H1.

You may collaborate with other students on the homework but you must submit your own individually written solution, identify your collaborators, and acknowledge any external sources that you consult. Do no submit a solution that you cannot explain to me. Please use *exactly 1 page* for your answers to 1,2,3. (Use `\newpage` to start a new page.)

PROBLEM 1 *Asymptotic Notation Review*

Arrange the functions below in order $g_1, g_2, \ldots, g_{12}$ so that $g_i = \Omega(g_{i+1})$.

$$n^{1/\log(n)} \quad n^{-3} \quad 3^{\sqrt{\log(n)}} \quad \log\log n \quad e^n \quad (\log n)^{\log n/\log\log n}$$
$$\sqrt{n} \quad \log^{\sqrt{e}}(n) \quad n2^n \quad 2^{1/n} \quad \sqrt{\sqrt{n^{\log n}}} \quad n^{\log\log n}$$

PROBLEM 2 *Solve the following recurrences by obtaining a Θ bound. You may assign a standard value for the base case terms $T(1), T(2), \ldots, T(k)$ for some small constant k. Prove your answer.*

1. $T(n) = T(n-5) + n$
2. $T(n) = 37T(\lceil n/23 \rceil) + n$
3. $T(n) = 2T(\lceil \sqrt{n} \rceil) + 2$
4. $T(n) = T(\lceil n/9 \rceil) + T(\lfloor 7n/10 \rfloor) + n$

PROBLEM 3 *Karatsuba Example*

Carry out the Karatsuba recursion for $41020125 * 20161334$. You can treat 2-digit multiplication is a unit operation (i.e., recursion stops when operands are two-digits).

PROBLEM 4 *Divide and Conquer*

The Greek Diogenes spent his life searching for an honest man. Consider the Greek system today with a group of n sorority sisters and fraternity brothers. We can test a pair—lets call them Alice and Bob for simplicity—by asking them whether the other is honest. Honest Greeks always report truthfully, but the dishonest can report arbitrarily. Thus, the following outcomes are possible:

Alice says	Bob says	
"Bob is honest"	"Alice is honest"	Either both are honest or both are dishonest
"Bob is honest"	"Alice is dishonest"	at least one is dishonest
"Bob is dishonest"	"Alice is honest"	at least one is dishonest
"Bob is dishonest"	"Alice is dishonest"	at least one is dishonest

1. A group of n Greeks is *moral* if more than half are honest.

   Suppose we start with a moral group of n Greeks. Describe a method that uses only $\lfloor n/2 \rfloor$ pair-wise tests between the Greeks to find a smaller moral group of at most $\lceil n/2 \rceil$ Greeks.

   Using this approach, devise an algorithm that classifies all Greeks as honest or dishonest using only $\Theta(n)$ pairwise tests. Prove the correctness of your algorithm, and prove that only $\Theta(n)$ tests are used.

2*. Society may fall apart if there is not an honest majority. Prove that a conspiracy of $\lfloor n/2 \rfloor + 1$ dishonest Greeks (who may, for example, know each other and share a plan) can foil any attempt to find an honest person.