L1 5880

Algorithms

abhi shelat

Jan 18 2022

let me intro myself

first goal: create an amazing learning experience second goal: share basic beautiful ideas from computer science third goal: help prepare you for a job in cs

what is this course about?

Theme 1

Small problems are easy to solve

Solve large problems by recasting them into smaller ones.

Theme 1

Small problems are easy to solve

Solve big problems by making them into smaller ones

Theme 2

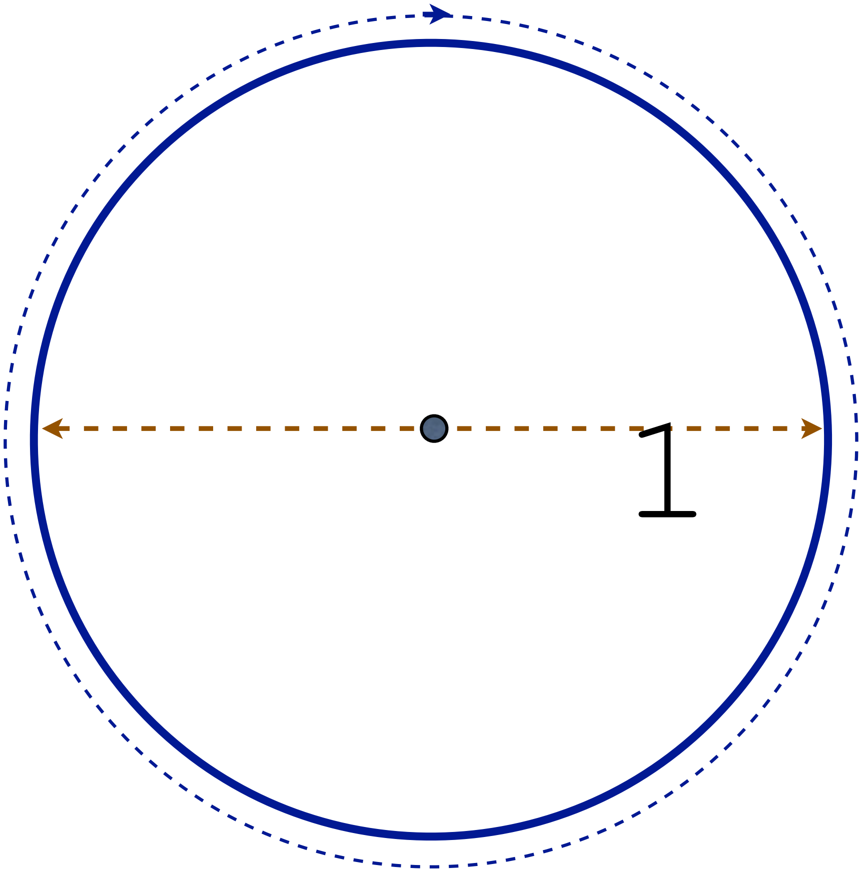
Learning how to convince through reason is a great mark of understanding

great pyramid at giza 2500bc

image from wikimedia

$$\pi$$

http://www.cupertino.org/inc/pdf/apple/Renderings.pdf



“how much granite/glass do i need?”

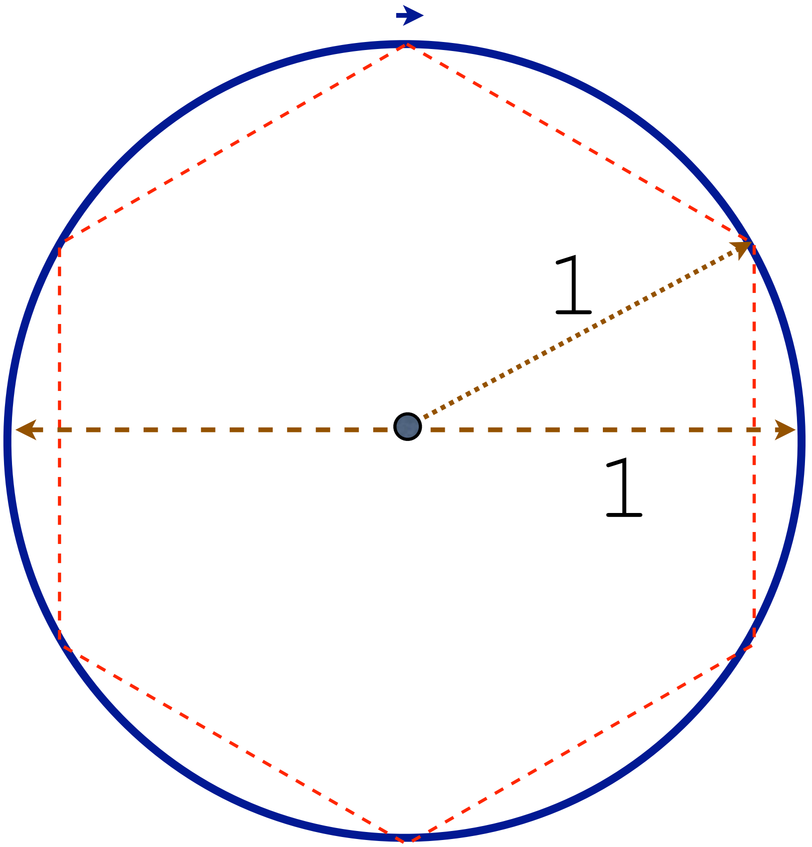
algorithm to compute

$$\pi$$

258 BCE

Written down by Archimedes

Idea: It is OK
to approximate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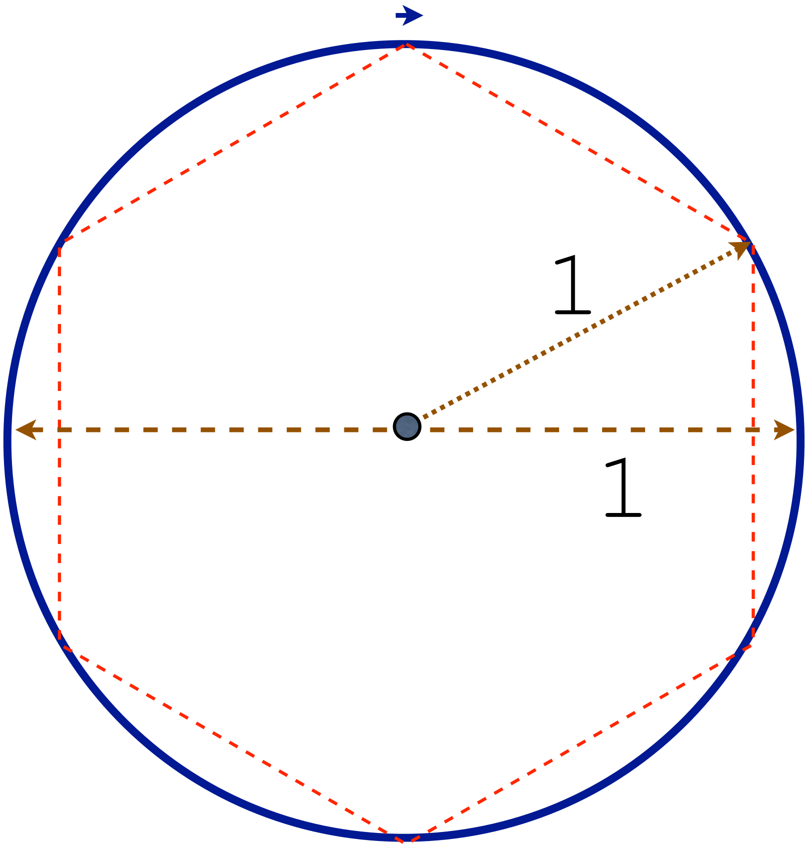
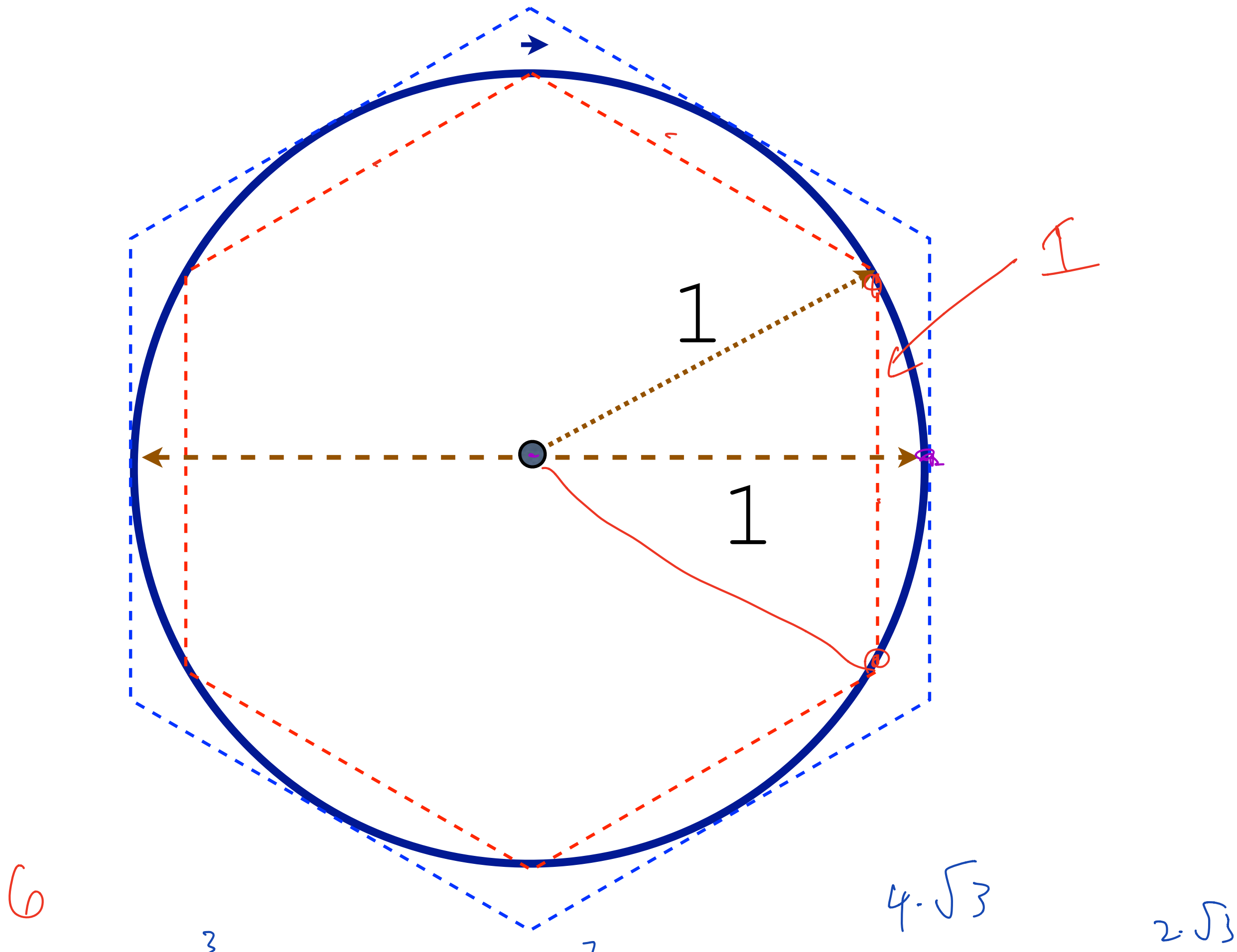
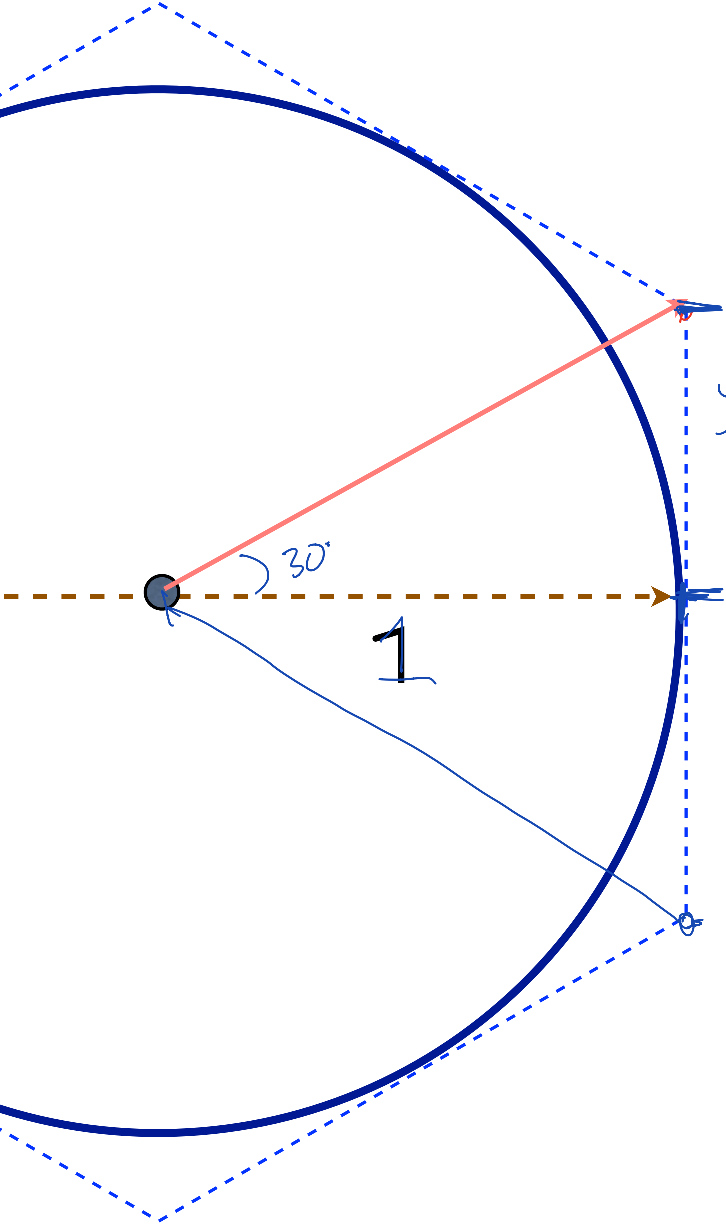


red perimeter $< \pi d$



red perimeter $< \pi d <$ blue perimeter



$$\tan(30) = \frac{y}{1} = \frac{1}{\sqrt{3}}$$

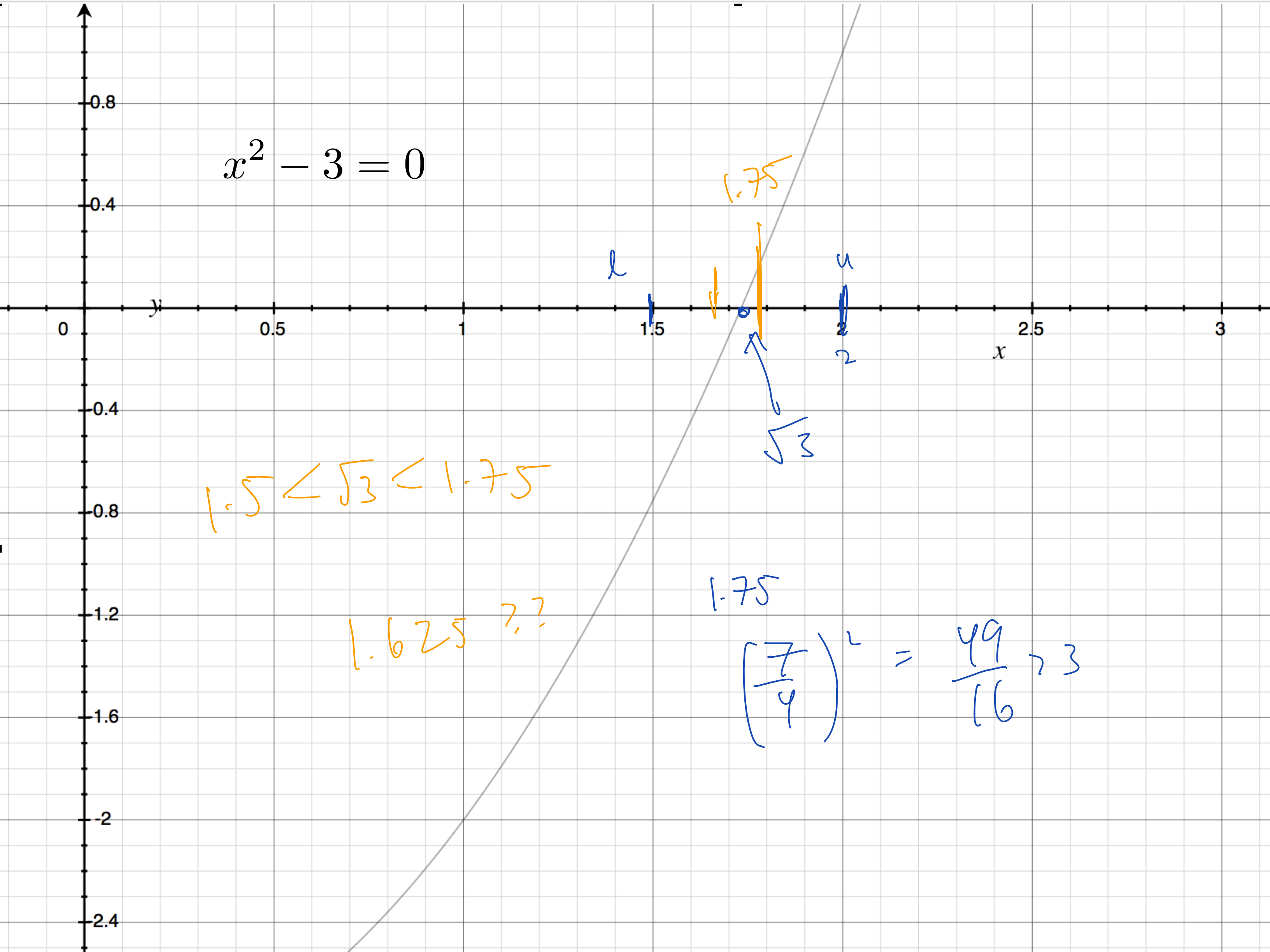
$$y = \frac{1}{\sqrt{3}}$$

entire blue perimeter =

$$\frac{12}{\sqrt{3}} = \underline{\underline{4 \cdot \sqrt{3}}}$$

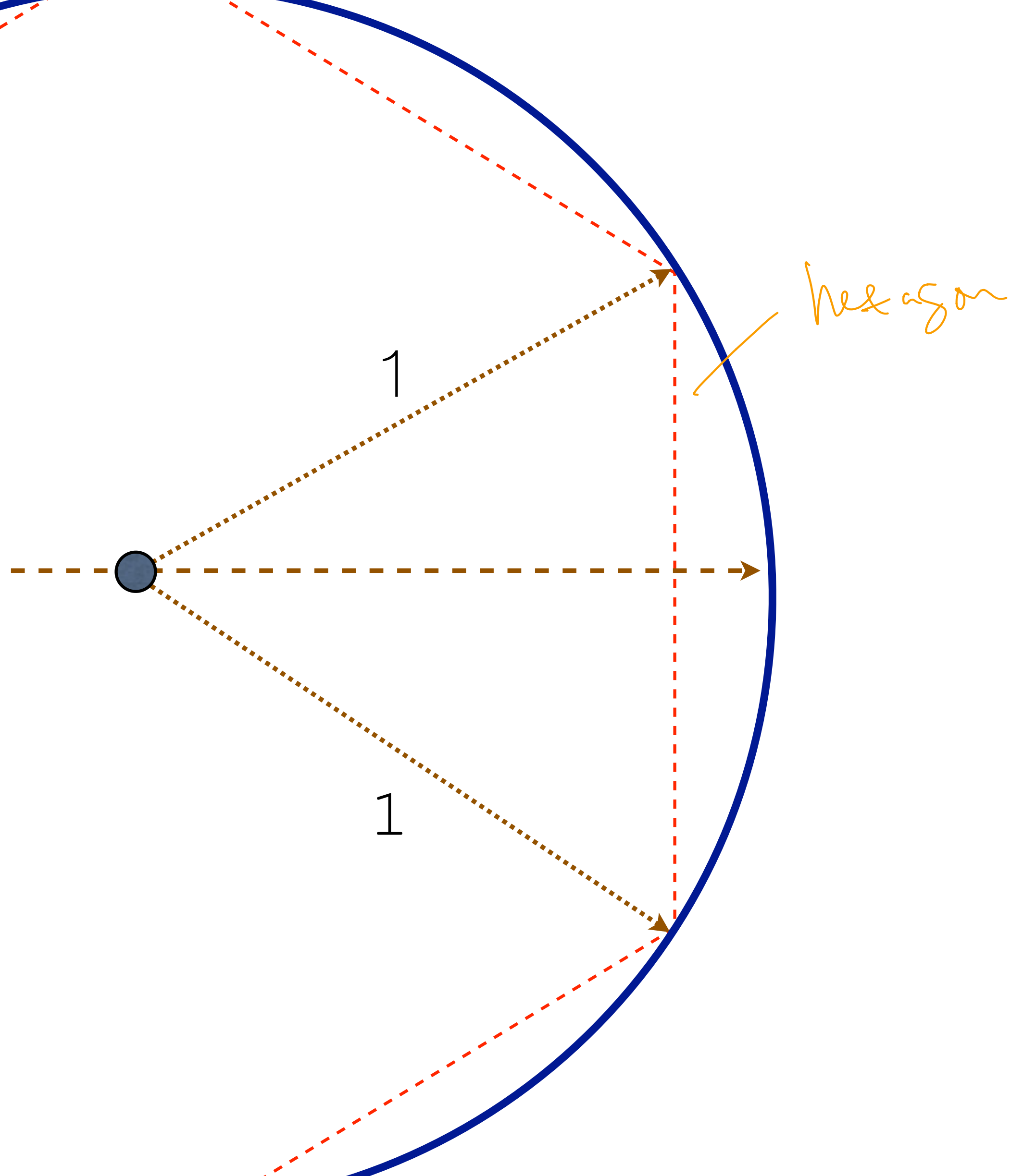
But what is $\sqrt{3}$

Theme1 : reduce the main problem to a simpler one



$x^2 - 3 = 0$
1.75
l
u
2
√3
1.5 < √3 < 1.75
1.625 ??
1.75
(7/4)^2 = 49/16 > 3
0
0.5
1
1.5
2
2.5
3
x
y
0.8
0.4
-0.4
-0.8
-1.2
-1.6
-2
-2.4

$$\frac{265}{153} \approx \sqrt{3}$$


1
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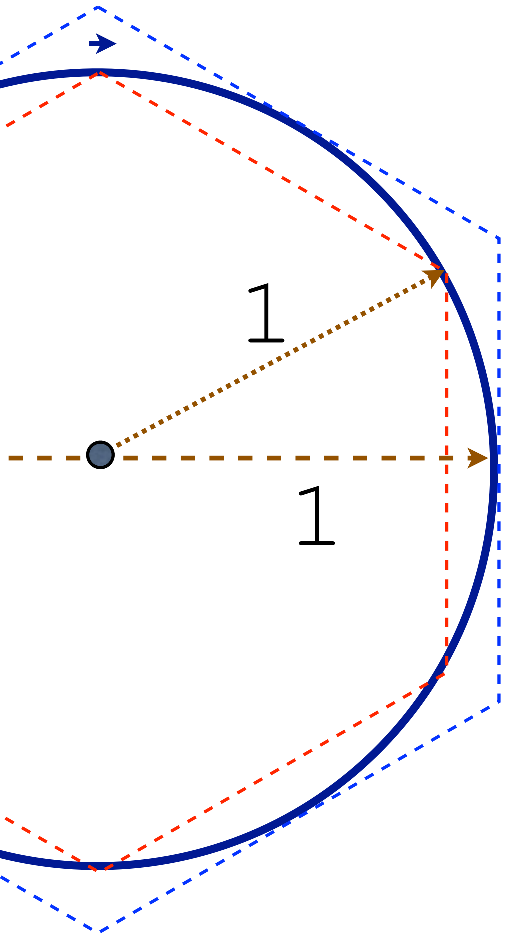
hexagon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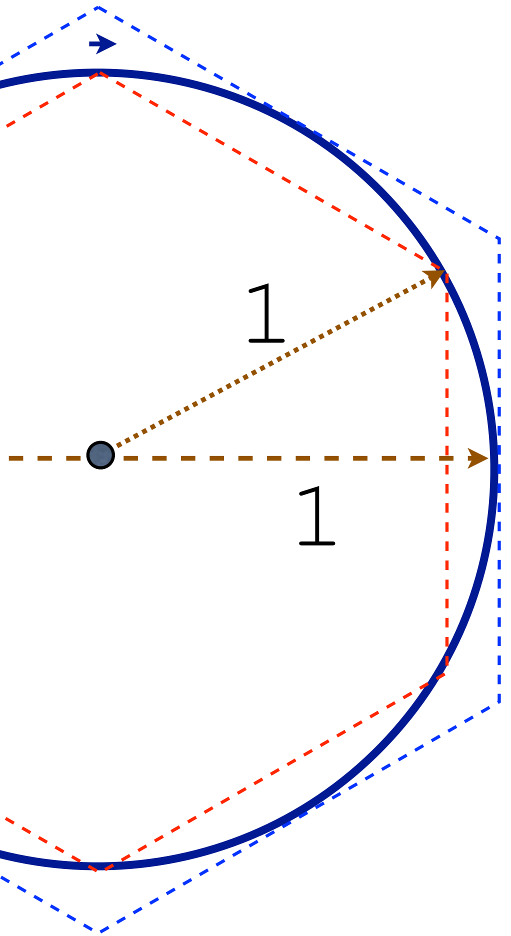
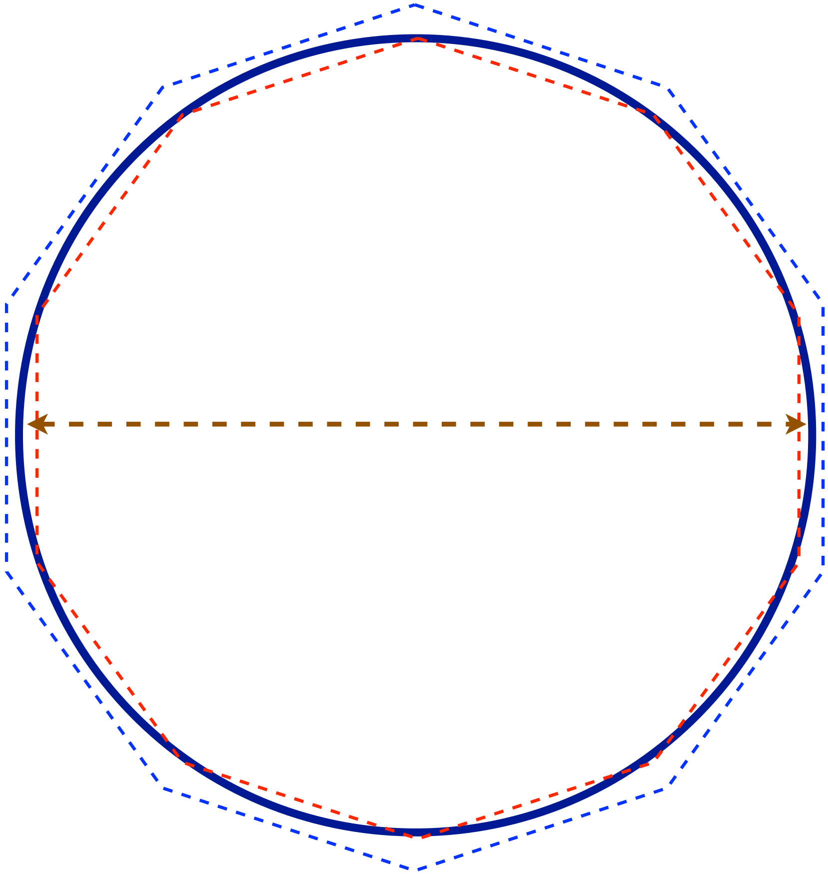


red perimeter $< \pi d <$ blue perimeter



3.1428   15   3.1408

$$3\frac{10}{70} > \pi > 3\frac{10}{71}$$

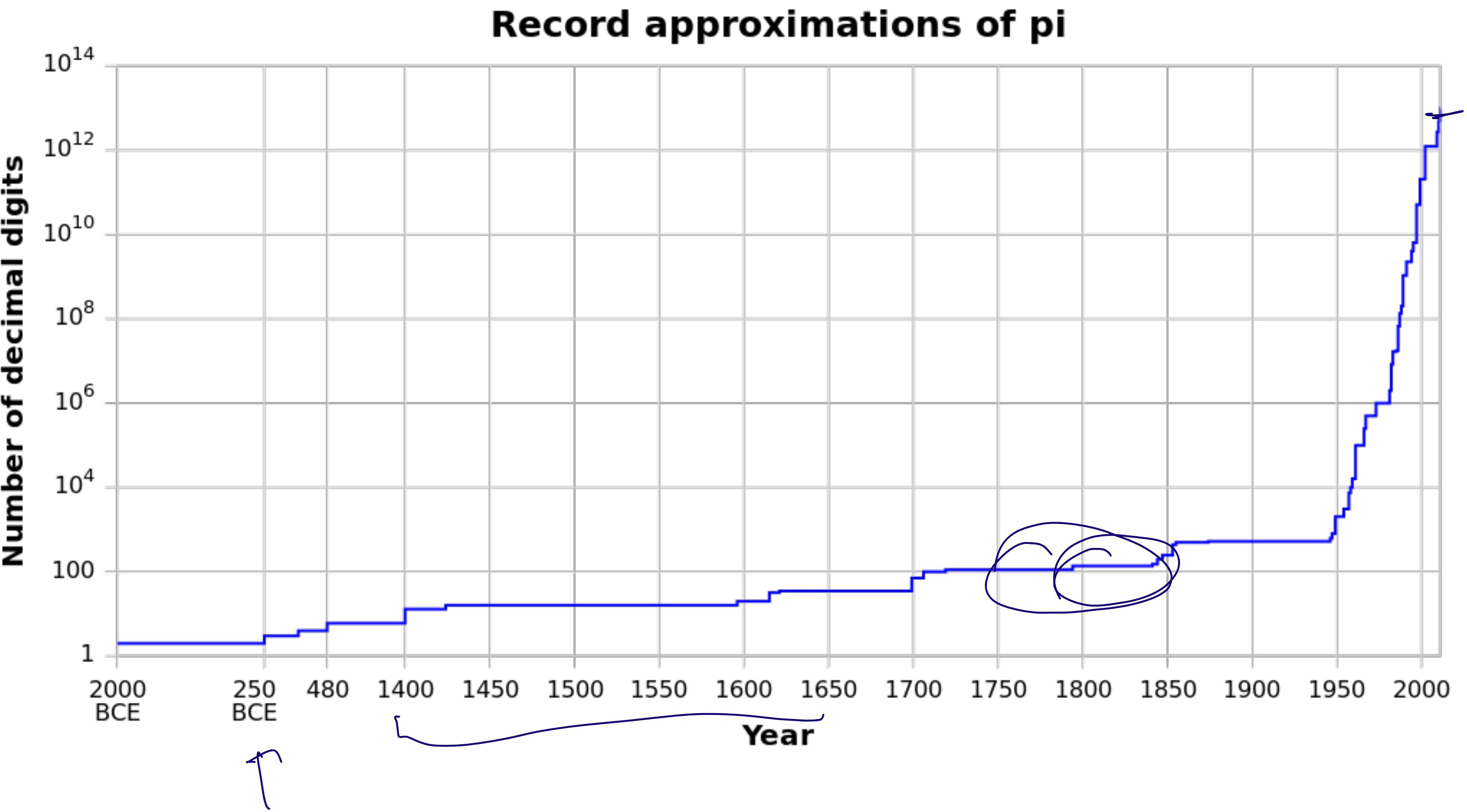
Using 96-gon, Archimedes

how to analyze this approach?

efficiency analysis.

– accuracy versus work

trade off



https://en.wikipedia.org/wiki/Approximations_of_%CF%80#/media/File:Record_pi_approximations.svg

Theme3: new insights lead to improved efficiency

$$\pi = \frac{9801}{\sqrt{8}} \left(\sum_{n=0}^{\infty} \frac{(4n)!(1103 + 26390n)}{(n!)^4 396^{4n}} \right)^{-1}$$

$$\pi = \frac{9801}{\sqrt{8}} \left(\sum_{n=0}^{\infty} \frac{(4n)!(1103 + 26390n)}{(n!)^4 396^{4n}} \right)^{-1}$$

n=0

$$\frac{(0!)(1103)}{0! \cdot 396^0}$$

$$\pi = \frac{9801}{\sqrt{8}} \cdot \left(\frac{1}{1103} \right)$$

$$\pi = \frac{9801}{\sqrt{8}} \left(\sum_{n=0}^{\infty} \frac{(4n)!(1103 + 26390n)}{(n!)^4 396^{4n}} \right)^{-1}$$

n=0

$$\pi \approx_0 \frac{9801}{\sqrt{8}} [1103]^{-1}$$

3.141592730013305760 17

$$\pi = \frac{9801}{\sqrt{8}} \left(\sum_{n=0}^{\infty} \frac{(4n)!(1103 + 26390n)}{(n!)^4 396^{4n}} \right)^{-1}$$

$$n = 1$$

$$\pi = \frac{9801}{\sqrt{8}} \left(\sum_{n=0}^{\infty} \frac{(4n)!(1103 + 26390n)}{(n!)^4 396^{4n}} \right)^{-1}$$

n=1

$$\pi \approx_1 \frac{9801}{\sqrt{8}} \left[1103 + \frac{24 \cdot 27493}{396^4} \right]^{-1}$$

3.14159265358979387799890582630

benefits?

much faster convergence for the amount of work.

good algorithms touch every aspect of our lives

good algorithms defend freedom

what skills do you need for this course?

- precision

- creativity

in•ge•nu•i•ty

how to learn
in this class

no cookbook

develop
general problem
solving
skills

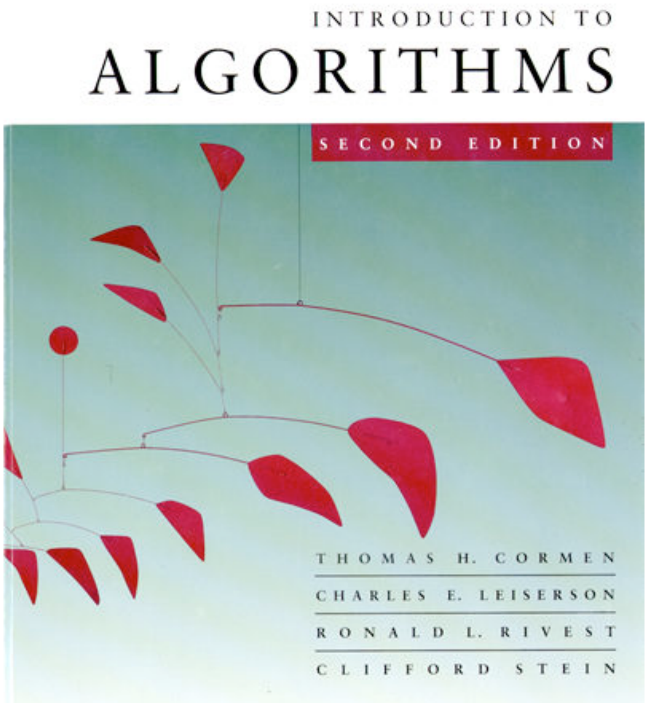
understand
known
techniques

#work with your peers

work with your peers

but do not copy

https://
shelat.khoury
.northeastern
.edu/22s-5800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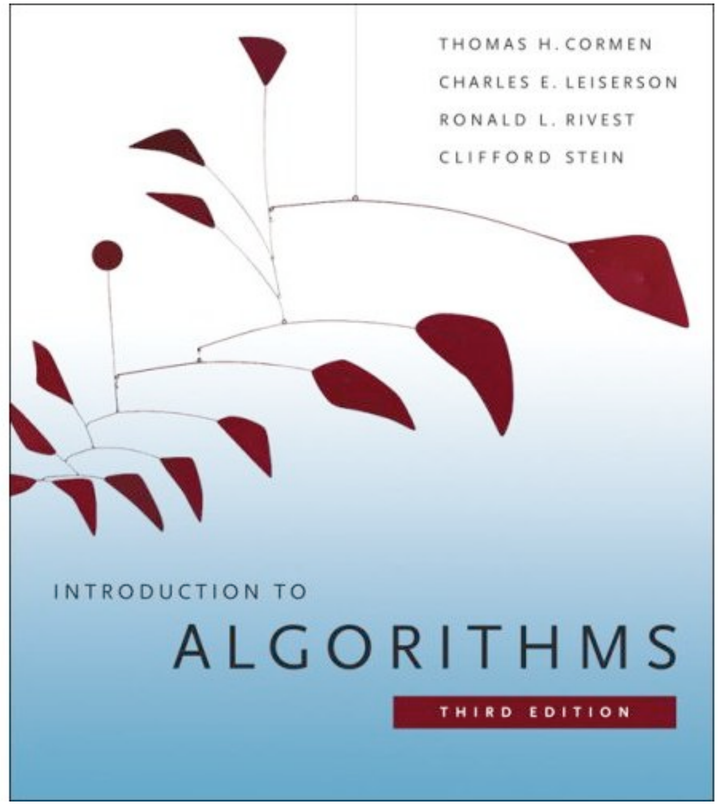
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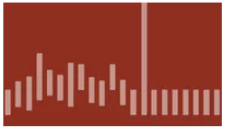
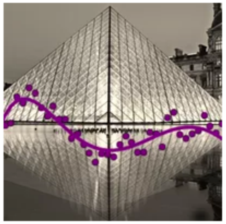


Algorithms, Part I

Princeton University



Approximation Algorithms Part I

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LaTeX

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LaTeX documentation

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by Tobias Oetiker

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hw0.tex

Typeset LaTeX Macros Tags Templates

```
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\def\yourname{}
% --- no need to change anything in this section ----------------------------------------------------
\def\homework{1} % 0 for solution, 1 for problem-set only
\def\duedate{fri jan 29, 2016 at 5p}
\def\duelocation{via \href{https://church.cs.virginia.edu/16s-4102/}{submission site}}
\def\hnumber{0}
\def\prof{abhi shelat}
\def\course{\href{https://www.cs.virginia.edu/~shelat/16s-4102}{cs4102 - algorithms - s'16}}

\documentclass[11pt]{article}
%------------------------------------------------------------------------------------------------

%% ================= no need to edit any of this stuff


%%% ==== standard installations of latex include all of the files that are referenced in this section.  However,
%%% ==== if you are having compile problems, consider commenting some of these commands out
\usepackage[colorlinks,urlcolor=blue]{hyperref}
\usepackage[osf]{mathpazo}
\usepackage{amsmath,amsfonts,graphicx}
\usepackage{latexsym}
\usepackage[top=1in,bottom=1.4in,left=1.5in,right=1.5in,centering]{geometry}
\usepackage{color}
\definecolor{mdb}{rgb}{0.3,0.02,0.02}
\definecolor{cit}{rgb}{0.05,0.2,0.45}
\markboth{\yourname}{\yourname}
%%% ==================================================================


%%% ============ should be no need to edit anything in this section ====================
\newenvironment{proof}{\par\noindent{\it Proof.}\hspace*{1em}}{$\Box$\bigskip}
\newcommand{\qed}{$\Box$}
\newcommand{\alg}[1]{\mathsf{#1}}
\newcommand{\handout}{
  \renewcommand{\thepage}{H\hnumber-\arabic{page}}%
  \noindent%
  \begin{center}%
    \vbox{%
   \hbox to \columnwidth {\sc{\course} --- abhi shelat \hfill}%
   \vspace{-2mm}%
   \hbox to \columnwidth {\sc due \MakeLowercase{\duedate} \duelocation\hfill {\Huge\color{mdb}H\hnumber.
\yourname}}%
     }
  \end{center}
```

hw0.pdf

Scale 115 Page 1 of 1

CS4102 - ALGORITHMS - S'16 — ABHI SHELAT
DUE FRI JAN 29, 2016 AT 5P VIA SUBMISSION SITE

You may collaborate with other students on the homework but
your own individually written solution, identify your collaborat
edge any external sources that you consult.

PROBLEM 1 *Passage*

Typeset your favorite passage from a book.

PROBLEM 2 *Asymptotic notation*

Let f be a function. Give a formal definition of the set $\Theta(f)$.

$$\Theta(f) = \{ \text{ functions } g \text{ such that } \ldots$$

Hint: Use the `\exists` command to make the "there exists" s
`\forall` command to make the "for all" symbol $\forall$.

PROBLEM 3 *includegraphics command*

Learn how to include drawings in your documents with the `\inc`
command by submitting a caricature of me.

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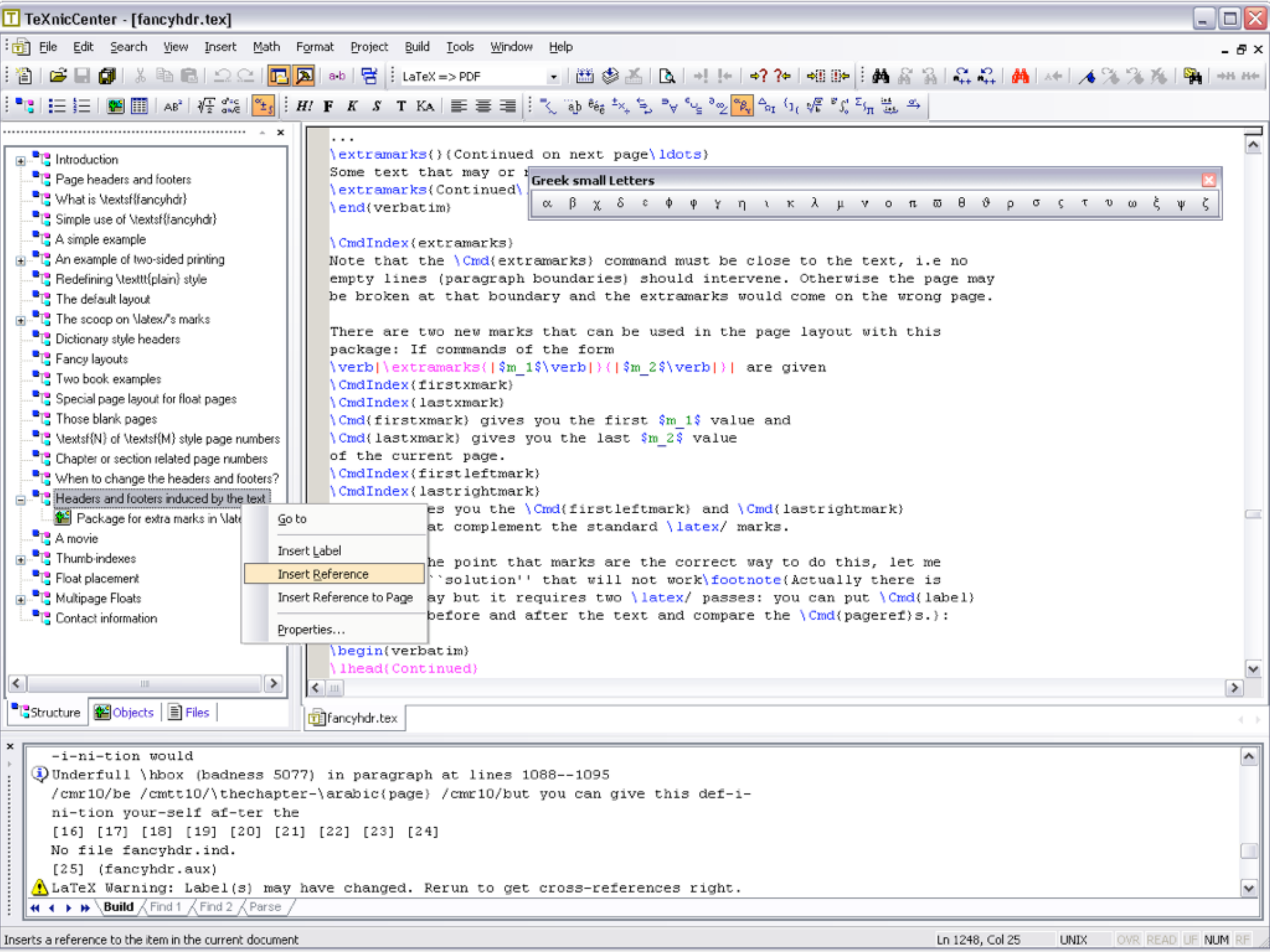
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A simple example
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\extramarks{}{Continued on next page\ldots}
Some text that may or
\extramarks{Continued\
\end{verbatim}
Greek small Letters
\CmdIndex{extramarks}
Note that the \Cmd{extramarks} command must be close to the text, i.e no
empty lines (paragraph boundaries) should intervene. Otherwise the page may
be broken at that boundary and the extramarks would come on the wrong page.
There are two new marks that can be used in the page layout with this
package: If commands of the form
\verb|\extramarks{|m_1\verb|}{|m_2\verb|}| are given
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\CmdIndex{lastxmark}
\Cmd{firstxmark} gives you the first m_1 value and
\Cmd{lastxmark} gives you the last m_2 value
of the current page.
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es you the \Cmd{firstleftmark} and \Cmd{lastrightmark}
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``solution'' that will not work\footnote{Actually there is
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before and after the text and compare the \Cmd{pageref}s.}:
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\lhead{Continued}
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Underfull \hbox (badness 5077) in paragraph at lines 1088--1095
/cmr10/be /cmtt10/\thechapter-\arabic{page} /cmr10/but you can give this def-i-
ni-tion your-self af-ter the
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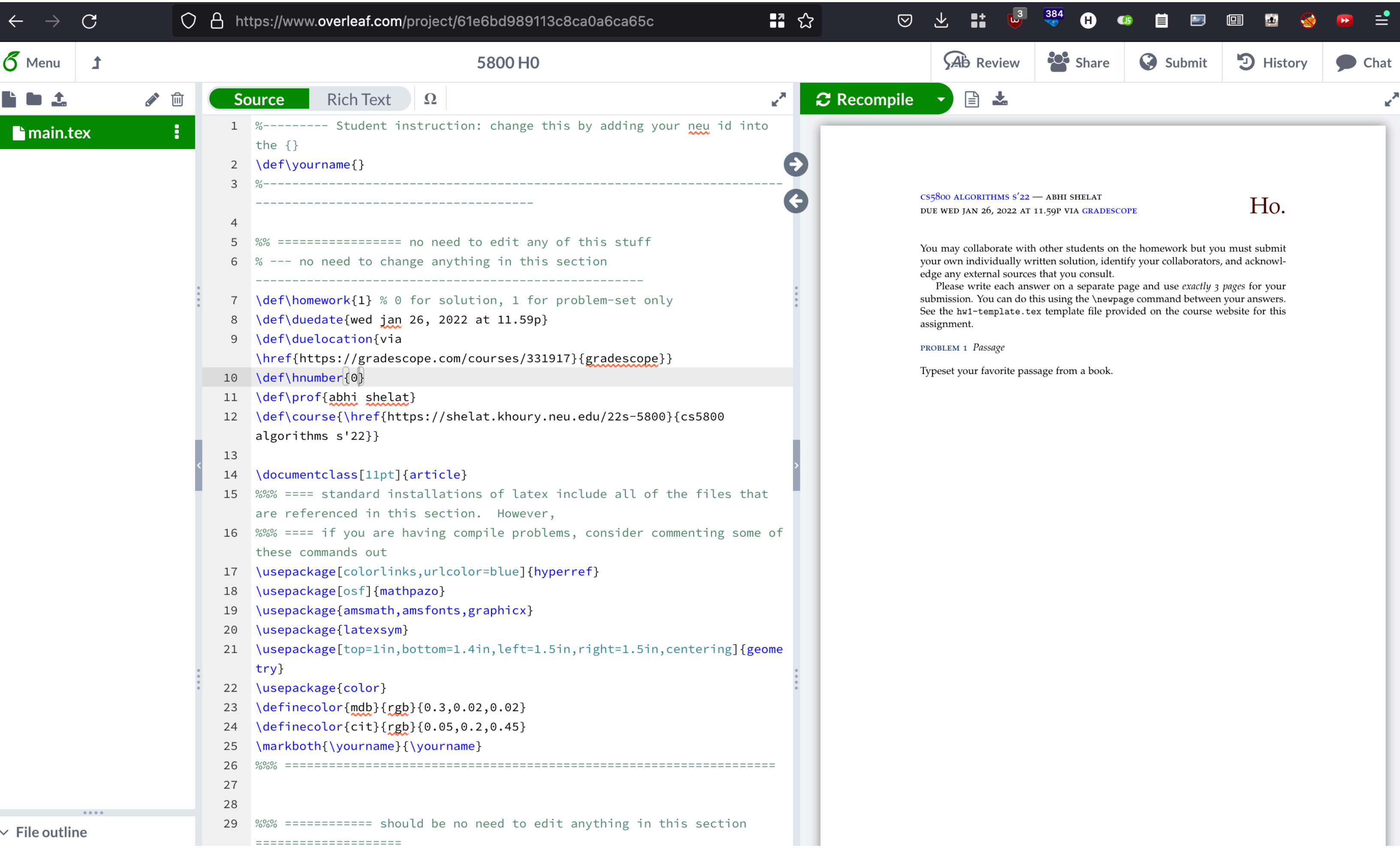

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DUE WED JAN 26, 2022 AT 11.59P VIA GRADESCOPE
H0.
You may collaborate with other students on the homework but you must submit your own individually written solution, identify your collaborators, and acknowledge any external sources that you consult.
Please write each answer on a separate page and use exactly 3 pages for your submission. You can do this using the \newpage command between your answers. See the hw1-template.tex template file provided on the course website for this assignment.
PROBLEM 1 Passage
Typeset your favorite passage from a book.

Submitting HW

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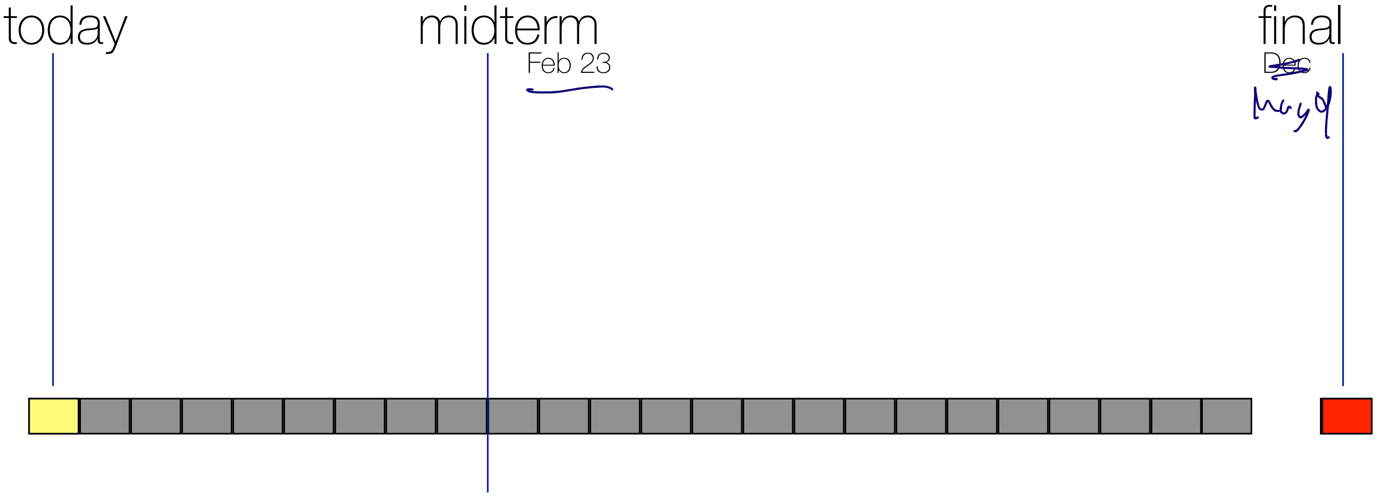
Honor Policy

I, _______, do hereby certify on my honor that during this course,

1. I shall write my answers entirely by myself, and neither share nor request text, code, or drawings.

2. I will not give or derive assistance from any unauthorized sources or the web.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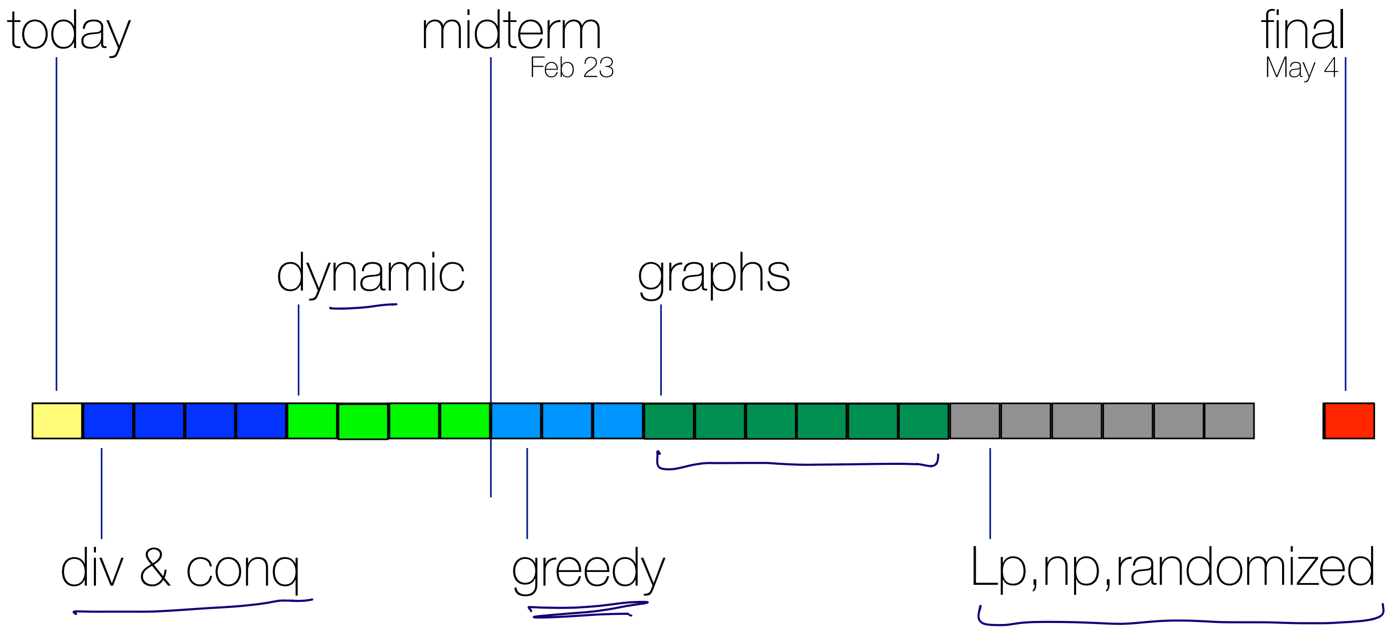
today
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Feb 23
final
May 4
dynamic
graphs
div & conq
greedy
Lp,np,randomized

counting

First example of an algorithmic pattern based on I1 and I2

❶ stand

1. stand
2. set your “number” to one

1. stand
2. set your “number” to one
3. greet your neighbor (pause if no partner)

1. stand
2. set your “number” to one
3. greet your neighbor (pause if no partner)
4. if you are older, give “number” and sit
   if you are younger, add “numbers”

1. stand
2. set your "number" to one
3. greet your neighbor (pause if no partner)
4. if you are older, give "number" and sit
   if you are younger, add "numbers"
5. if you are standing & you have a neighbor,
   goto 3

1 stand

2 set

3 greet

4 sit/add

5 repeat

lets analyze this alg

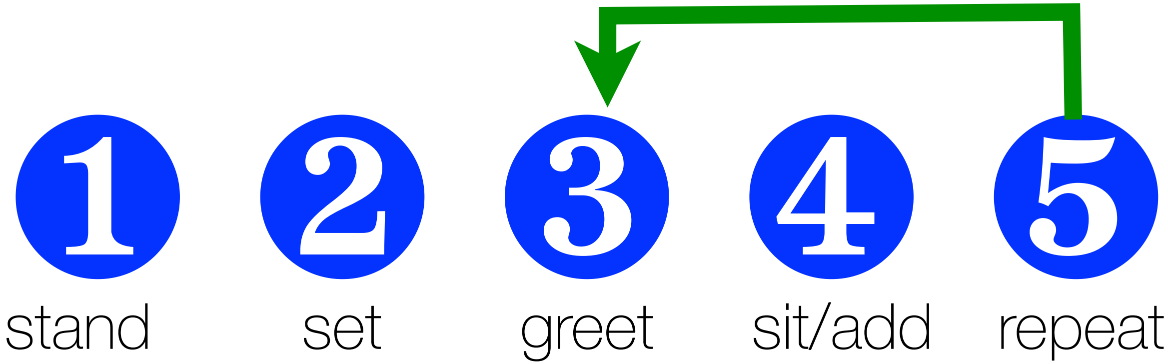
Our model of computation

Basic op: 1 unit

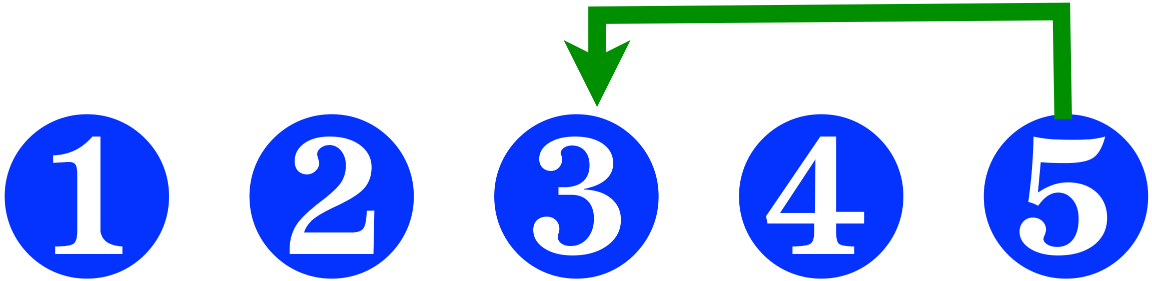
Set, Greet, add, compare, sit

Simplify: in each round, every standing person can do 1 op

Lets count # of rounds until we finish



how fast does it work:

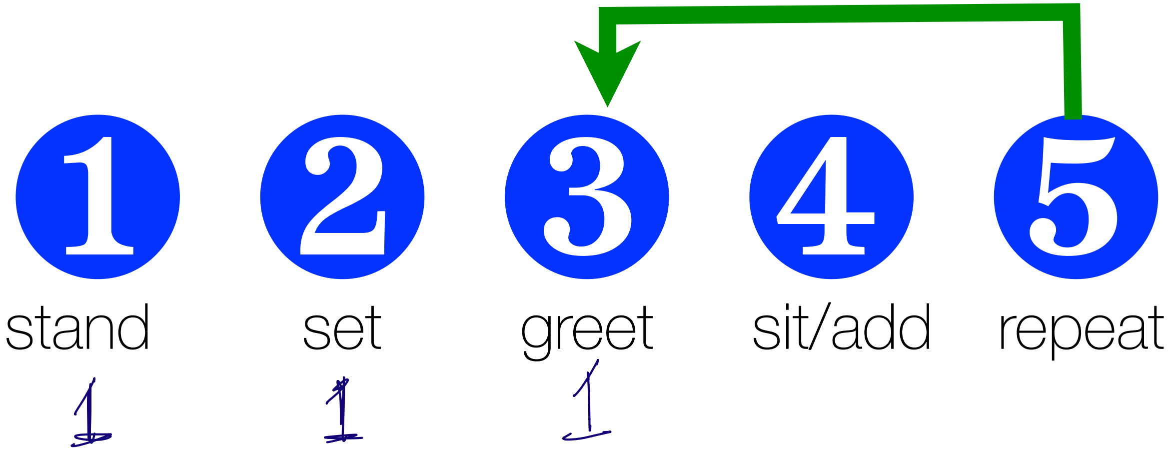


stand set greet sit/add repeat

how fast does it work:

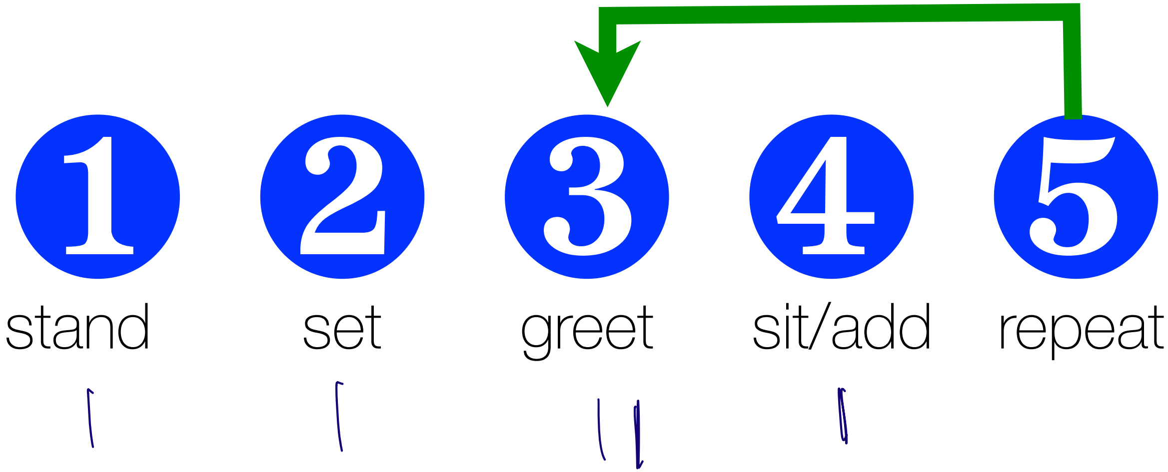
$$T(n)$$

rounds to finish in a room with n people



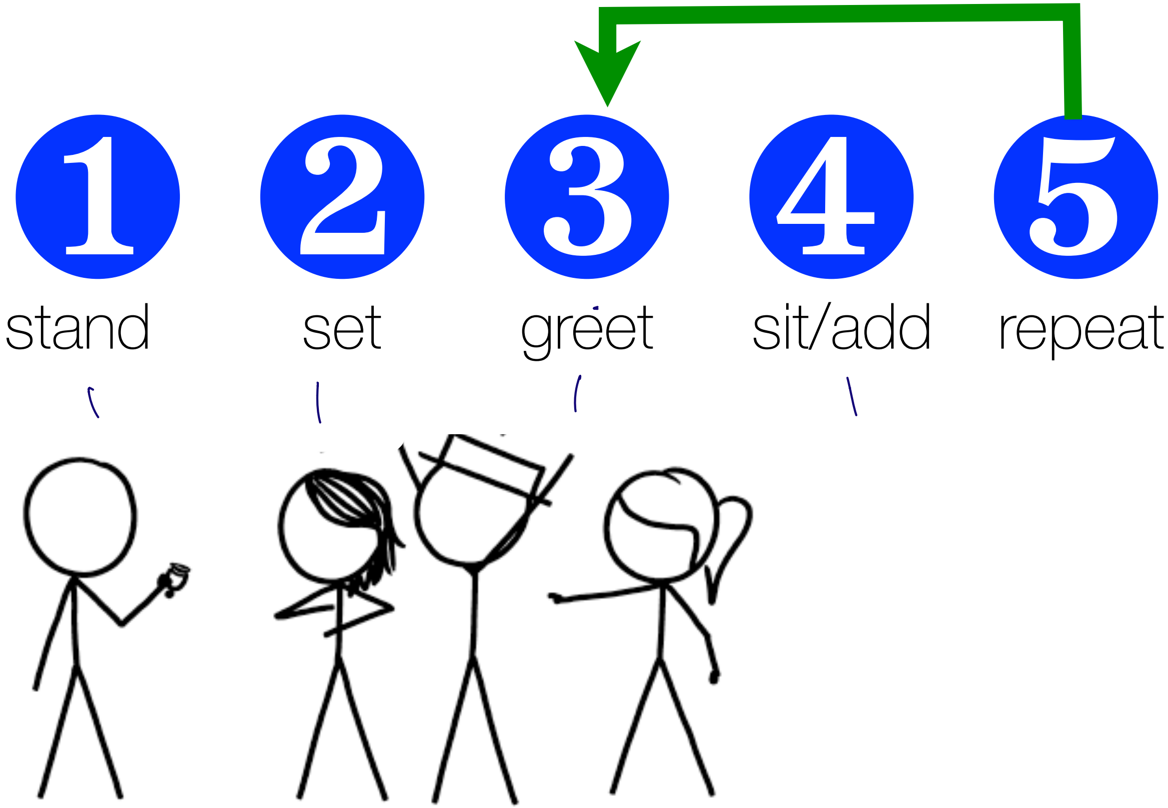
Simple case: 1 person

$$T(1) = 3$$

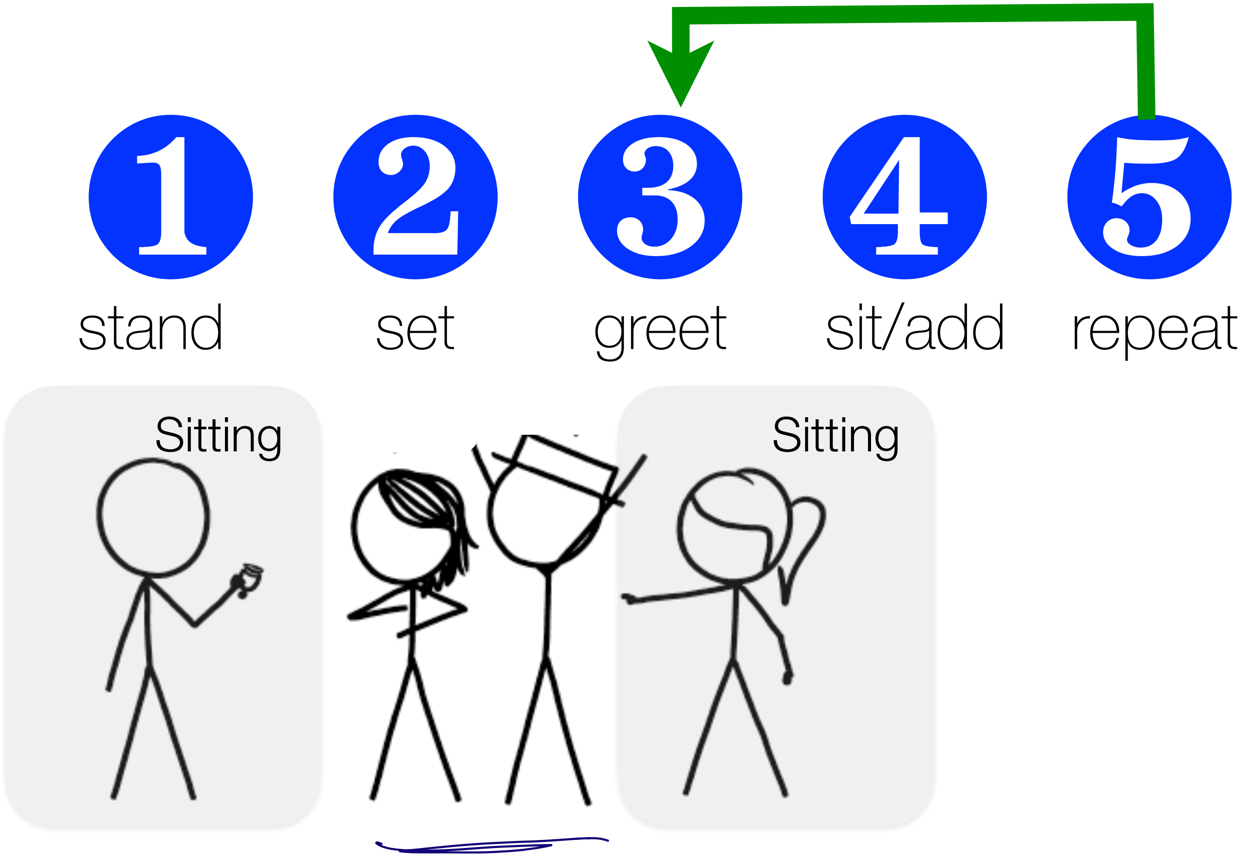


Simple case: 2 people

$$T(2) = 5$$



$$T(4) = 4 + T(2)$$



After step 4

$$T(4) =$$

1 2

stand set

3 4 5

greet sit/add repeat

These steps only happen once.

What about these?

I1:Approx is OK

1 stand 2 set 3 greet 4 sit/add 5 repeat

how fast does it work:

$$T(n) = 1 + 1 + T(\lceil n/2 \rceil)$$

1 stand 2 set

3 greet 4 sit/add 5 repeat

how fast does it work:

$$T(n) = 1 + 1 + T(\lceil n/2 \rceil)$$

$$T(1) = 3$$

This is a recurrence

$$T(n) = T(\lceil n/2 \rceil) + 2$$

$$T(1) = 3$$

solve a simpler case when n is a power of 2.

$$T(2^k) = 2 + T(2^{k-1})$$

$$T(2^k) = 2 + T(2^{k-1})$$

$$
\begin{aligned}
T(2^k) &= 2 + T(2^{k-1}) \\
&= 2 + 2 + T(2^{k-2})
\end{aligned}
$$

$$T(2^k) = 2 + T(2^{k-1})$$

$$= 2 + 2 + T(2^{k-2})$$

$$= \overbrace{2 + 2 + \cdots + 2}^{k} + T(2^0)$$

$$
\begin{aligned}
T(2^k) &= 2 + T(2^{k-1}) \\
&= 2 + 2 + T(2^{k-2}) \\
&= \overbrace{2 + 2 + \cdots + 2}^{k} + T(2^0) \\
&= 2k + T(1)
\end{aligned}
$$

“intuition here”

$$T(2^k) = 2 + T(2^{k-1})$$

Other cases?

“intuition here”

$$T(2^k) = 2 + T(2^{k-1})$$
$$= 2 + 2 + T(2^{k-2})$$

Other cases?

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$$T(2^k) = 2 + T(2^{k-1})$$

$$= 2 + 2 + T(2^{k-2})$$

$$= \overbrace{2 + 2 + \cdots + 2}^{k} + T(2^0)$$

Other cases?

“intuition here”

$$T(2^k) = 2 + T(2^{k-1})$$

$$= 2 + 2 + T(2^{k-2})$$

$$= \overbrace{2 + 2 + \cdots + 2}^{k} + T(2^0)$$

$$= 2k + T(1)$$

Other cases?

Idea1 : It is OK to approximate

Asymptotic notation

$$O(g)$$

This notation represents a set

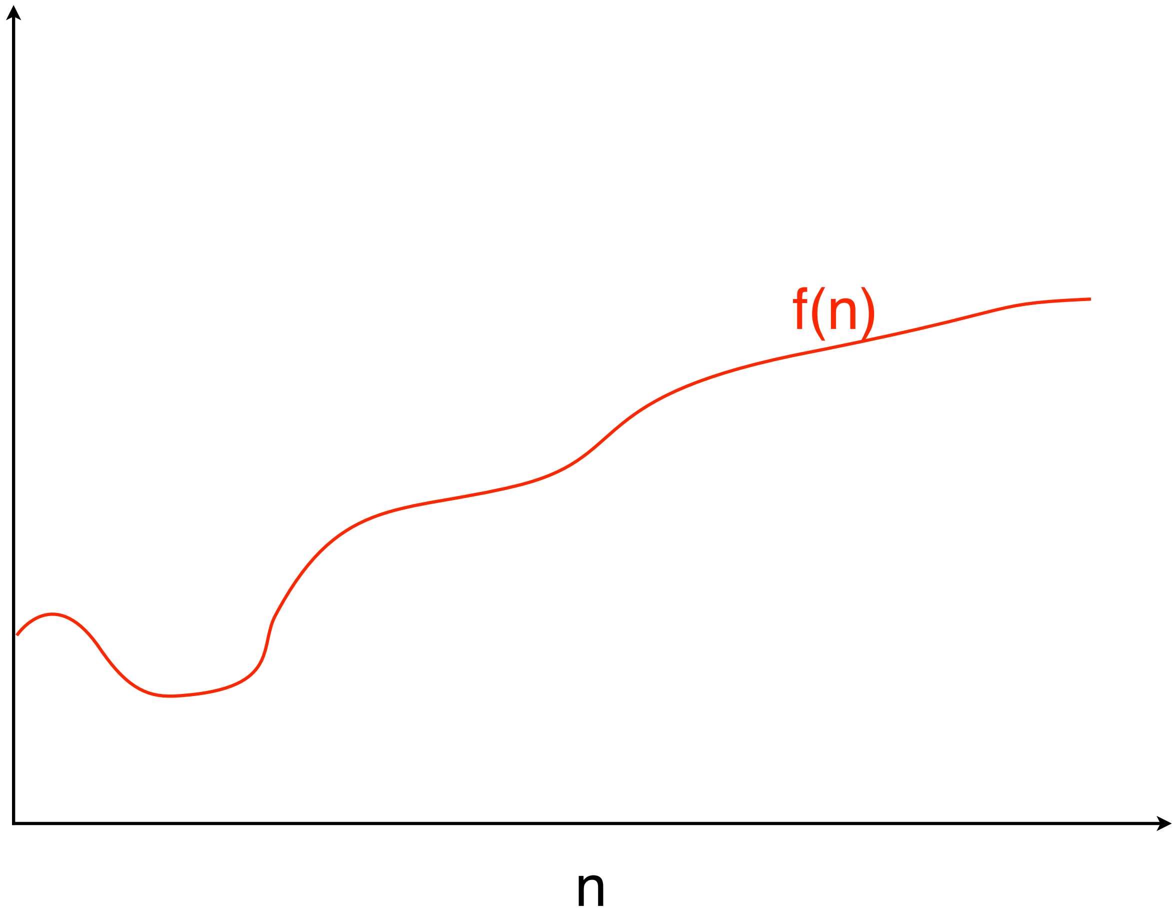
Asymptotic notation

$O(g)$ Set of functions that are at most within const of g for large n

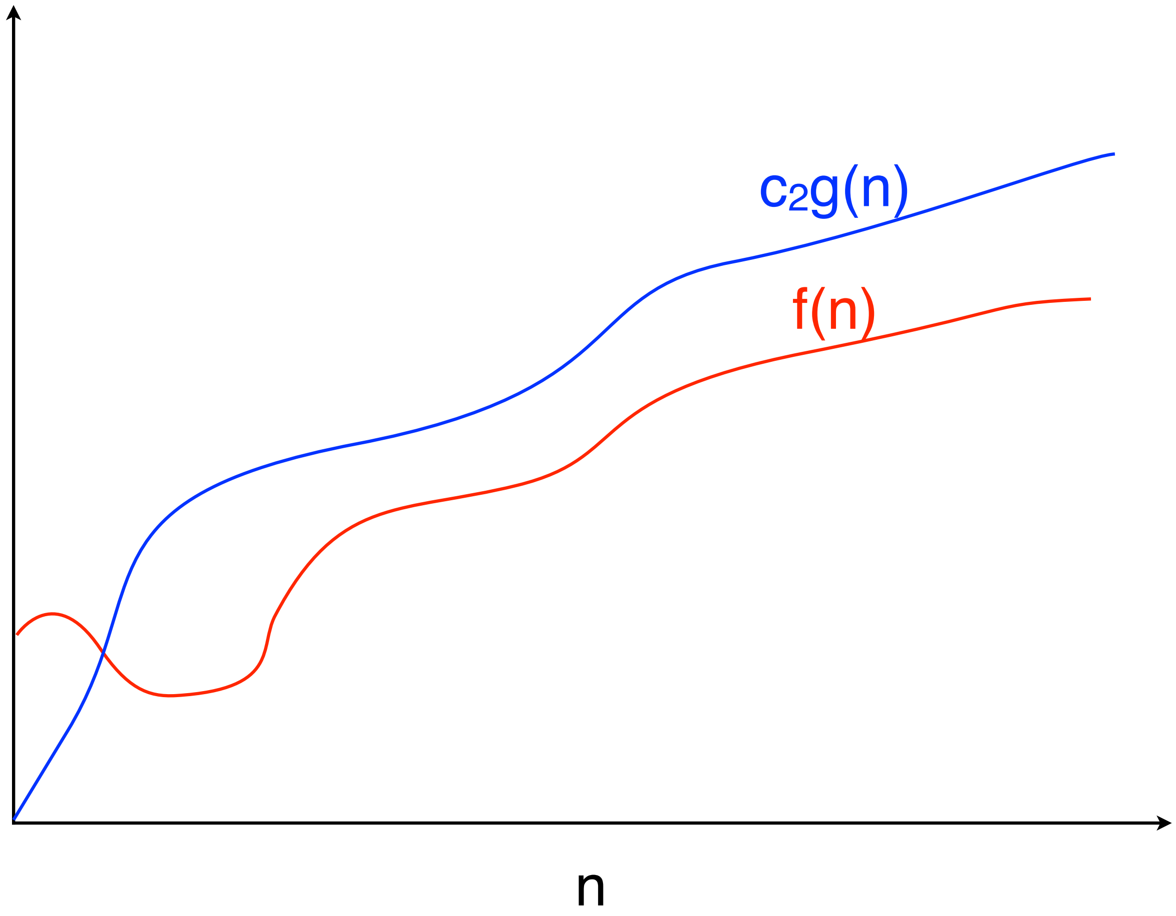
Asymptotic notation

$O(g)$	at most within const of g	for large n
$\Omega(g)$	at least within const of g	for large n
$\Theta(g)$	within a const of g	for large n

Omega sandwich

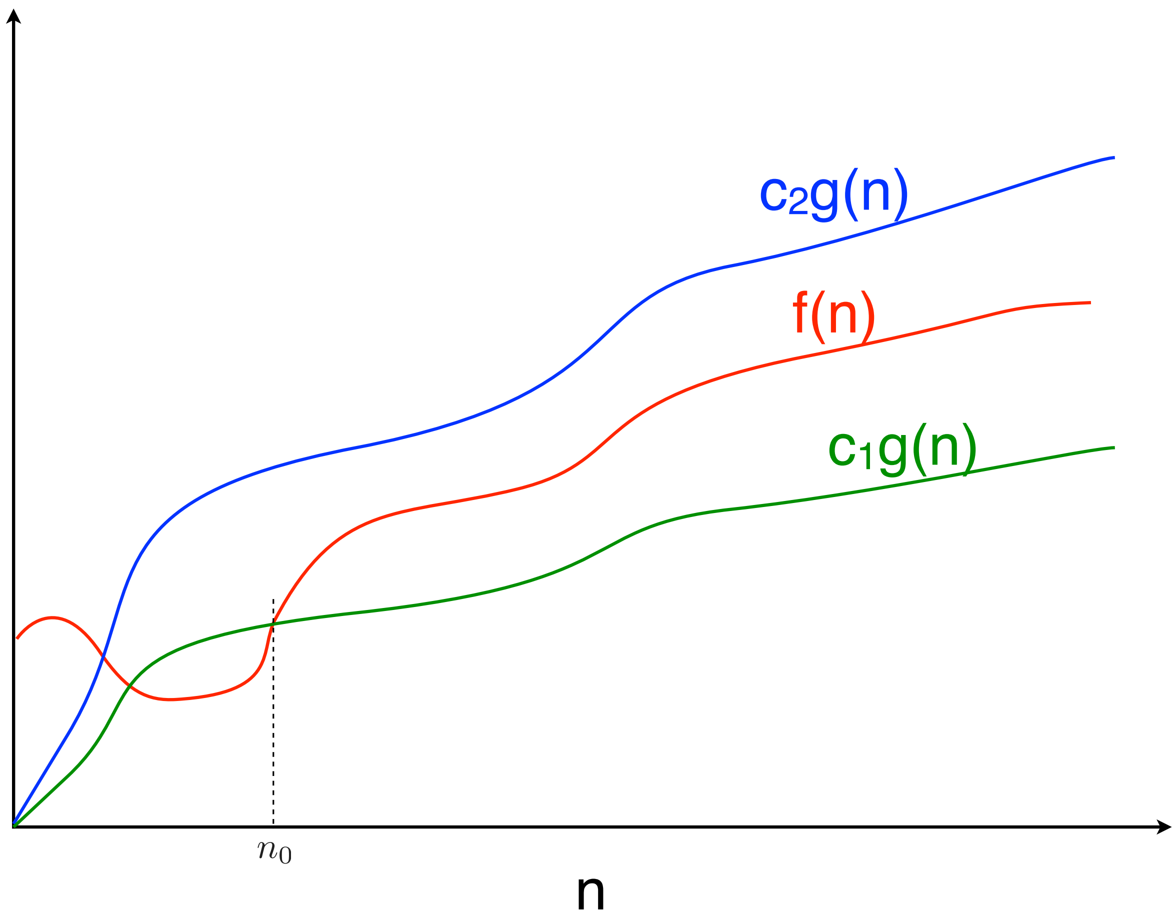


Omega sandwich



$$f(n) = O(g(n))$$

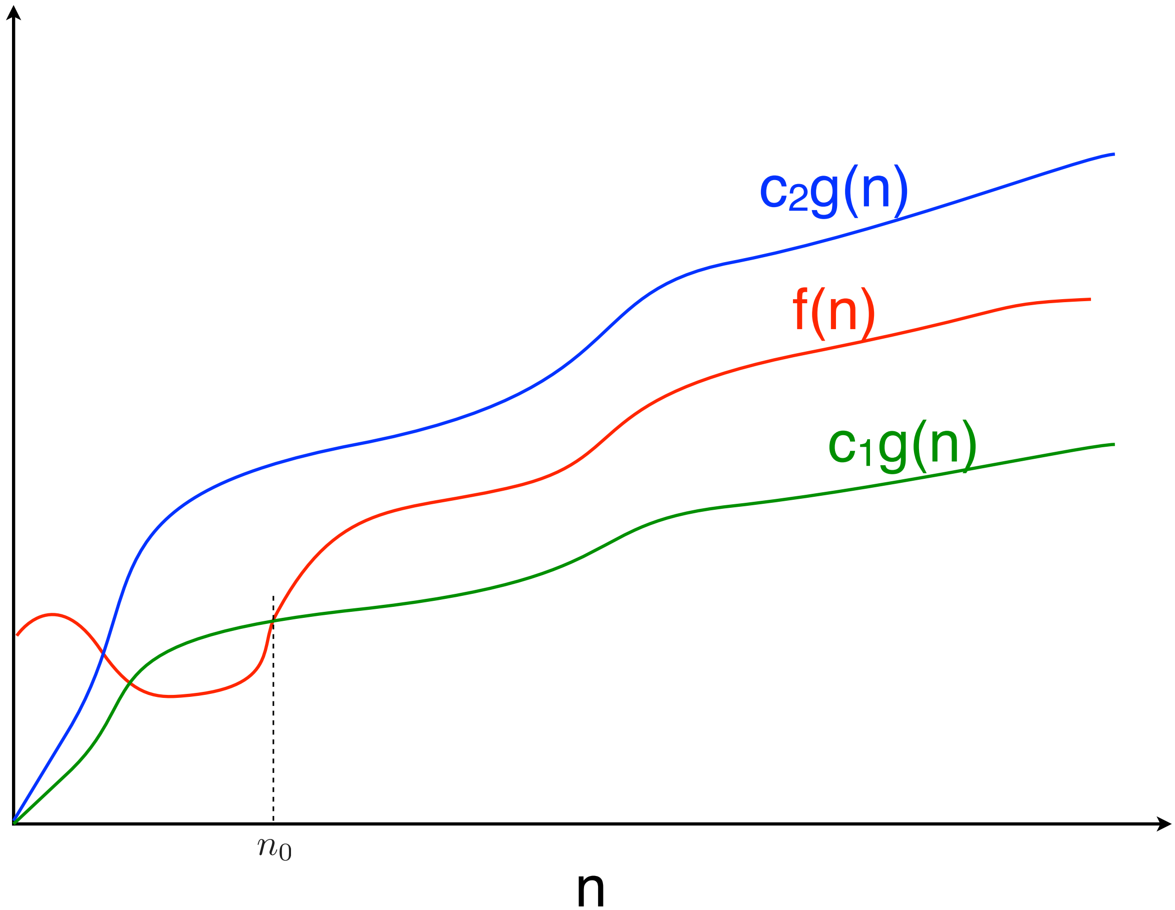
Omega sandwich



$$f(n) = O(g(n))$$

$$f(n) = \Omega(g(n))$$

Omega sandwich



$$f(n) = O(g(n))$$

$$f(n) = \Theta(g(n)$$

$$f(n) = \Omega(g(n))$$

“intuition here”

$$T(2^k) = 2 + T(2^{k-1})$$

$$= 2 + 2 + T(2^{k-2})$$

$$= \overbrace{2 + 2 + \cdots + 2}^{k} + T(2^0)$$

$$= 2k + T(1) \; = O(\log(2^k))$$

“intuition here”

$$T(2^k) = 2 + T(2^{k-1})$$

$$= 2 + 2 + T(2^{k-2})$$

$$= \overbrace{2 + 2 + \cdots + 2}^{k} + T(2^0)$$

$$= 2k + T(1) = O(\log(2^k))$$

$$\forall 0 < n < m, T(n) \leq T(m)$$

$$T(m) \leq T(2^{\lceil \log(m) \rceil}) = 2\lceil \log(m) \rceil + 2$$

“intuition here”

$$T(2^k) = 2 + T(2^{k-1})$$

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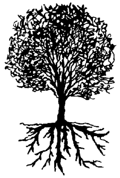
$$\forall 0 < n < m, T(n) \leq T(m)$$

$$T(m) \leq T(2^{\lceil \log(m) \rceil}) = 2\lceil \log(m) \rceil + 2$$

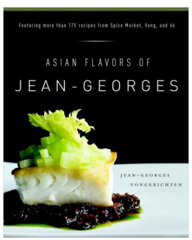
$$T(m) = \Omega(\log(m))$$

$$= \Theta(\log(m))$$

How to solve recurrence relations



?-✓



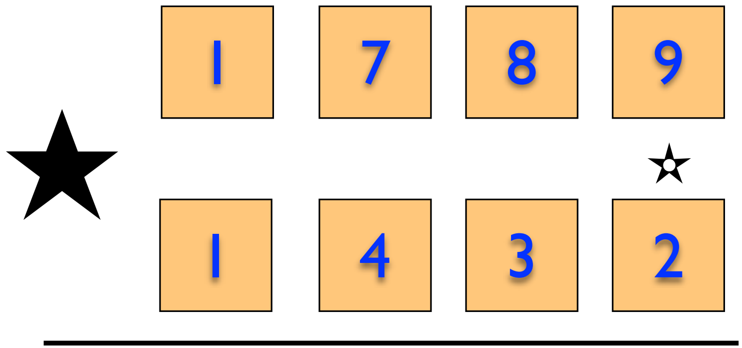
http://www.drblank.com/law301.jpg

Multiplication

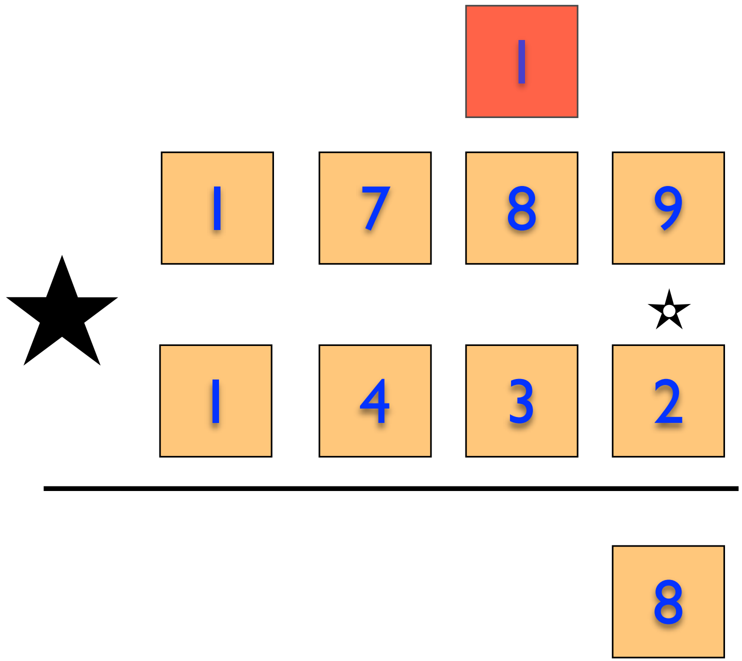
How much work does it take?

	1	7	8	9
★	1	4	3	2

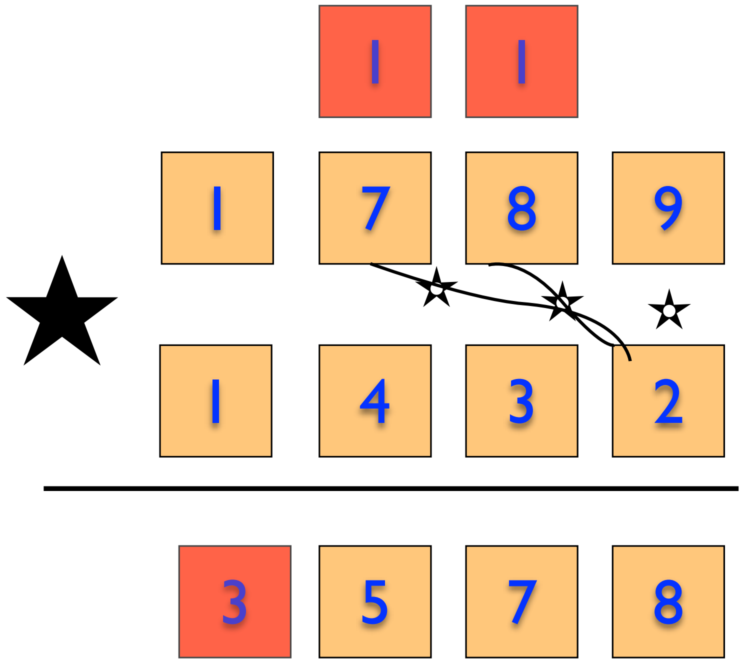
$(n-1)(n+1) +$



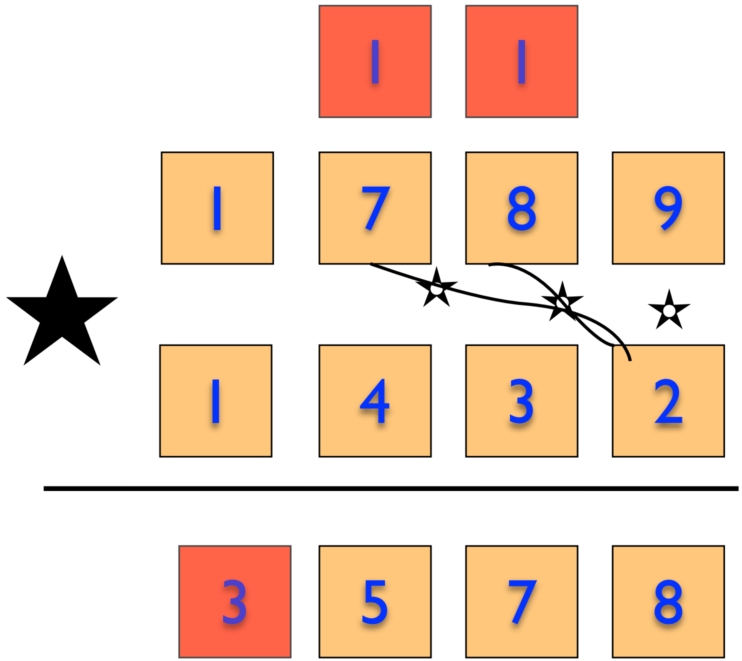
(n-1)(n+1) +



(n-1)(n+1) +

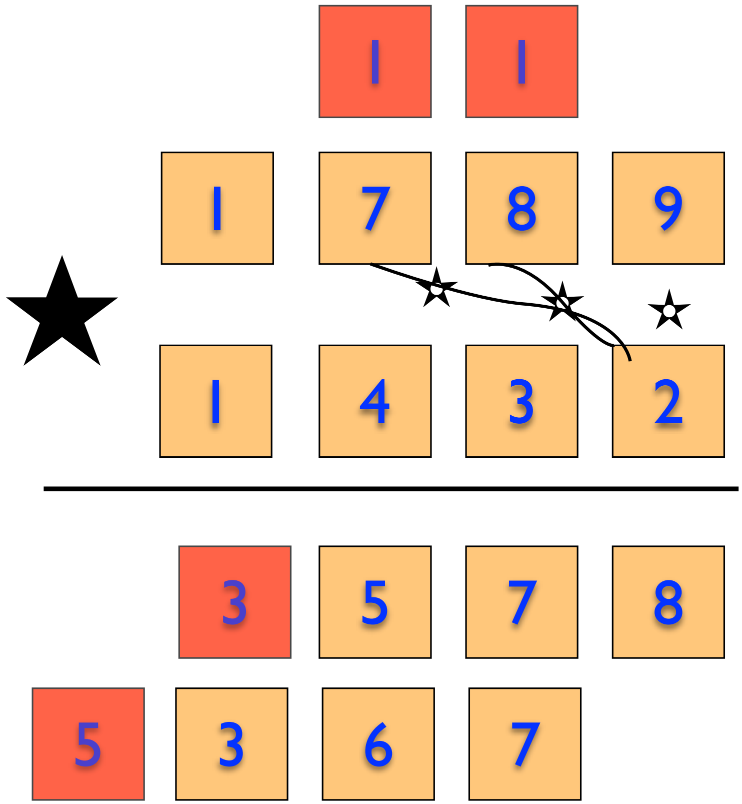


(n-1)(n+1) +



(n-1)(n+1) ✢

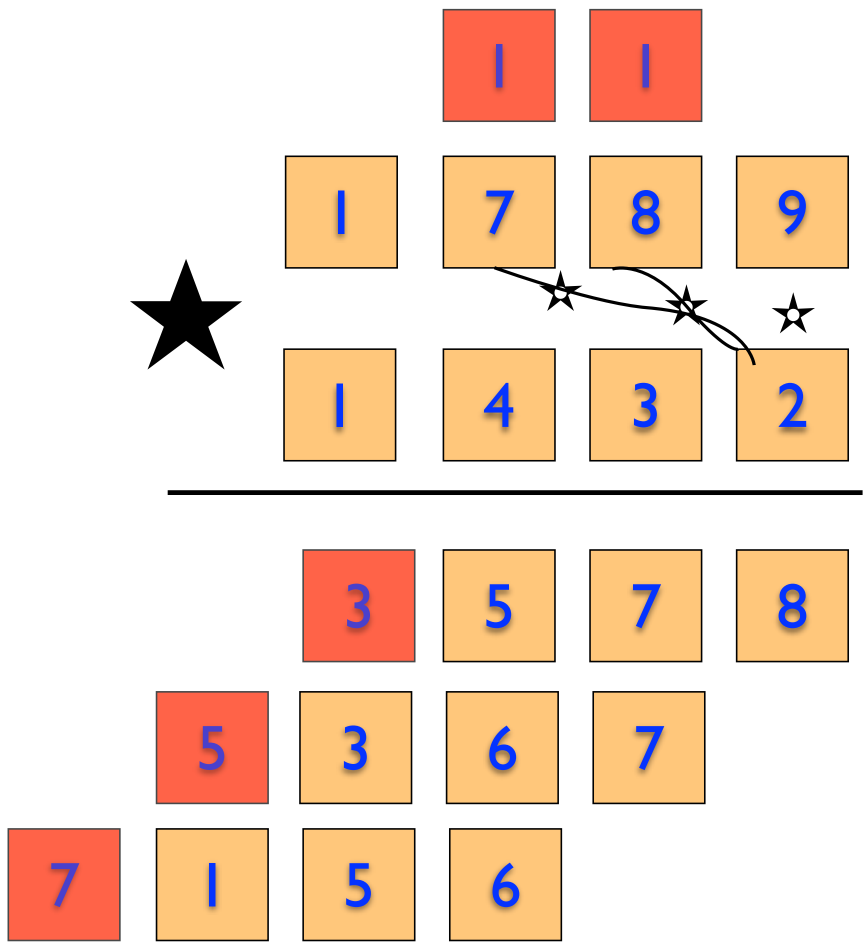
n✫ n-1 ✢



(n-1)(n+1) ✢

n✩ n-1 ✢

n✩ n-1 ✢

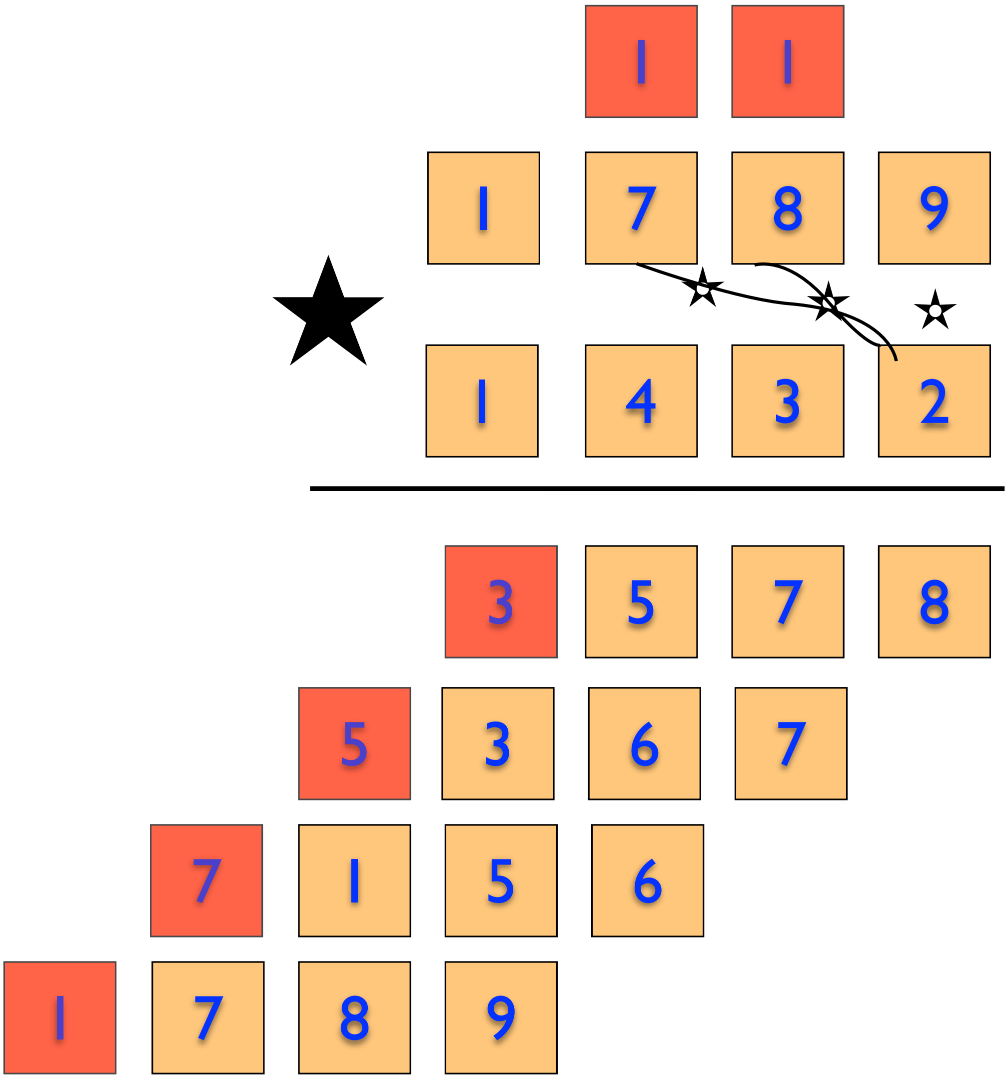


(n-1)(n+1) ✢

n☆ n-1 ✢

n☆ n-1 ✢

n☆ n-1 ✢



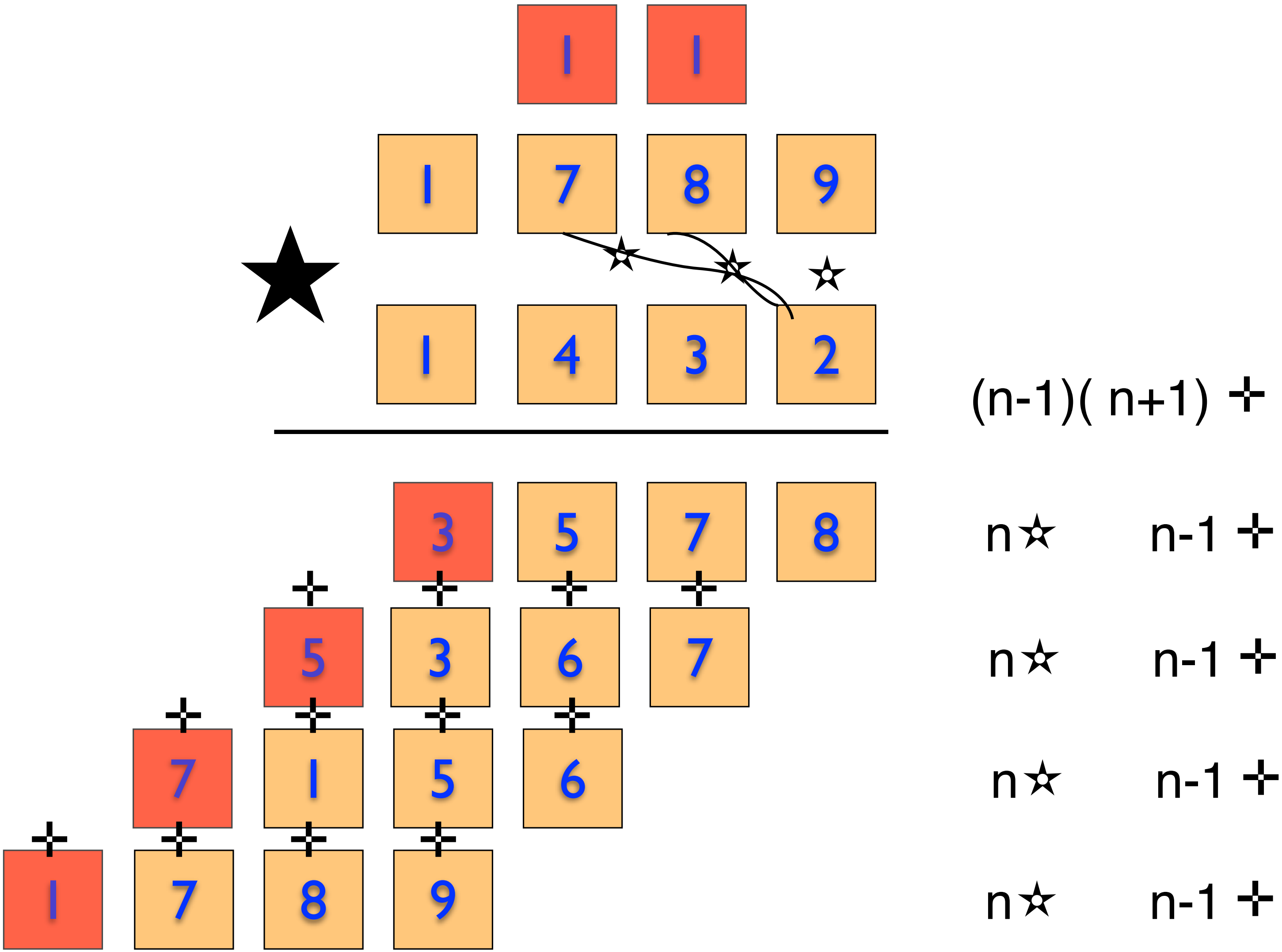
(n-1)(n+1) ✛

n☆ n-1 ✛

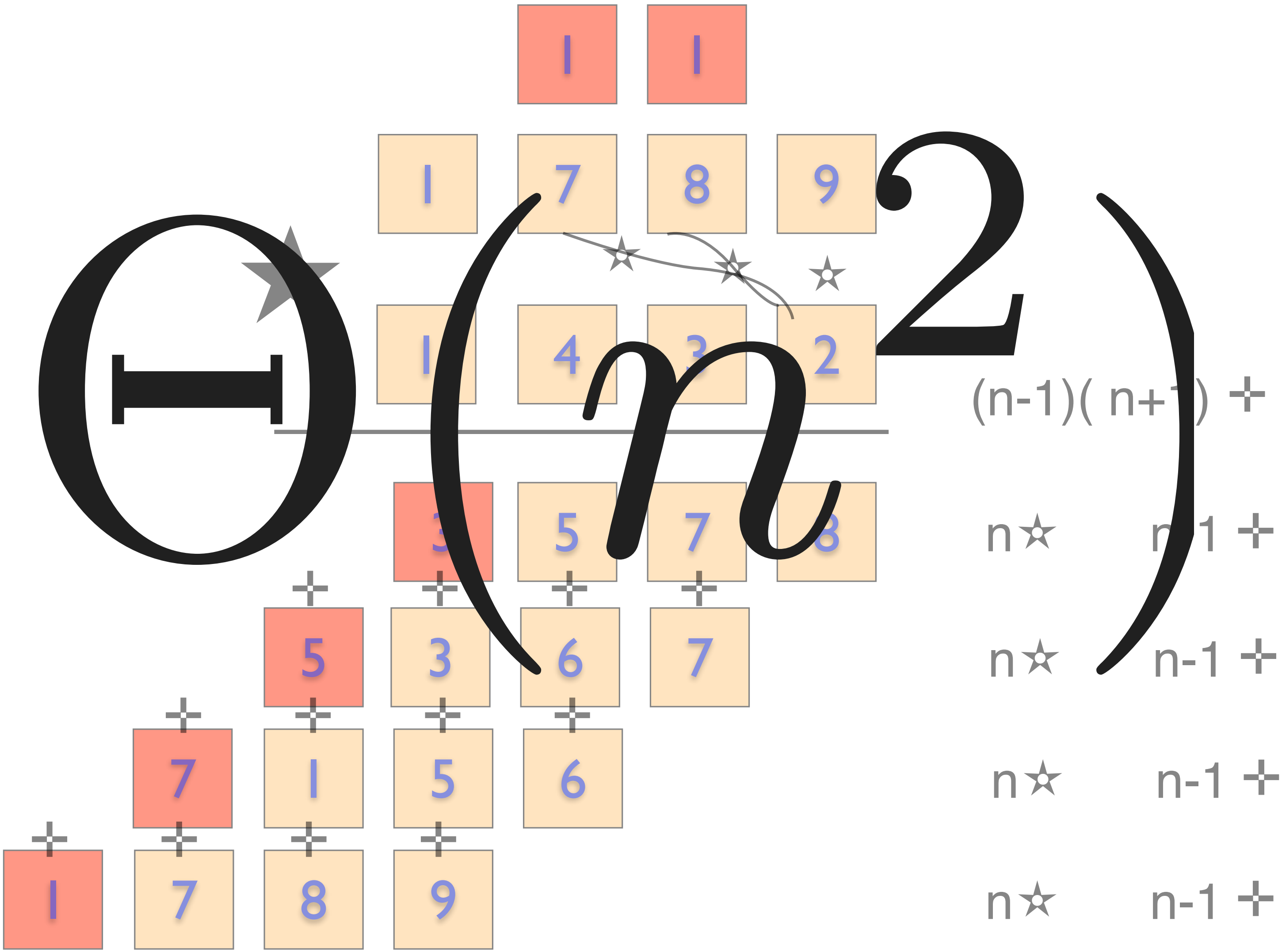
n☆ n-1 ✛

n☆ n-1 ✛

n☆ n-1 ✛


1 1
1 7 8 9
1 4 3 2
3 5 7 8
5 3 6 7
7 1 5 6
1 7 8 9
(n-1)(n+1) ✛
n☆ n-1 ✛
n☆ n-1 ✛
n☆ n-1 ✛
n☆ n-1 ✛

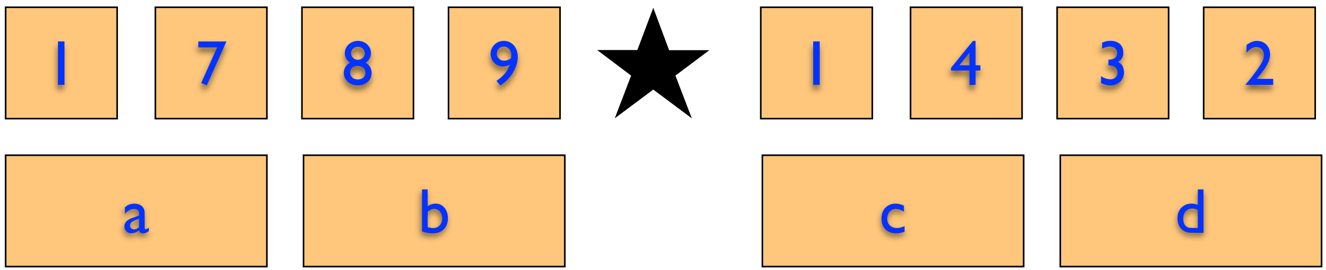
$$\Theta(n^2)$$

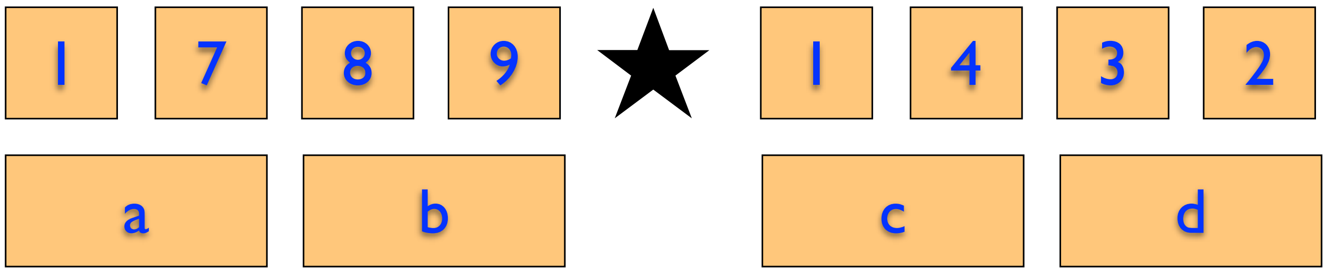


Theme 1

A first attempt…

1	7	8	9	★	1	4	3	2

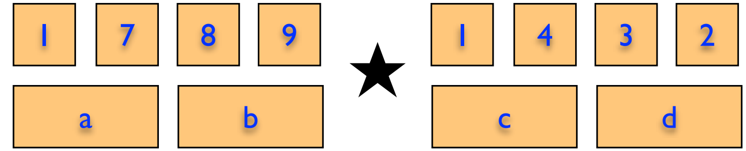

1
7
8
9
1
4
3
2
a
b
c
d



$$ac100^2 + (ad + bc)100 + bd$$

n-digit inputs

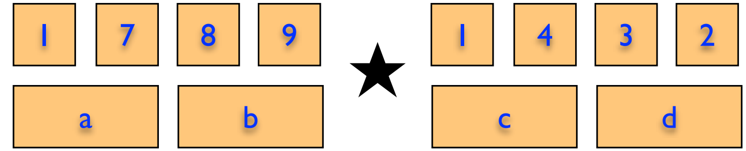
Mult(ab, cd)



$$ac100^2 + (ad + bc)100 + bd$$

Base case: return b*d if inputs are 1-digit

Mult(ab, cd)



$$ac100^2 + (ad + bc)100 + bd$$

Base case: return b*d if inputs are 1-digit

Compute x = Mult(a,c)

Compute y = Mult(a,d)

Compute z = Mult(b,c)

Compute w = Mult(b,d)

Return r = $x*100^2 + (y+z)100 + w$

$$T(n) = 4T(n/2) + 3O(n)$$

calculations: