2550 Intro to cybersecurity

L6: Crypto: IND-CPA, AES, MACs

abhi shelat

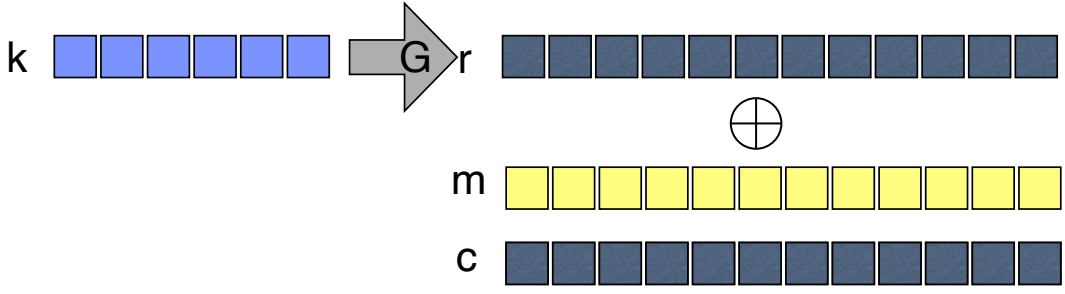
An encryption scheme

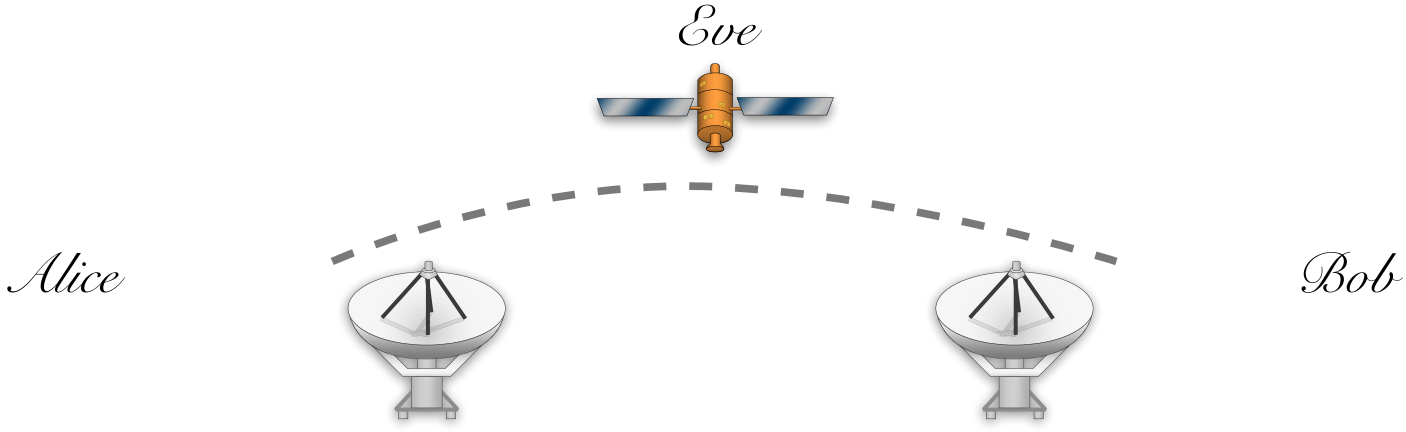
Gen(1^n) $k \leftarrow U_{n/2}$ small key (key generation)

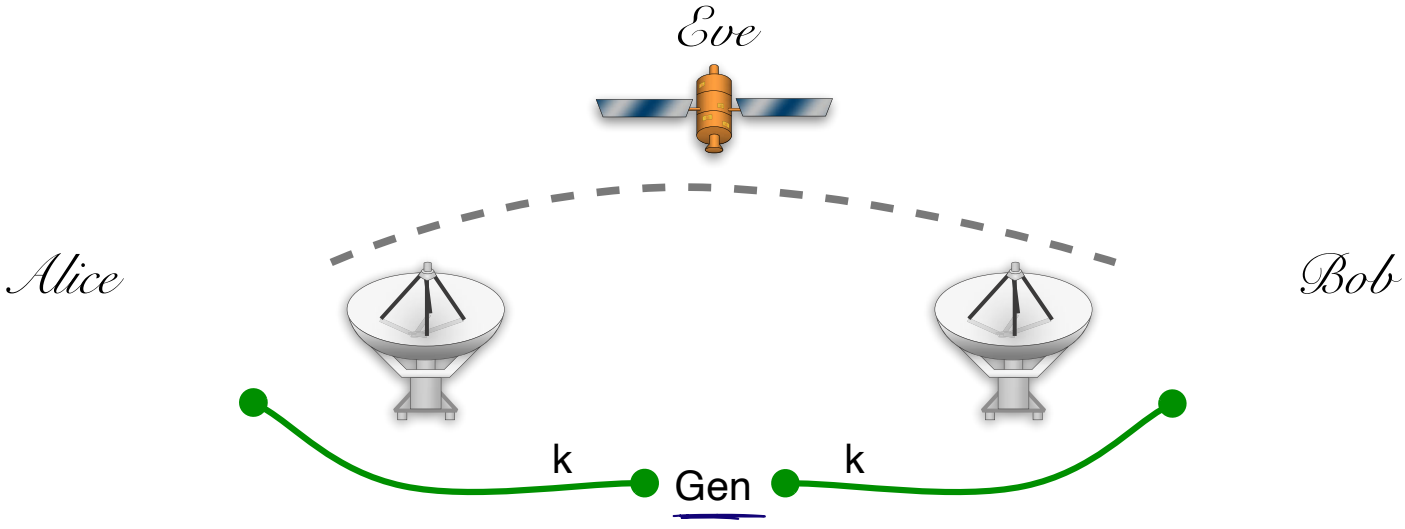
$\text{Enc}_k(m)$ $r \leftarrow G(k)$ $|r| = n$ (encryption)

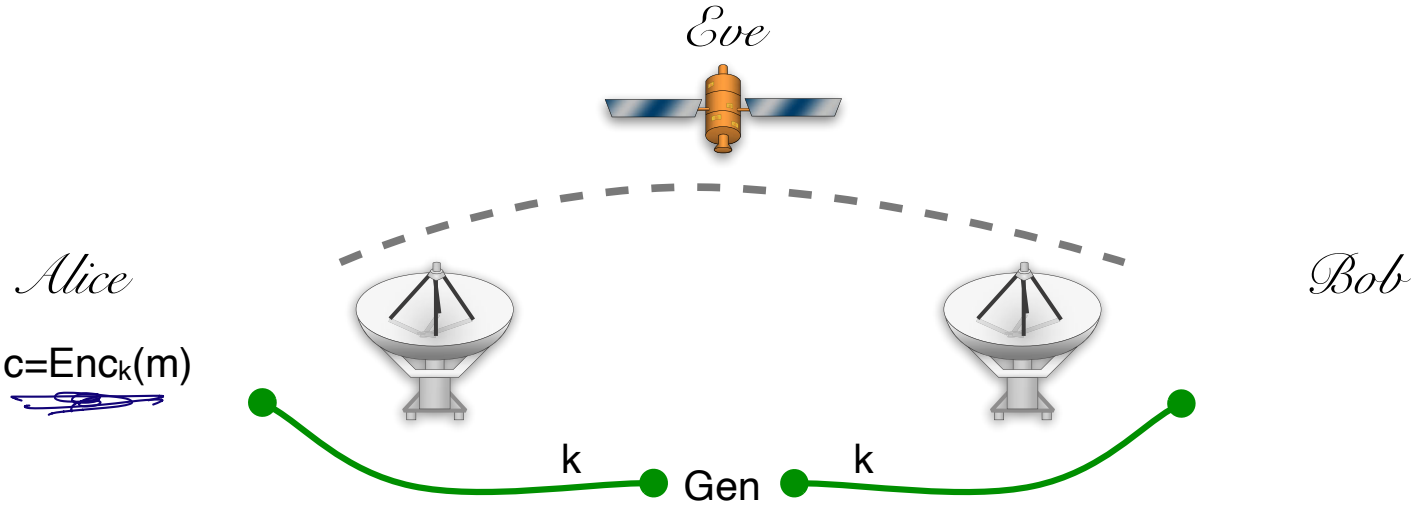
output $m \oplus r$

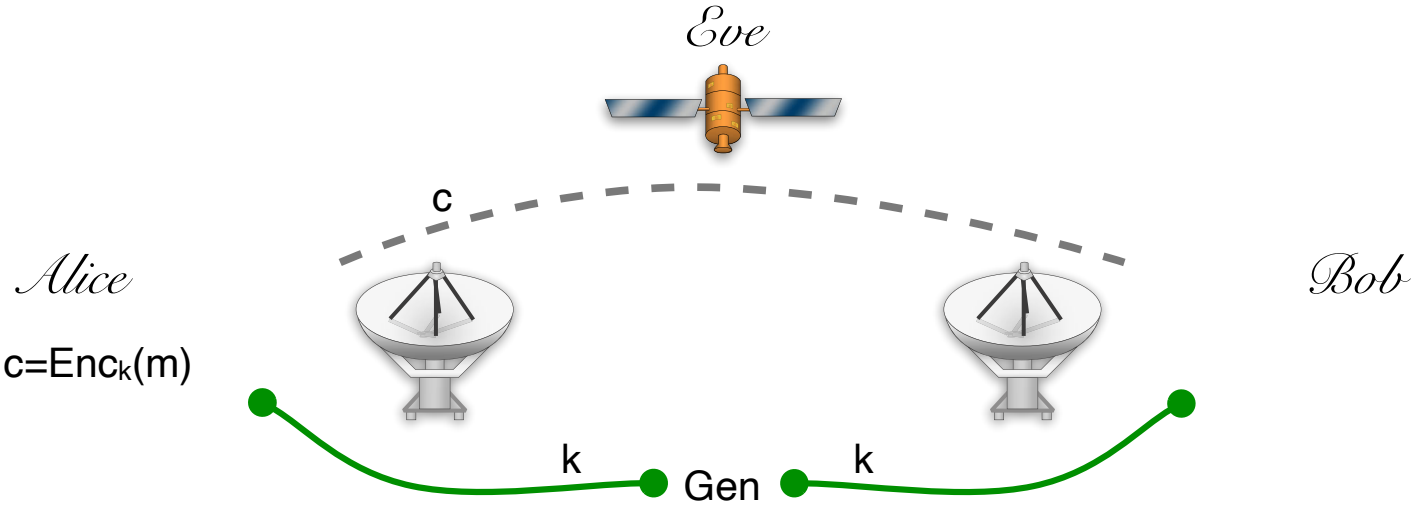
$\text{Dec}_k(c)$ (decryption)

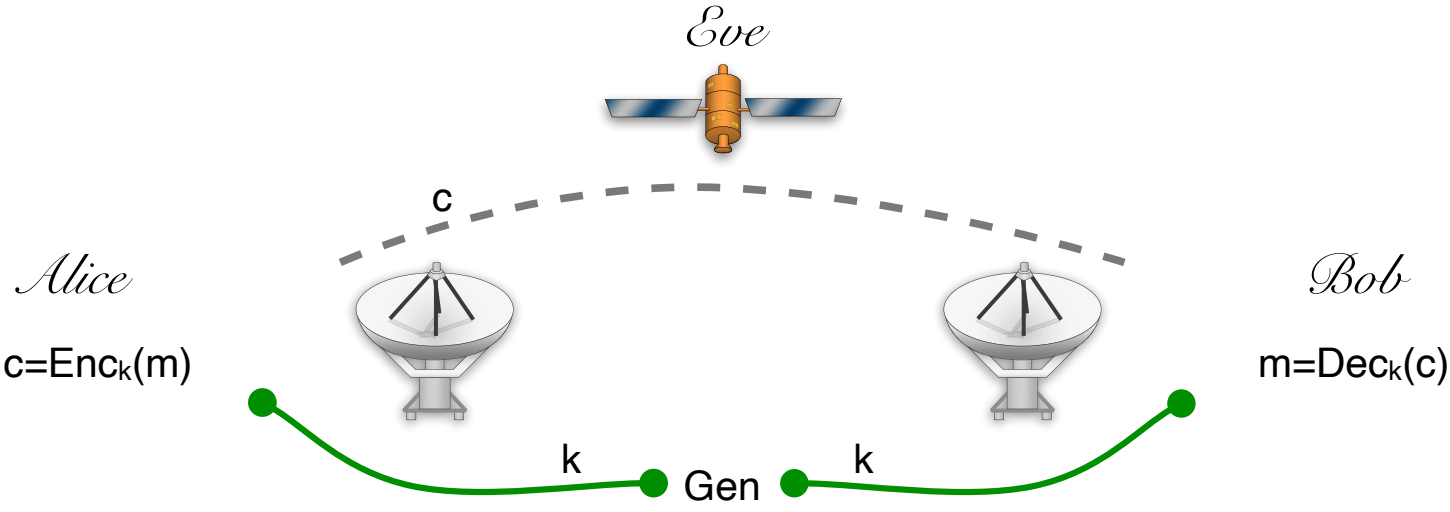


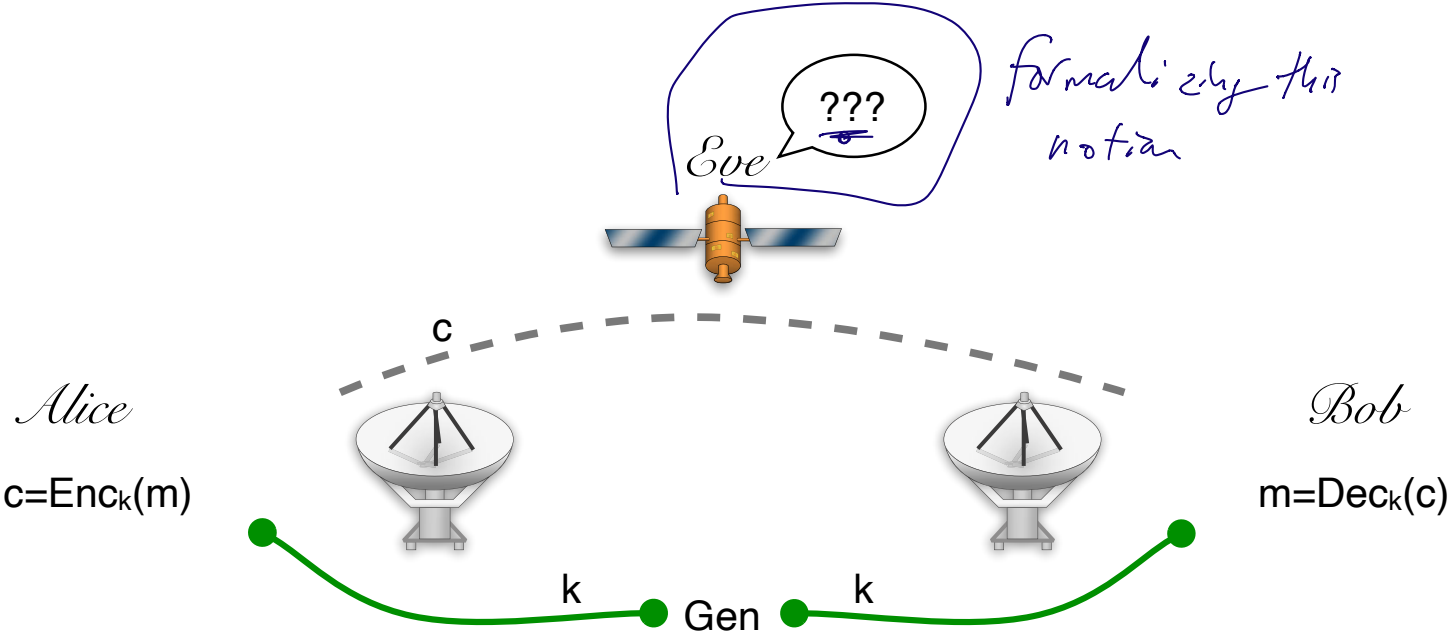

Eve
Alice
Bob


Eve
Alice
Bob
k
Gen
k


Eve
Alice
Bob
c=Enc$_k$(m)
k
Gen
k


Eve
c
Alice
Bob
c=Enc$_k$(m)
k
Gen
k


Eve
c
Alice
Bob
c=Enc$_k$(m)
m=Dec$_k$(c)
k
Gen
k


???
Eve
formalizing this notion
c
Alice
Bob
c=Enc_k(m)
m=Dec_k(c)
k
Gen
k

computational secrecy

$(\mathsf{Gen}, \mathsf{Enc}, \mathsf{Dec}, \mathcal{M}, \mathcal{K})$

is said to be computationally secure if

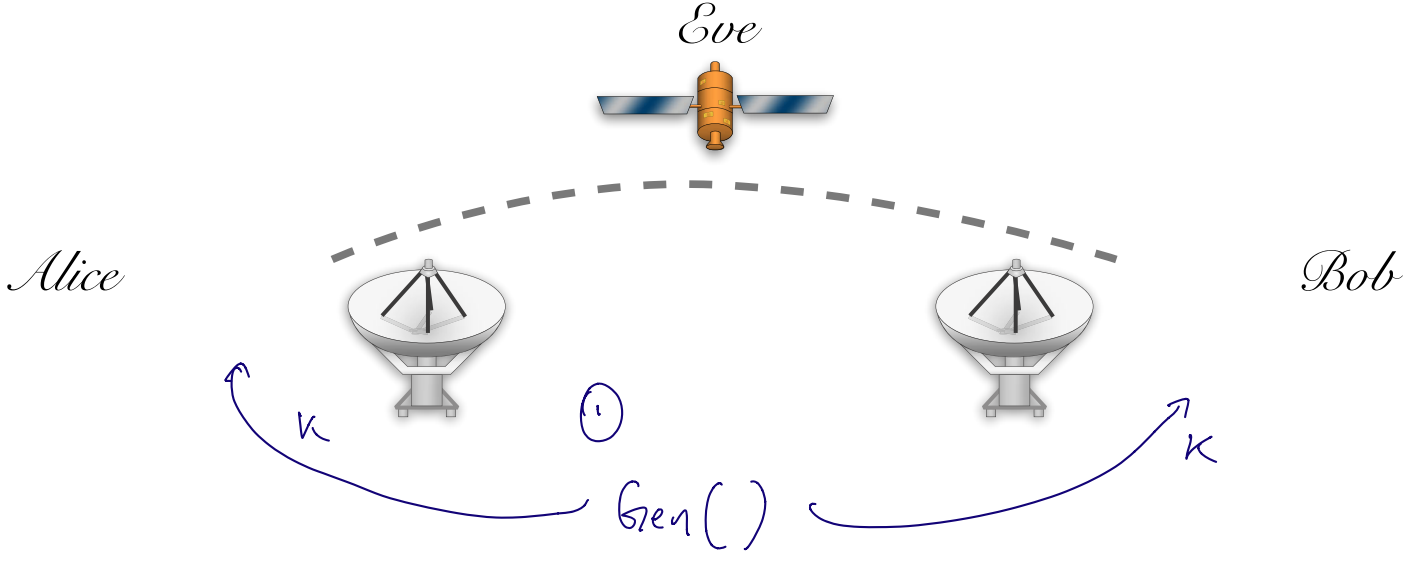
$$\forall m_1, m_2 \in \mathcal{M} \text{ s.t. } |m_1| = |m_2|, \forall c$$

$$\{k \leftarrow \mathrm{Gen}(1^n) \ : \ Enc_k(m_1)\}$$

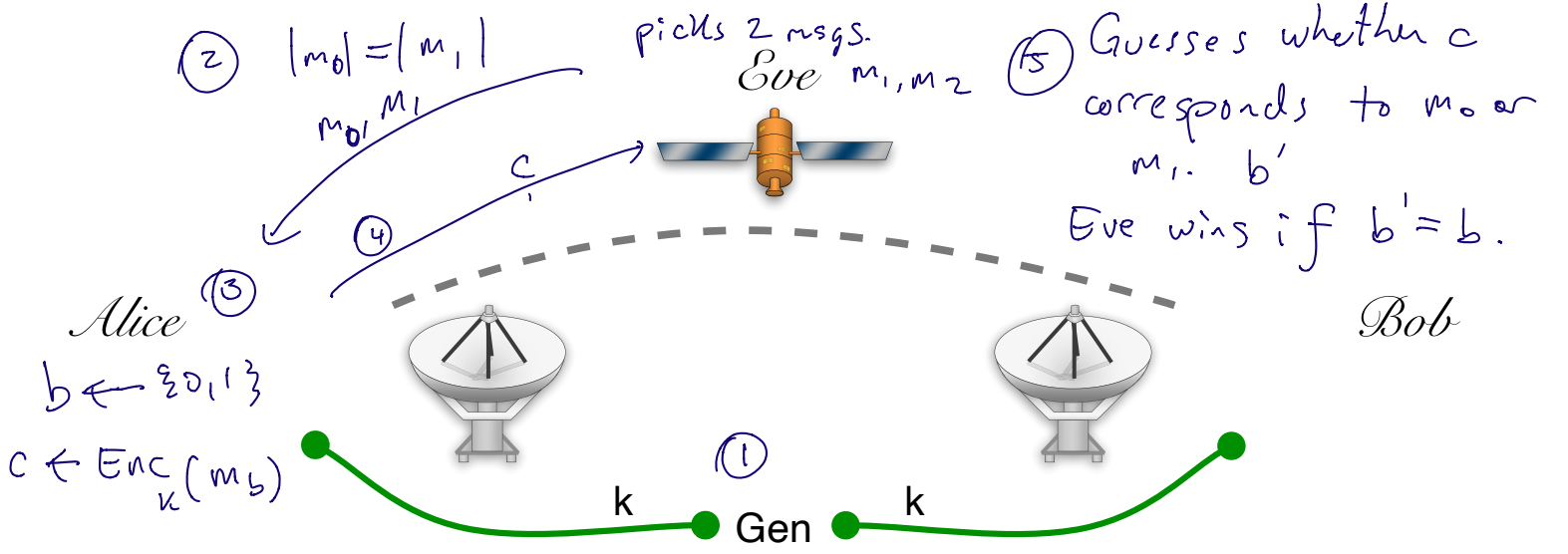
$$\approx$$

$$\{k \leftarrow \mathrm{Gen}(1^n) \ : \ Enc_k(m_2)\}$$

Simple security game for Enc

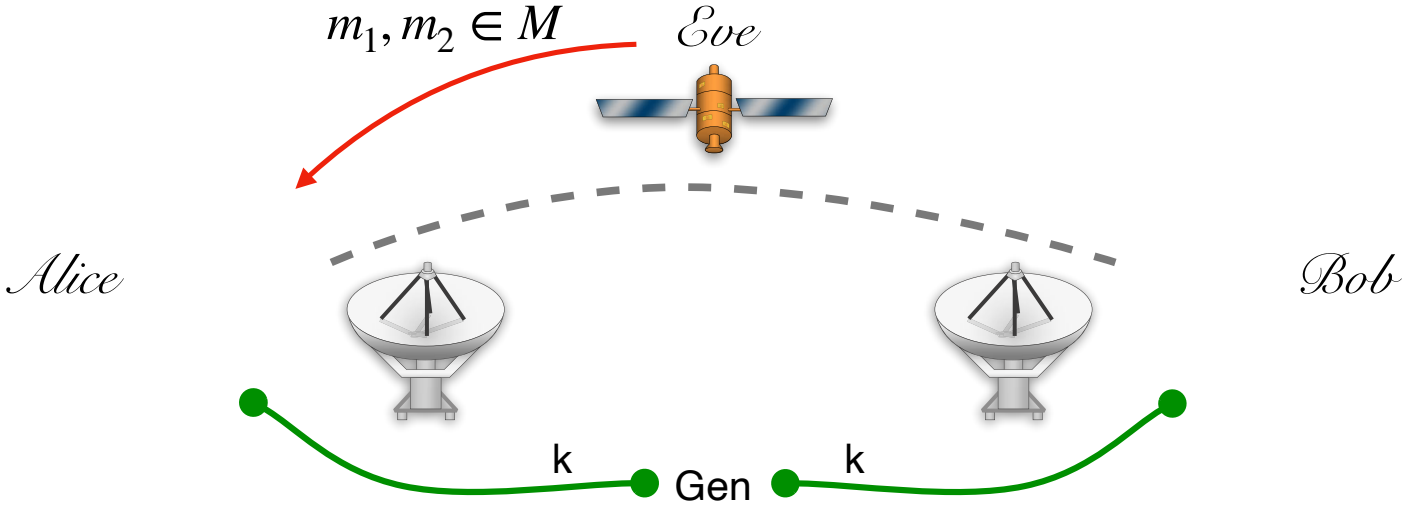


Simple security game for Enc

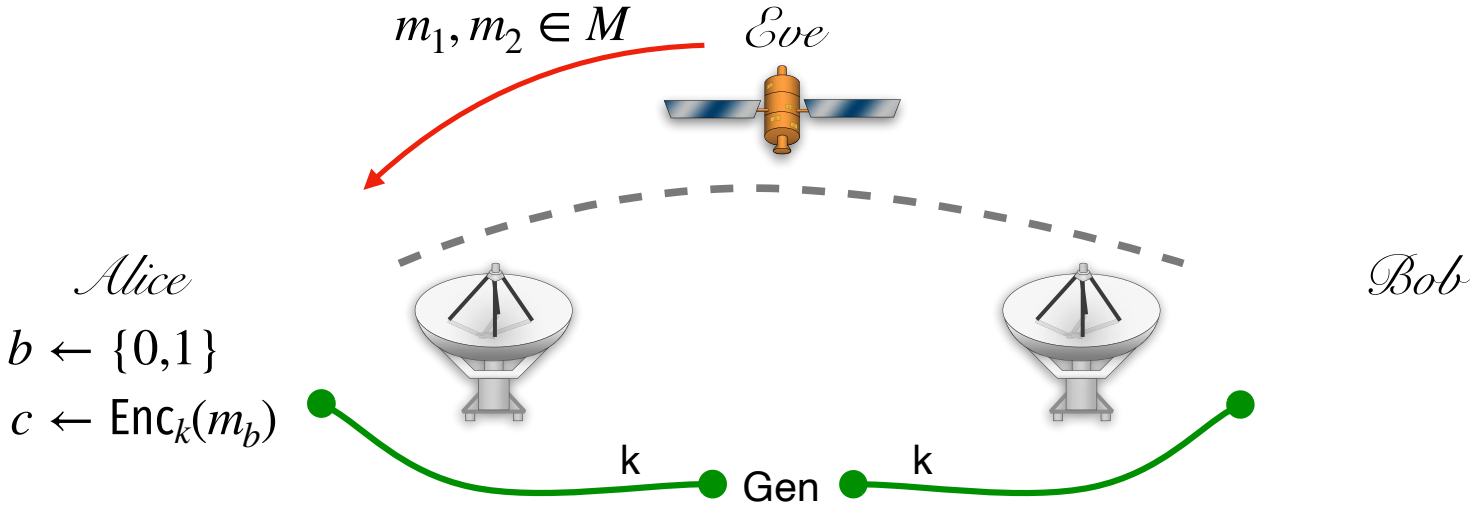


CLAIM: $Pr[\text{that Eve wins this game}] \approx 1/2$

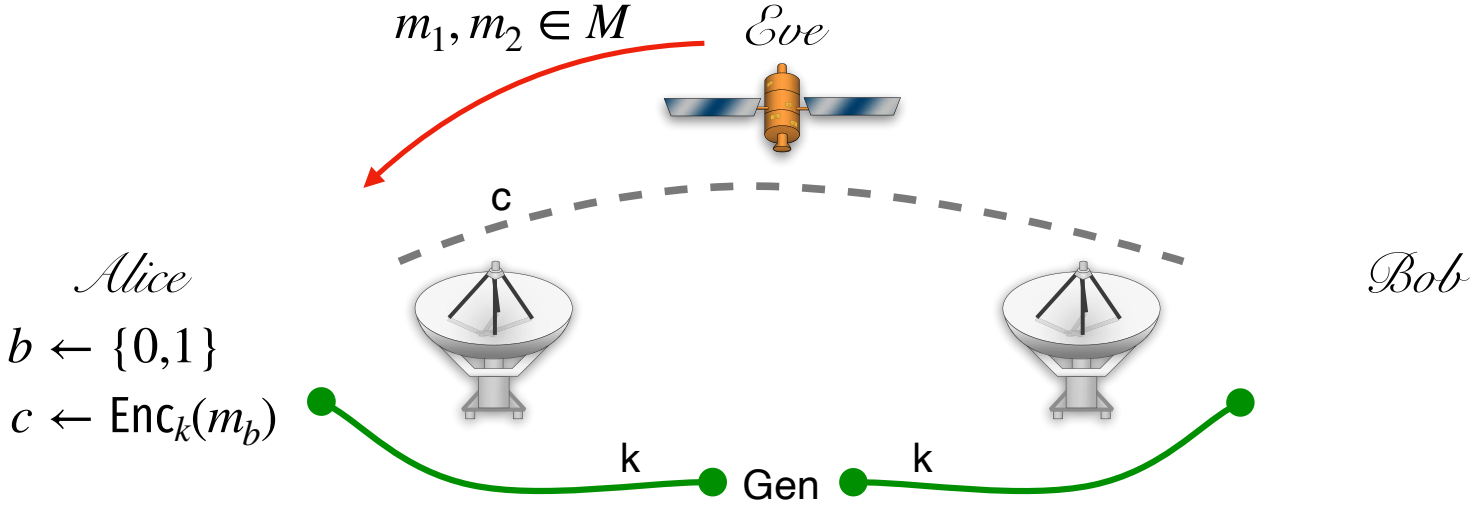
Simple security game for Enc



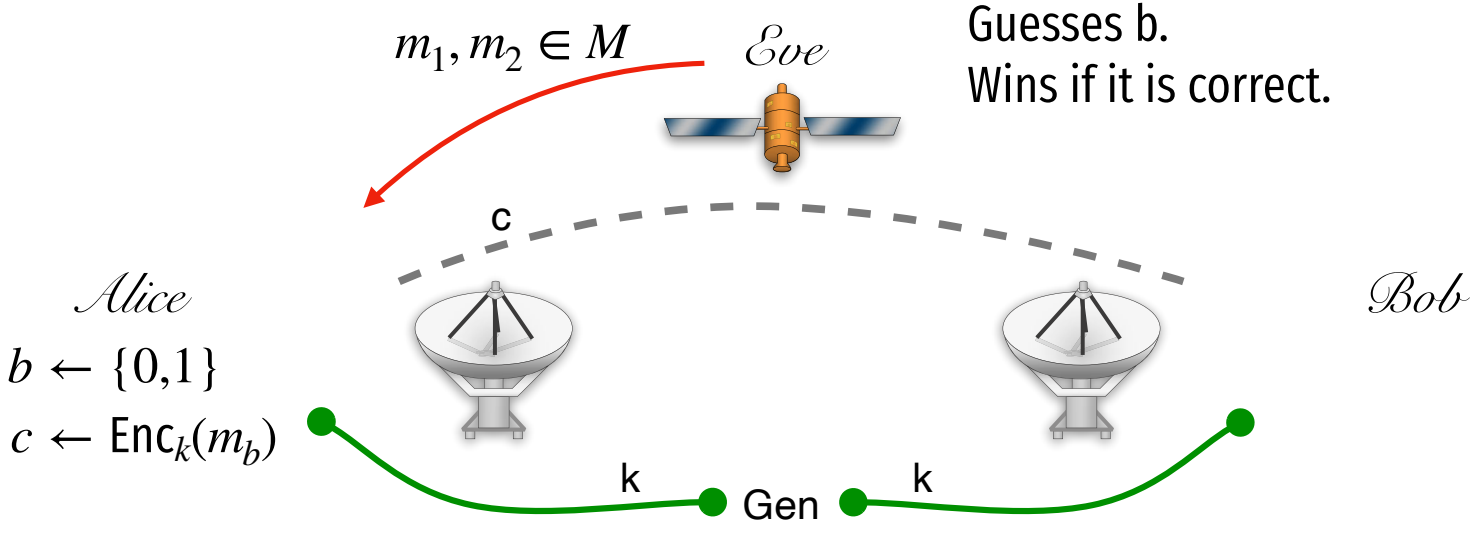
Simple security game for Enc



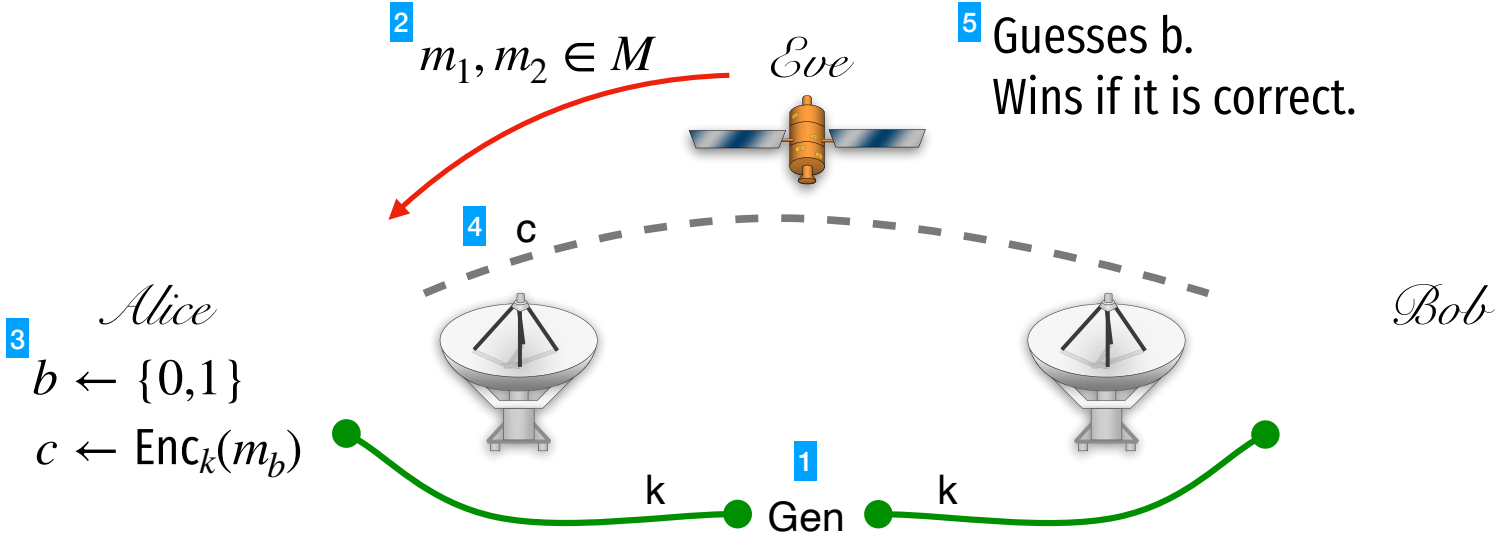
Simple security game for Enc



Simple security game for Enc

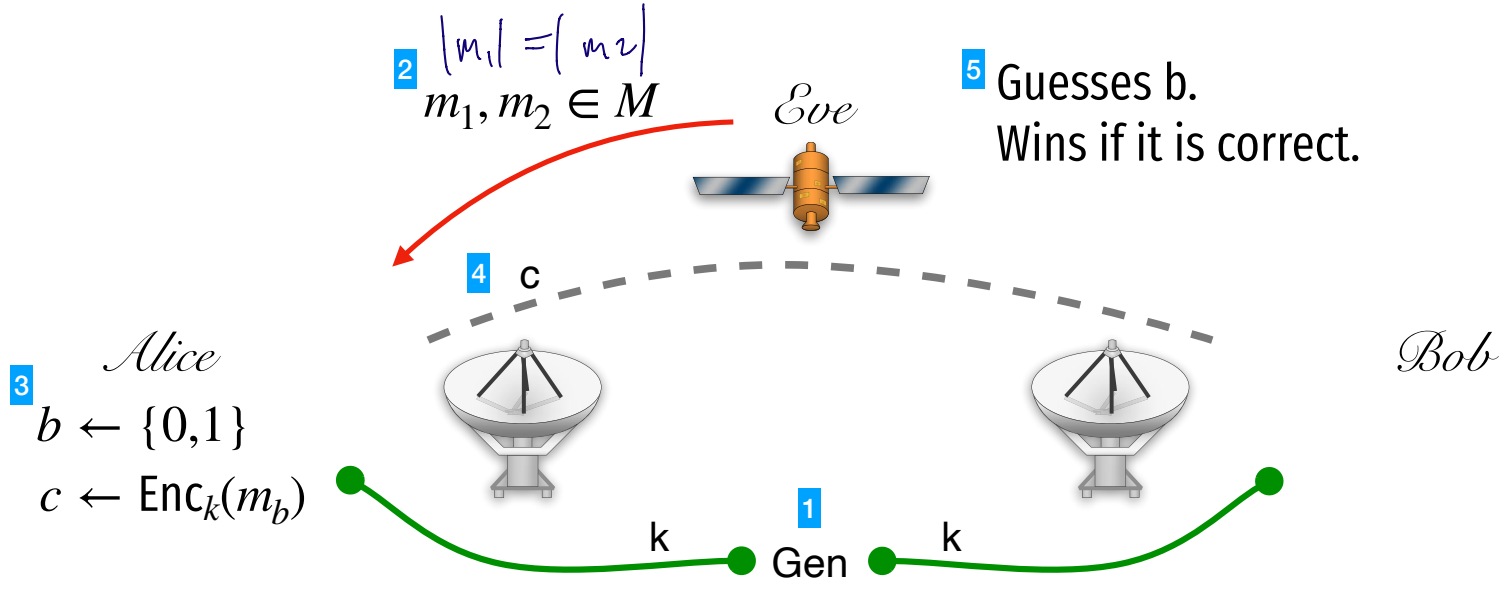


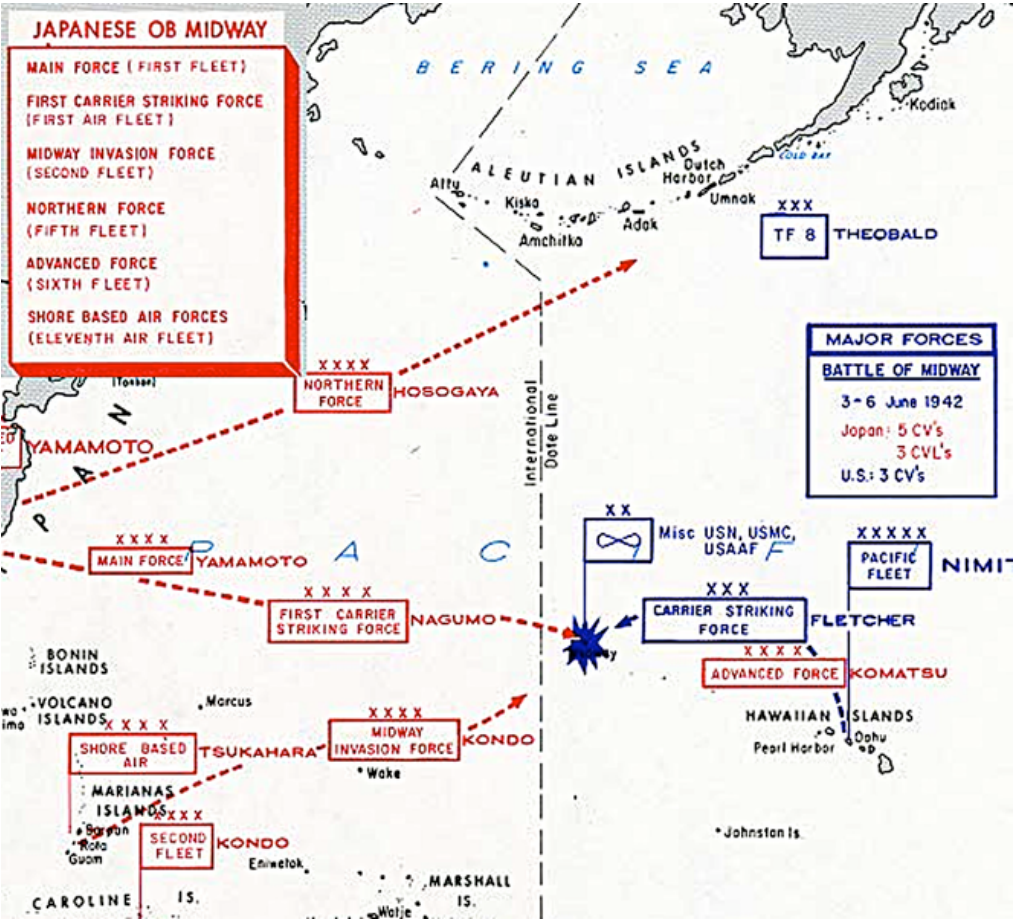
Simple security game for Enc



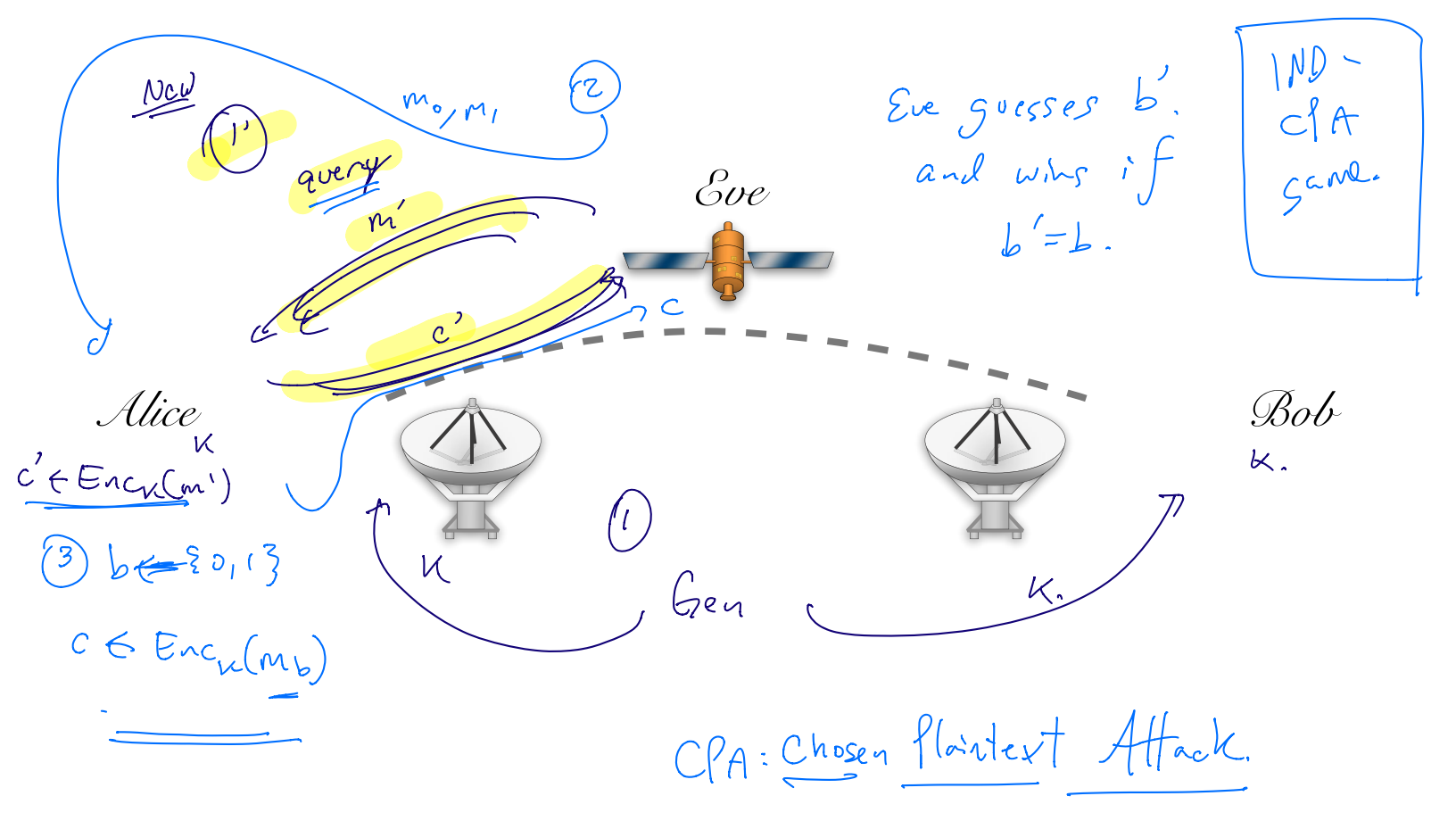
If PRG is secure (i.e. discrete log is hard),
then (Gen, Enc, Dec) described earlier is secure in this game.

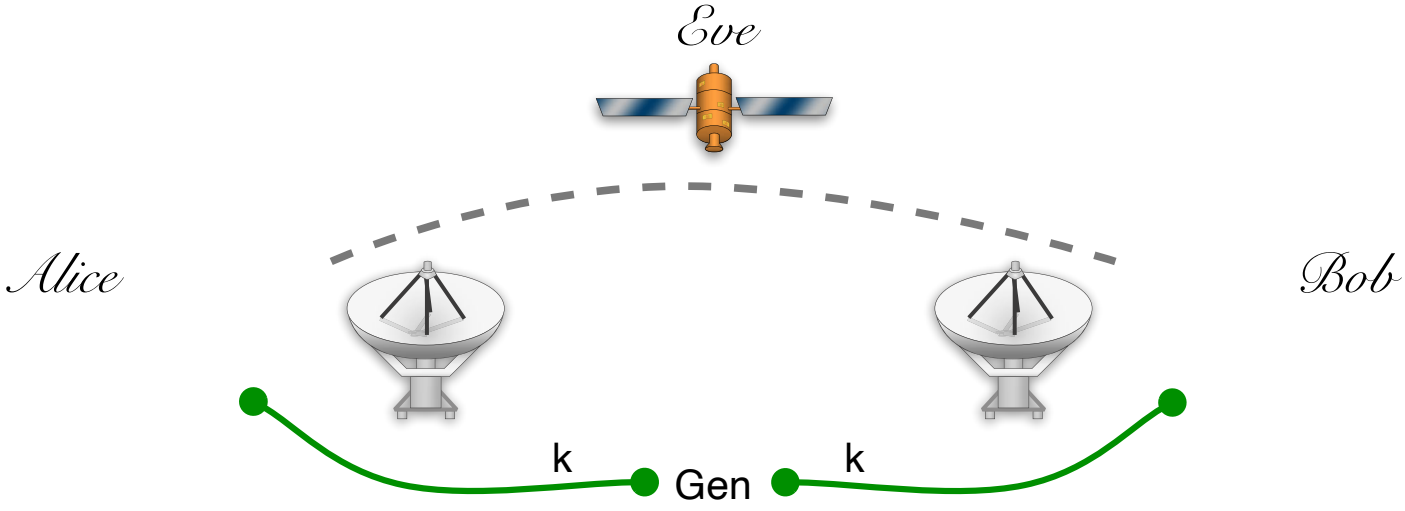
Is this game strong enough to capture all feasible attacks?

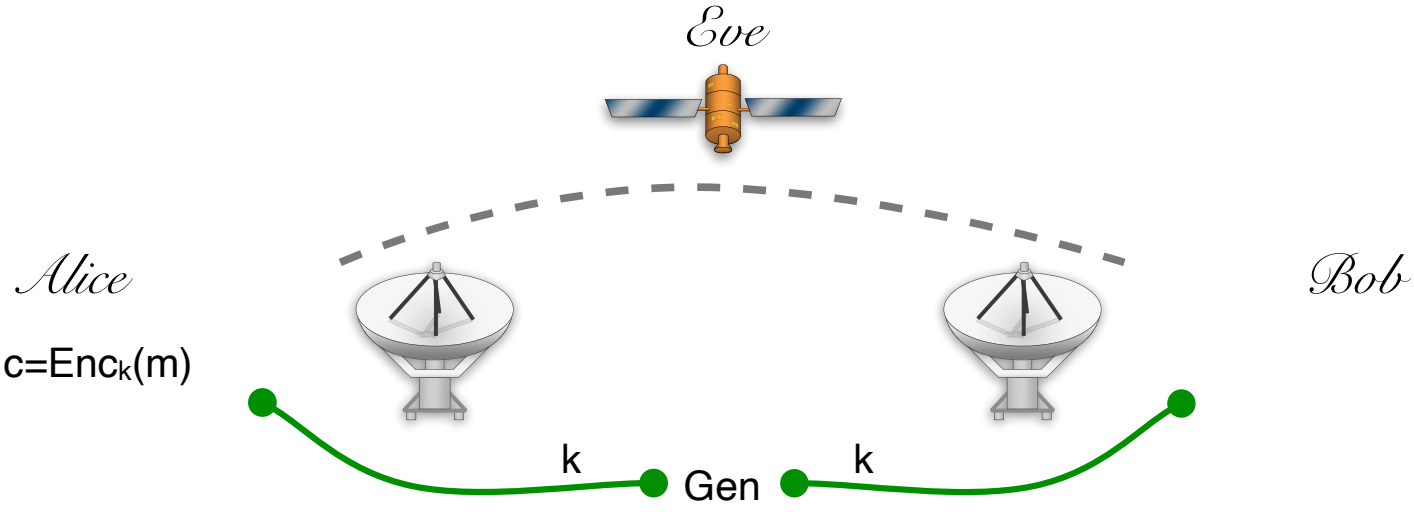


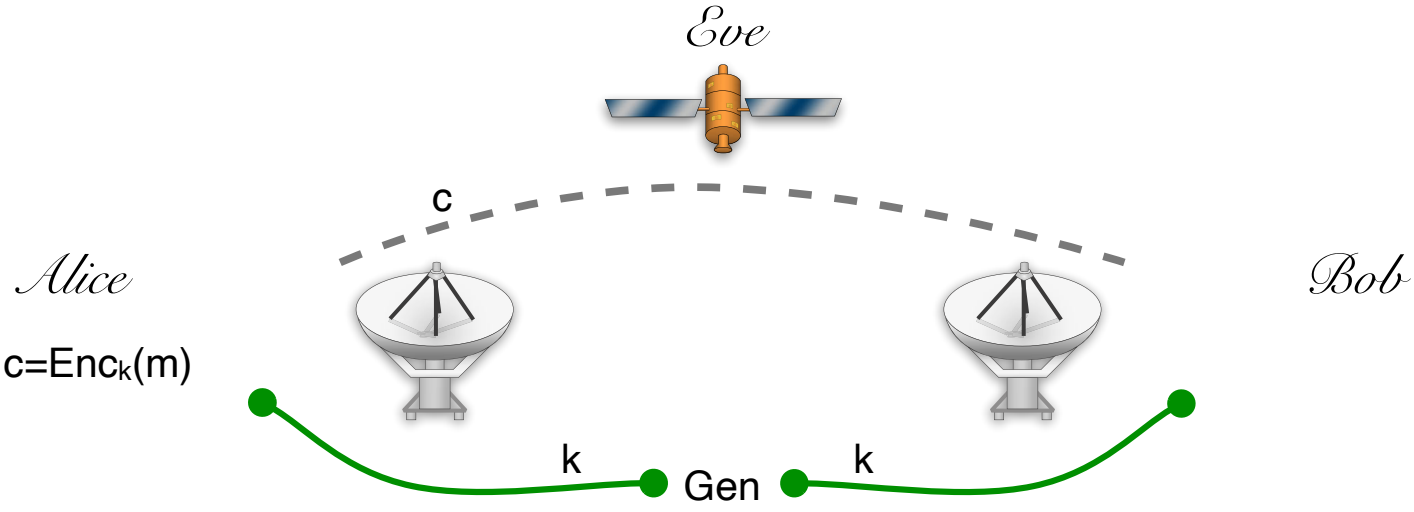


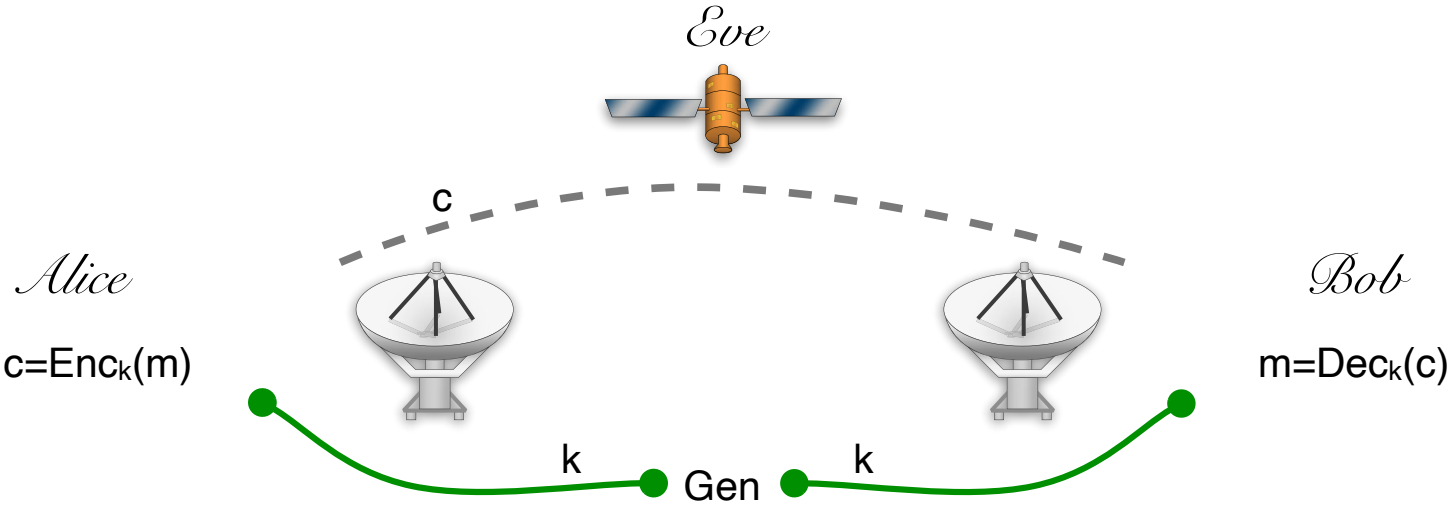
Eve was able to "sweep the Japanese ene scheme"

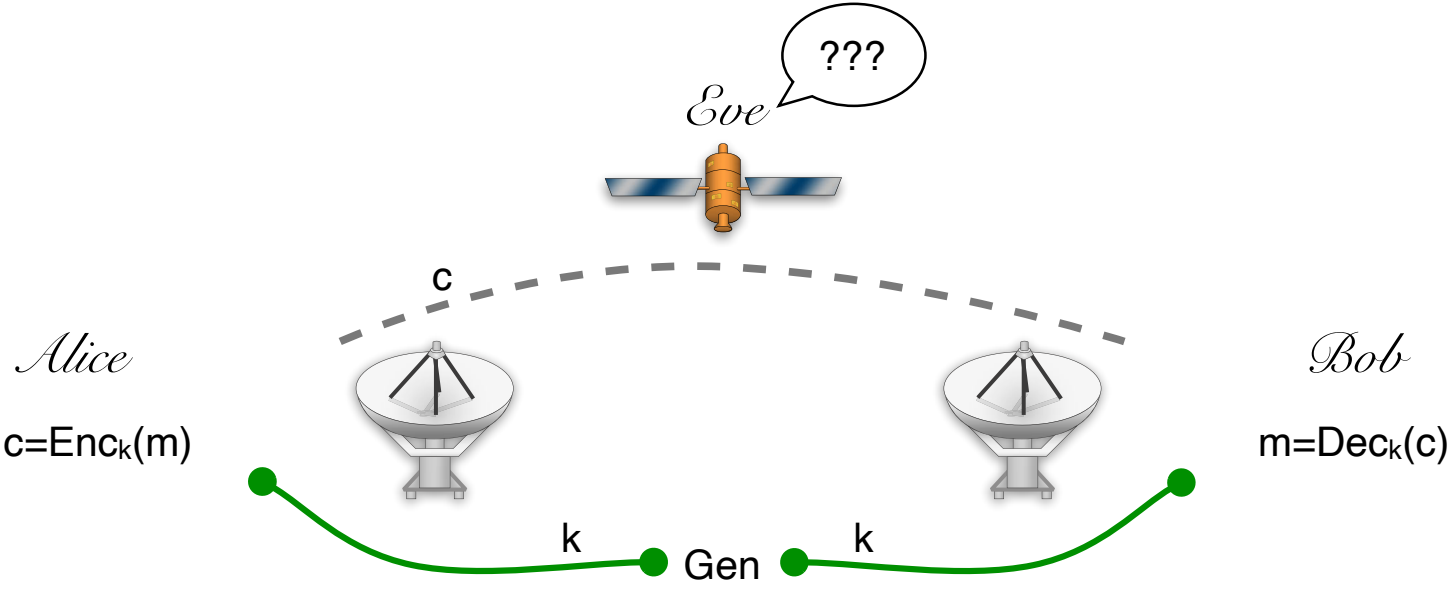

New
(1')
query
m'
c'
m_0, m_1
(2)
c
Eve
Alice
k
$c' \leftarrow Enc_k(m')$
(3) $b \leftarrow \{0,1\}$
$c \leftarrow Enc_k(m_b)$
Bob
k.
(1)
k
Gen
k.
Eve guesses b'.
and wins if
b' = b.
IND-CPA game.
CPA: Chosen Plaintext Attack.


Eve
Alice
Bob
k
Gen
k


Eve
Alice
Bob
c=Enc$_k$(m)
k
Gen
k

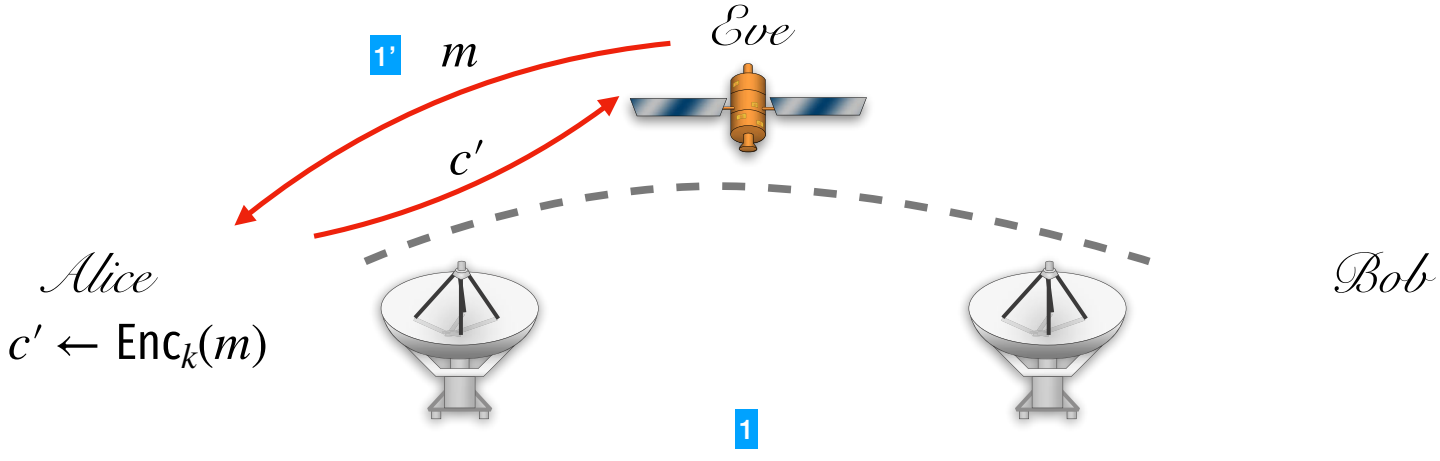

Eve
c
Alice
Bob
c=Enc$_k$(m)
k
Gen
k


Eve
c
Alice
Bob
c=Enc$_k$(m)
m=Dec$_k$(c)
k
Gen
k

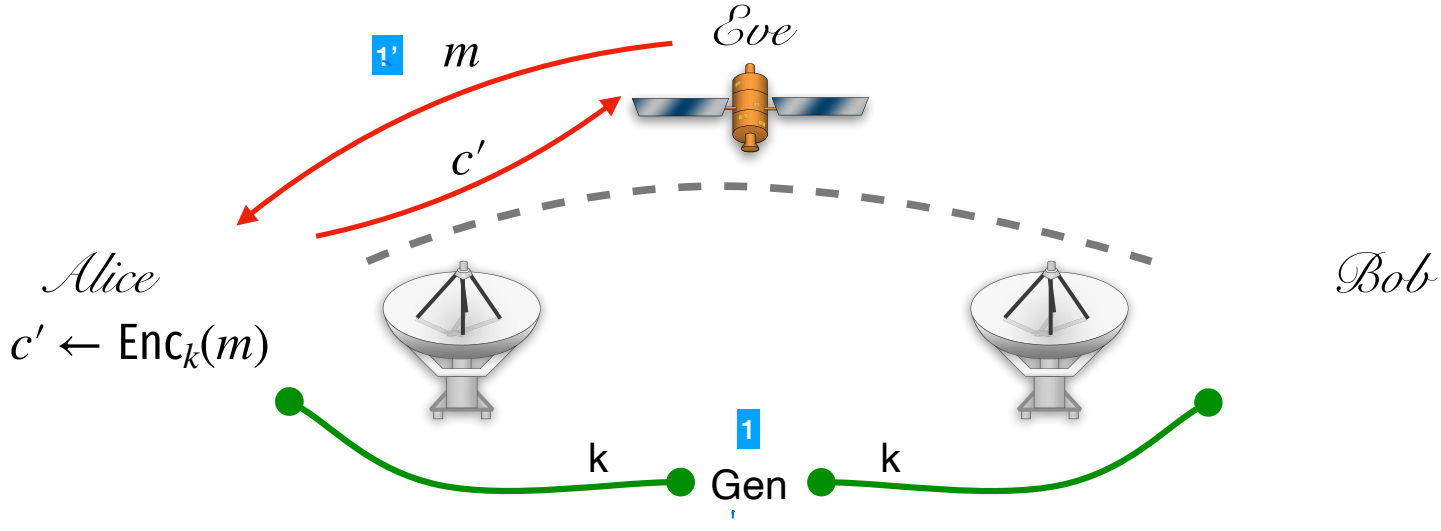

???
Eve
c
Alice
Bob
c=Enc$_k$(m)
m=Dec$_k$(c)
k
Gen
k

IND-CPA attack for Symmetric Enc

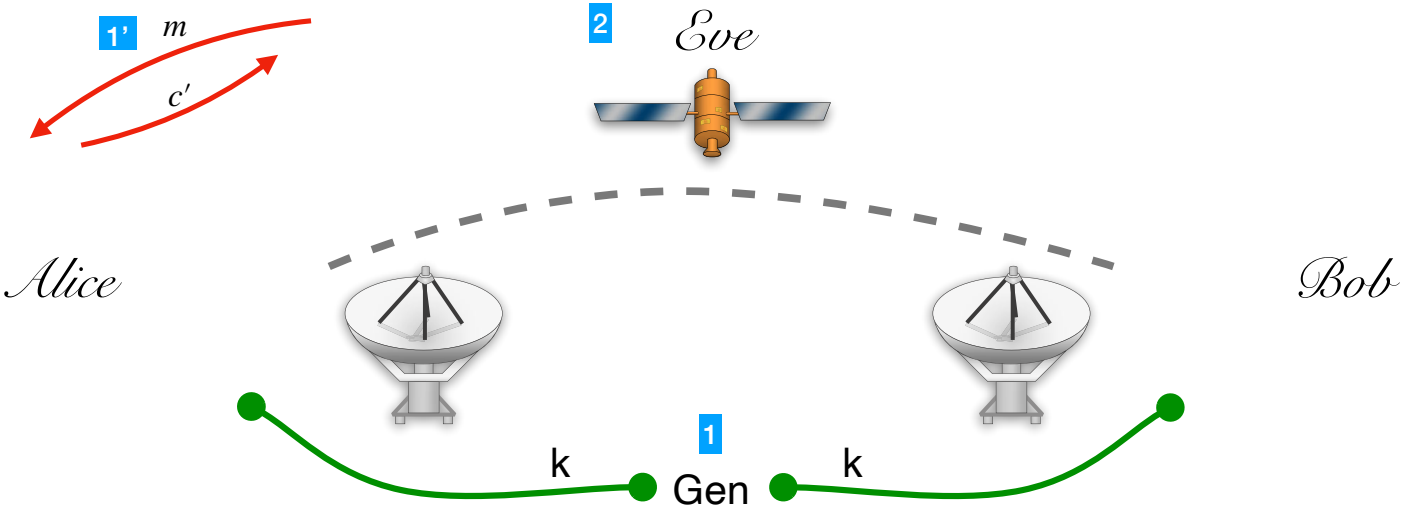
IND-CPA attack for Symmetric Enc



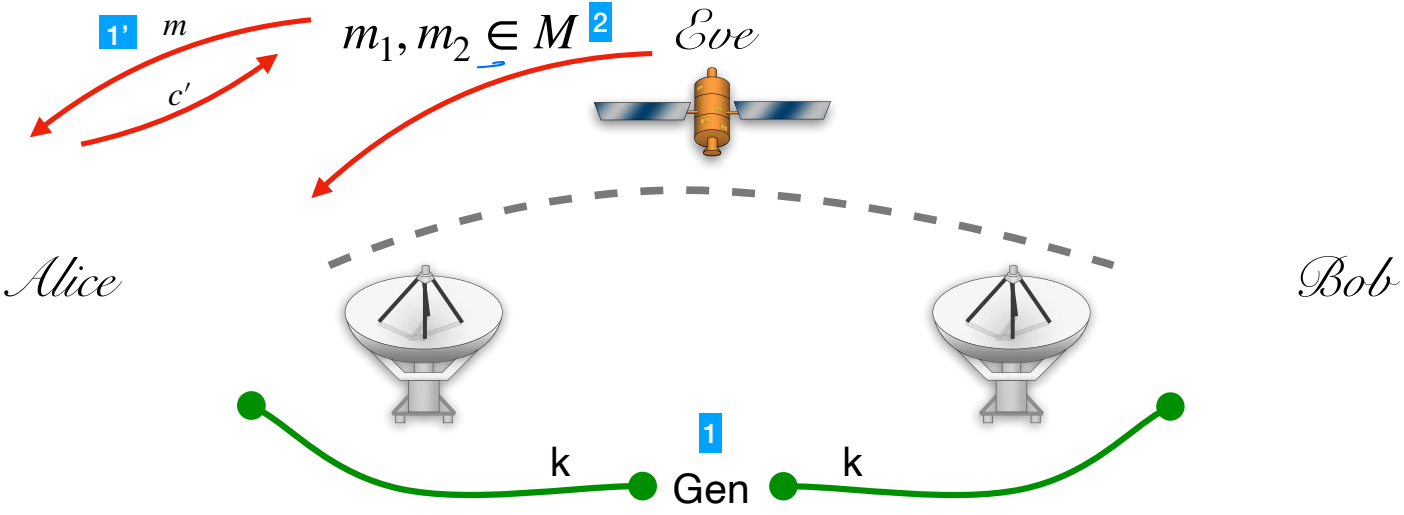
IND-CPA attack for Symmetric Enc



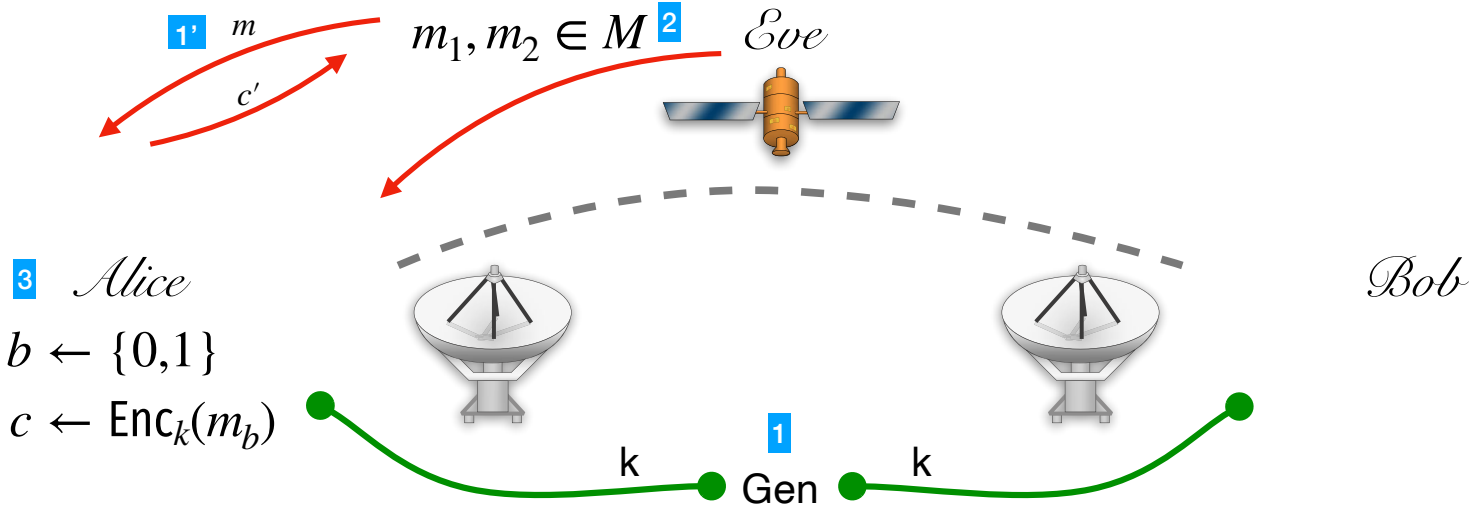
IND-CPA attack for Symmetric Enc



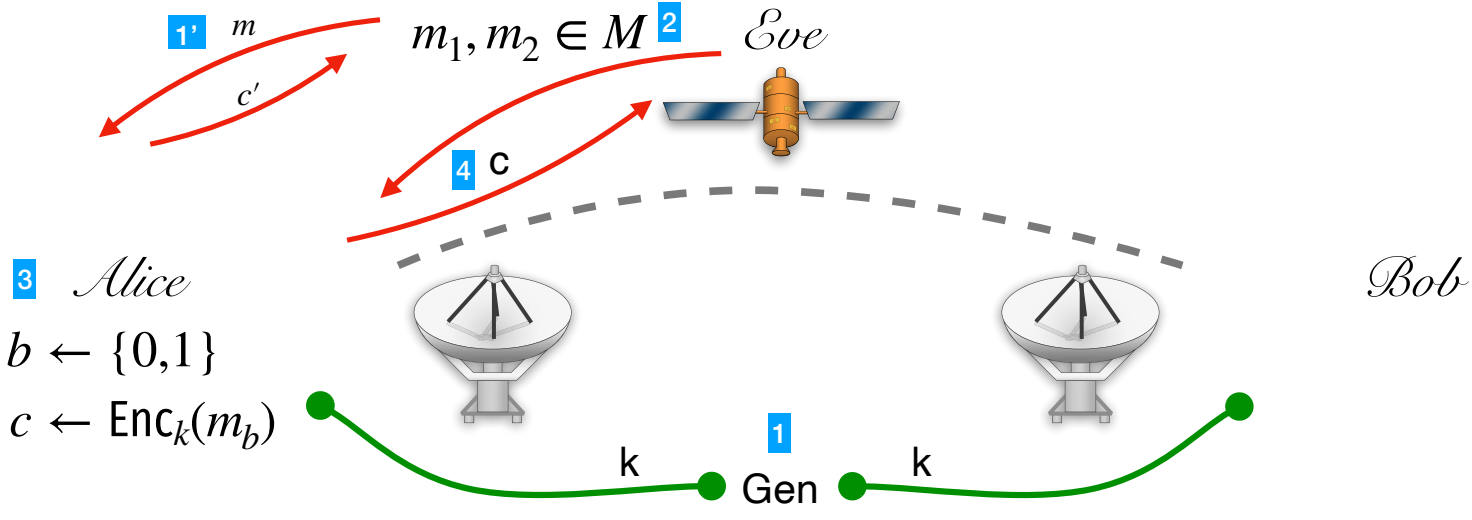
IND-CPA attack for Symmetric Enc



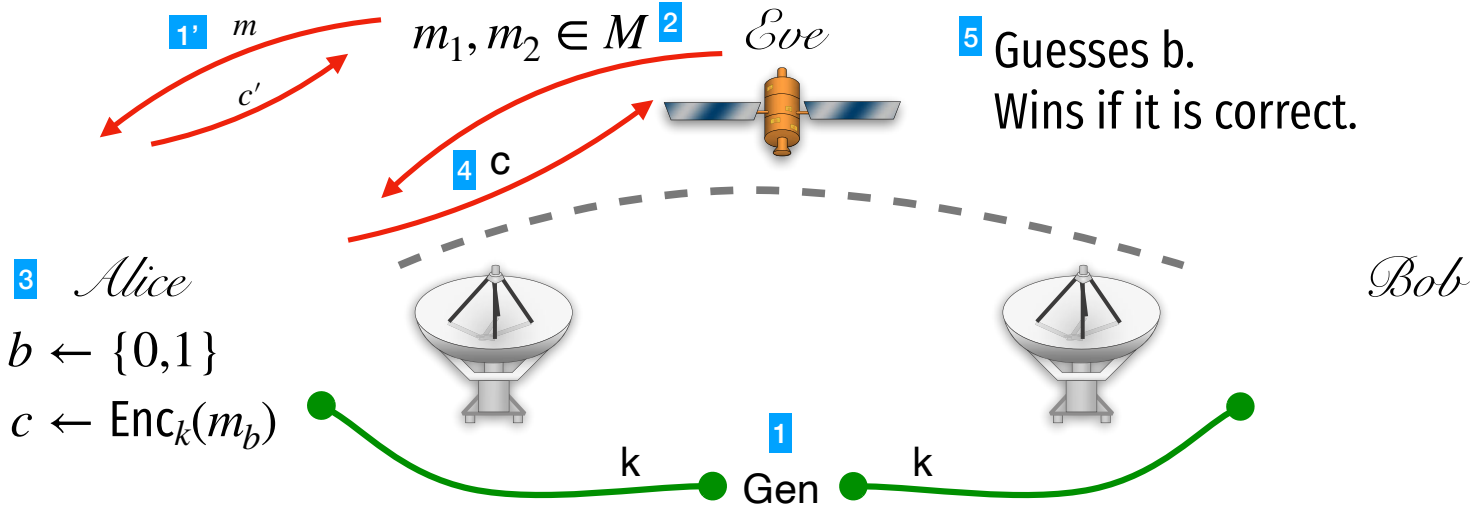
IND-CPA attack for Symmetric Enc



IND-CPA attack for Symmetric Enc

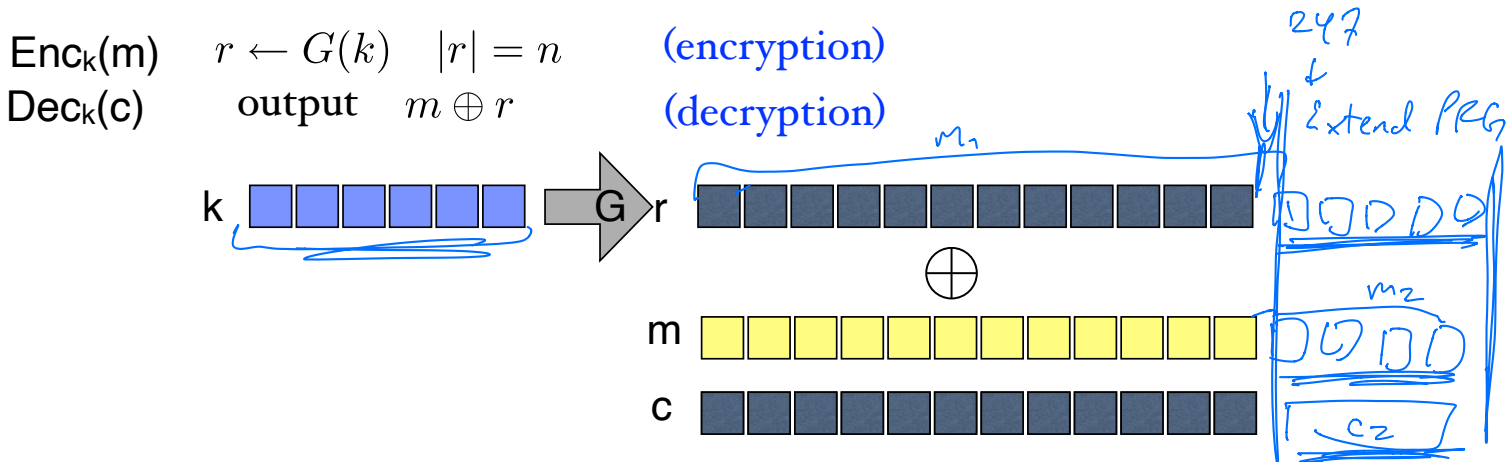


IND-CPA attack for Symmetric Enc



(5 steps of the IND-CPA game.)

Our construction can satisfy this notion if both Alice and Bob maintain a counter of how much random tape they have used.



Theorem: If One-way functions exist, Then IND-CPA secure symmetric encryption exists.

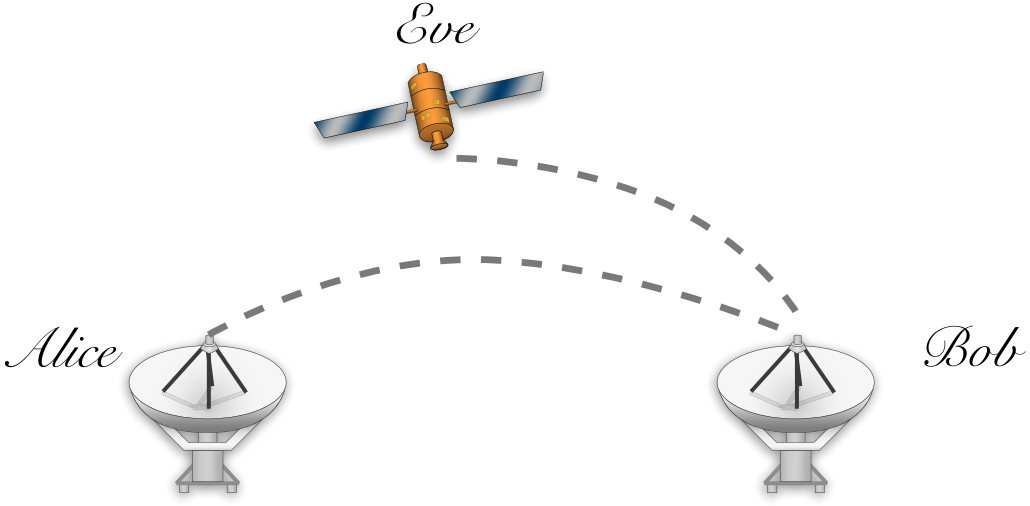
HASTAD, IMPAGLIAZZO, LUBY, LEVIN. (HILL)

① What happens if Alice & Bob get out of sync ??

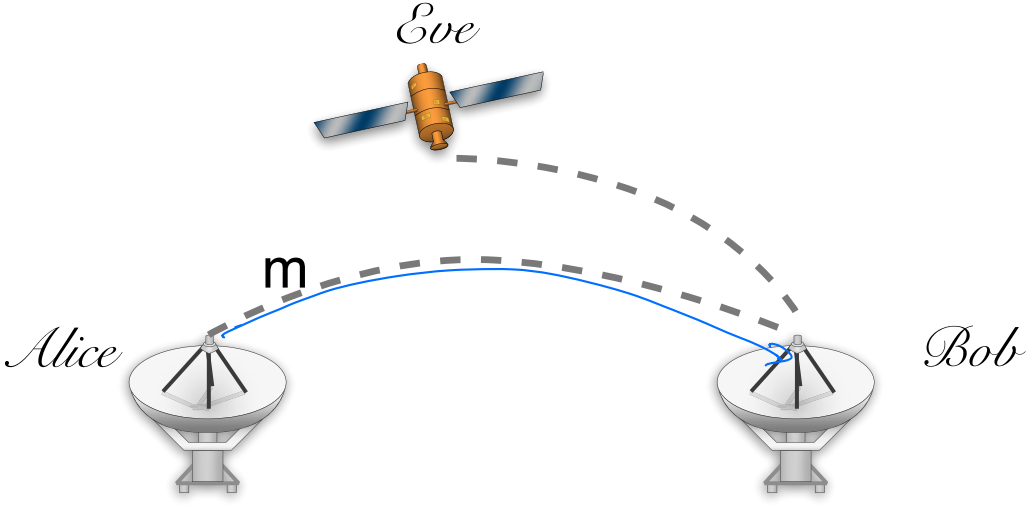
Authentication game

How do we know that
Alice sent us this message??

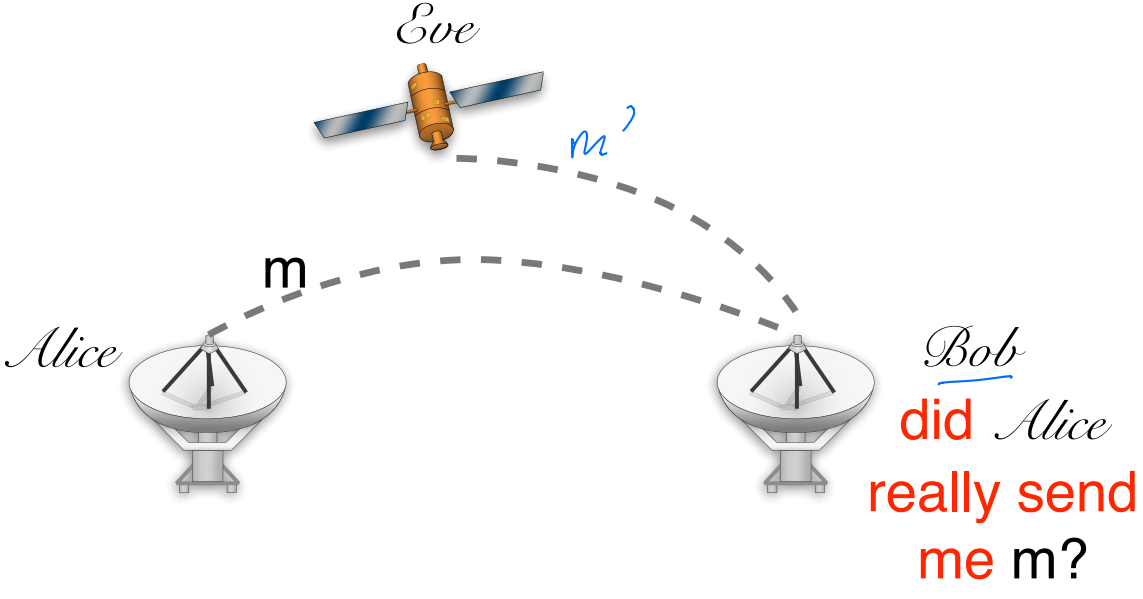
New Problem



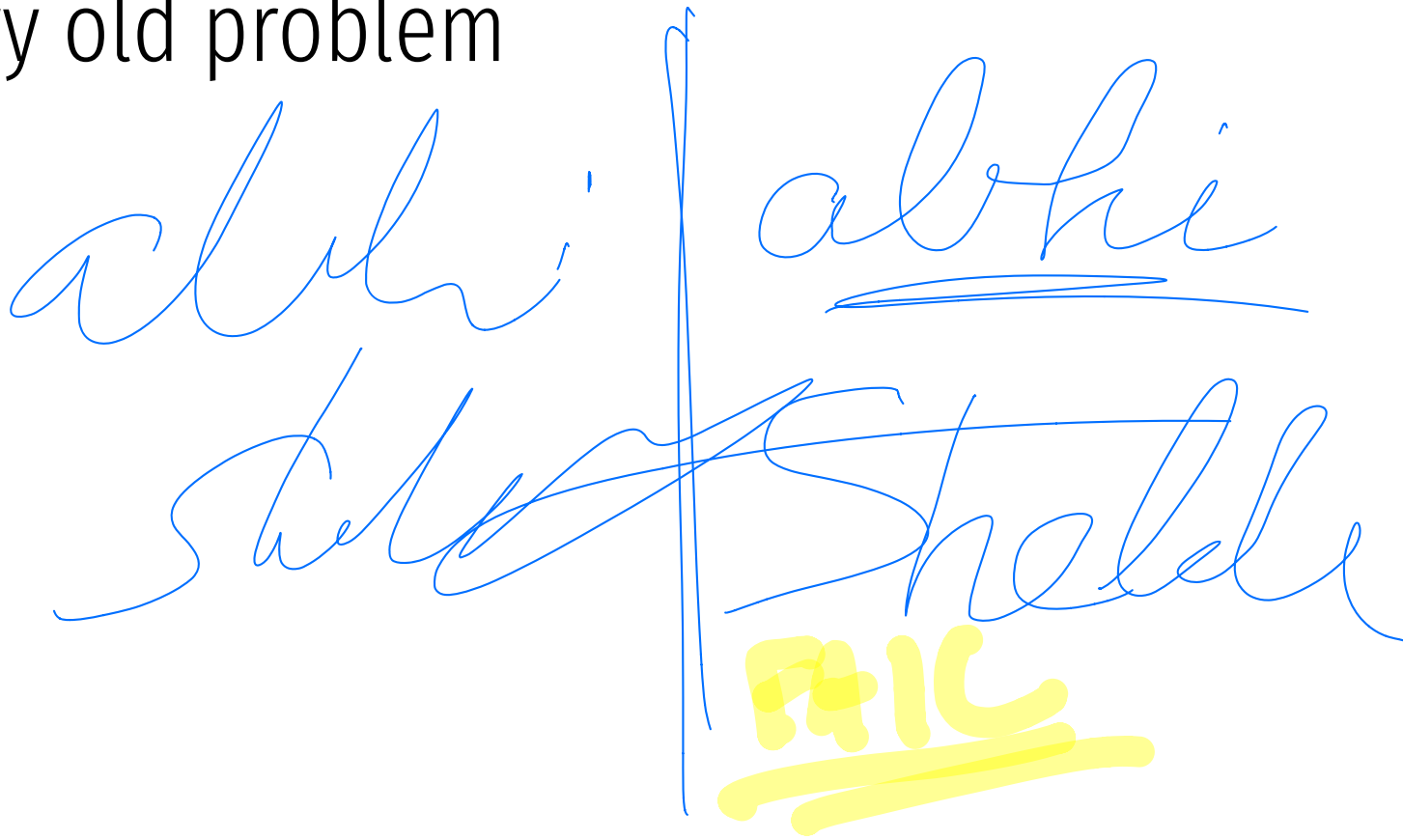
New Problem

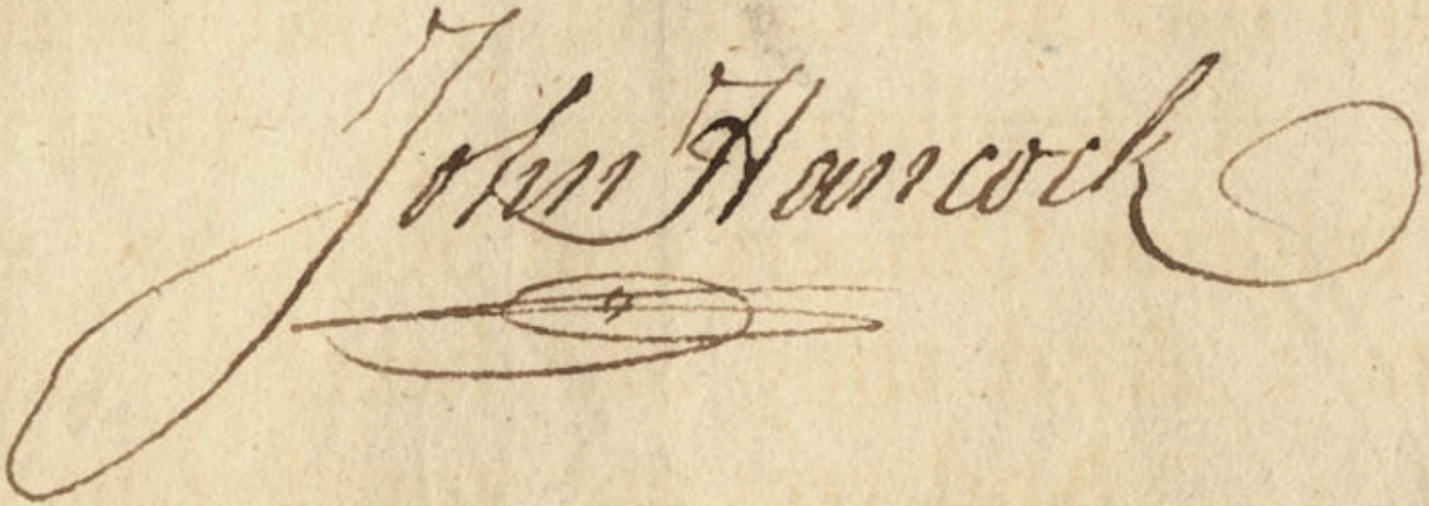


New Problem

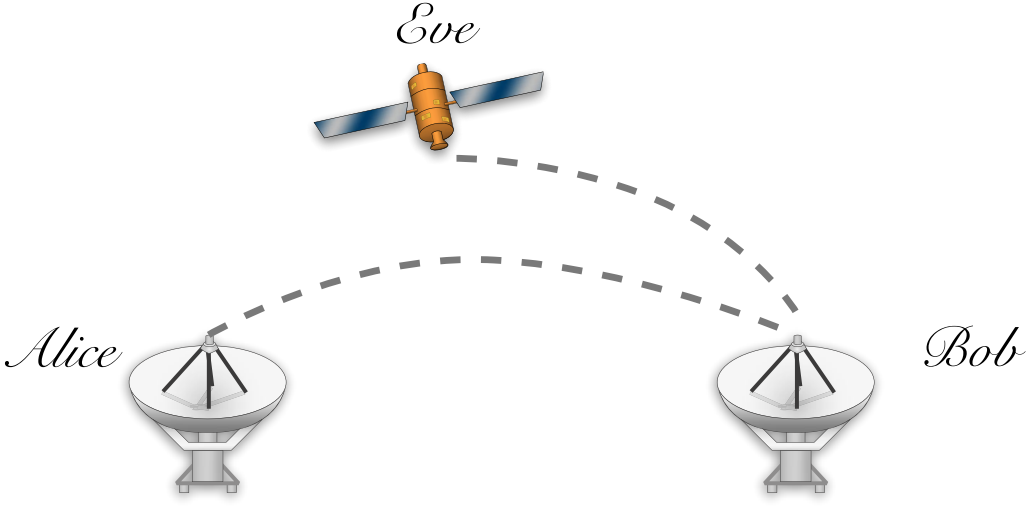


Very old problem

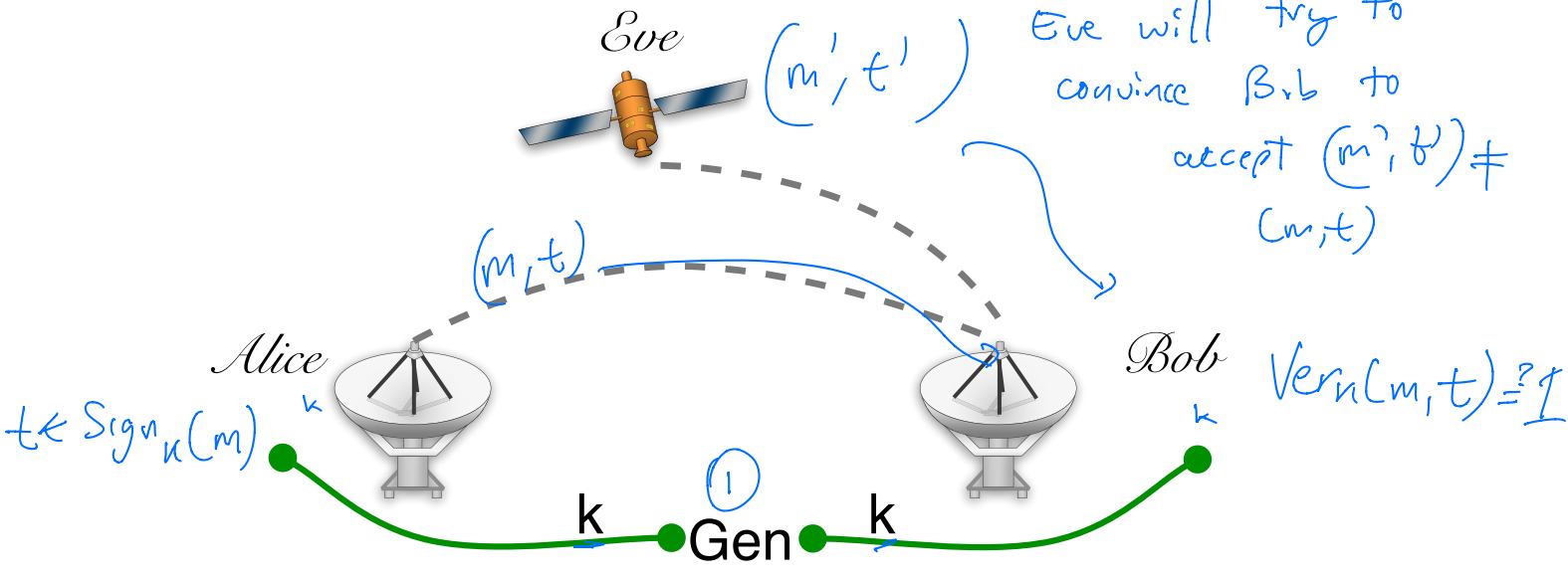




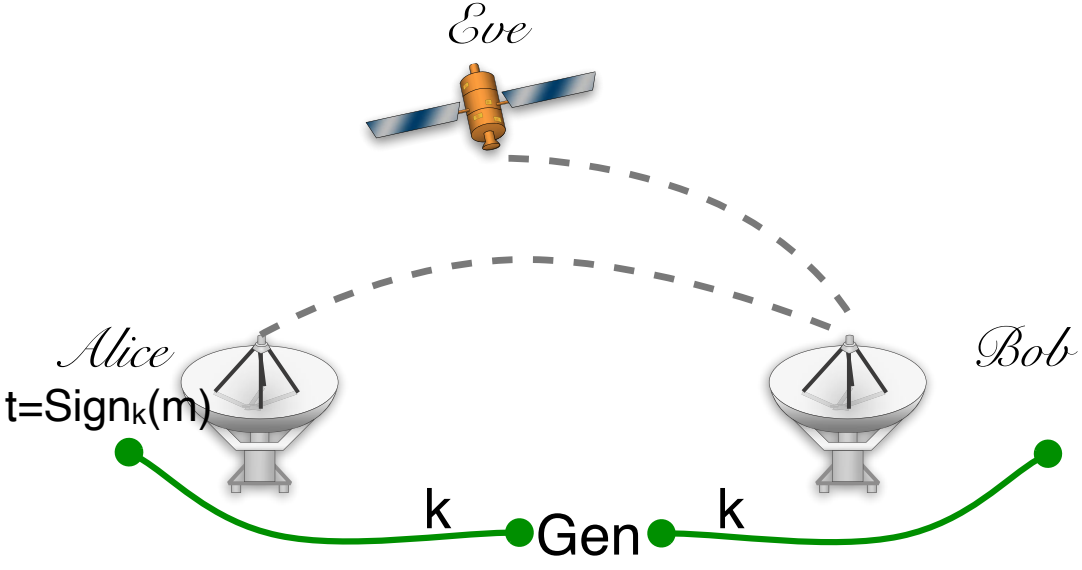
Message Authentication codes (MAC)



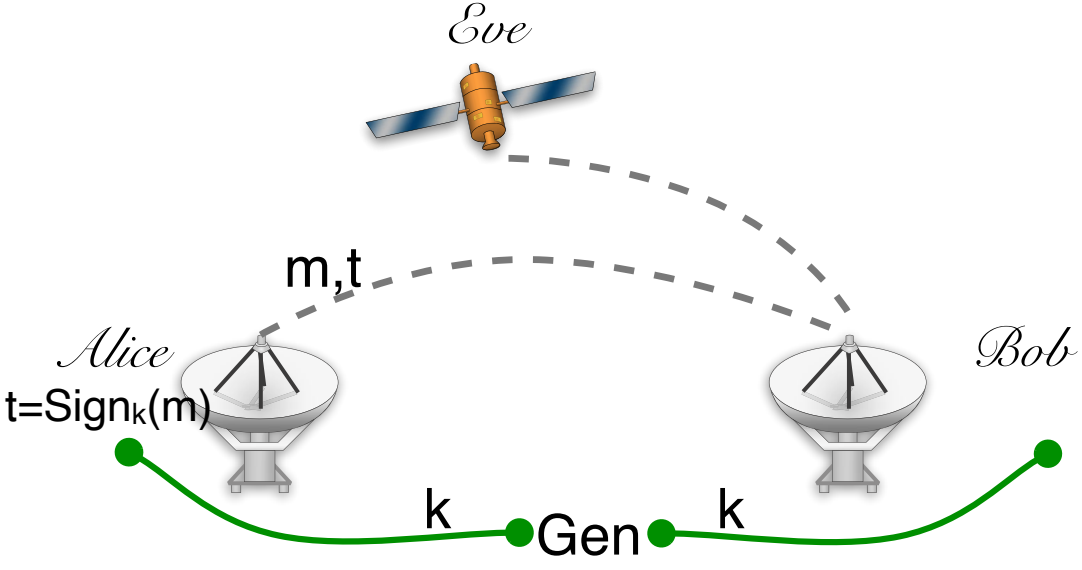
Message Authentication codes



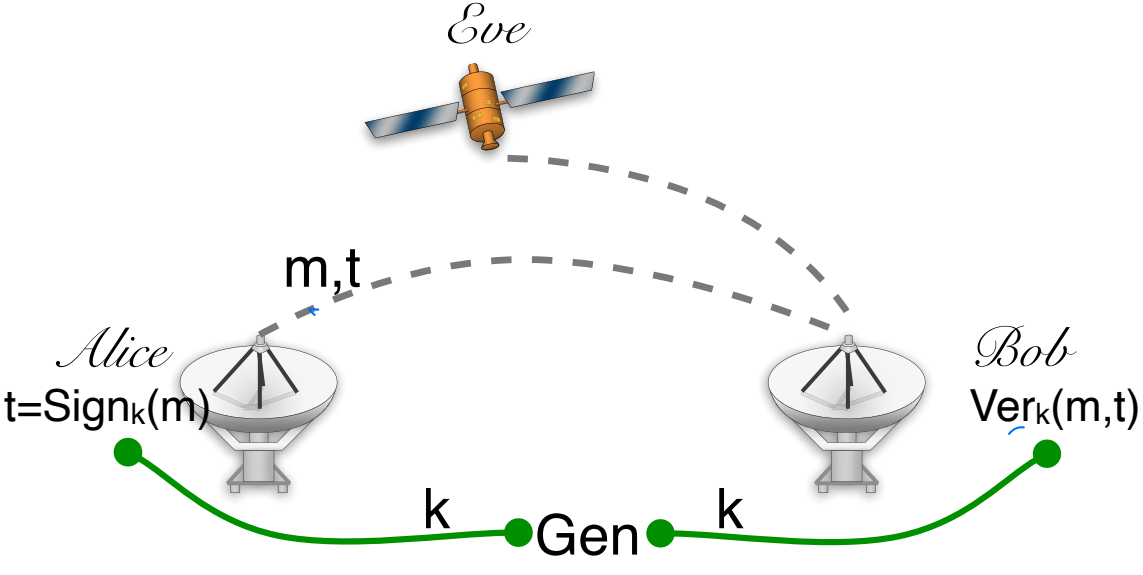
Message Authentication codes



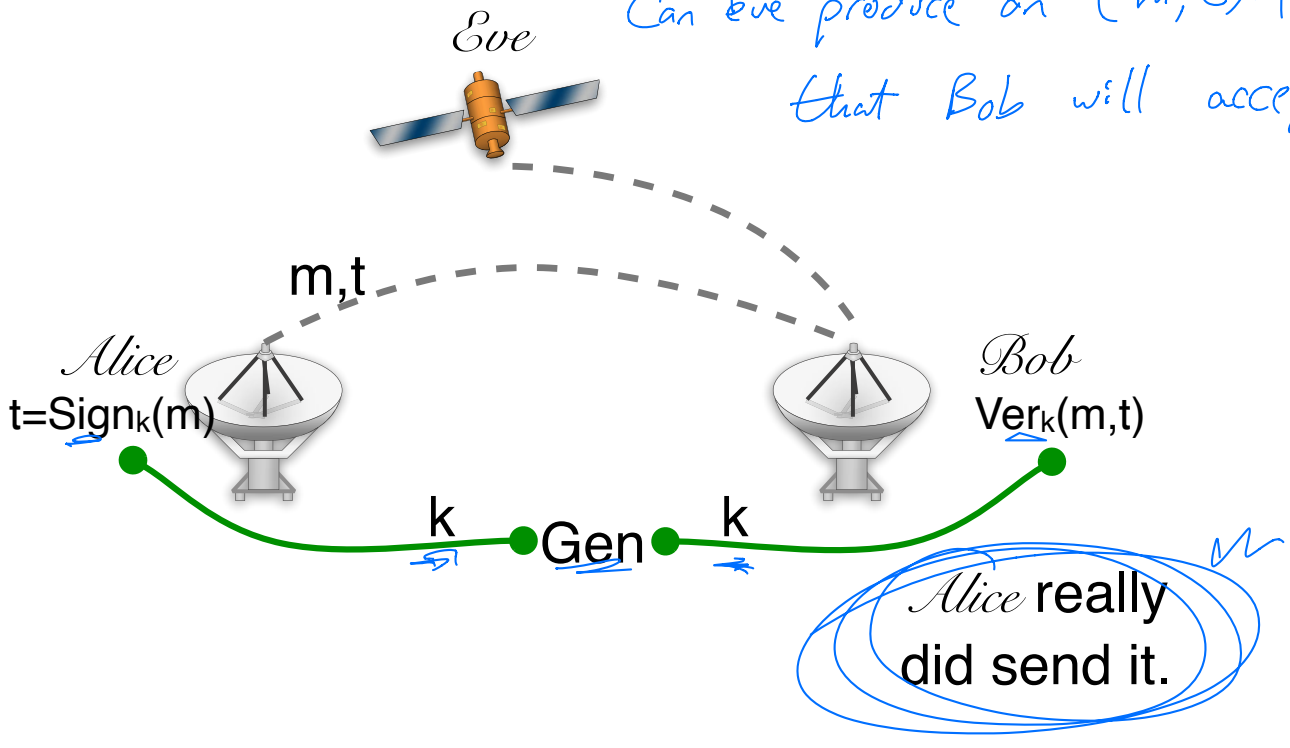
Message Authentication codes



Message Authentication codes



Message Authentication codes



MAC

message space $\{\mathcal{M}\}_n$

$\mathsf{Gen}(1^n)$ makes a key

$\mathsf{Sign}_k(m)$ – produces a "tag" for a message
signature

$\mathsf{Ver}_k(m,t)$ – verifies a tag on a message

mac

message space

$\mathrm{Gen}(1^n)$

$\mathrm{Sign}_k(m)$ $m \in \mathcal{M}_n$

$\mathrm{Ver}_k(m,t)$

mac

message space

$\text{Gen}(1^n)$ generates a key k

$\text{Sign}_k(m)$ $m \in \mathcal{M}_n$

$\text{Ver}_k(m,t)$

mac

message space

$\text{Gen}(1^n)$ generates a key k

$\text{Sign}_k(m)$ generates a tag t for $m \in \mathcal{M}_n$

$\text{Ver}_k(m,t)$

mac

message space

$\text{Gen}(1^n)$ generates a key k

$\text{Sign}_k(m)$ generates a tag t for $m \in \mathcal{M}_n$

$\text{Ver}_k(m,t)$ accepts or rejects a msg,tag

mac

message space

Gen(1^n) generates a key k

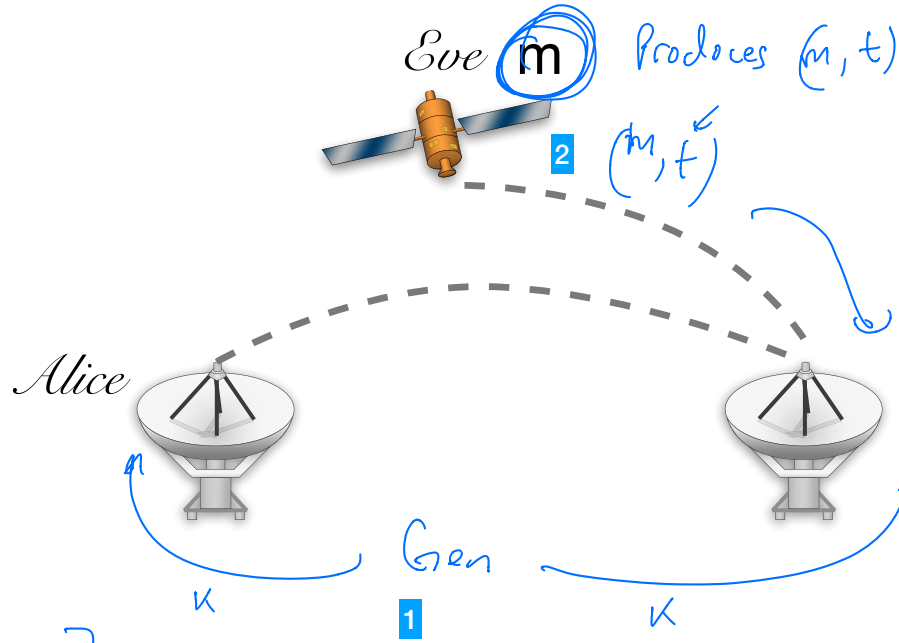
$\text{Sign}_k(m)$ generates a tag t for $m \in \mathcal{M}_n$

$\text{Ver}_k(m,t)$ accepts or rejects a msg,tag

$$\Pr[k \leftarrow Gen(1^n) : Ver_k(m, Tag_k(m)) = 1] = 1$$

Secure MAC

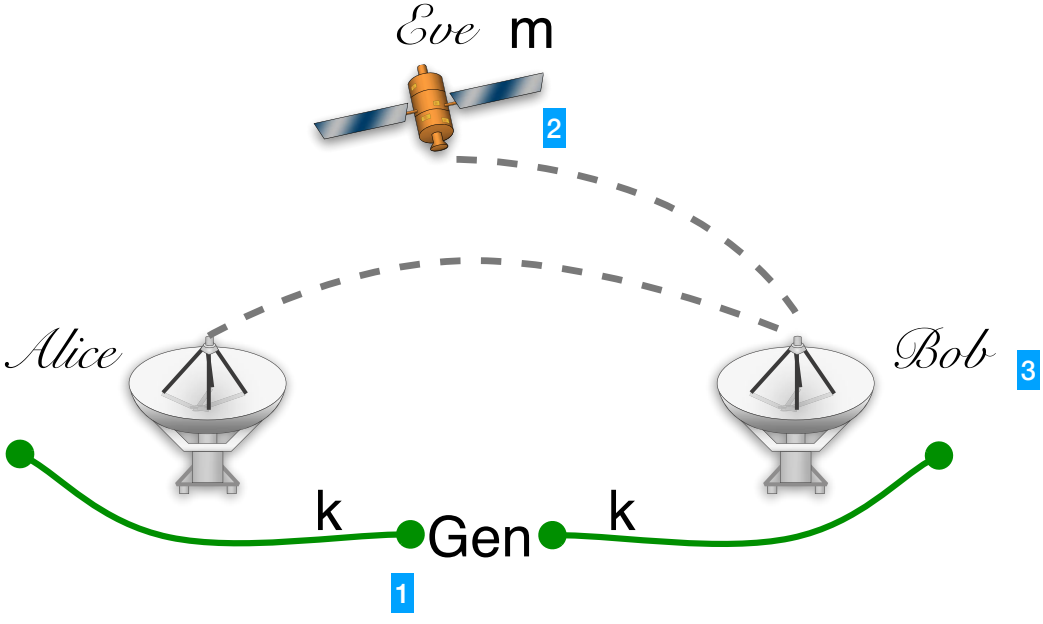
"an adversary cannot forge a tag for a given message"



$Pr[\text{Eve wins}] \leq 2^{-\gamma}$ ← security parameter

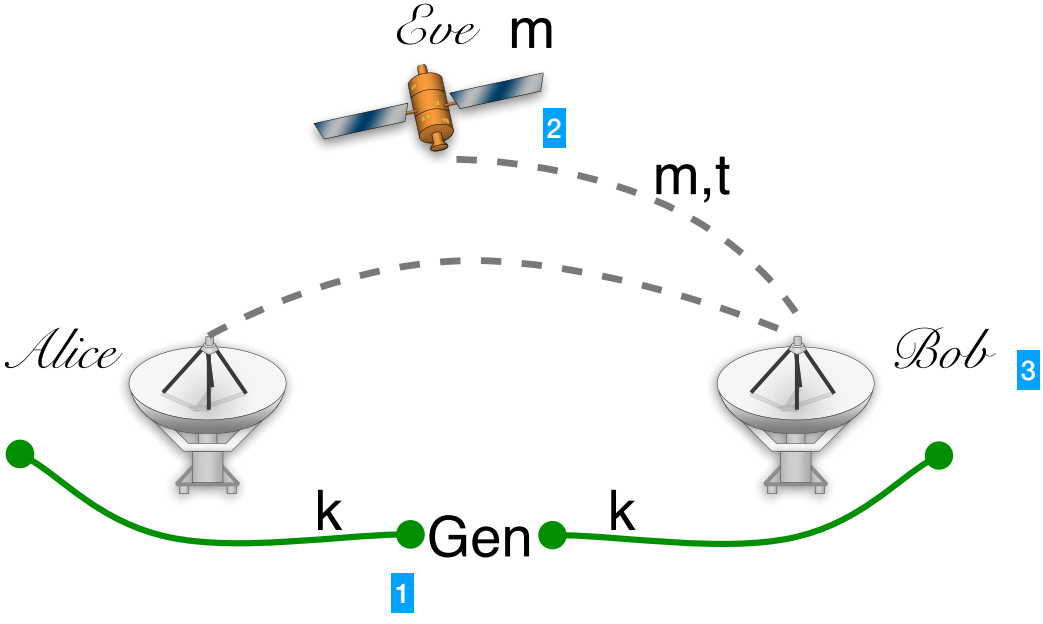
Secure MAC

"an adversary cannot forge a tag for a given message"



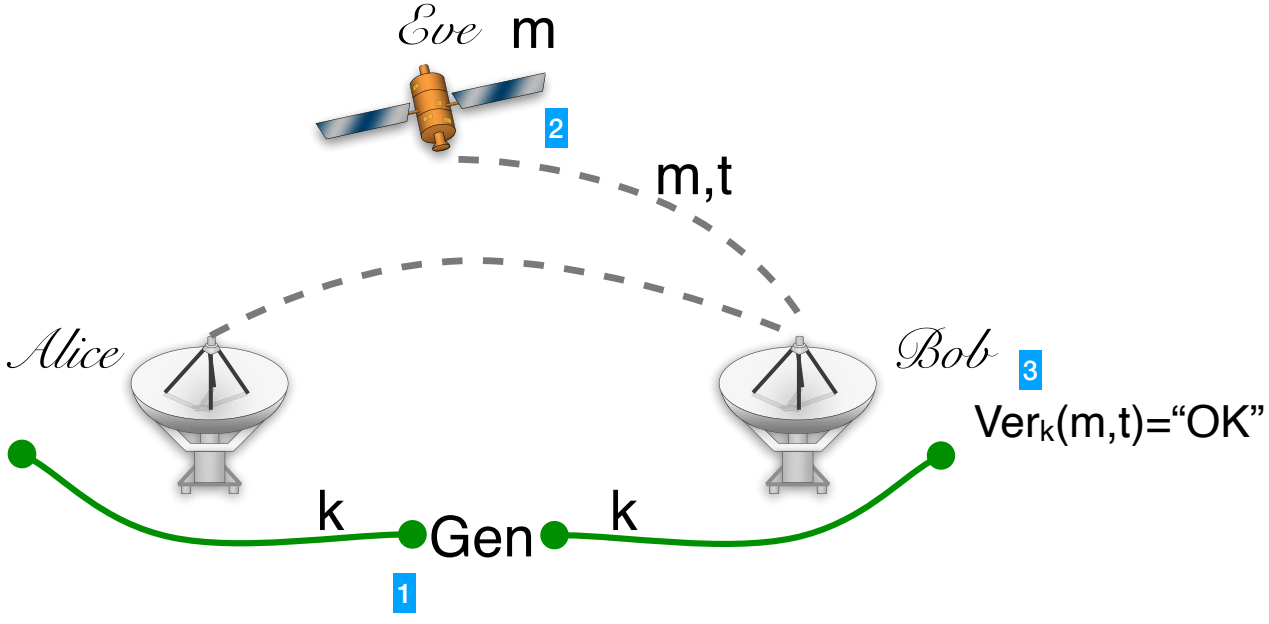
Secure MAC

"an adversary cannot forge a tag for a given message"



Secure MAC

"an adversary cannot forge a tag for a given message"

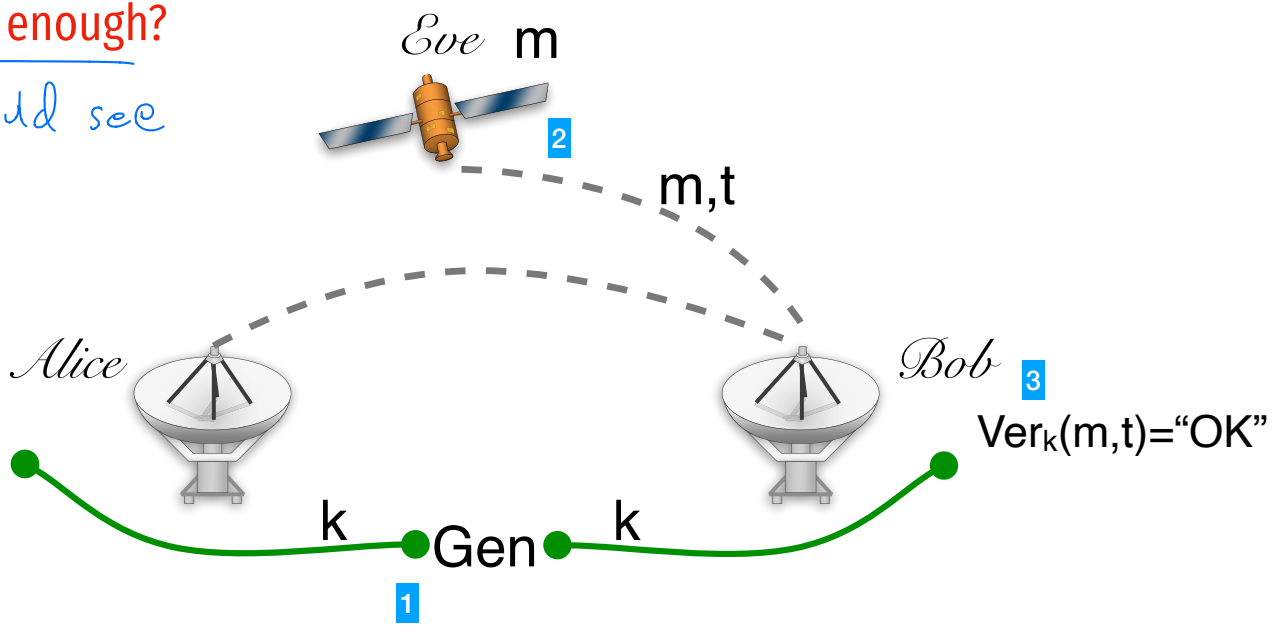


Secure MAC

"an adversary cannot forge a tag for a given message"

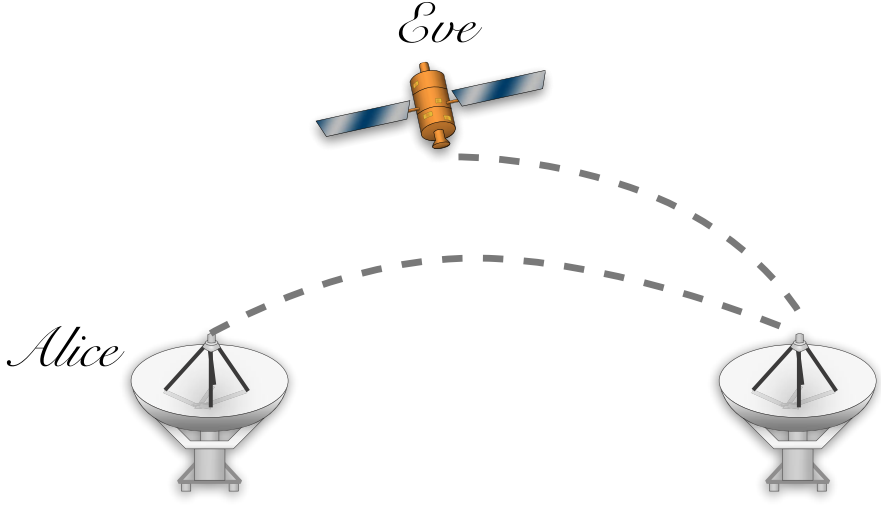
Is this strong enough?

(1) Eve should see examples



Secure MAC

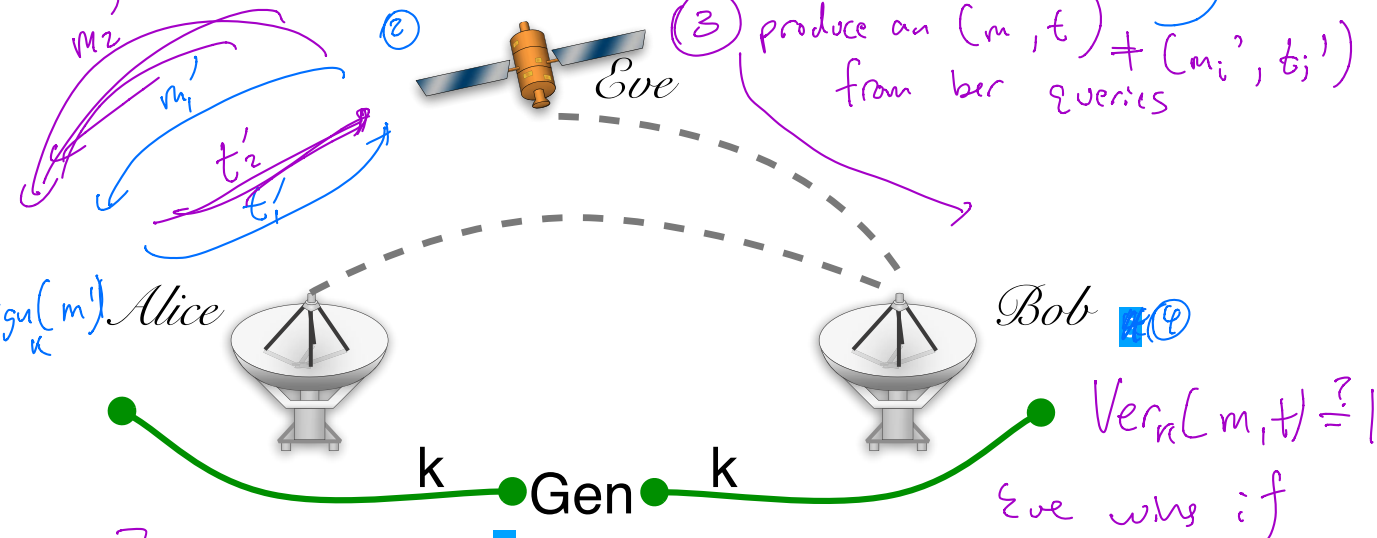
"an adversary cannot forge a tag for any message of its choosing "



Secure MAC

"even when given a tag oracle, an adversary cannot forge a tag for any message of its choosing"

SIGN

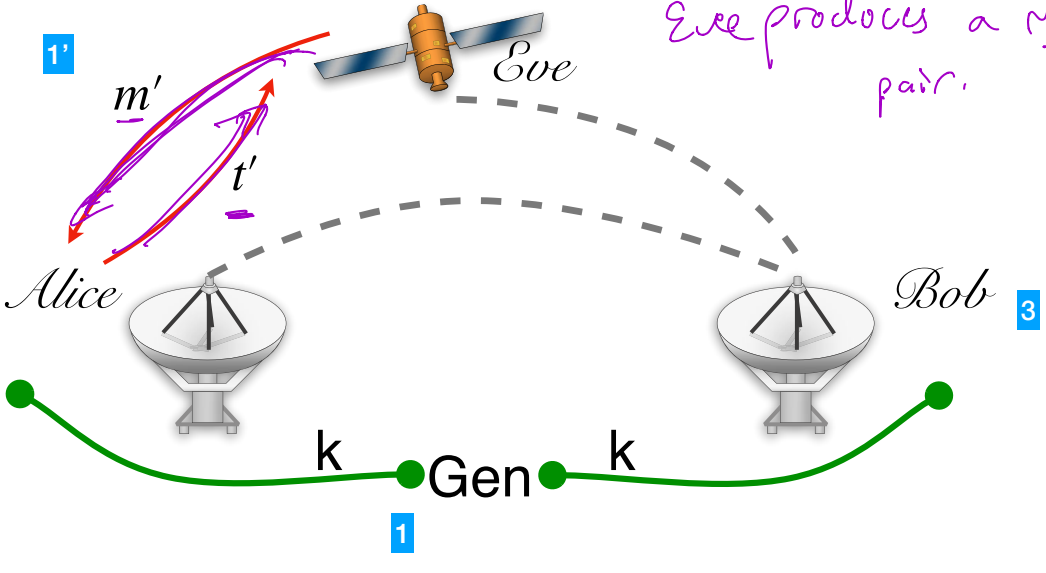


Secure MAC

"even when given a tag oracle, an adversary cannot forge a tag for any message of its choosing "

Eve queries Alice to sign several messages.

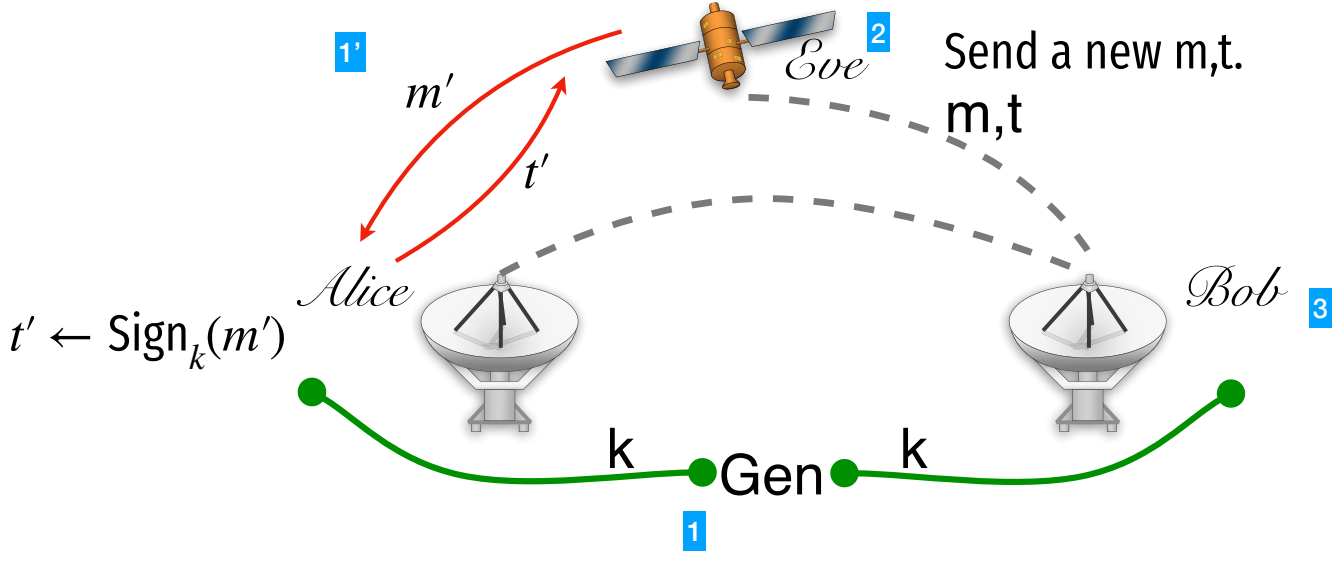
Eve produces a new (m, t) pair.



$t' \leftarrow \mathrm{Sign}_k(m')$

Secure MAC

"even when given a tag oracle,
an adversary cannot forge a tag for
any message of its choosing "



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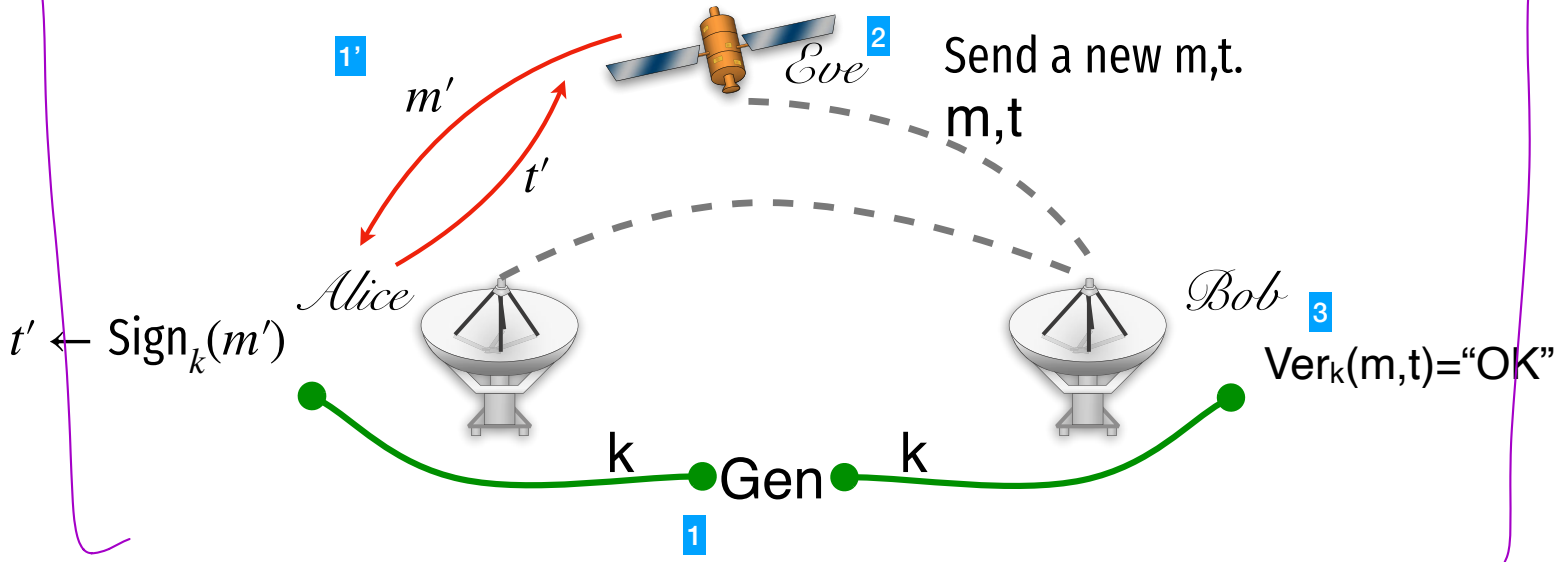

Security game cartoon..

security def for mac

for all non-uniform ppt A

security def for mac

for all non-uniform ppt A

$$\Pr\Big[\qquad\qquad\Big] < \mu(n)$$

For all efficient adversaries A

$$k \leftarrow Gen(1^n); (m,t) \leftarrow A^{Tag_k(\cdot)} :$$

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and A didn't receive (m,t)

For all efficient adversaries A

signing oracle

$$\Pr\left[\begin{array}{l} k \leftarrow Gen(1^n); (m,t) \leftarrow A^{Tag_k(\cdot)} : \\ Ver_k(m,t) = 1 \\ \text{and A didn't receive (m,t)} \end{array}\right] < \mu(n)$$

from the oracle

negligible

Construction of a mac

$Gen(1^n)$:

$Sign_k(m)$:

$Ver_k(m,t)$:

Random functions

$$R : \{0,1\}^n \rightarrow \{0,1\}^n$$

(length preserving)

In	Out
0	010...00
1	110...110
2	001..100
3	110...001
...	
2^n-1	100...111

flip random coins.

How many random functions are there?

$$R : \{0,1\}^n \rightarrow \{0,1\}^n$$

In	Out
0	010...00
1	110...110
2	001..100
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total space of table:

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...	
2^n-1	100...111

total space of table: $2^n n$

total # of random f: $2^{2^n n}$

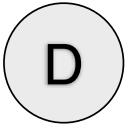
Defining pseudo-random functions

Families of functions

$\{RF_n\}_n$ uniform distribution over all n-bit random functions

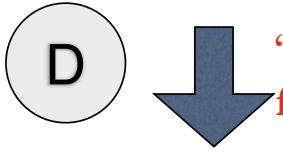
$\{F_n\}_n$ uniform distribution over all n-bit random functions

Defining a prf



distinguisher is given one
of these two functions
and must guess which

Defining a prf



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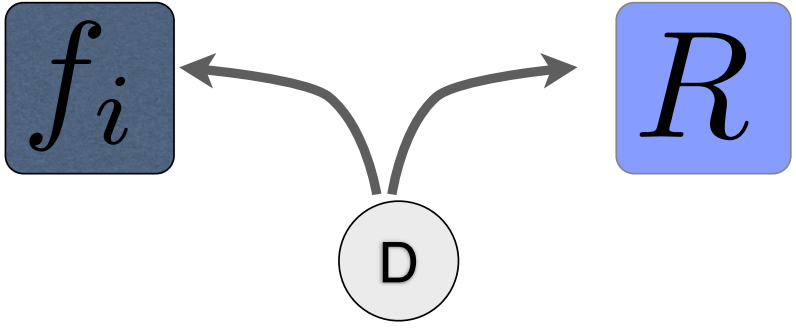
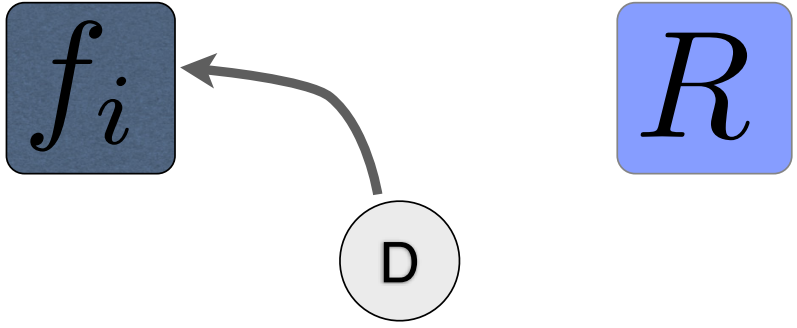
Defining a prf



distinguisher is given oracle
access to one of these two functions
and must guess which

Defining a prf



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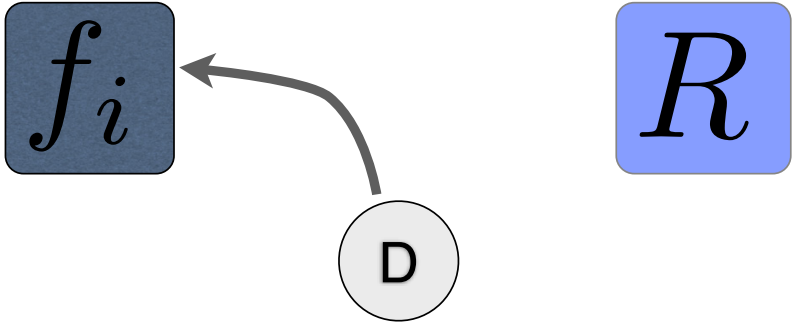
Defining a prf



distinguisher is given oracle
access to one of these two functions
and must guess which

$$\Pr[f_k \leftarrow F_n;\ D^{f_k}(1^n) = 1] - \Pr[R \leftarrow RF_n;\ D^{R}(1^n) = 1] < \epsilon(n)$$

A function family $F_n = \{f_k\}$

is a pseudo-random function family if

A function family $F_n = \{f_k\}$

is a pseudo-random function family if

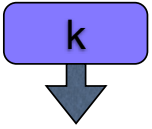
F_n can be sampled in p.p.t.

f_k:{0,1}^{n} to {0,1}^{n} can be computed in p.p.t.

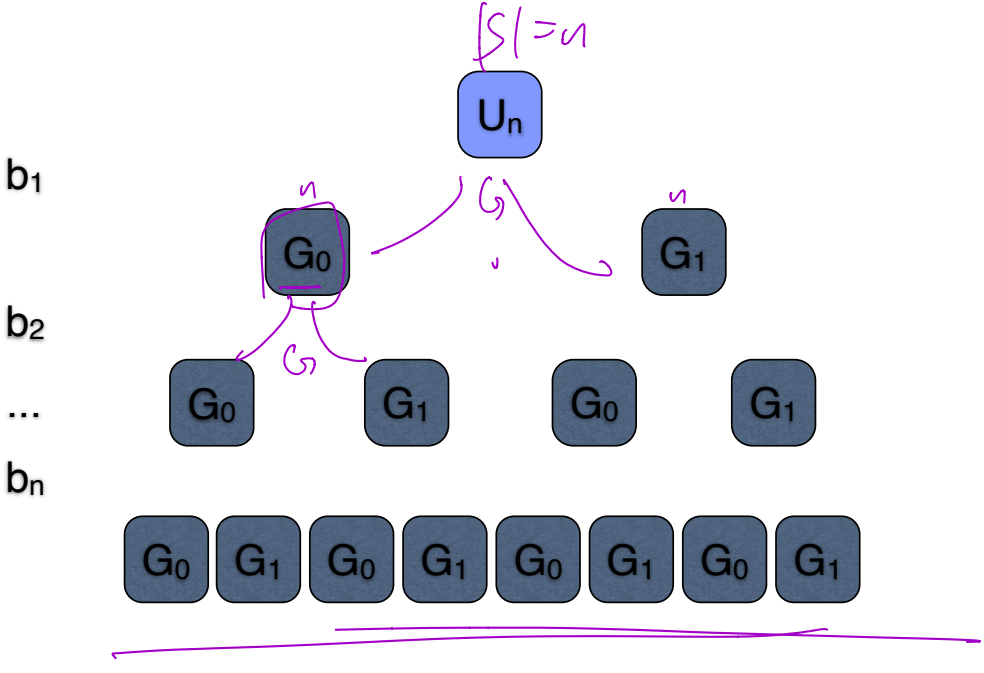
$|f_k(x)| = |x|$

$$\Pr[f_k \leftarrow F_n;\ D^{f_k}(1^n) = 1] - \Pr[R \leftarrow RF_n;\ D^R(1^n) = 1] < \epsilon(n)$$

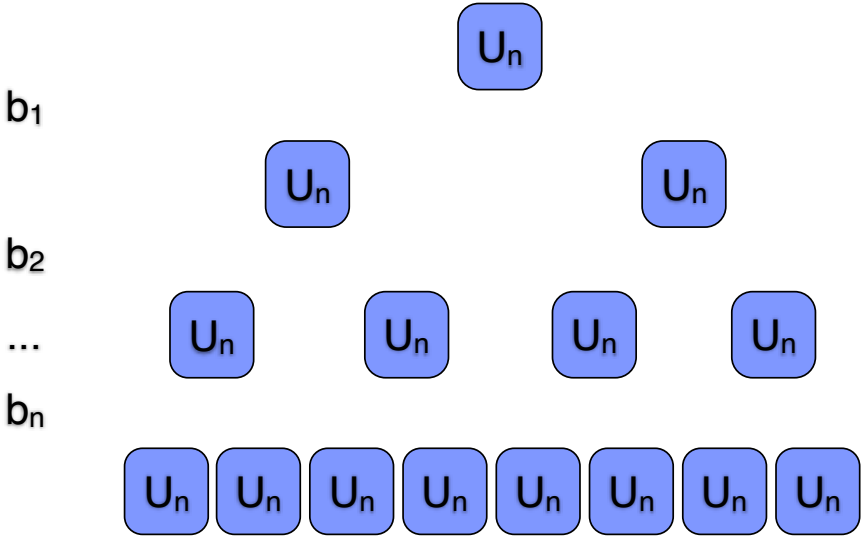
How to construct a PRF from a PRG



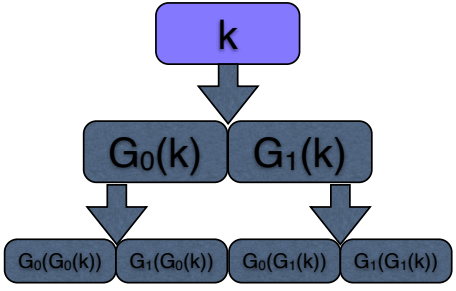
PRF



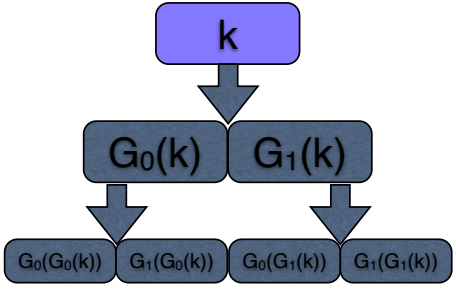
Random function

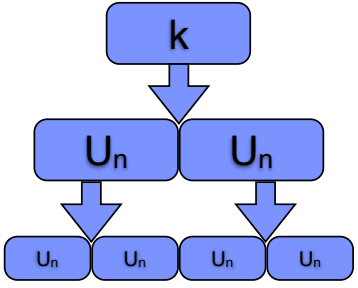


Why is this secure?



Why is this secure?





Construction of a MAC

$\text{Gen}(1^n)$:

$\text{Sign}_k(m)$:

$\text{Ver}_k(m,t)$:

Construction of a MAC

let $\{F_k\}$ be a prf family

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Gen(1^n): $k \leftarrow U_n$

Sign$_k$(m):

Ver$_k$(m,t):

Construction of a MAC

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Gen(1^n): $k \leftarrow U_n$

Sign$_k$(m): $t \leftarrow F_k(m)$

Ver$_k$(m,t):

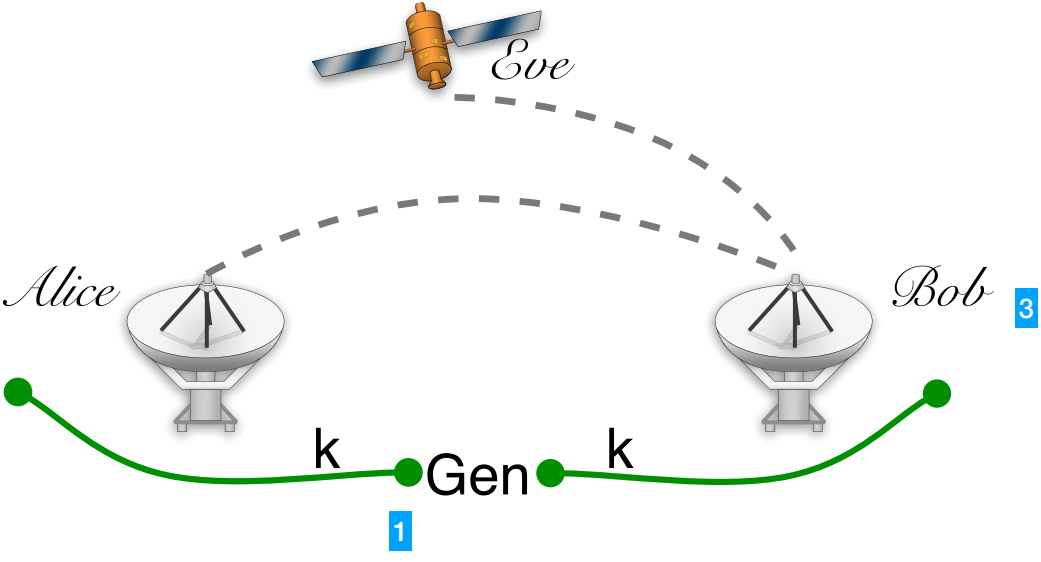
Construction of a MAC

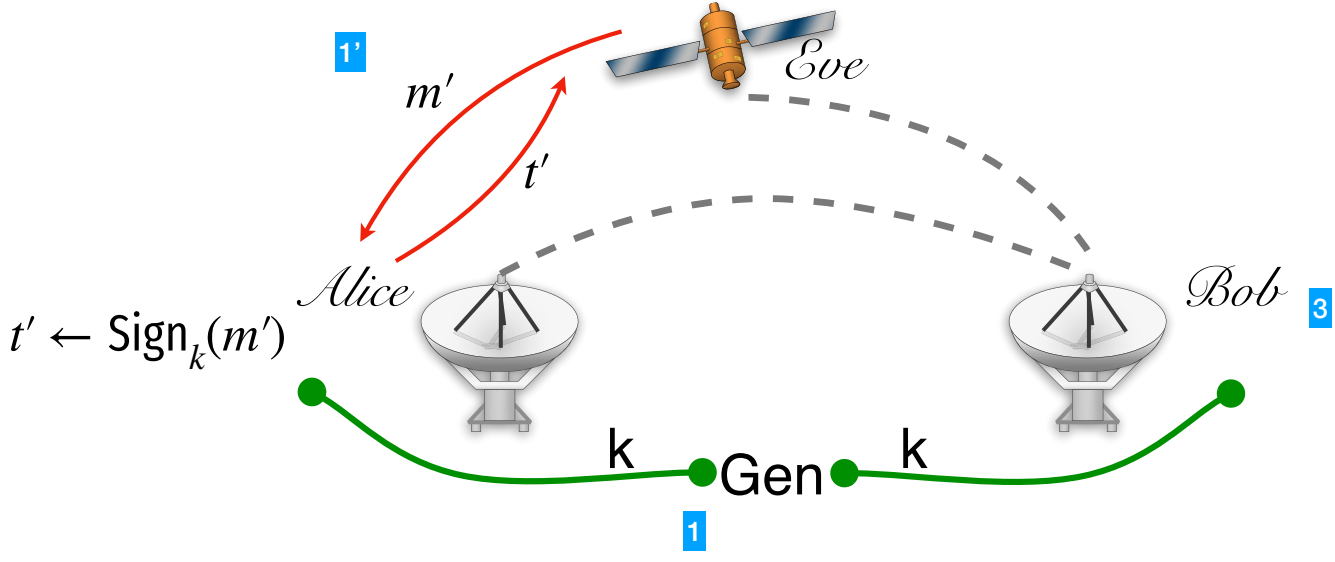
let $\{F_k\}$ be a prf family

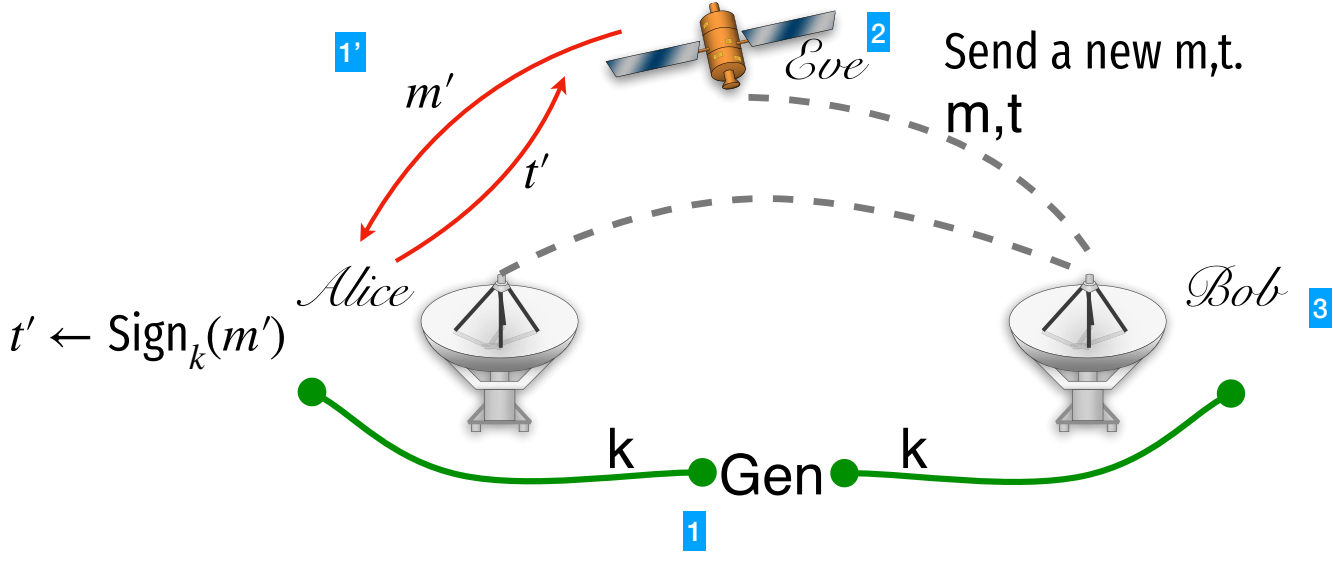
Gen(1^n): $k \leftarrow U_n$

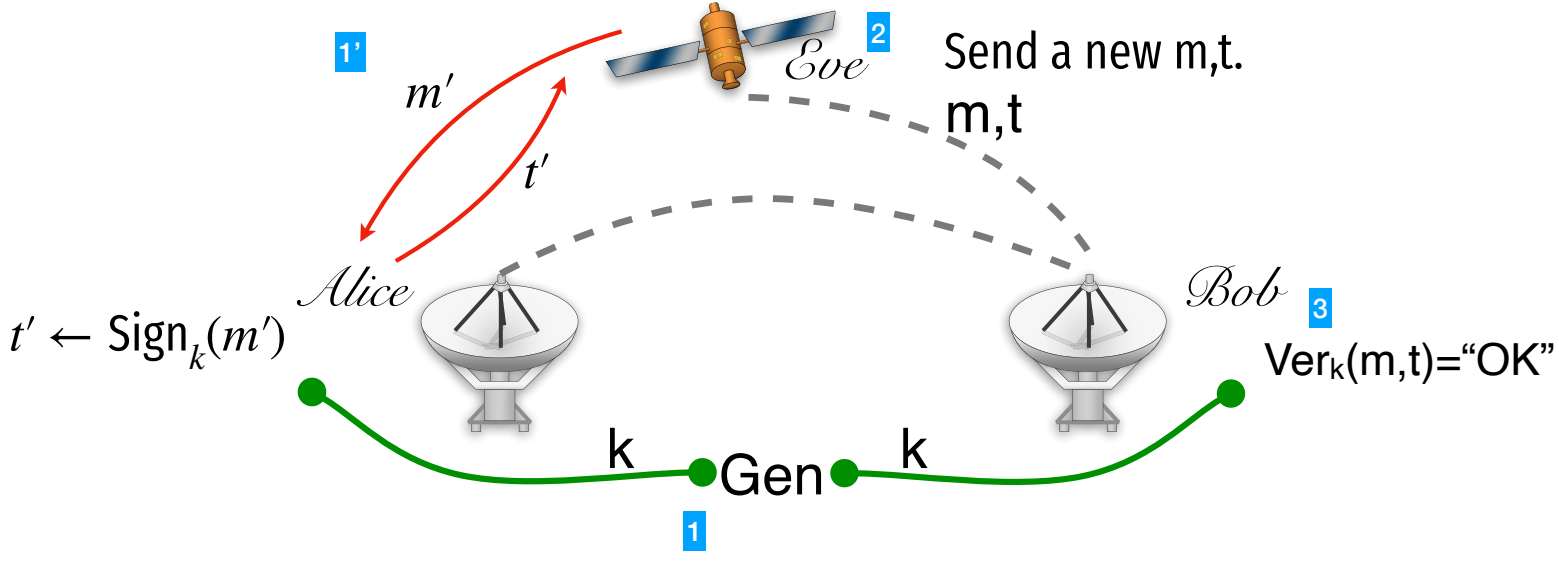
Sign$_k$(m): $t \leftarrow F_k(m)$

Ver$_k$(m,t): accept if $t \stackrel{?}{=} F_k(m)$


Eve
Alice
Bob
3
k
Gen
k
1


1'
m'
t'
Eve
Alice
Bob
3
t' ← Sign_k(m')
k
Gen
k
1


1'
m′
t′
Eve
2
Send a new m,t.
m,t
Alice
Bob
3
t′ ← Sign_k(m′)
k
Gen
k
1


1'
m′
t′
Eve
2
Send a new m,t.
m,t
Alice
Bob
3
t′ ← Sign_k(m′)
Ver_k(m,t)="OK"
k
Gen
k
1

security proof

let $\{F_k\}$ be a prf family

Gen(1^n): $k \leftarrow U_n$ Tag$_k$(m): $t \leftarrow F_k(m)$

Security proof idea

let $\{F_k\}$ be a prf family

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$$\Pr\left[\begin{array}{l} k \leftarrow Gen(1^n); (m,t) \leftarrow A^{Tag_k(\cdot)} : \\ \quad Ver_k(m,t) = 1 \\ \quad \text{and A didn't query m} \end{array}\right]$$

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$$< \mu(n) + 2^{-n}$$

Efficiency: AES as a PRF

AES(k,m):

Expand k (16 bytes) into 11 round keys bytes

Add round key 1 into m

For i=1...9:

SubBytes: apply a map to all bytes
ShiftRows: permute the bytes
MixColumns: permute columns
AddRoundKey i+1

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ShiftRows: permute the bytes
AddRoundKey i+1

Main security comes from s-box

AES S-Box

	00	01	02	03	04	05	06	07	08	09	0a	0b	0c	0d	0e	0f
00	63	7c	77	7b	f2	6b	6f	c5	30	01	67	2b	fe	d7	ab	76
10	ca	82	c9	7d	fa	59	47	f0	ad	d4	a2	af	9c	a4	72	c0
20	b7	fd	93	26	36	3f	f7	cc	34	a5	e5	f1	71	d8	31	15
30	04	c7	23	c3	18	96	05	9a	07	12	80	e2	eb	27	b2	75
40	09	83	2c	1a	1b	6e	5a	a0	52	3b	d6	b3	29	e3	2f	84
50	53	d1	00	ed	20	fc	b1	5b	6a	cb	be	39	4a	4c	58	cf
60	d0	ef	aa	fb	43	4d	33	85	45	f9	02	7f	50	3c	9f	a8
70	51	a3	40	8f	92	9d	38	f5	bc	b6	da	21	10	ff	f3	d2
80	cd	0c	13	ec	5f	97	44	17	c4	a7	7e	3d	64	5d	19	73
90	60	81	4f	dc	22	2a	90	88	46	ee	b8	14	de	5e	0b	db
a0	e0	32	3a	0a	49	06	24	5c	c2	d3	ac	62	91	95	e4	79
b0	e7	c8	37	6d	8d	d5	4e	a9	6c	56	f4	ea	65	7a	ae	08
c0	ba	78	25	2e	1c	a6	b4	c6	e8	dd	74	1f	4b	bd	8b	8a
d0	70	3e	b5	66	48	03	f6	0e	61	35	57	b9	86	c1	1d	9e
e0	e1	f8	98	11	69	d9	8e	94	9b	1e	87	e9	ce	55	28	df
f0	8c	a1	89	0d	bf	e6	42	68	41	99	2d	0f	b0	54	bb	16

The column is determined by the least significant nibble, and the row by the most significant nibble. For example, the value `0x9a` is converted into `0xb8`.

AES is very fast.

https://calomel.org/aesni_ssl_performance.html

Cipher Performance per CPU core

AES Performance per CPU core for TLS v1.2 Ciphers
(Higher is Better, Speeds in Megabytes per Second)

	ChaCha20	AES-128-GCM	AES-256-GCM	AES-128-CBC	AES-256-CBC	Total Score
AMD Ryzen 7 1800X	573	3006	2642	1513	1101	= 8835
Intel W-2125	565	2808	2426	1698	1235	= 8732
Intel i7-6700	585	2607	2251	1561	1131	= 8135
AMD EPYC 7551	355	2213	1962	1114	811	= 6455
Intel i5-6500	410	1729	1520	1078	783	= 5520
Intel i7-4750HQ	369	1556	1353	688	499	= 4465
AMD FX 8350	367	1453	1278	716	514	= 4328
AMD FX 8150	347	1441	1273	716	515	= 4292
Intel E5-2650 v4	404	1479	1286	652	468	= 4289
Intel i7-2700K	382	1353	1212	763	552	= 4262
Intel i7-3840QM	373	1279	1143	725	520	= 4040
Intel i5-2500K	358	1274	1140	728	522	= 4022
AMD FX 6100	326	1344	1186	671	481	= 4008
AMD A10-7850K	321	1303	1176	685	499	= 3984
AMD A8-7600 Kaveri	306	1246	1108	648	470	= 3778
Intel E5-2640 v3	303	1286	1126	585	419	= 3719
AMD Opteron 6380	293	1203	1063	589	423	= 3571
AMD Opteron 6378	282	1138	986	561	406	= 3373
AMD Opteron 6274	232	1054	926	524	376	= 3112
Intel Xeon E5-2630	247	962	864	541	394	= 3008
Intel Xeon E5645	262	817	717	727	524	= 3047

Efficiency: SALSA 20