CS4102 - ALGORITHMS - S'16 — ABHI SHELAT
DUE FRI APR 22, 2016 AT 5P VIA COURSE WEBSITE

H7.

You may collaborate with other students on the homework but you must submit your own individually written solution, identify your collaborators, and acknowledge any external sources that you consult. Do no submit a solution that you cannot explain to me.

PROBLEM 1 *Edmonds-Karp shortest paths*

In class, we stated that in the Edmonds-Karp maxflow algorithm, the length of shortest paths in G are monotonically increasing. However, this is not obvious because as we add augmenting paths, new edges are introduce to the graph. In this problem, prove that for any $j > i$ and for any $u \in V$, we have $\delta_j(s,u) \geq \delta_i(s,u)$. (Hint, consider a proof by contradiction. Pick the node that is closest to s whose distance to s decreases between steps j and i. Identify the conditions under which its distance to s can decrease. Explain how this leads to a contradiction. The proof should take 6 sentences.)

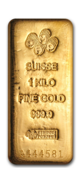
PROBLEM 2 *Classrooms*

Before the start of the Spring semester, the Registrar must assign each class to a time and a classroom. The classroom must be larger than the class it holds to properly seat all the students. Suppose there are n classes such that class i has s_i students enrolled. The university has m rooms, and room j can hold r_j students. Finally, there are non-overlapping time slots $t_1, \ldots, t_k$ for the classes. For example t_1 is "MW9-10.15" and t_2 is "MW10.30-11.45" and so on. Given all this data, namely, given $(s_1, \ldots, s_n), (r_1, \ldots, r_m), (t_1, \ldots, t_k)$, design an efficient algorithm that assigns classes to times and classrooms. Analyze the running time and argue correctness.

PROBLEM 3 *Gold Bullion face*

In the ruins of Pompeii, I remember seeing the House of the Tragic Poet with a famous mosaic floor proclaiming visitors to "Beware of the Dog." In Charlottesville, a less tragic and wealthier poet has commissioned a mosaic using 1kg bars of solid gold, specifically the type CreditSuisse mints in the dimension 80mmx40mm.

Design an algorithm that takes as input an orange and white grid of squares—each representing a 40mmx40mm region—and determines if the orange squares in the design can be *entirely covered* with gold bullion bars. Note that gold bars can never be split in half (that would destroy their value [sic])! Each gold bar covers exactly two of the squares. As an example, consider the gold bars on the left, and the pixel art on the right. Can the UVA logo be covered in gold?



(Hint: formulate the question as a type of bipartite matching problem.)

PROBLEM 4 *TA assignment*

Your task is to assign TAs to office hour slots for a very large undergraduate class. Your goal is to have continuous TA coverage for the entire week. Specifically, you have divided the week into time-slots $t_1, \ldots, t_k$ and you want every slot to have a TA. Moreover, for some popular slots, you want more than 1 TA. To express this, you have value $c_i > 0$ which indicates how many TAs you want for slot t_i. All of the TAs have their own class scheduling conflicts which makes this process tedious. Suppose there are n TAs, and each TA i gives you a list of possible time slots $S_i = \{t_1, t_2, \ldots, t_k\}$ when they are free for office hours. From experience, you know that the TAs only report a subset of the actuals slots that are free for them. Therefore, your goal is to produce an office-hours schedule such that all time slots are covered with the appropriate coverage, and each TA i is assigned *at most* $B > 0$ time slots that are not in her list S_i. Design an efficient algorithm that solves this problem, analyze its running time and correctness.