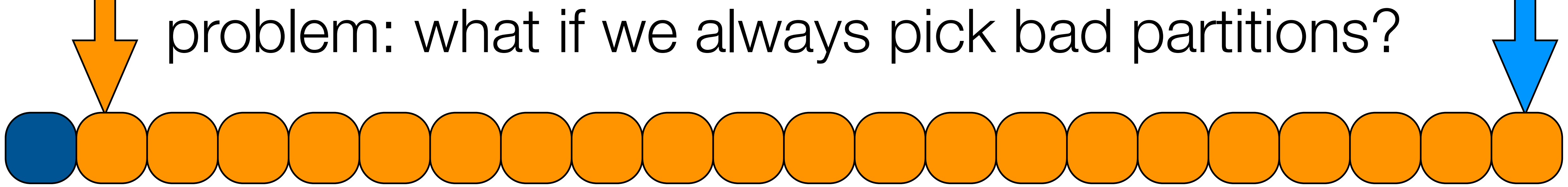
$$T(n) = T(n/2) + O(n)$$

$$\Theta(n)$$

problem: what if we always pick bad partitions?



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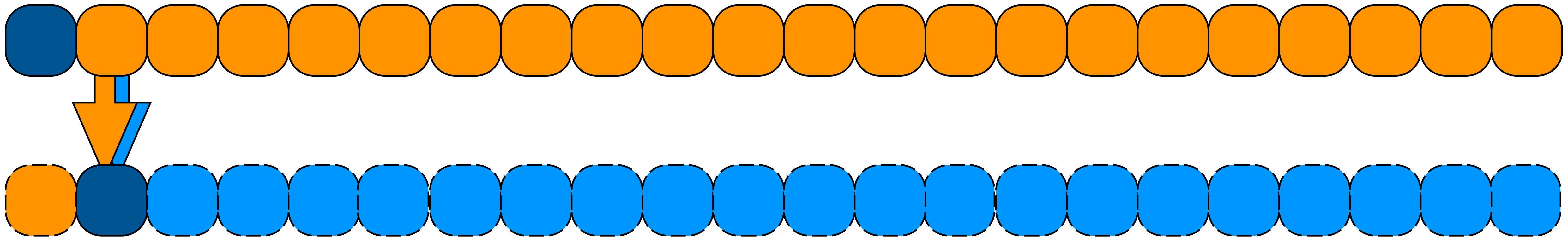

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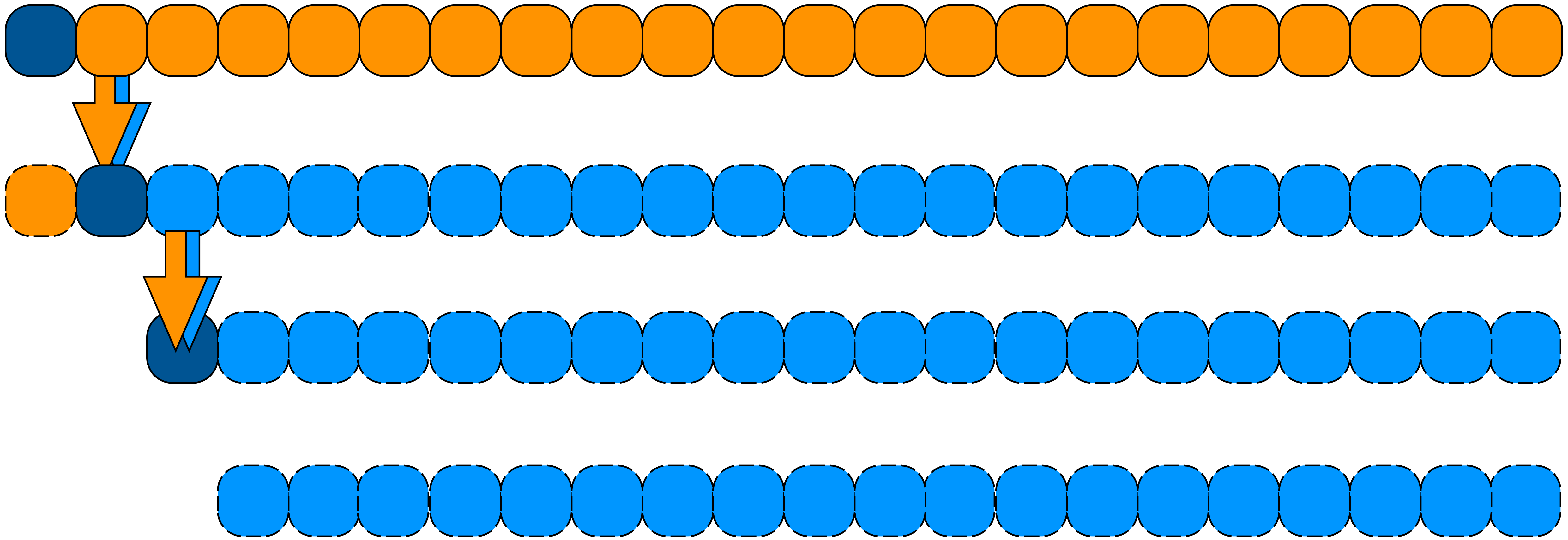

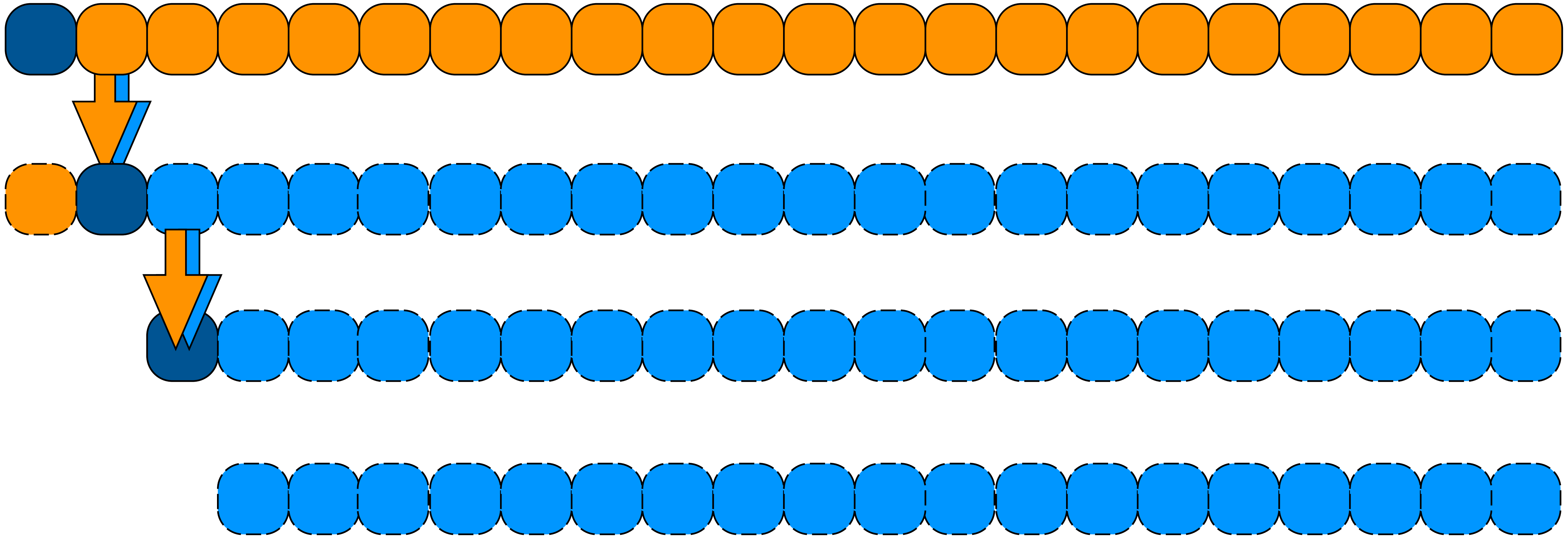

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else select $((i-p-1), A[p+1, \ldots, n])$

$$T(n) = T(n-1) + O(n)$$

$$\Theta(n^2)$$

Needed:

a good partition element

partition $(A[1, \ldots, n])$

Needed:

a good partition element

partition $(A[1, \ldots, n])$

produce an element where
30% smaller, 30% larger

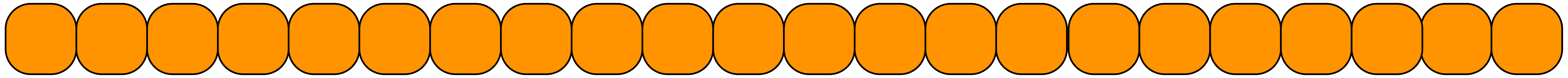
solution: bootstrap

image: gucci

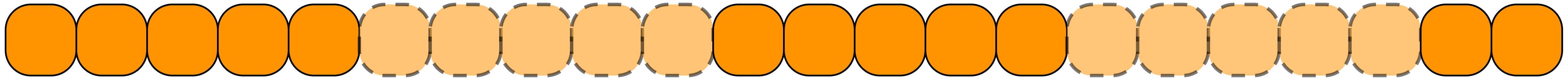
image: mark nason

image: d&g

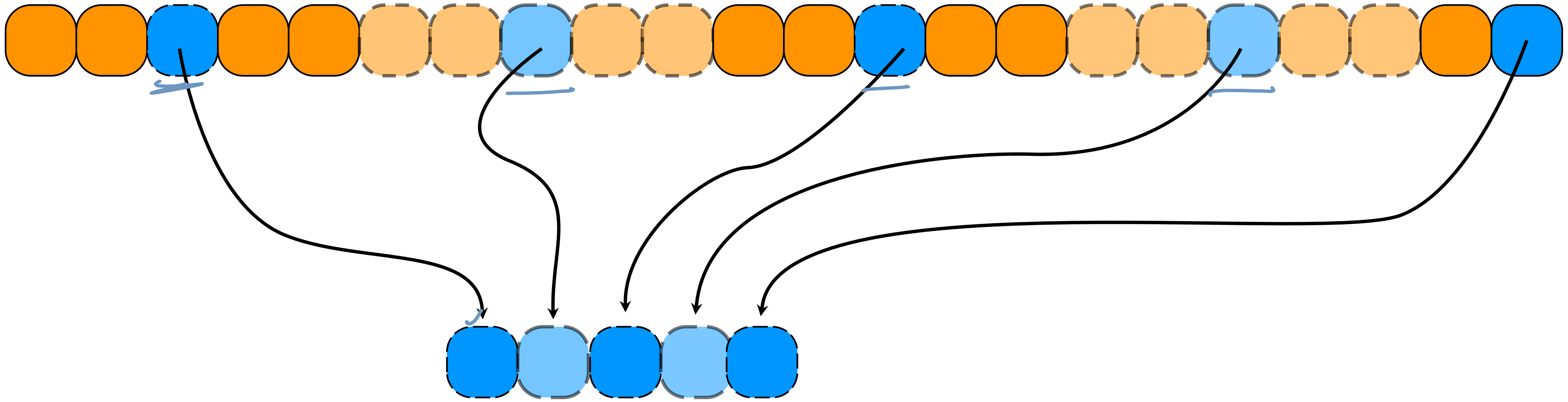
partition $(A[1, \ldots, n])$

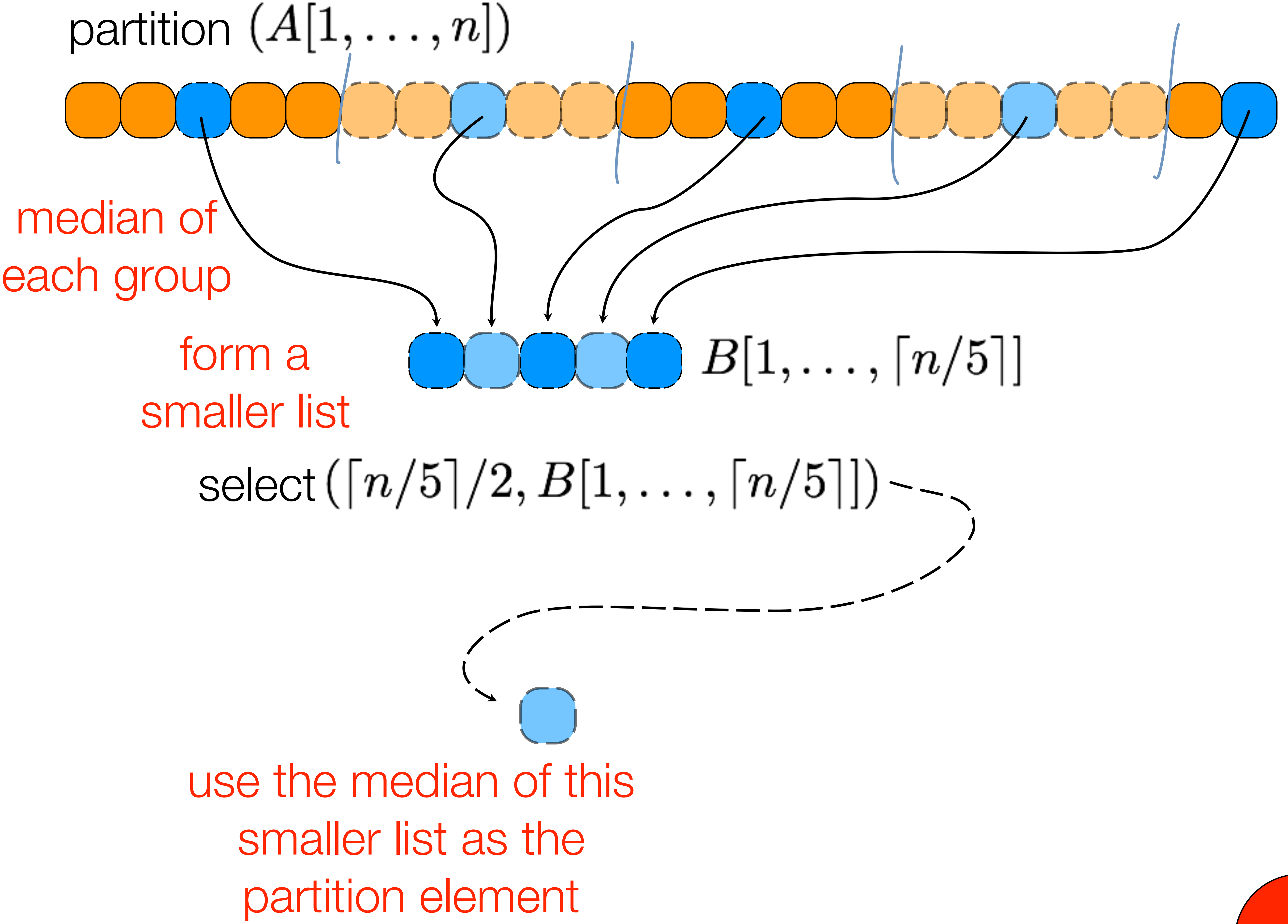


partition $(A[1, \ldots, n])$

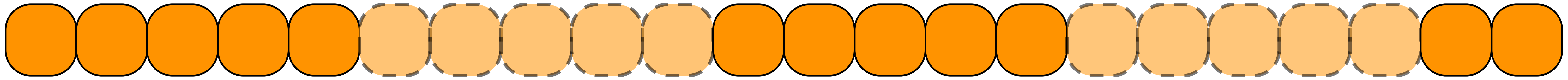


partition $(A[1,\ldots,n])$



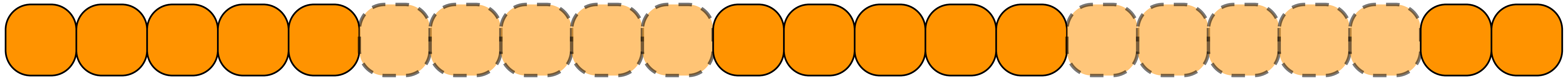

partition $(A[1, \ldots, n])$
median of each group
form a smaller list
$B[1, \ldots, \lceil n/5 \rceil]$
select $(\lceil n/5 \rceil / 2, B[1, \ldots, \lceil n/5 \rceil])$
use the median of this smaller list as the partition element

partition $(A[1, \ldots, n])$



1.
2.
3.
4.
5.

partition $(A[1,\ldots,n])$



divide list into groups of 5 elements

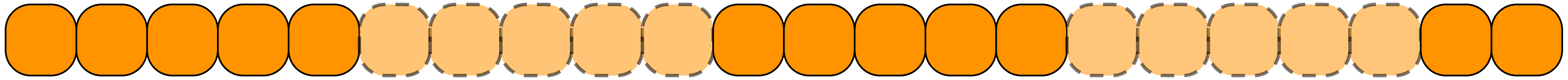
find median of each small list

gather all medians

call select(...) on this sublist to find median

return the result

partition $(A[1, \ldots, n])$



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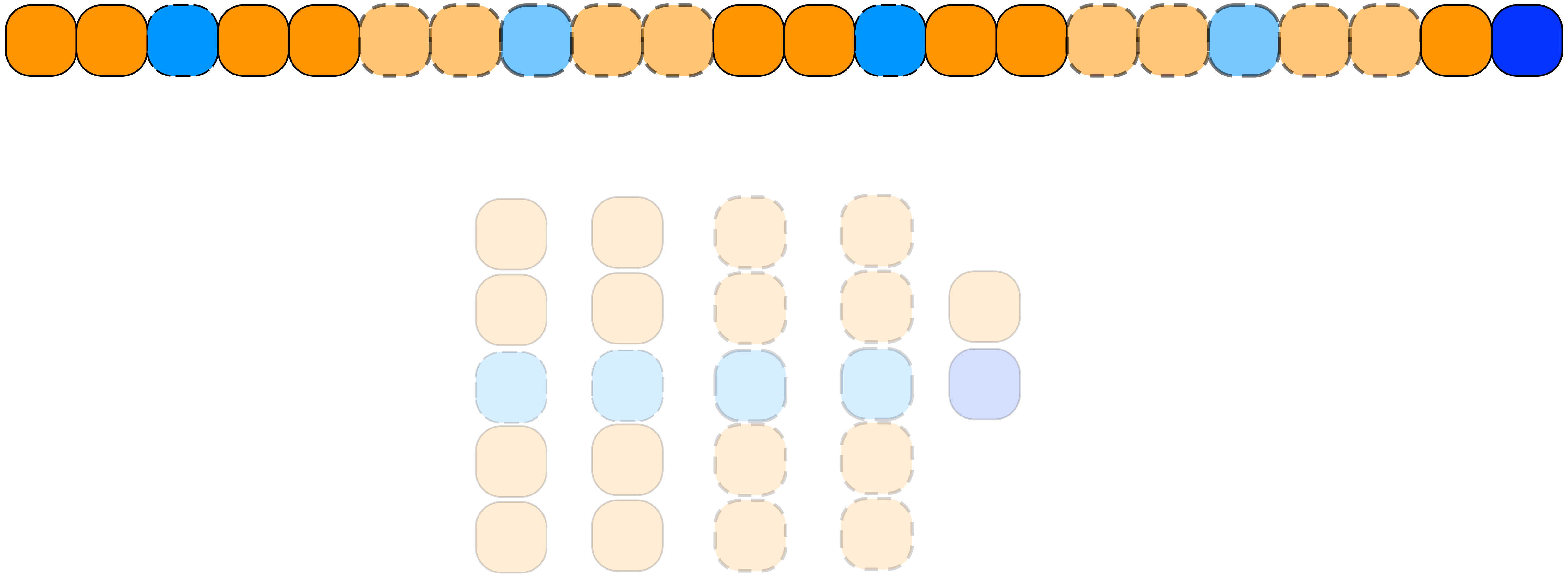
$$P(n) = S(\lceil n/5 \rceil) + O(n)$$

a nice property of our partition



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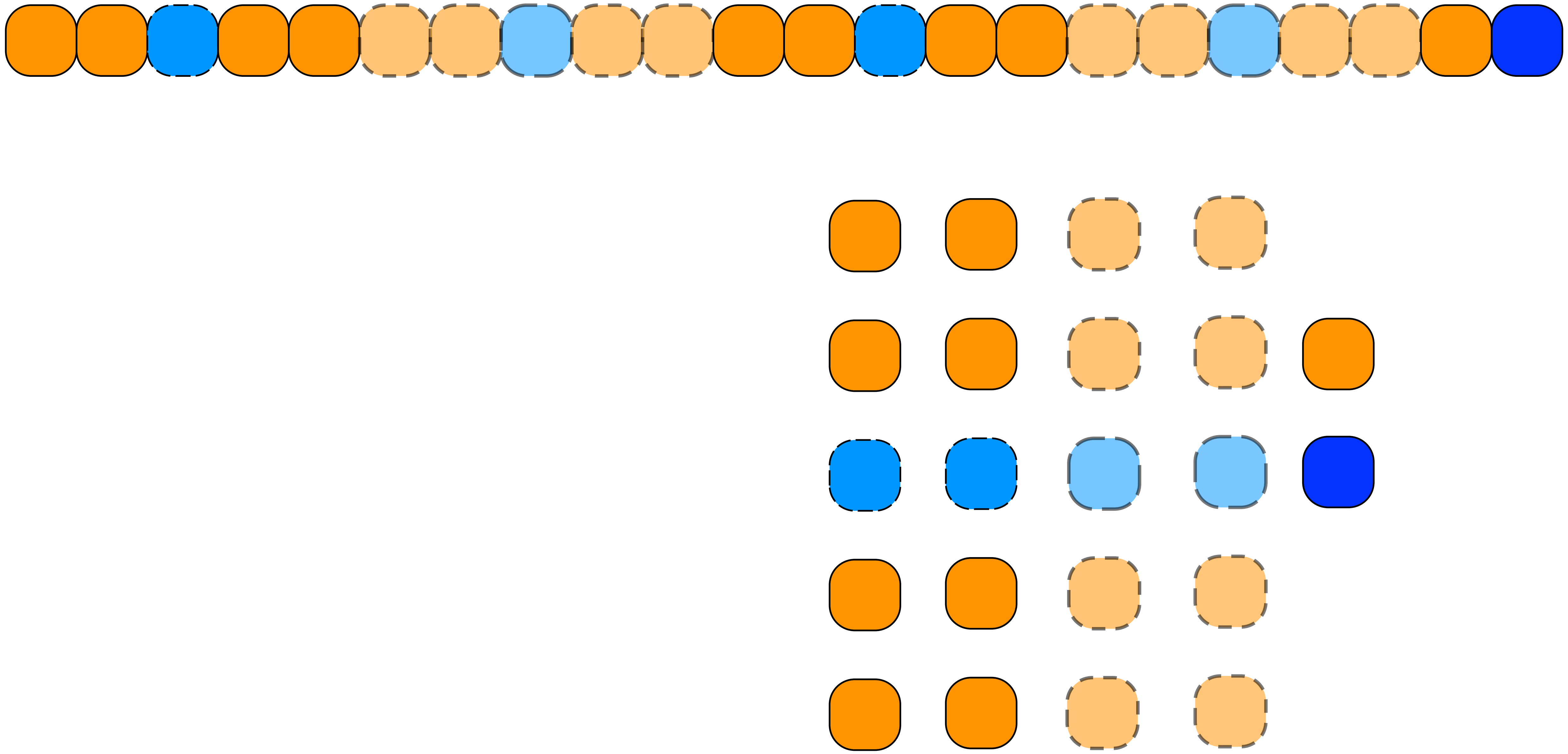

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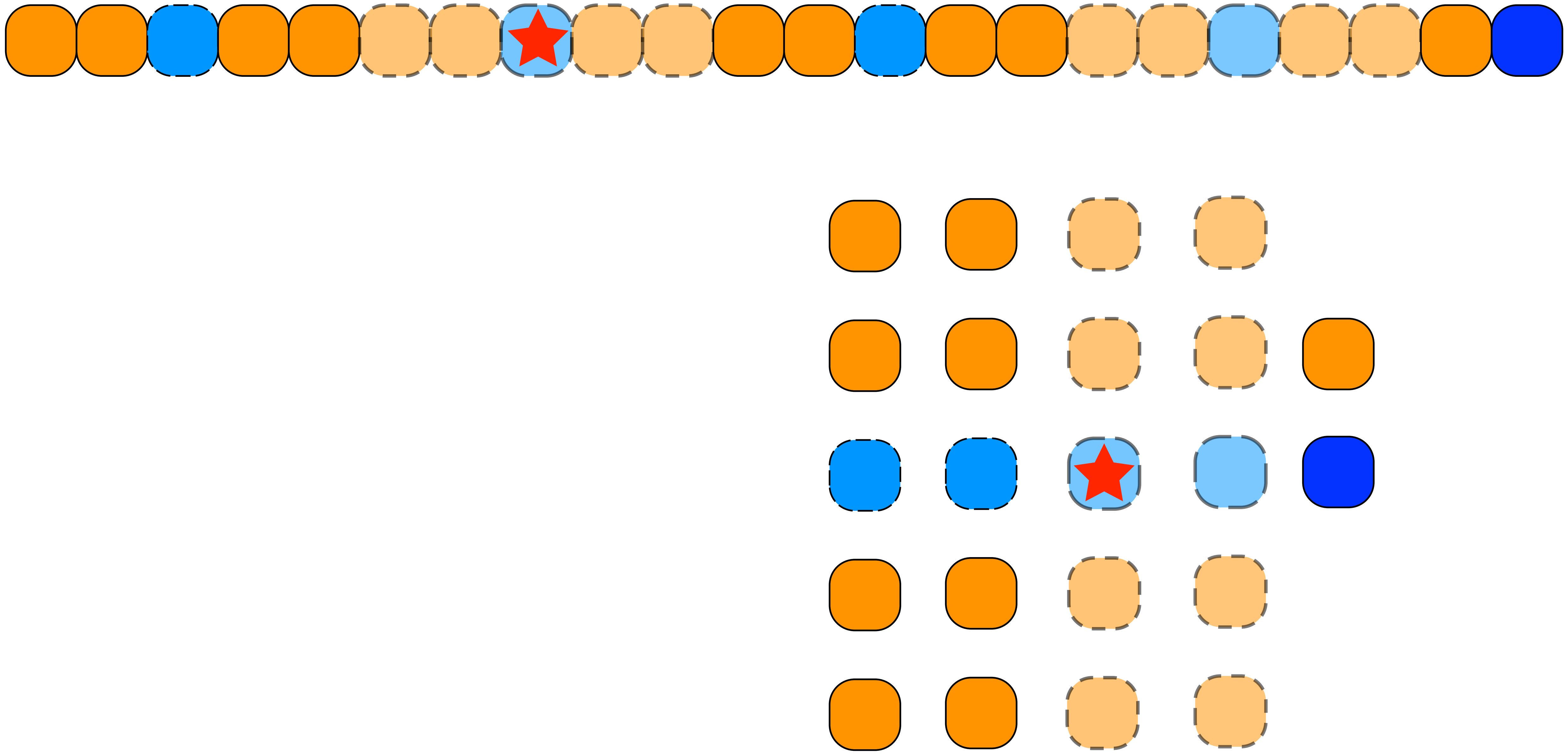
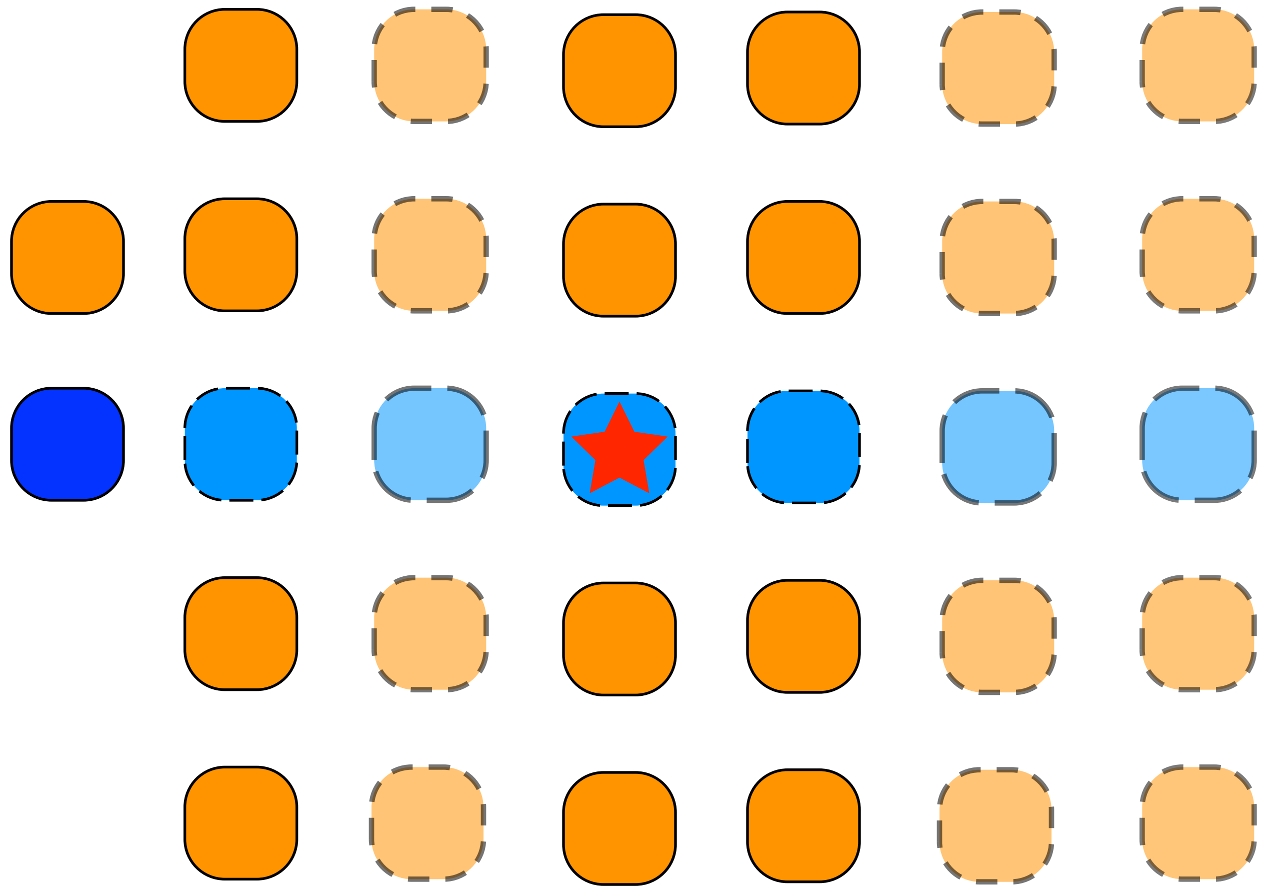
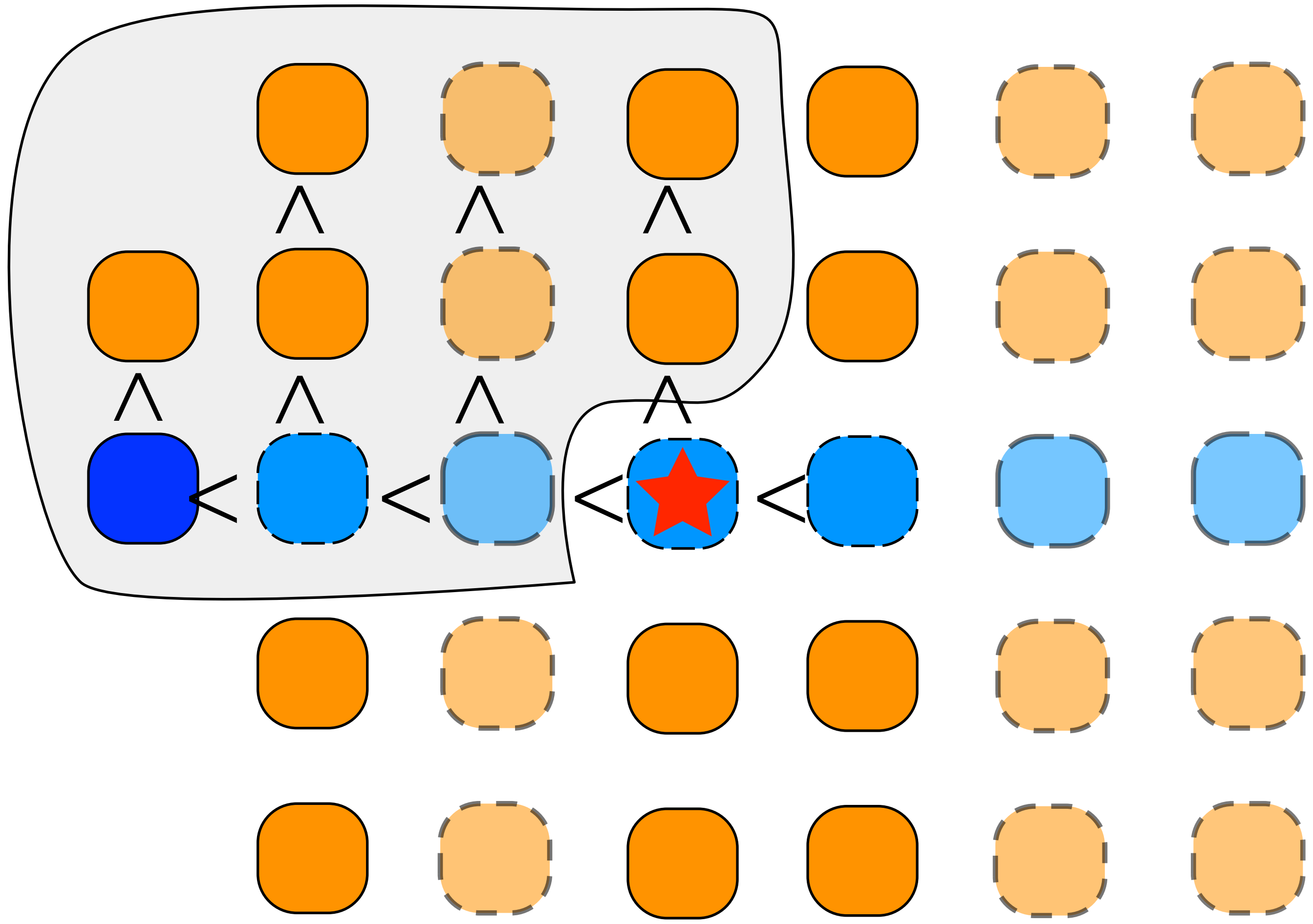


SWITCH TO A BIGGER EXAMPLE



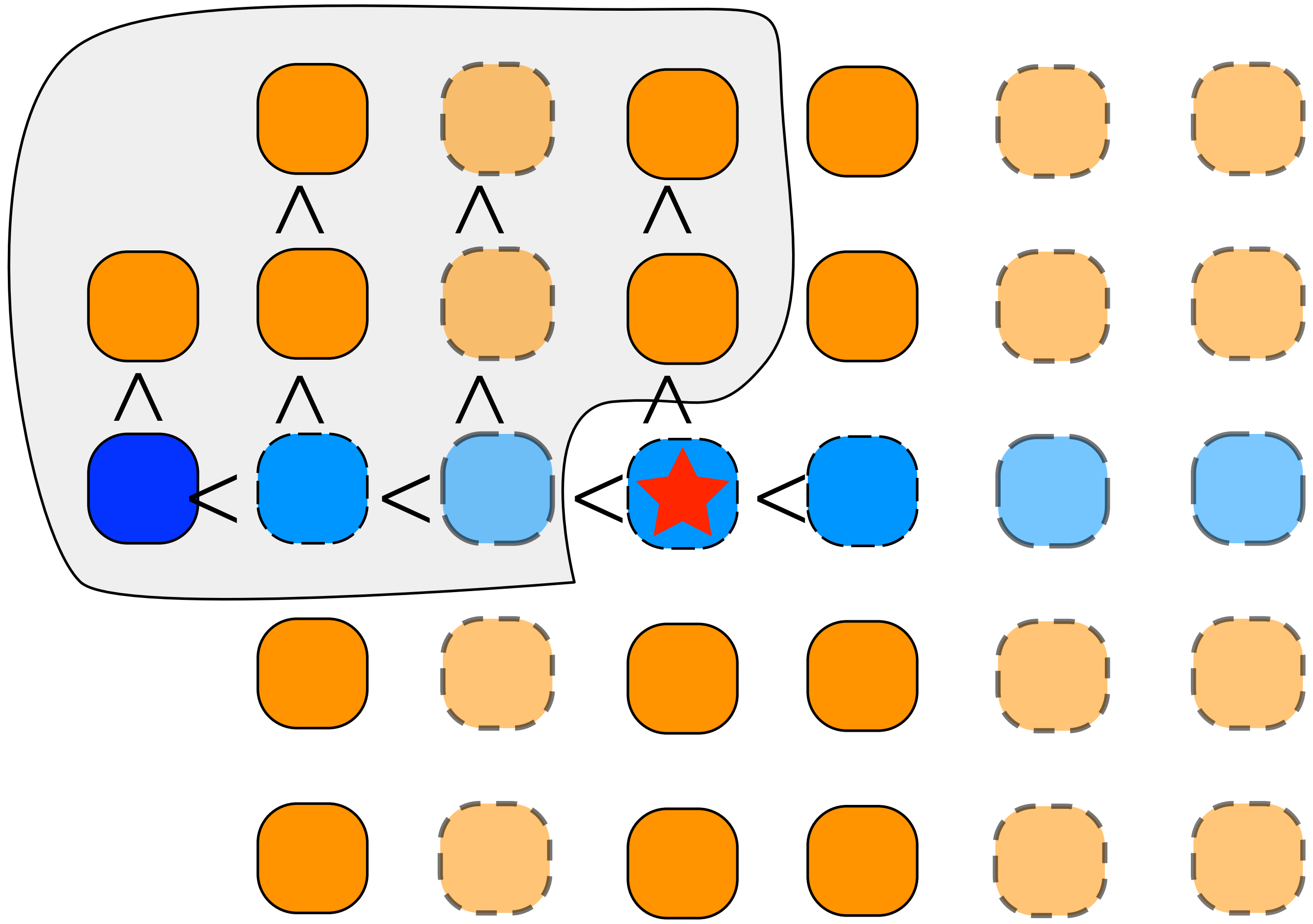
a nice property of our partition



a nice property of our partition

$$3\left(\left\lceil \frac{1}{2}\lceil n/5\rceil \right\rceil - 2\right)$$

$$\geq \frac{3n}{10} - 6$$

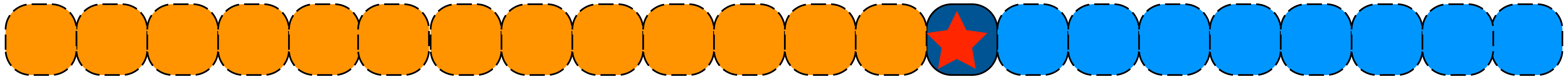


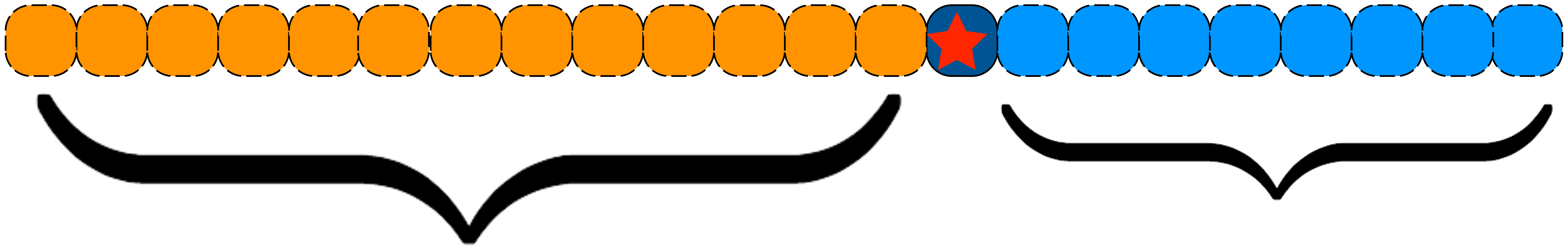
this implies there are

at most $\frac{7n}{10} + 6$ numbers

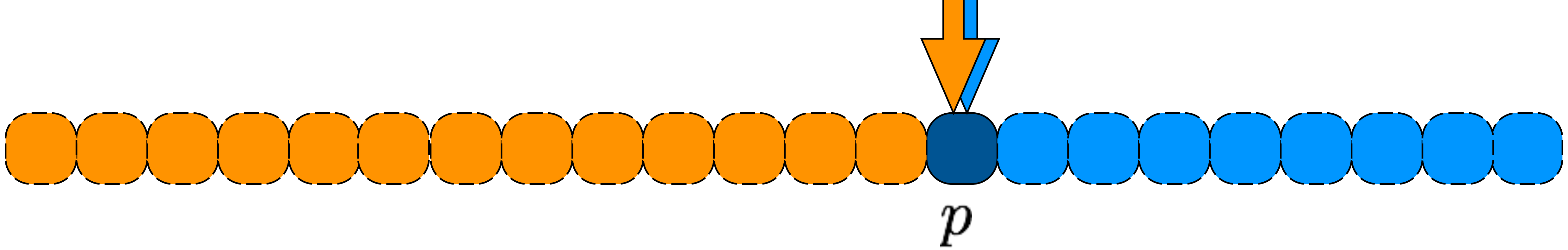
larger than ★
/smaller

a nice property of our partition

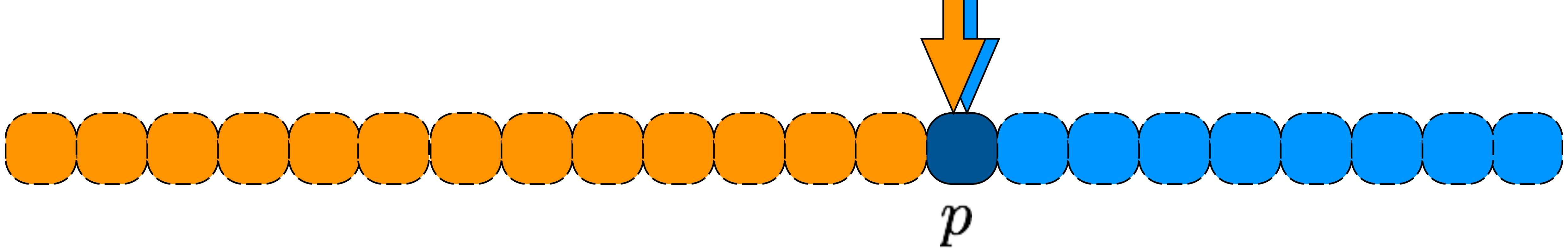




$$\leq \frac{7n}{10} + 6 \qquad\qquad \leq \frac{7n}{10} + 6$$



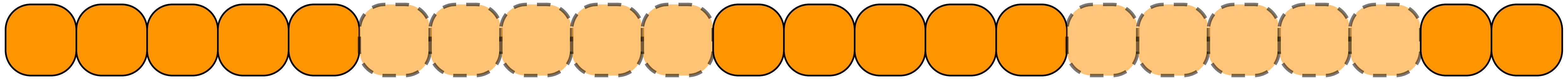
select $(i, A[1, \ldots, n])$



select $(i, A[1, \ldots, n])$

handle base case for small list
else pivot = FindPartitionValue(A,n)
partition list about pivot
if pivot is position **i**, return pivot
else if pivot is in position > **i** select $(i, A[1, \ldots, p-1])$
else select $((i-p-1), A[p+1, \ldots, n])$

FindPartition $(A[1, \ldots, n])$



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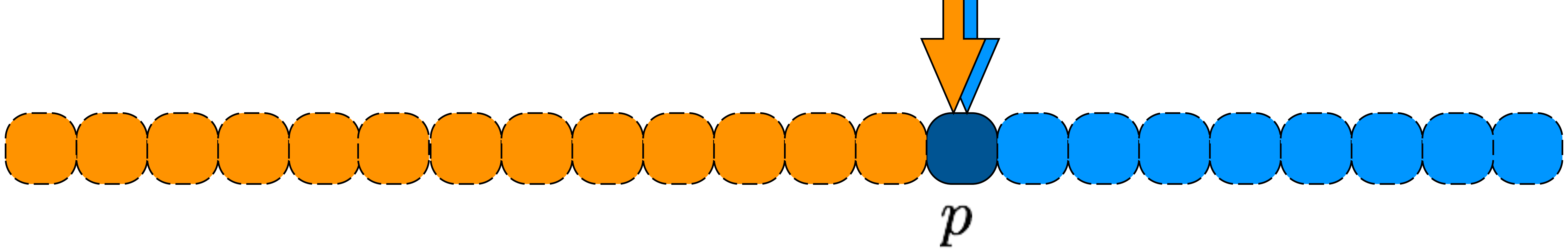
return the result

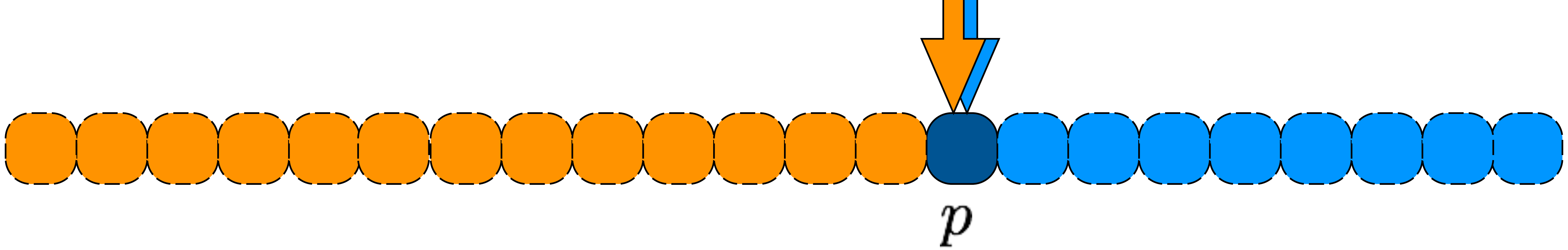
$$P(n) = S(\lceil n/5 \rceil) + O(n)$$



select $(i, A[1,\ldots,n])$

handle base case for small list
else pivot = FindPartitionValue(A,n)
partition list about pivot
if pivot is position **i**, return pivot
else if pivot is in position > **i** select $(i, A[1,\ldots,p-1])$
else select $((i-p-1), A[p+1,\ldots,n])$

$$S(n) = S(\lceil n/5 \rceil) + O(n) + S(7n/10 + 6)$$



select $(i, A[1, \ldots, n])$

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else pivot = FindPartitionValue(A,n)
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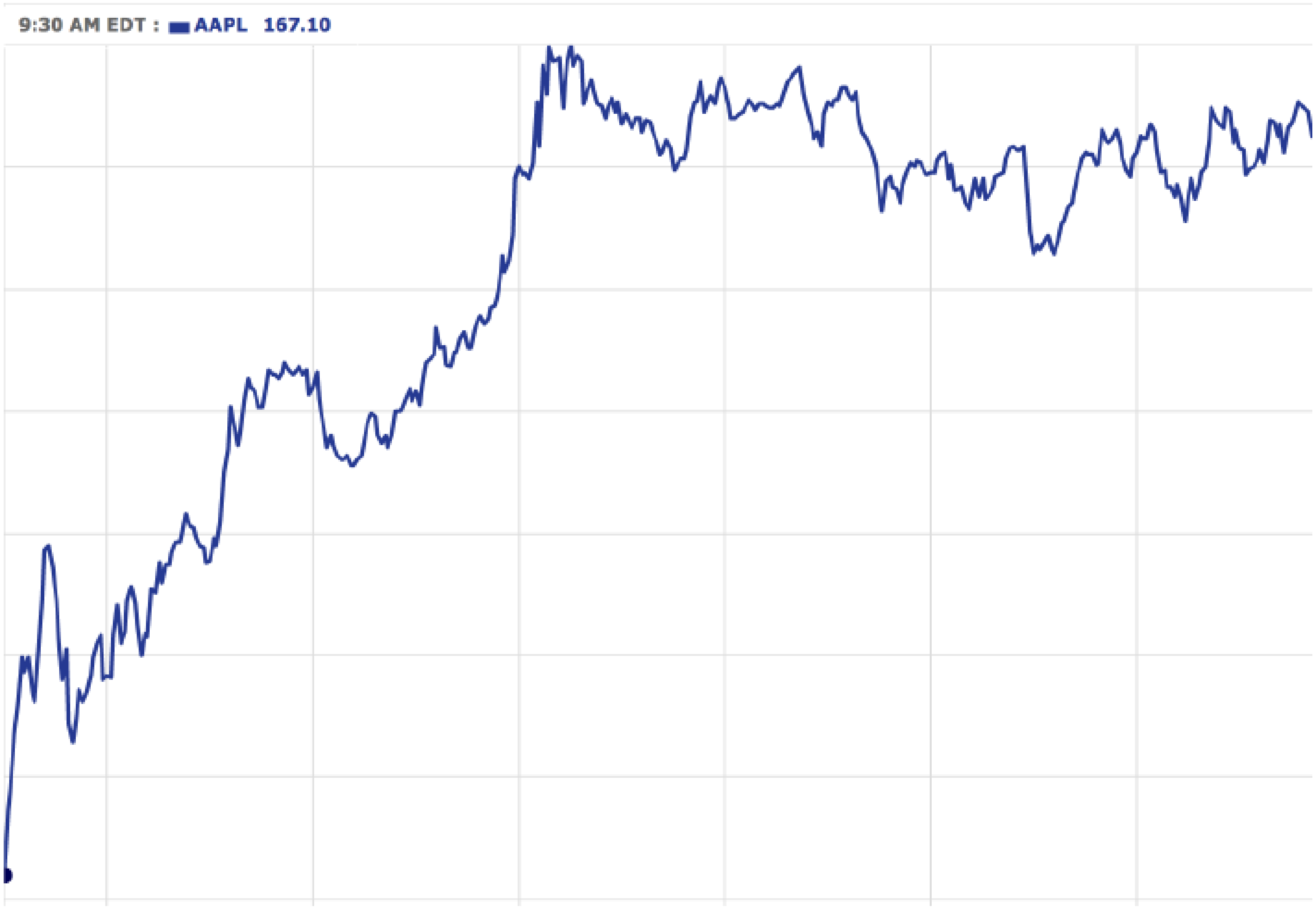
if pivot is position **i**, return pivot
else if pivot is in position > **i** select $(i, A[1, \ldots, p-1])$
else select $((i-p-1), A[p+1, \ldots, n])$

$$S(n) = S(\lceil n/5 \rceil) + O(n) + S(7n/10 + 6)$$

$$\Theta(n)$$

arbitrage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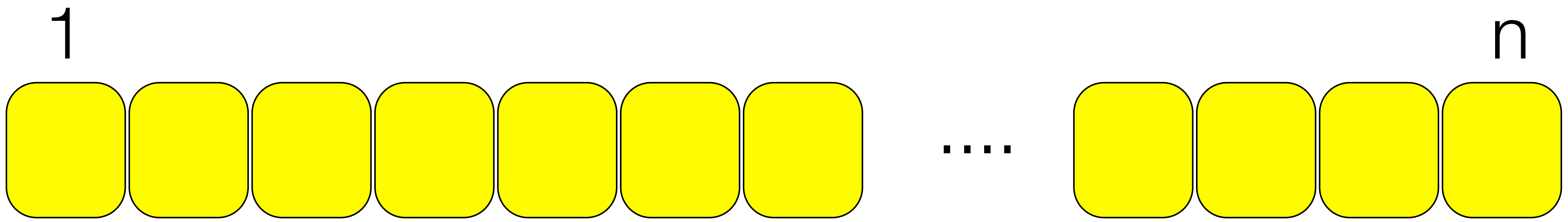
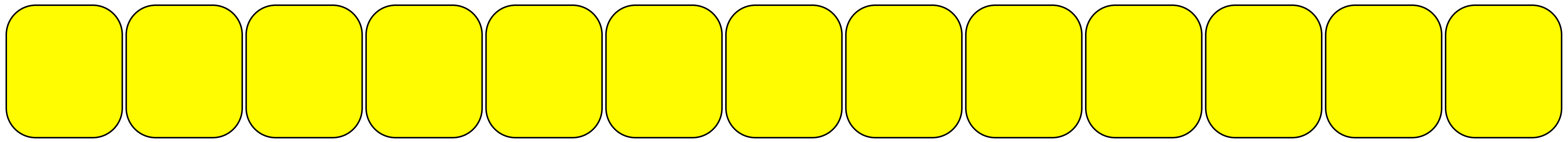
input: array of n numbers

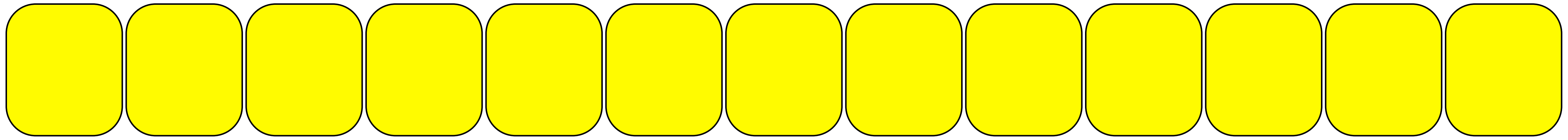


goal:

first attempt

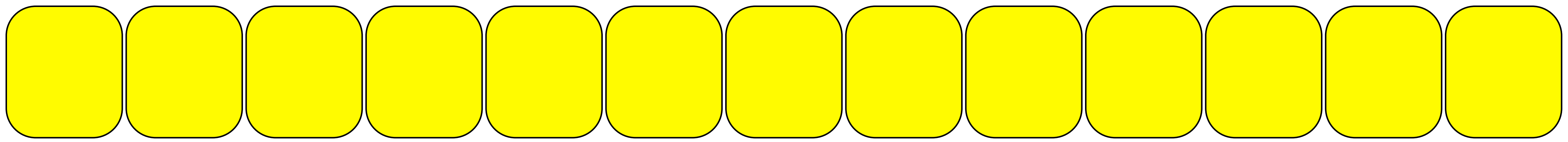


first attempt



```
arbit(A[1...n])
```

first attempt



```
arbit(A[1...n])
    base case if |A|=1
    lg = arbit(left(A))
    rg = arbit(right(A))
    minl = min(left(A))
    maxr = max(right(A))
    return max{maxr-minl,lg,rg}
```

better approach

second attempt

```
arbit+(A[1...n])
   base case if |A|=1
```

second attempt

```
arbit+(A[1...n])
   base case if |A|=1
   (lg,minl,max) = arbit(left(A))
   (rg,mi,maxr) = arbit(right(A))

   return max{maxr-minl,lg,rg}
```