L1 5800 Algorithms

abhi shelat

Jan 18 2022

let me intro myself

first goal: create an amazing learning experience second goal: share basic beautiful ideas from computer science third goal: help prepare you for a job in cs

what is this course about?

Theme 1

Small problems are easy to solve

Theme 1

Small problems are easy to solve

Solve big problems by making them into smaller ones

Theme 2

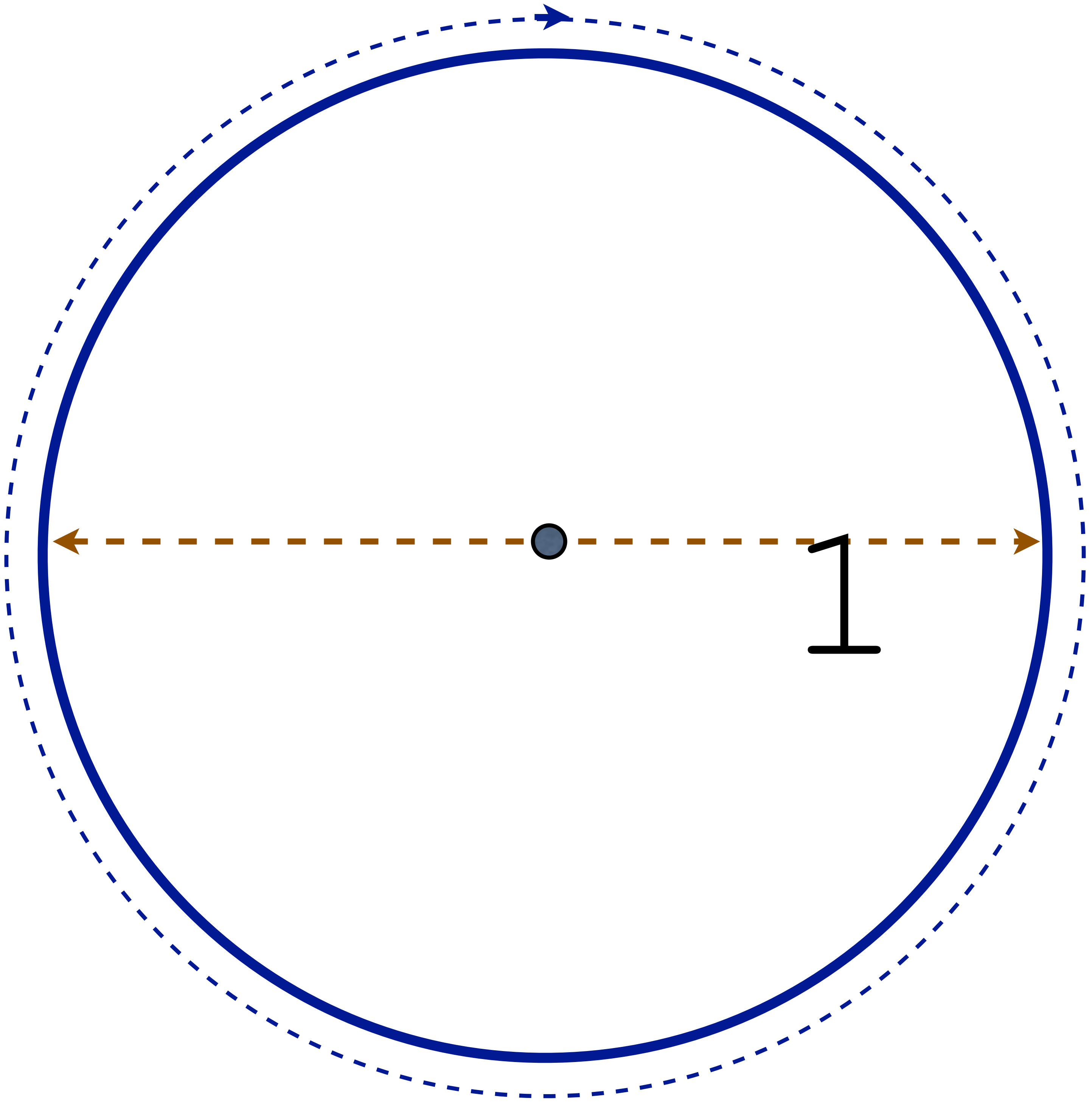
Learning how to convince through reason is a great mark of understanding

great pyramid at giza 2500bc

image from wikimedia

$$\pi$$

http://www.cupertino.org/inc/pdf/apple/Renderings.pdf



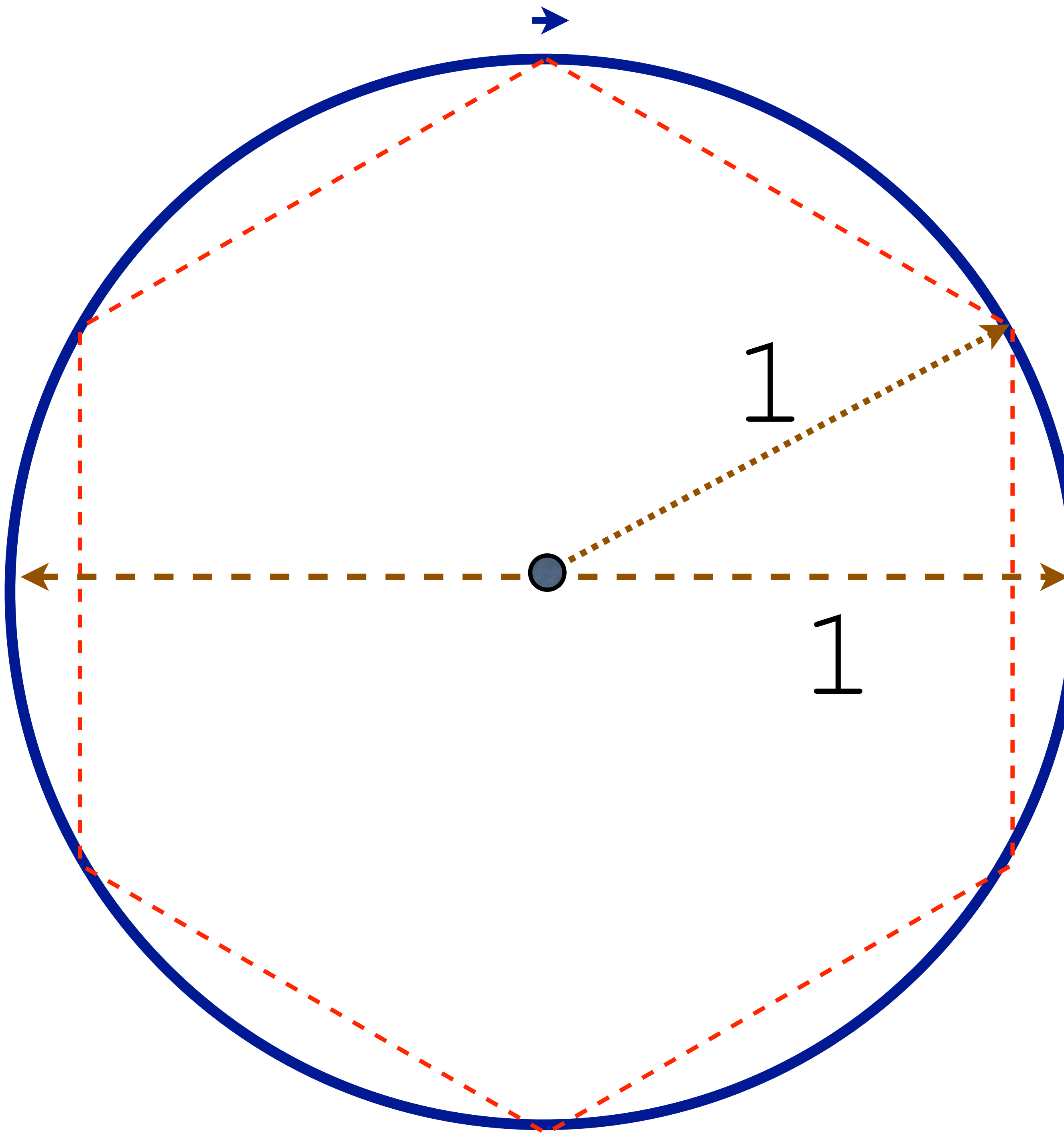
“how much granite/glass do i need?”

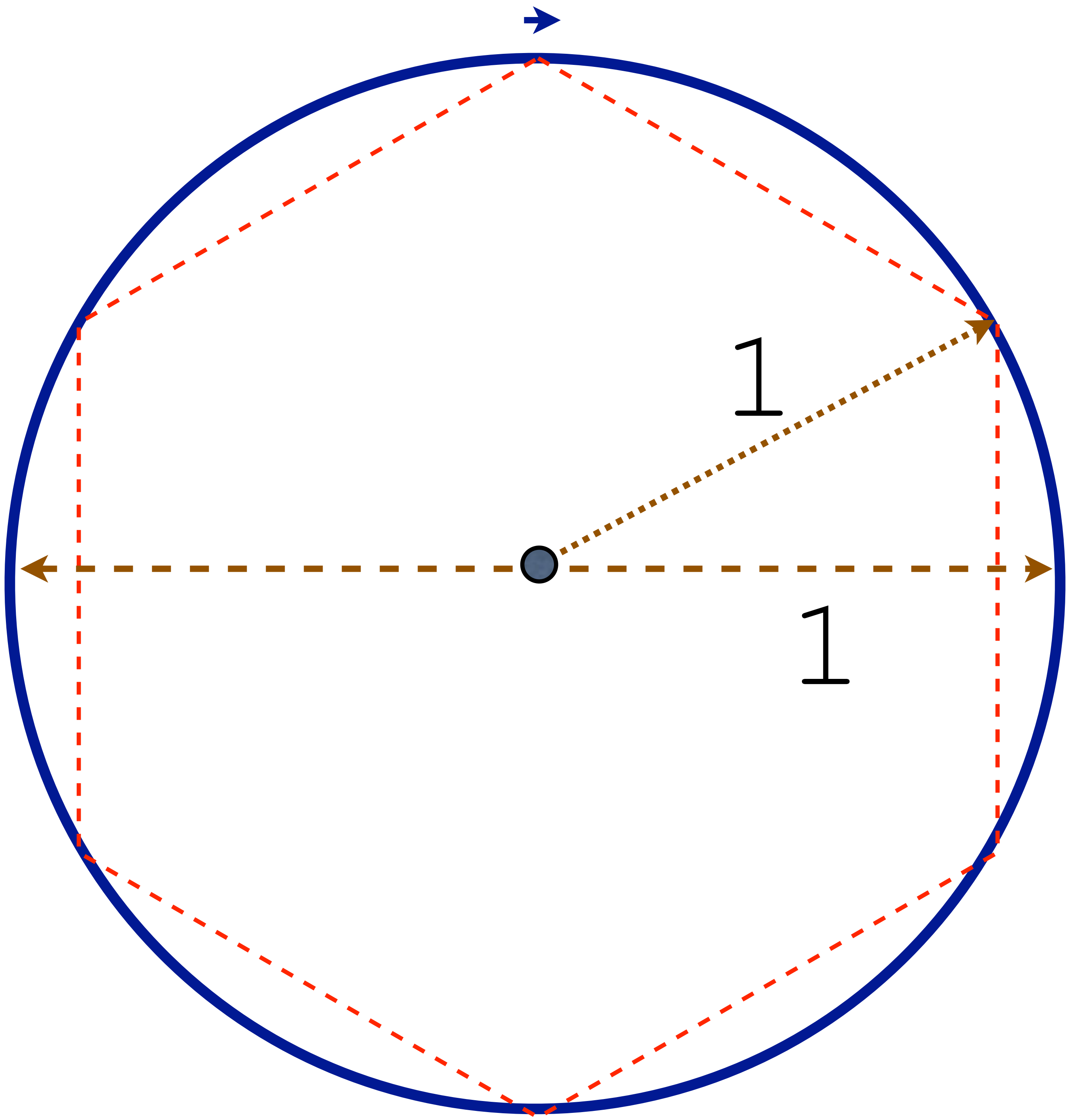
algorithm to compute

$$\pi$$

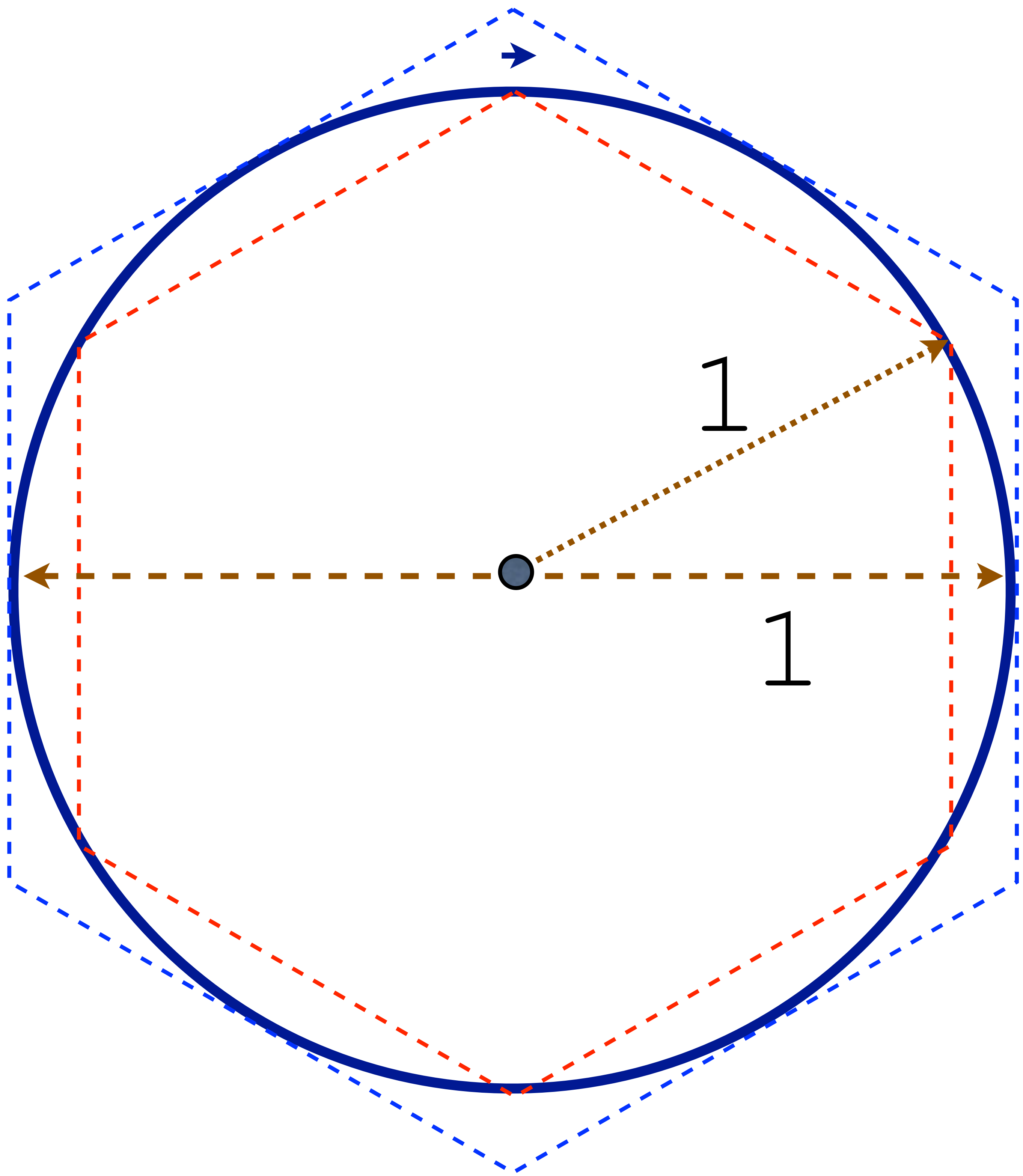
Written down by Archimedes

Idea: It is OK to approximate

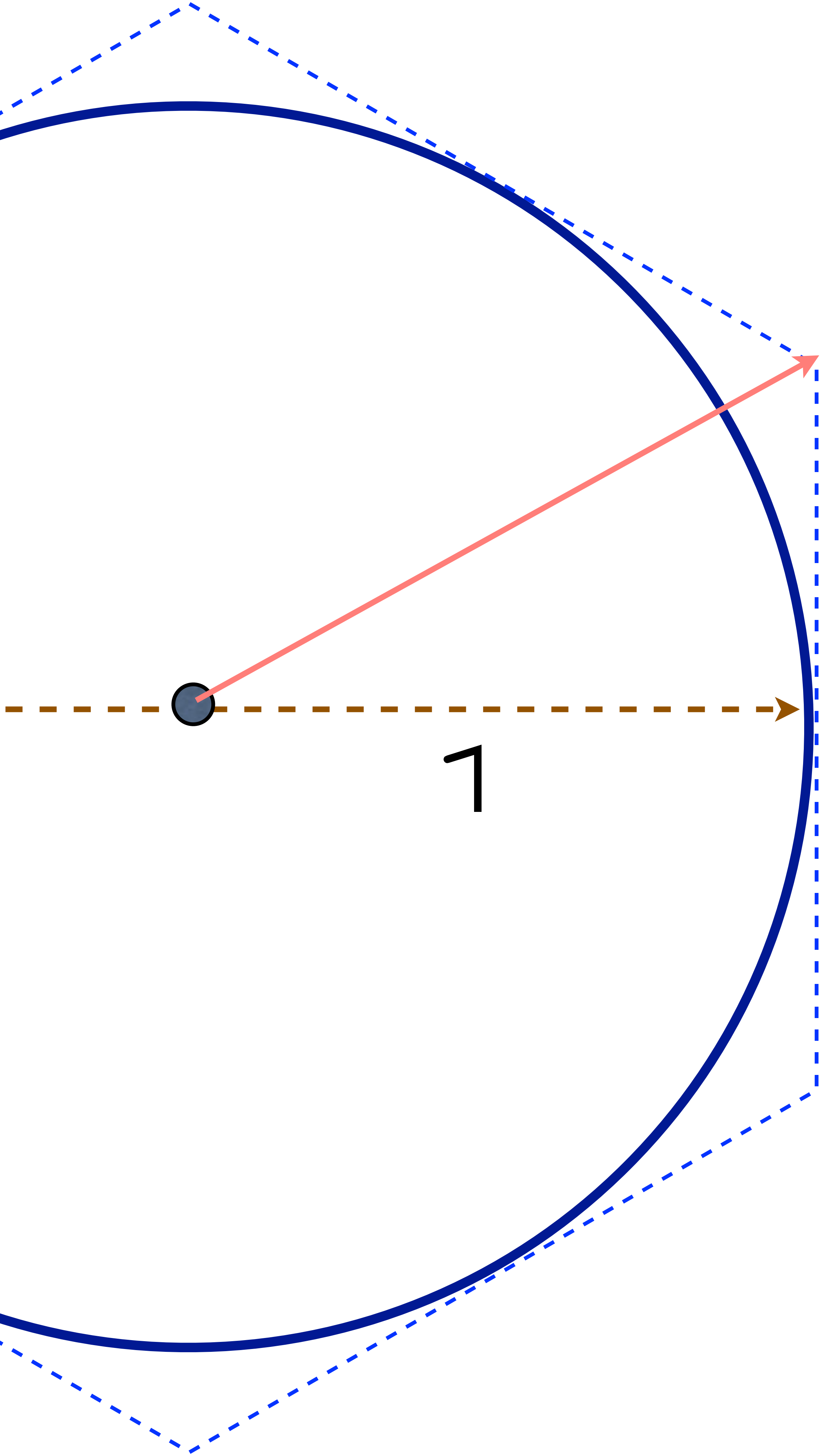

1
1



red perimeter $< \pi d$



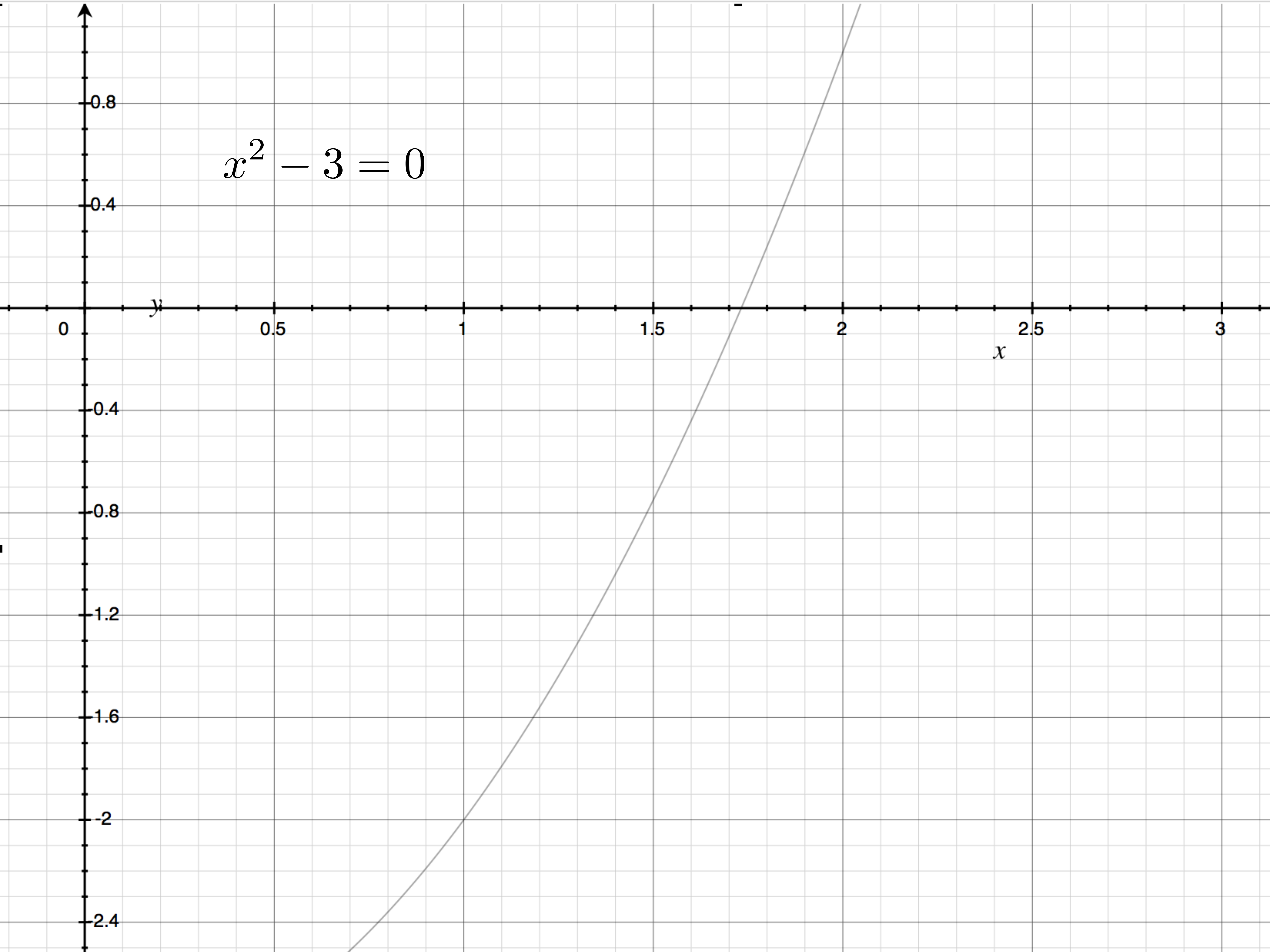
red perimeter $< \pi d <$ blue perimeter


1

But what is

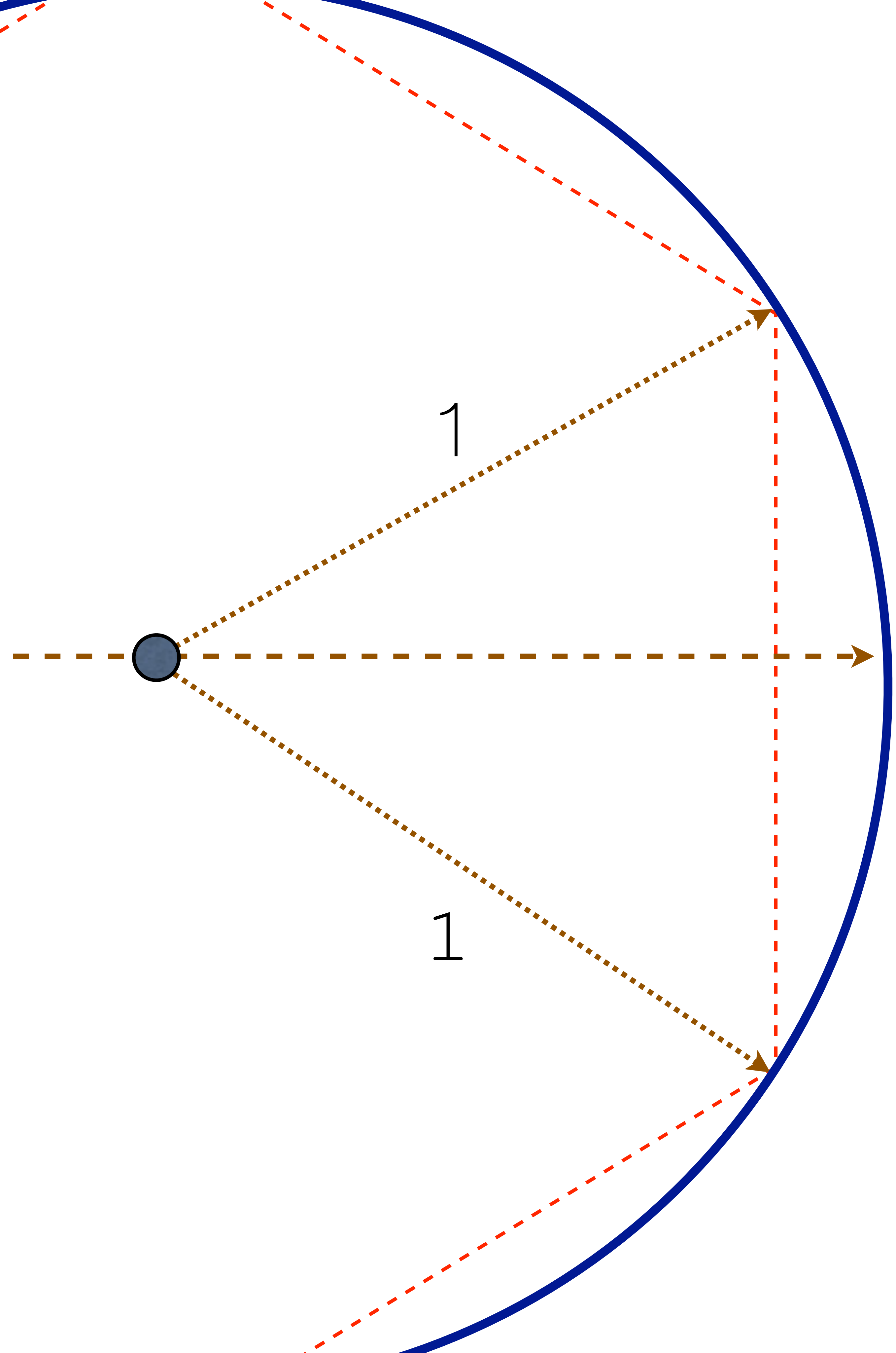
$$\sqrt{3}$$

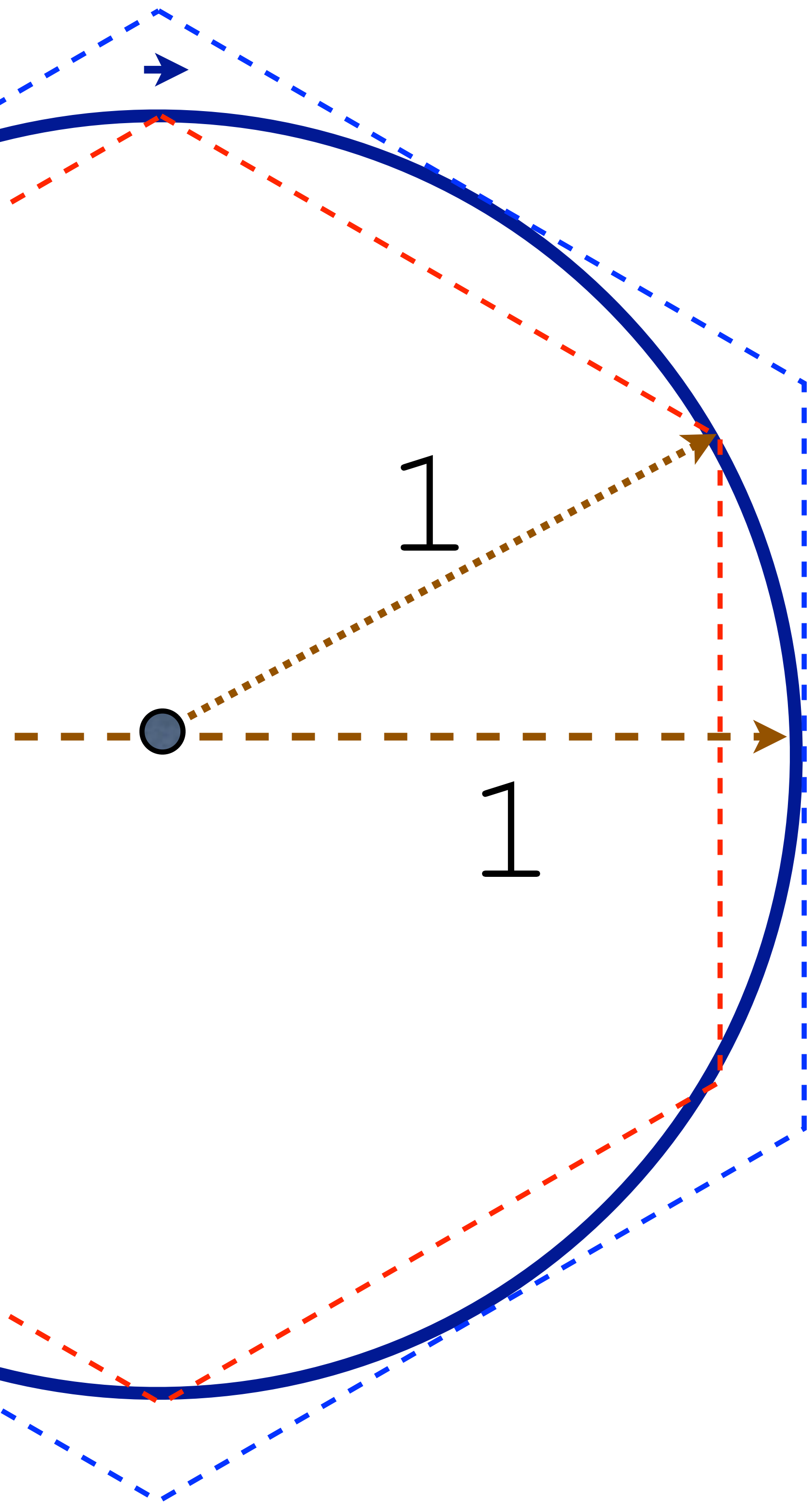
Theme1 : reduce the main problem to a simpler one

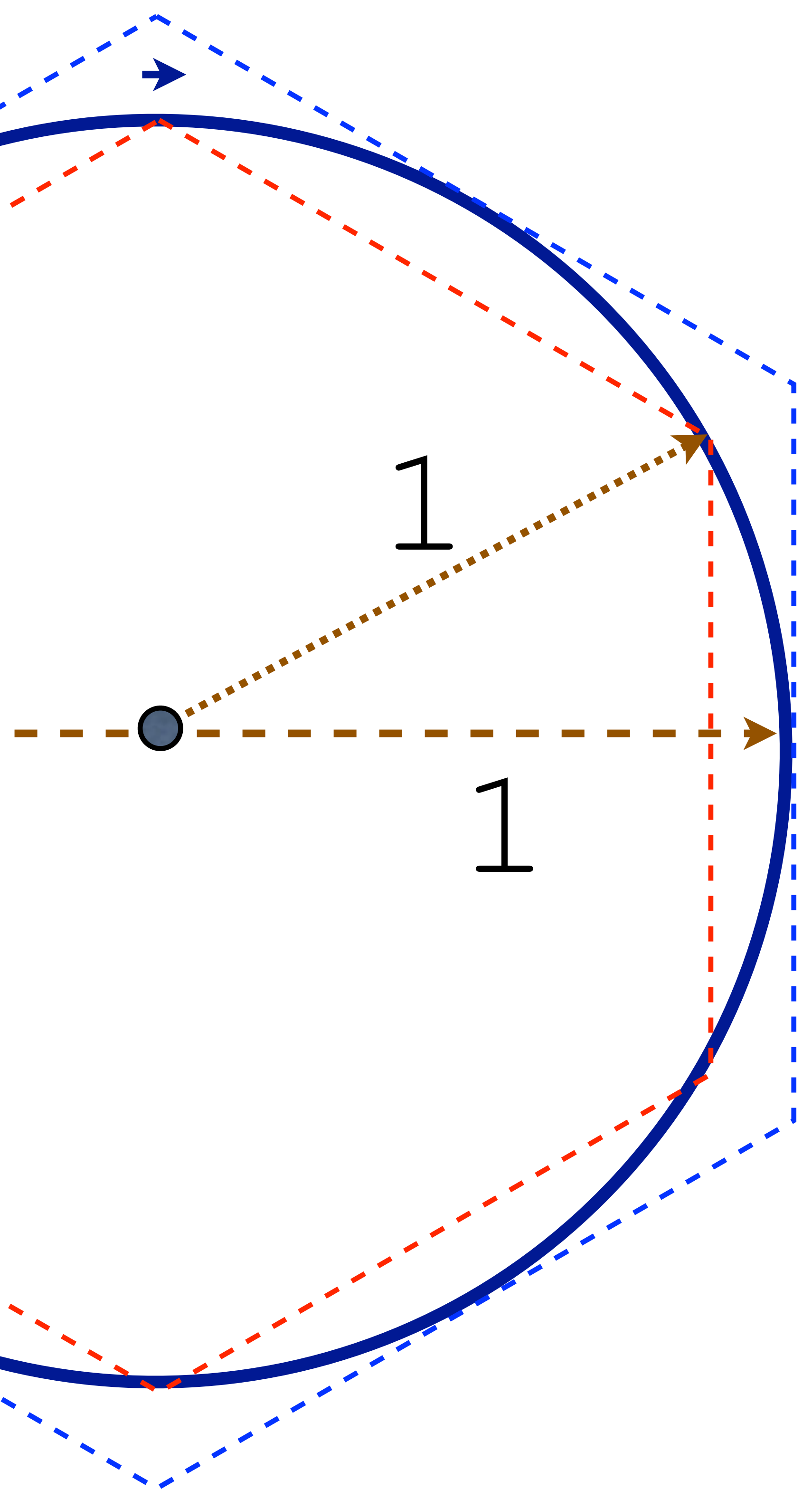


$x^2 - 3 = 0$
0.8
0.4
-0.4
-0.8
-1.2
-1.6
-2
-2.4
0
0.5
1
1.5
2
2.5
3
x
y

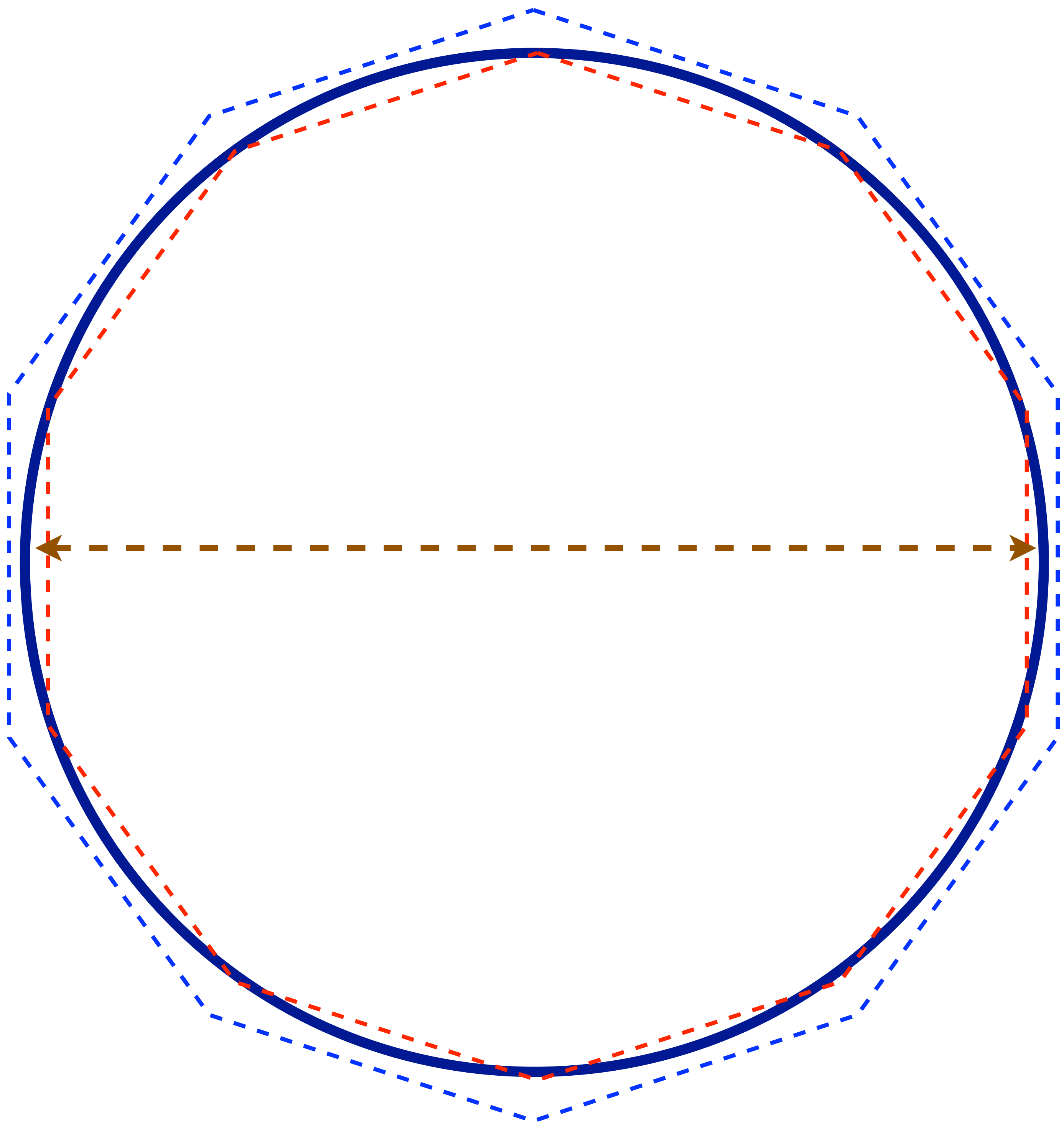
$$\frac{265}{153} \approx \sqrt{3}$$


1
1


1
1



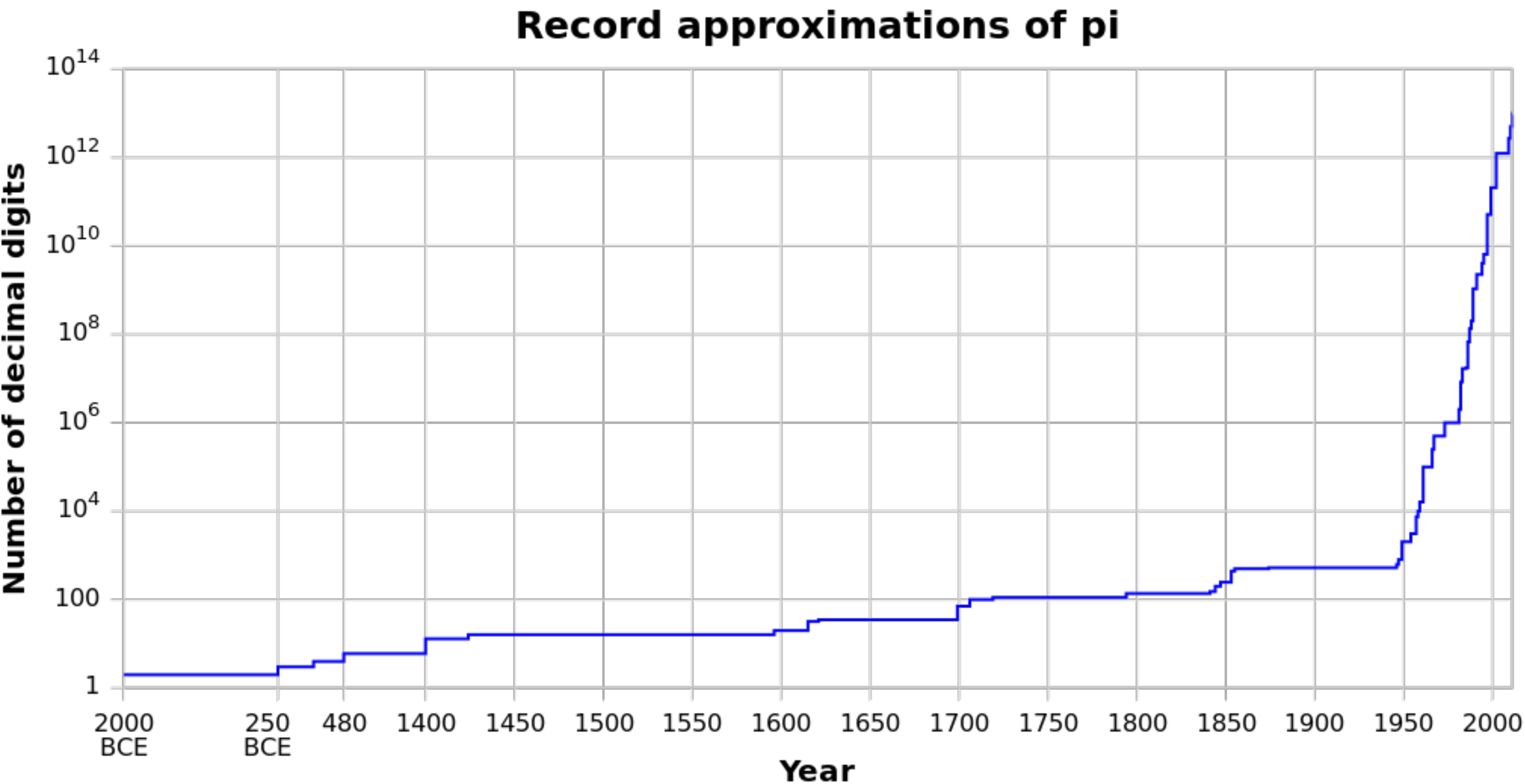
red perimeter $< \pi d <$ blue perimeter



$$3\frac{10}{70} > \pi > 3\frac{10}{71}$$

Using 96-gon,
Archimedes

how to analyze this approach?



https://en.wikipedia.org/wiki/Approximations_of_%CF%80#/media/File:Record_pi_approximations.svg

Theme3: new insights lead to improved efficiency

$$\pi = \frac{9801}{\sqrt{8}} \left(\sum_{n=0}^{\infty} \frac{(4n)!(1103 + 26390n)}{(n!)^4 396^{4n}} \right)^{-1}$$

$$\pi = \frac{9801}{\sqrt{8}} \left(\sum_{n=0}^{\infty} \frac{(4n)!(1103 + 26390n)}{(n!)^4 396^{4n}} \right)^{-1}$$

n=0

$$\pi = \frac{9801}{\sqrt{8}} \left(\sum_{n=0}^{\infty} \frac{(4n)!(1103 + 26390n)}{(n!)^4 396^{4n}} \right)^{-1}$$

n=0

$$\pi \approx_0 \frac{9801}{\sqrt{8}} [1103]^{-1}$$

3.141592730013305760174

$$\pi = \frac{9801}{\sqrt{8}} \left(\sum_{n=0}^{\infty} \frac{(4n)!(1103 + 26390n)}{(n!)^4 396^{4n}} \right)^{-1}$$

n=1

$$\pi = \frac{9801}{\sqrt{8}} \left(\sum_{n=0}^{\infty} \frac{(4n)!(1103 + 26390n)}{(n!)^4 396^{4n}} \right)^{-1}$$

n=1

$$\pi \approx_1 \frac{9801}{\sqrt{8}} \left[1103 + \frac{24 \cdot 27493}{396^4} \right]^{-1}$$

3.14159265358979387799890582630

benefits?

good algorithms touch every aspect of our lives

Google

FedEx

TESLA MOTORS

amazon

NETFLIX

at&t

good algorithms defend freedom

what skills do you need for this course?

precision

creativity

in·ge·nu·i·ty

how to learn in this class

no cookbook

develop
general problem
solving
skills

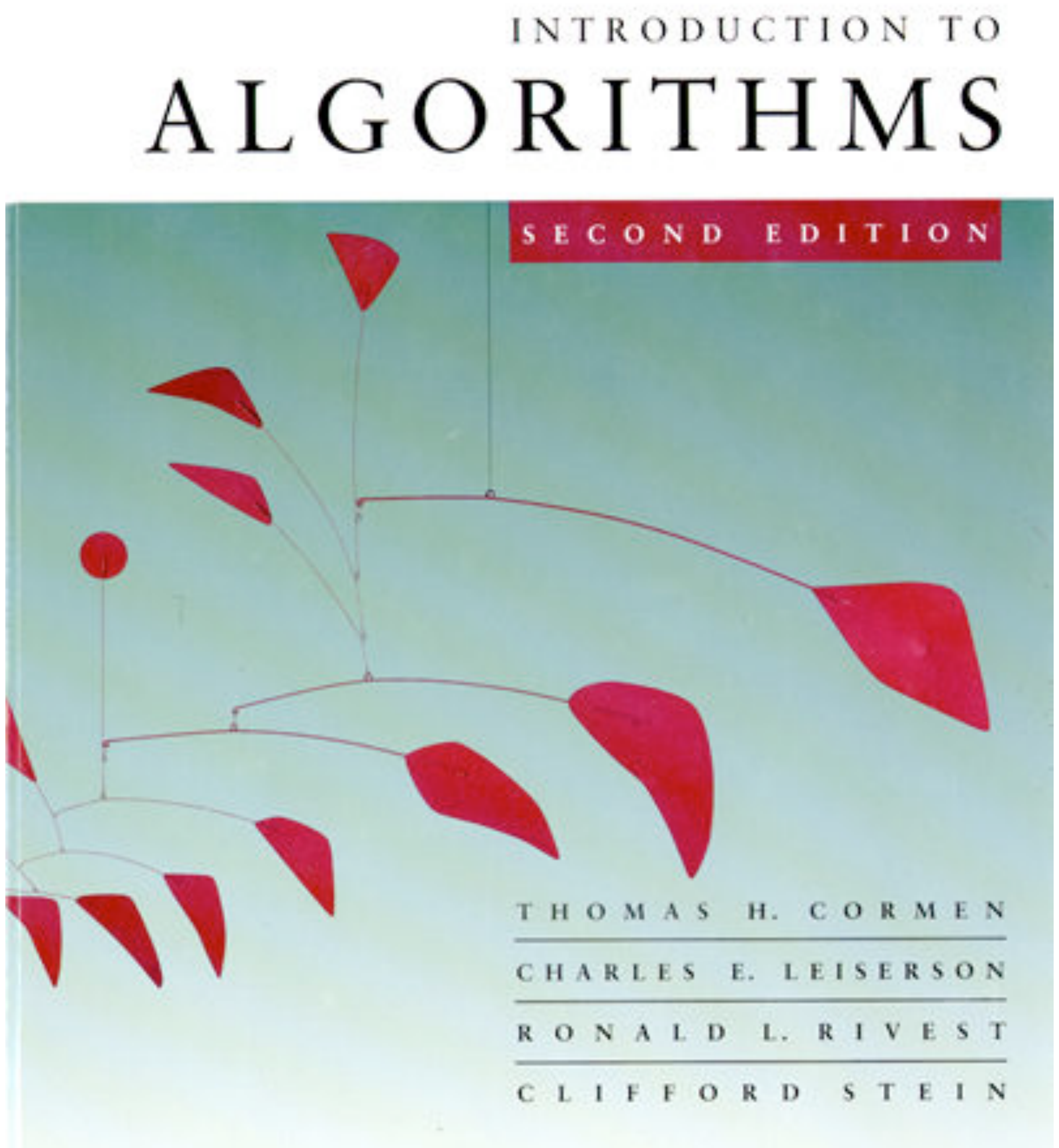
understand
known
techniques

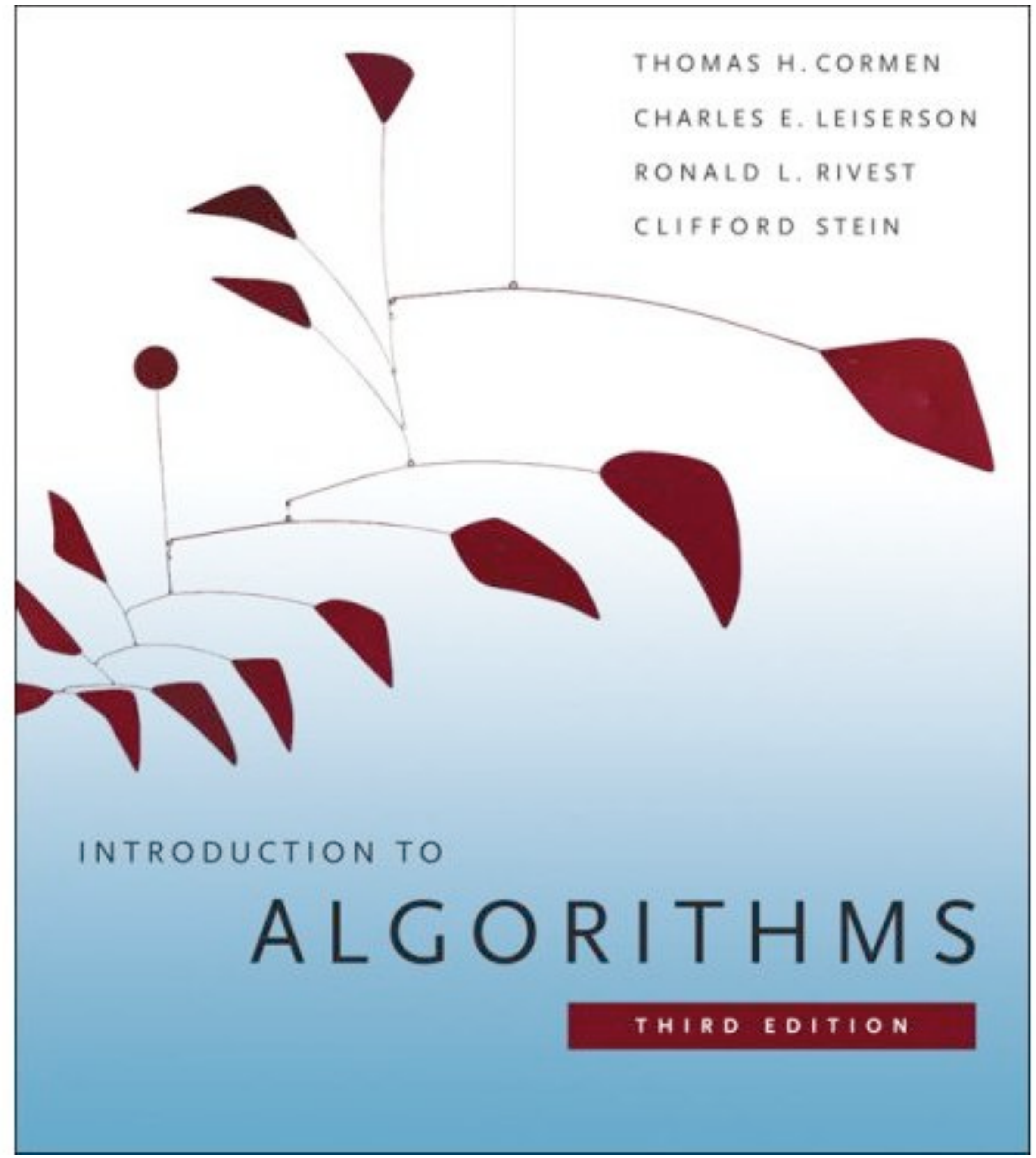
work with your peers

work with your peers

but do not copy

https://
shelat.khoury
.northeastern
.edu/22s-5800


INTRODUCTION TO
ALGORITHMS
SECOND EDITION
THOMAS H. CORMEN
CHARLES E. LEISERSON
RONALD L. RIVEST
CLIFFORD STEIN


THOMAS H. CORMEN
CHARLES E. LEISERSON
RONALD L. RIVEST
CLIFFORD STEIN
INTRODUCTION TO
ALGORITHMS
THIRD EDITION

coursera

Catalog | algorithms

Instituti

You searched for **algorithms.** 118 matches

Availability

- [ ] This Month 53
- [ ] 1-2 months 2
- [ ] 2-3 months 1

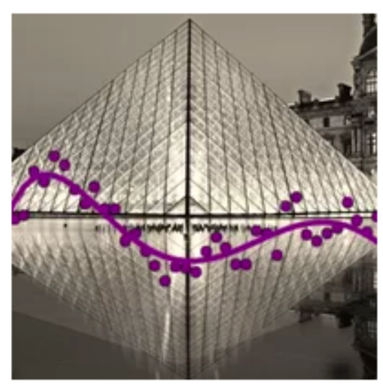
Show More

All Topics

- [ ] Computer Science 54
- [ ] Data Science 41
- [ ] Life Sciences 13

Algorithms, Part I

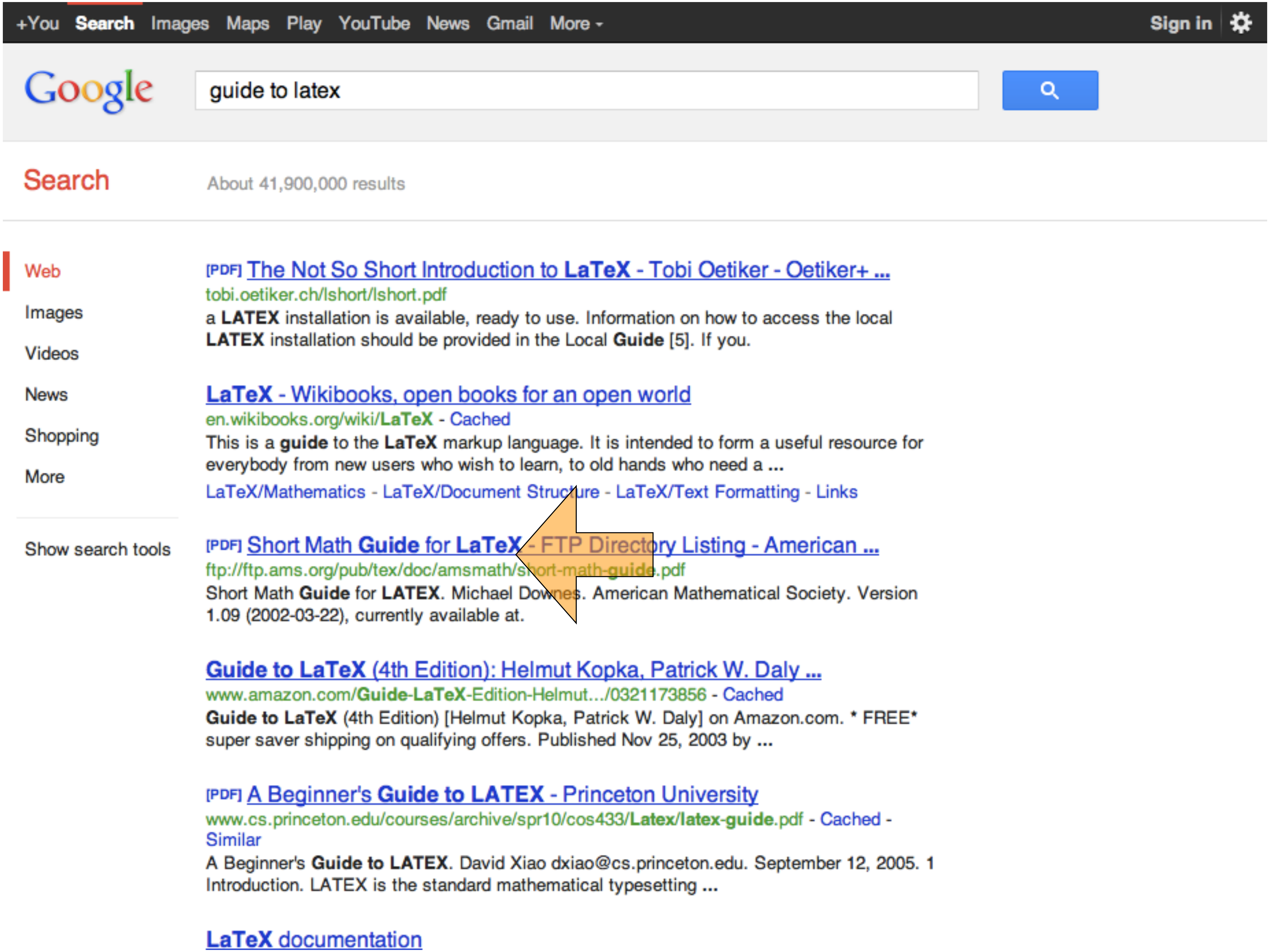
Princeton University



Approximation Algorithms Part I

École normale supérieure

LaTeX


+You Search Images Maps Play YouTube News Gmail More
Sign in
Google
guide to latex
Search
About 41,900,000 results
Web
Images
Videos
News
Shopping
More
Show search tools
[PDF] The Not So Short Introduction to LaTeX - Tobi Oetiker - Oetiker+ ...
tobi.oetiker.ch/lshort/lshort.pdf
a LATEX installation is available, ready to use. Information on how to access the local LATEX installation should be provided in the Local Guide [5]. If you.
LaTeX - Wikibooks, open books for an open world
en.wikibooks.org/wiki/LaTeX - Cached
This is a guide to the LaTeX markup language. It is intended to form a useful resource for everybody from new users who wish to learn, to old hands who need a ...
LaTeX/Mathematics - LaTeX/Document Structure - LaTeX/Text Formatting - Links
[PDF] Short Math Guide for LaTeX - FTP Directory Listing - American ...
ftp://ftp.ams.org/pub/tex/doc/amsmath/short-math-guide.pdf
Short Math Guide for LATEX. Michael Downes. American Mathematical Society. Version 1.09 (2002-03-22), currently available at.
Guide to LaTeX (4th Edition): Helmut Kopka, Patrick W. Daly ...
www.amazon.com/Guide-LaTeX-Edition-Helmut.../0321173856 - Cached
Guide to LaTeX (4th Edition) [Helmut Kopka, Patrick W. Daly] on Amazon.com. * FREE* super saver shipping on qualifying offers. Published Nov 25, 2003 by ...
[PDF] A Beginner's Guide to LATEX - Princeton University
www.cs.princeton.edu/courses/archive/spr10/cos433/Latex/latex-guide.pdf - Cached - Similar
A Beginner's Guide to LATEX. David Xiao dxiao@cs.princeton.edu. September 12, 2005. 1 Introduction. LATEX is the standard mathematical typesetting ...
LaTeX documentation

The Not So Short Introduction to $\LaTeX\,2_\varepsilon$

Or $\LaTeX\,2_\varepsilon$ in 157 minutes

by Tobias Oetiker

TeXShop File Edit Source Macros Typeset Preview Window Help 100%

hw0.tex

Typeset LaTeX Macros Tags Templates

```latex
%--------change this by adding your uva id into the {}
\def\yourname{}
% --- no need to change anything in this section ------------------------------------------------------
\def\homework{1} % 0 for solution, 1 for problem-set only
\def\duedate{fri jan 29, 2016 at 5p}
\def\duelocation{via \href{https://church.cs.virginia.edu/16s-4102/}{submission site}}
\def\hnumber{0}
\def\prof{abhi shelat}
\def\course{\href{https://www.cs.virginia.edu/~shelat/16s-4102}{cs4102 - algorithms - s'16}}

\documentclass[11pt]{article}
%------------------------------------------------------------------------------------------------------

%% ================ no need to edit any of this stuff


%%% ==== standard installations of latex include all of the files that are referenced in this section.  However,
%%% ==== if you are having compile problems, consider commenting some of these commands out
\usepackage[colorlinks,urlcolor=blue]{hyperref}
\usepackage[osf]{mathpazo}
\usepackage{amsmath,amsfonts,graphicx}
\usepackage{latexsym}
\usepackage[top=1in,bottom=1.4in,left=1.5in,right=1.5in,centering]{geometry}
\usepackage{color}
\definecolor{mdb}{rgb}{0.3,0.02,0.02}
\definecolor{cit}{rgb}{0.05,0.2,0.45}
\markboth{\yourname}{\yourname}
%%% ==================================================================


%%% =========== should be no need to edit anything in this section ===================
\newenvironment{proof}{\par\noindent{\it Proof.}\hspace*{1em}}{$\Box$\bigskip}
\newcommand{\qed}{$\Box$}
\newcommand{\alg}[1]{\mathsf{#1}}
\newcommand{\handout}{
  \renewcommand{\thepage}{H\hnumber-\arabic{page}}%
  \noindent%
  \begin{center}%
    \vbox{%
   \hbox to \columnwidth {\sc{\course} --- abhi shelat \hfill}%
   \vspace{-2mm}%
   \hbox to \columnwidth {\sc due \MakeLowercase{\duedate} \duelocation\hfill {\Huge\color{mdb}H\hnumber.
\yourname}}%
    }
  \end{center}
```

hw0.pdf

Scale 115 Page 1 of 1

CS4102 - ALGORITHMS - S'16 — ABHI SHELAT
DUE FRI JAN 29, 2016 AT 5P VIA SUBMISSION SITE

You may collaborate with other students on the homework but
your own individually written solution, identify your collaborat
edge any external sources that you consult.

PROBLEM 1 *Passage*

Typeset your favorite passage from a book.

PROBLEM 2 *Asymptotic notation*

Let f be a function. Give a formal definition of the set $\Theta(f)$.

$$\Theta(f) = \{ \text{ functions } g \text{ such that } \ldots$$

Hint: Use the `\exists` command to make the "there exists" s
`\forall` command to make the "for all" symbol $\forall$.

PROBLEM 3 *includegraphics command*

Learn how to include drawings in your documents with the `\inc`
command by submitting a caricature of me.

Google

texshop

All Videos Images News Shopping More Search tools

About 189,000 results (0.37 seconds)

TeXShop

pages.uoregon.edu/koch/**texshop**/ University of Oregon

TeXShop (v 3.59) Release 01/01/2016. (Mountain Lion or Higher Strongly Recommended). (for Lion, Mountain Lion, Mavericks, Yosemite, El Capitan) ...

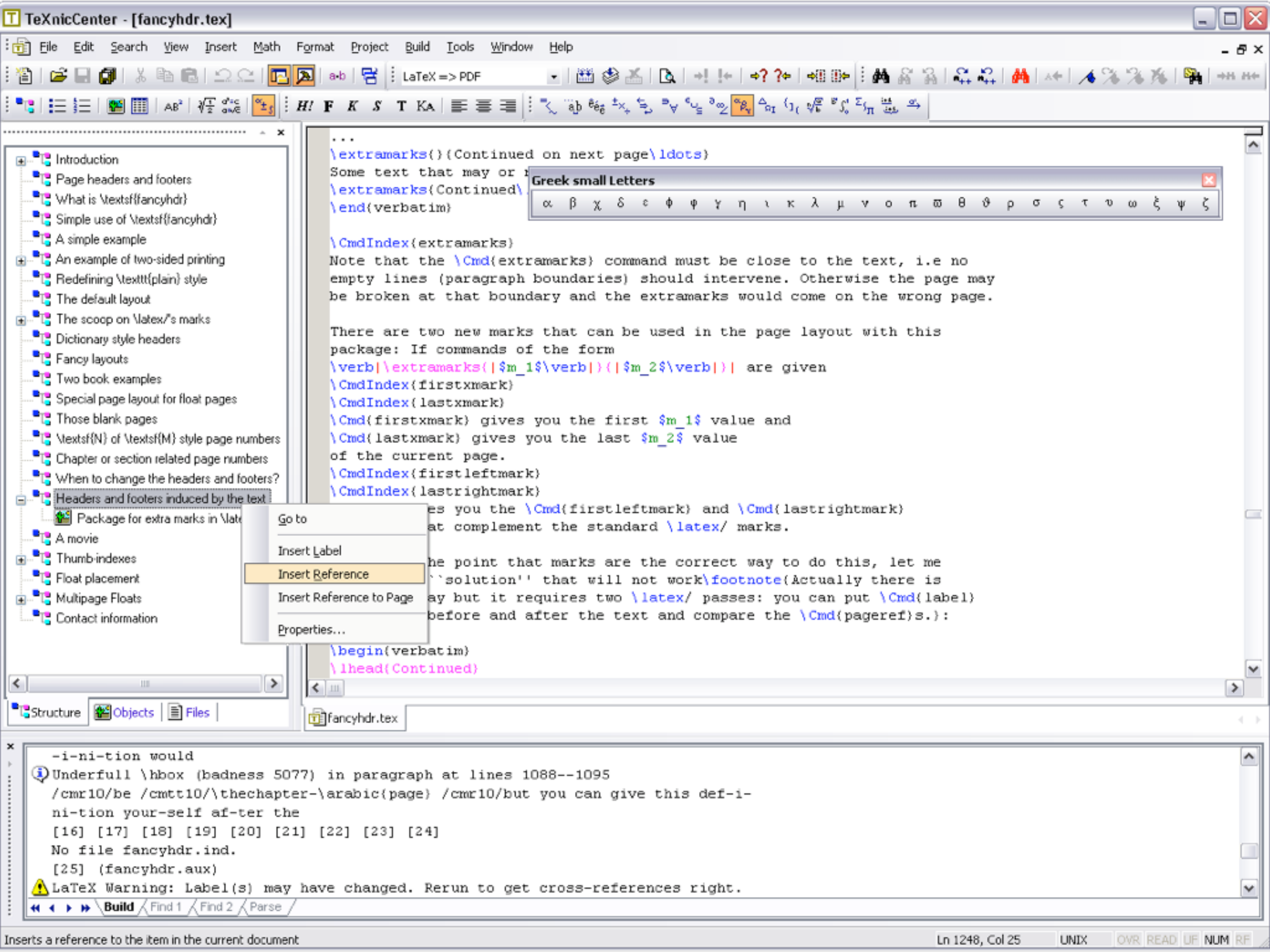
Obtaining TeXShop

Obtaining TeXShop. If you just want to upgrade to the latest ...

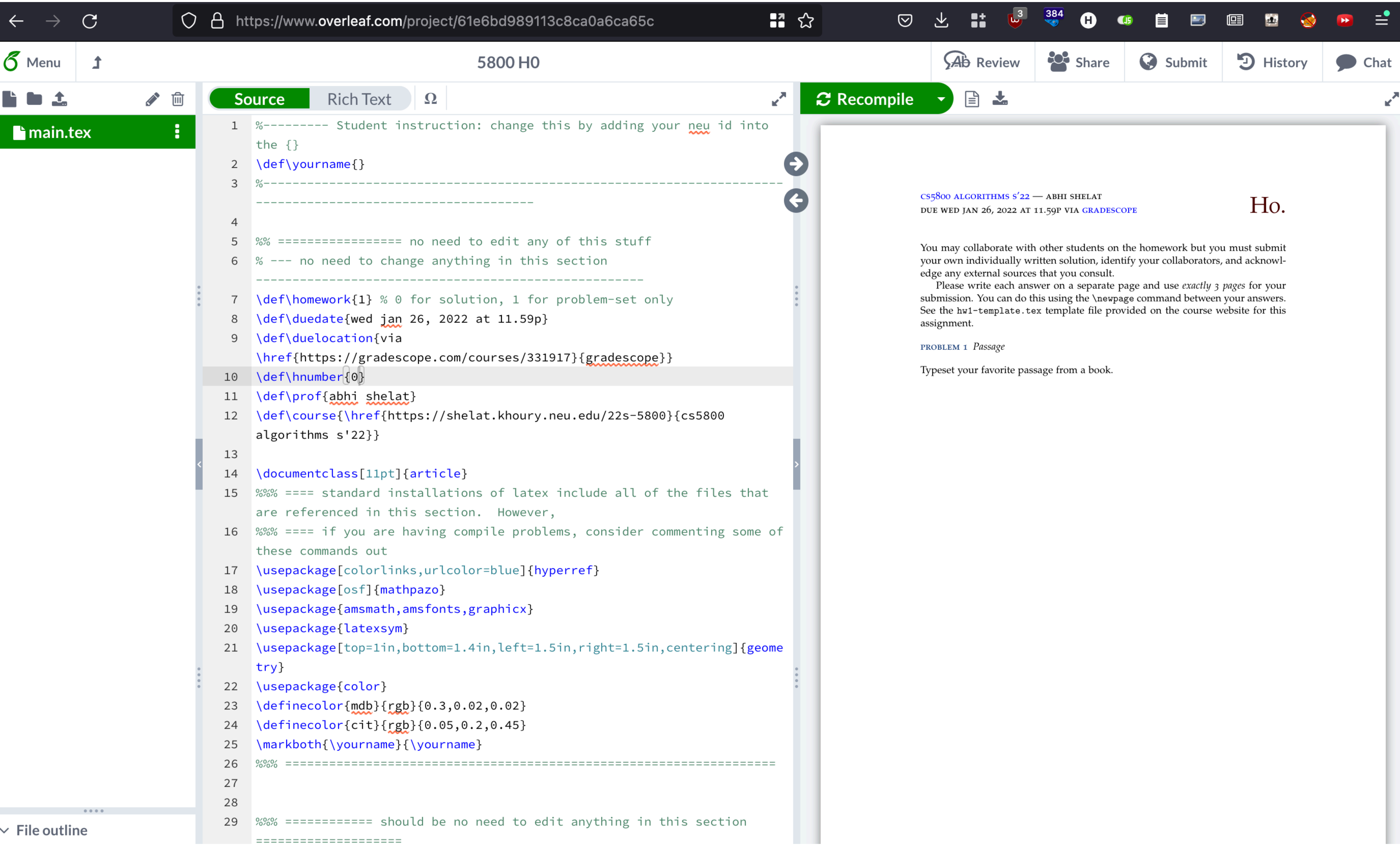
Installing

Direct Download: TeXShop 3 for Lion | Lion Source | TeXShop 2 ...

More results from uoregon.edu »


TeXnicCenter - [fancyhdr.tex]
File Edit Search View Insert Math Format Project Build Tools Window Help
LaTeX => PDF
Introduction
Page headers and footers
What is \textsf{fancyhdr}
Simple use of \textsf{fancyhdr}
A simple example
An example of two-sided printing
Redefining \texttt{plain} style
The default layout
The scoop on \latex/'s marks
Dictionary style headers
Fancy layouts
Two book examples
Special page layout for float pages
Those blank pages
\textsf{N} of \textsf{M} style page numbers
Chapter or section related page numbers
When to change the headers and footers?
Headers and footers induced by the text
Package for extra marks in \late
A movie
Thumb-indexes
Float placement
Multipage Floats
Contact information
Go to
Insert Label
Insert Reference
Insert Reference to Page
Properties...
Structure
Objects
Files
Greek small Letters
...
\extramarks{}{Continued on next page\ldots}
Some text that may or n
\extramarks{Continued\
\end{verbatim}
\CmdIndex{extramarks}
Note that the \Cmd{extramarks} command must be close to the text, i.e no
empty lines (paragraph boundaries) should intervene. Otherwise the page may
be broken at that boundary and the extramarks would come on the wrong page.
There are two new marks that can be used in the page layout with this
package: If commands of the form
\verb|\extramarks{|m_1\verb|}{|m_2\verb|}| are given
\CmdIndex{firstxmark}
\CmdIndex{lastxmark}
\Cmd{firstxmark} gives you the first m_1 value and
\Cmd{lastxmark} gives you the last m_2 value
of the current page.
\CmdIndex{firstleftmark}
\CmdIndex{lastrightmark}
es you the \Cmd{firstleftmark} and \Cmd{lastrightmark}
at complement the standard \latex/ marks.
he point that marks are the correct way to do this, let me
``solution'' that will not work\footnote{Actually there is
ay but it requires two \latex/ passes: you can put \Cmd{label}
before and after the text and compare the \Cmd{pageref}s.}:
\begin{verbatim}
\lhead{Continued}
fancyhdr.tex
-i-ni-tion would
Underfull \hbox (badness 5077) in paragraph at lines 1088--1095
/cmr10/be /cmtt10/\thechapter-\arabic{page} /cmr10/but you can give this def-i-
ni-tion your-self af-ter the
[16] [17] [18] [19] [20] [21] [22] [23] [24]
No file fancyhdr.ind.
[25] (fancyhdr.aux)
LaTeX Warning: Label(s) may have changed. Rerun to get cross-references right.
Build
Find 1
Find 2
Parse
Inserts a reference to the item in the current document
Ln 1248, Col 25
UNIX
OVR
READ
UF
NUM
RF

Overleaf



https://www.overleaf.com/project/61e6bd989113c8ca0a6ca65c
Menu
5800 H0
Review
Share
Submit
History
Chat
Source
Rich Text
Recompile
main.tex
File outline
%--------- Student instruction: change this by adding your neu id into the {}
\def\yourname{}
%---
%% ================ no need to edit any of this stuff
% --- no need to change anything in this section ---
\def\homework{1} % 0 for solution, 1 for problem-set only
\def\duedate{wed jan 26, 2022 at 11.59p}
\def\duelocation{via \href{https://gradescope.com/courses/331917}{gradescope}}
\def\hnumber{0}
\def\prof{abhi shelat}
\def\course{\href{https://shelat.khoury.neu.edu/22s-5800}{cs5800 algorithms s'22}}
\documentclass[11pt]{article}
%%% ==== standard installations of latex include all of the files that are referenced in this section. However,
%%% ==== if you are having compile problems, consider commenting some of these commands out
\usepackage[colorlinks,urlcolor=blue]{hyperref}
\usepackage[osf]{mathpazo}
\usepackage{amsmath,amsfonts,graphicx}
\usepackage{latexsym}
\usepackage[top=1in,bottom=1.4in,left=1.5in,right=1.5in,centering]{geometry}
\usepackage{color}
\definecolor{mdb}{rgb}{0.3,0.02,0.02}
\definecolor{cit}{rgb}{0.05,0.2,0.45}
\markboth{\yourname}{\yourname}
%%% ==
%%% =========== should be no need to edit anything in this section ===================
CS5800 ALGORITHMS S'22 — ABHI SHELAT
DUE WED JAN 26, 2022 AT 11.59P VIA GRADESCOPE
H0.
You may collaborate with other students on the homework but you must submit your own individually written solution, identify your collaborators, and acknowledge any external sources that you consult.
Please write each answer on a separate page and use exactly 3 pages for your submission. You can do this using the \newpage command between your answers. See the hw1-template.tex template file provided on the course website for this assignment.
PROBLEM 1 Passage
Typeset your favorite passage from a book.

Submitting HW

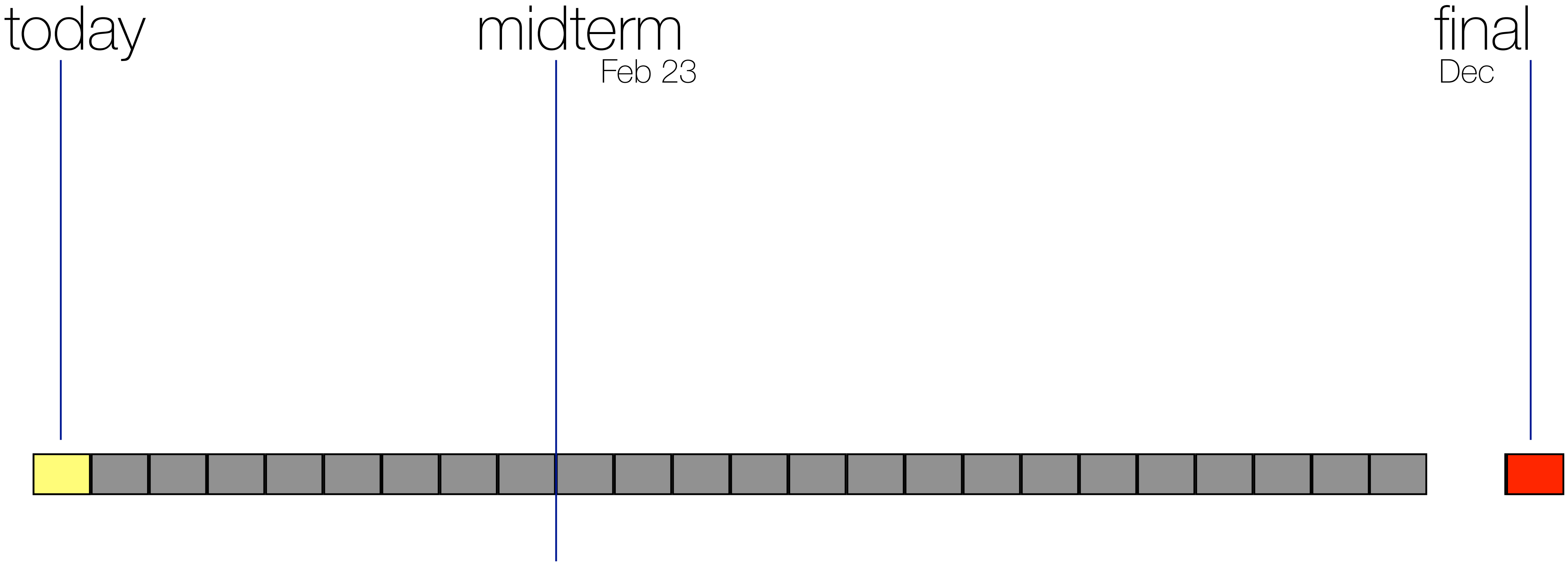
[gradescope]

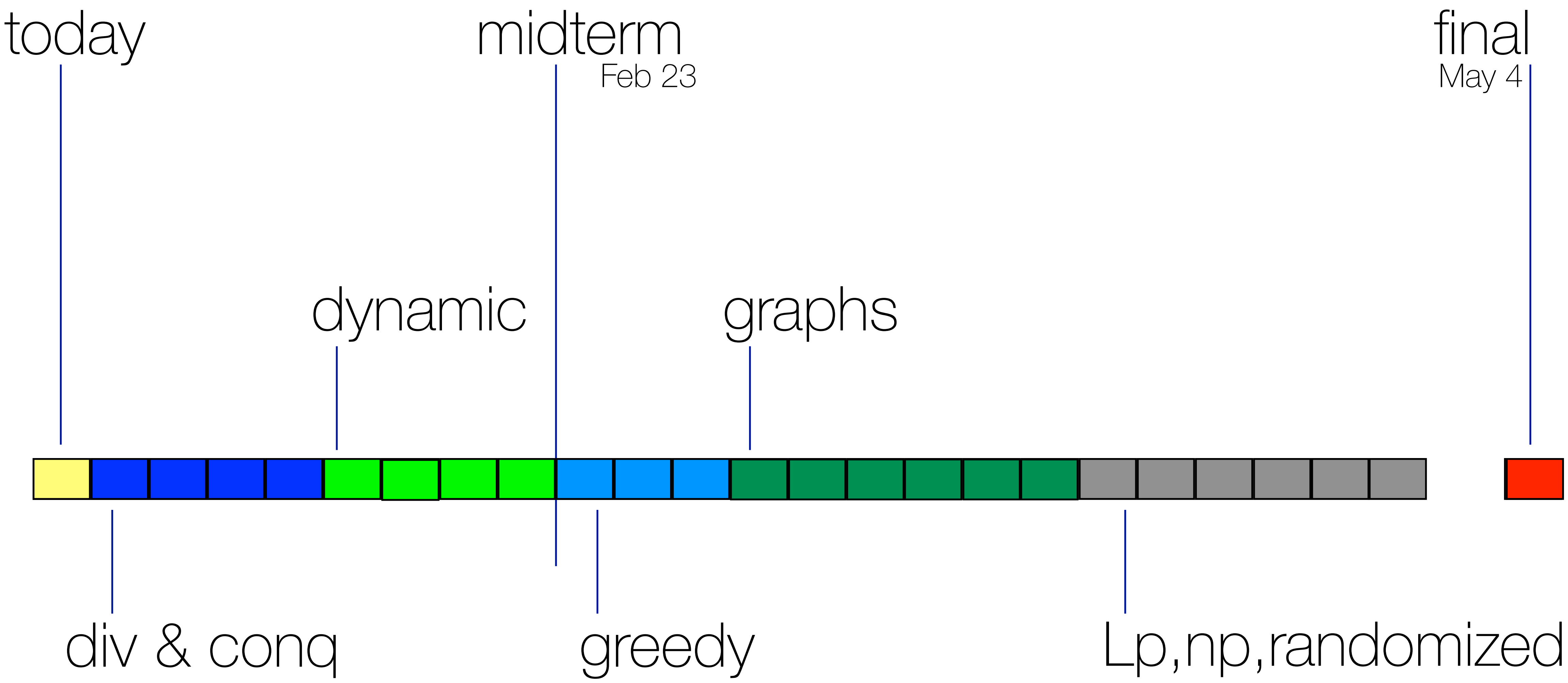
Honor Policy

I, _______, do hereby certify on my honor that during this course,

1. I shall write my answers entirely by myself, and neither share nor request text, code, or drawings.

2. I will not give or derive assistance from any unauthorized sources or the web.


today
midterm
Feb 23
final
Dec


today
midterm
Feb 23
final
May 4
dynamic
graphs
div & conq
greedy
Lp,np,randomized

counting

First example of an algorithmic pattern based on l1 and l2

❶ stand

1. stand
2. set your “number” to one

1. stand
2. set your “number” to one
3. greet your neighbor (pause if no partner)

1. stand
2. set your “number” to one
3. greet your neighbor (pause if no partner)
4. if you are older, give “number” and sit
   if you are younger, add “numbers”

1. stand
2. set your “number” to one
3. greet your neighbor (pause if no partner)
4. if you are older, give “number” and sit
   if you are younger, add “numbers”
5. if you are standing & you have a neighbor, goto 3

1. stand
2. set
3. greet
4. sit/add
5. repeat

lets analyze this alg

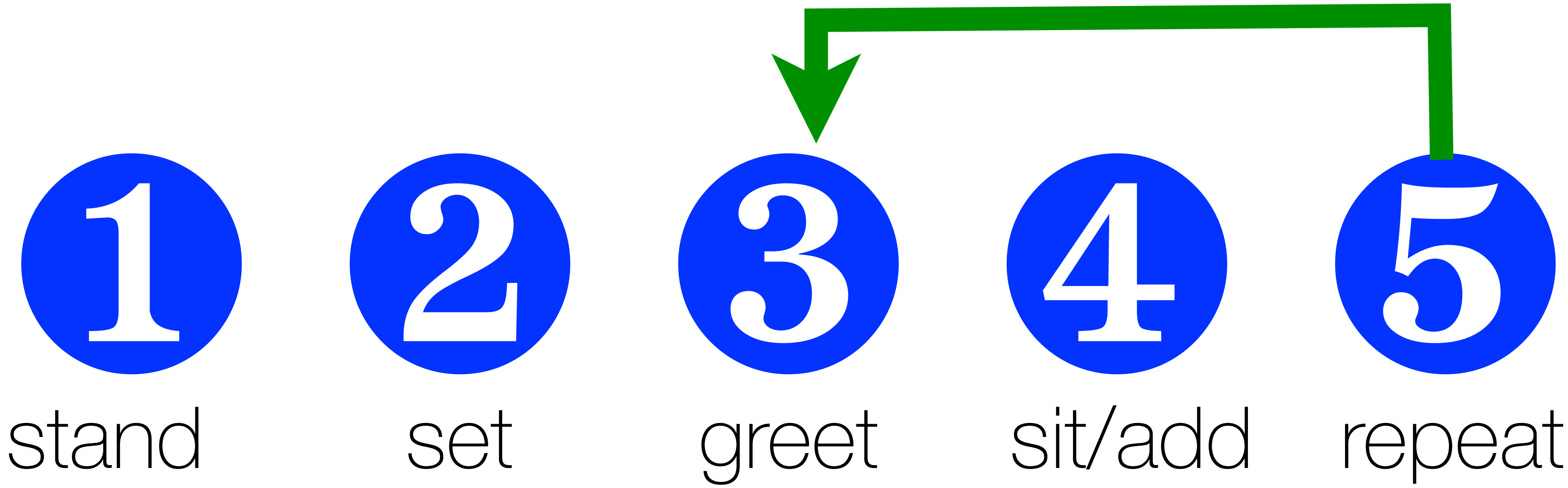
Our model of computation

Basic op: 1 unit

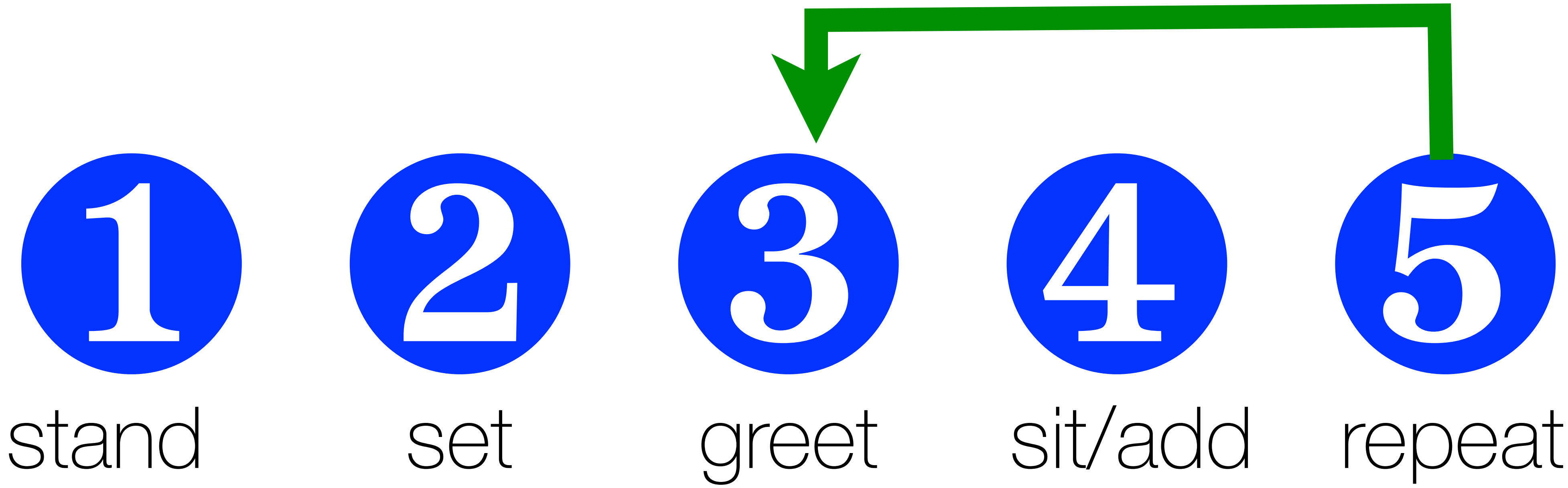
Set, Greet, add, compare, sit

Simplify: in each round, every standing person can do 1 op

Lets count # of rounds until we finish



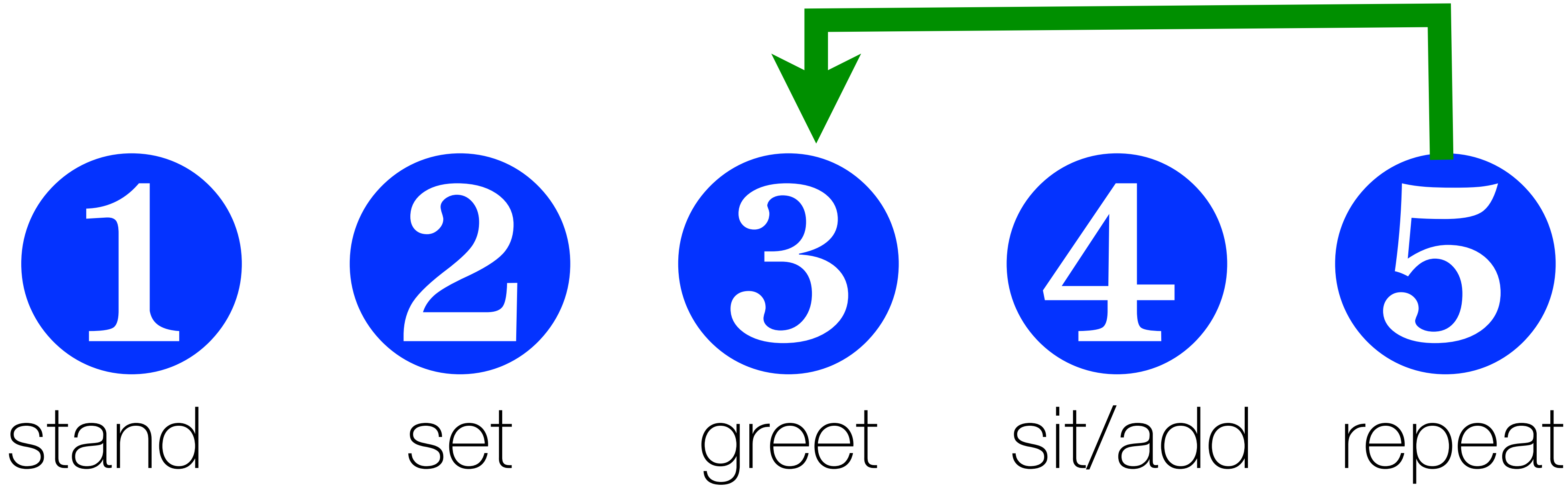
how fast does it work:



how fast does it work:

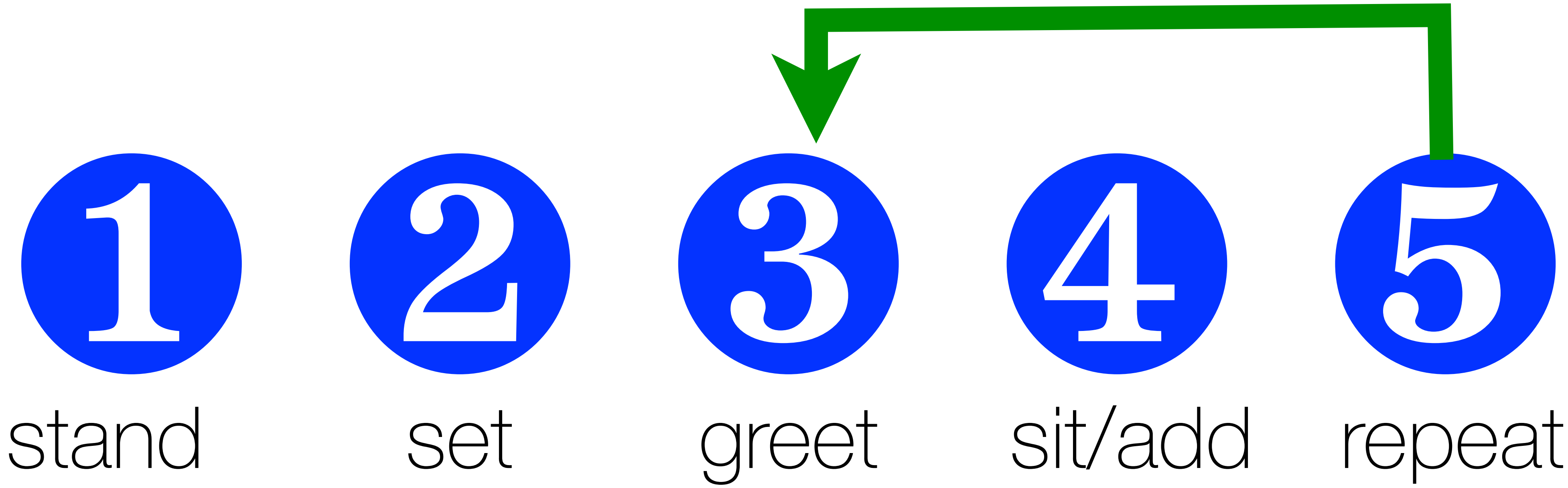
$$T(n)$$

rounds to finish in a room with n people



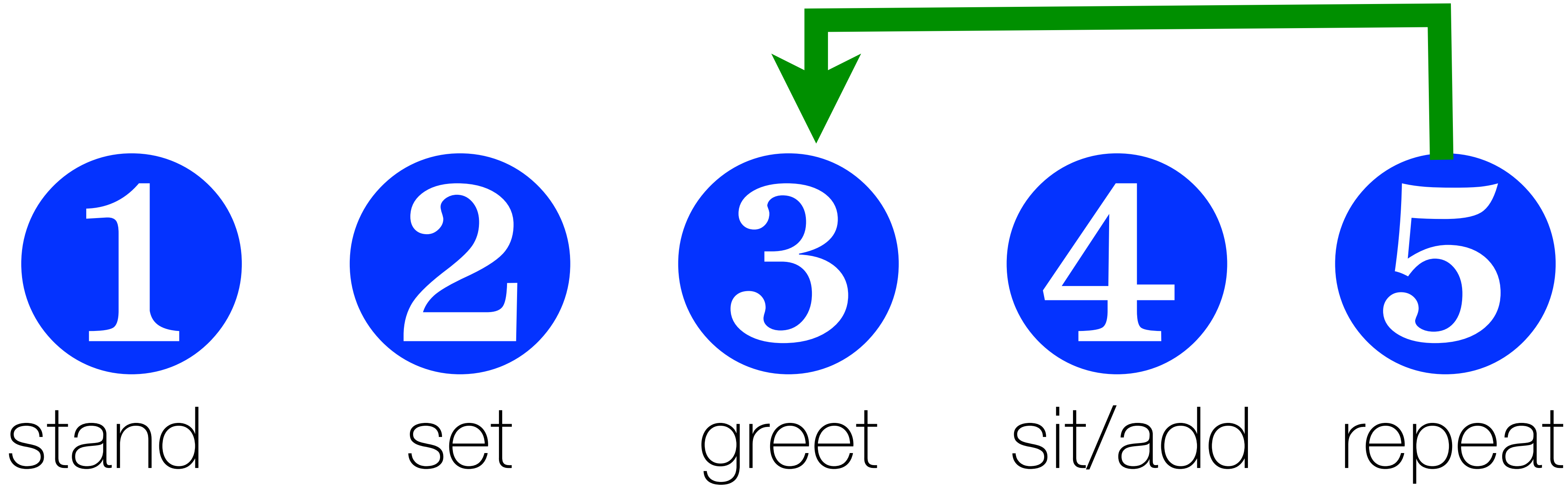
Simple case: 1 person

$$T(1) =$$

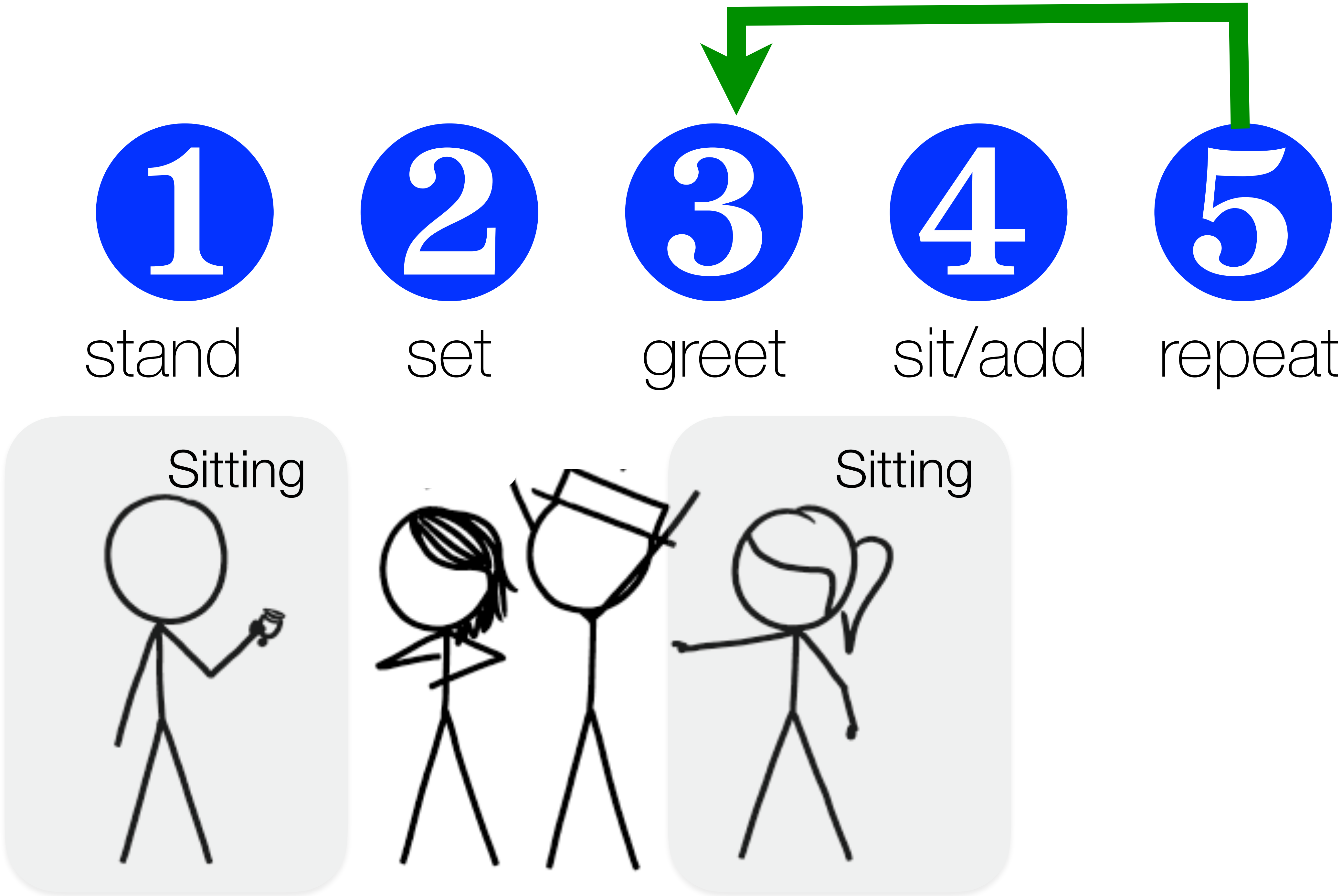


Simple case: 2 people

$$T(2) =$$



$$T(4) =$$



After step 4

$$T(4) =$$

1 stand

2 set

3 greet

4 sit/add

5 repeat

These steps only happen once.

What about these?

I1:Approx is OK

1 stand

2 set

3 greet

4 sit/add

5 repeat

how fast does it work:

$$T(n) = 1 + 1 + T(\lceil n/2 \rceil)$$

1 stand 2 set

3 greet 4 sit/add 5 repeat

how fast does it work:

$$T(n) = 1 + 1 + T(\lceil n/2 \rceil)$$

$$T(1) = 3$$

This is a recurrence

$$T(n) = T(\lceil n/2 \rceil) + 2$$

$$T(1) = 3$$

solve a simpler case when n is a power of 2.

$$T(2^k) = 2 + T(2^{k-1})$$

$$T(2^k) = 2 + T(2^{k-1})$$

$$\begin{aligned} T(2^k) &= 2 + T(2^{k-1}) \\ &= 2 + 2 + T(2^{k-2}) \end{aligned}$$

$$T(2^k) = 2 + T(2^{k-1})$$

$$= 2 + 2 + T(2^{k-2})$$

$$= \overbrace{2 + 2 + \cdots + 2}^{k} + T(2^0)$$

$$
\begin{aligned}
T(2^k) &= 2 + T(2^{k-1}) \\
&= 2 + 2 + T(2^{k-2}) \\
&= \overbrace{2 + 2 + \cdots + 2}^{k} + T(2^0) \\
&= 2k + T(1)
\end{aligned}
$$

“intuition here”

$$T(2^k) = 2 + T(2^{k-1})$$

Other cases?

“intuition here”

$$T(2^k) = 2 + T(2^{k-1})$$
$$= 2 + 2 + T(2^{k-2})$$

Other cases?

“intuition here”

$$T(2^k) = 2 + T(2^{k-1})$$

$$= 2 + 2 + T(2^{k-2})$$

$$= \overbrace{2 + 2 + \cdots + 2}^{k} + T(2^0)$$

Other cases?

“intuition here”

$$
\begin{aligned}
T(2^k) &= 2 + T(2^{k-1}) \\
&= 2 + 2 + T(2^{k-2}) \\
&= \overbrace{2 + 2 + \cdots + 2}^{k} + T(2^0) \\
&= 2k + T(1)
\end{aligned}
$$

Other cases?

Idea1 : It is OK to approximate

Asymptotic notation

$O(g)$

This notation represents a set

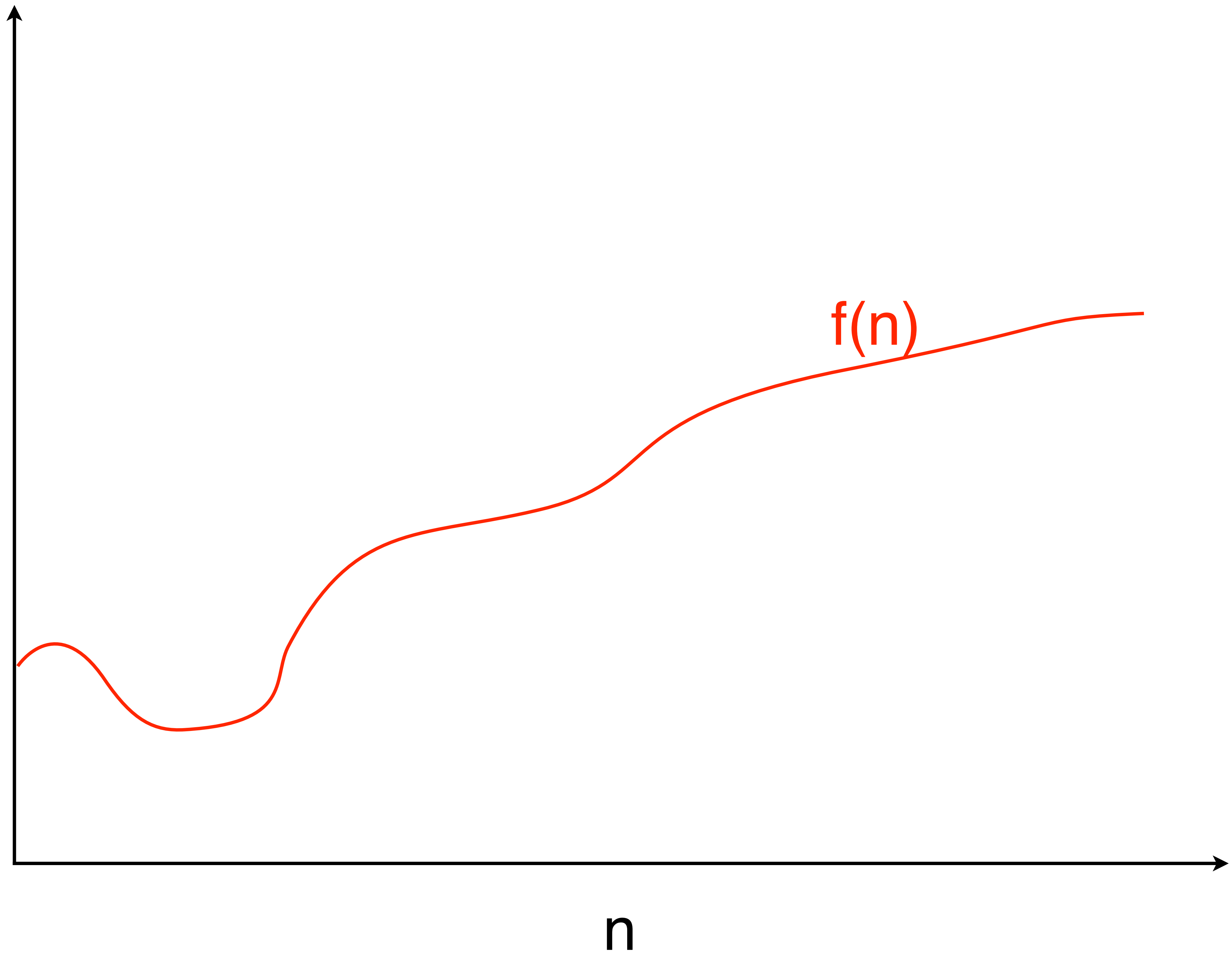
Asymptotic notation

$O(g)$ Set of functions that are at most within const of g for large n

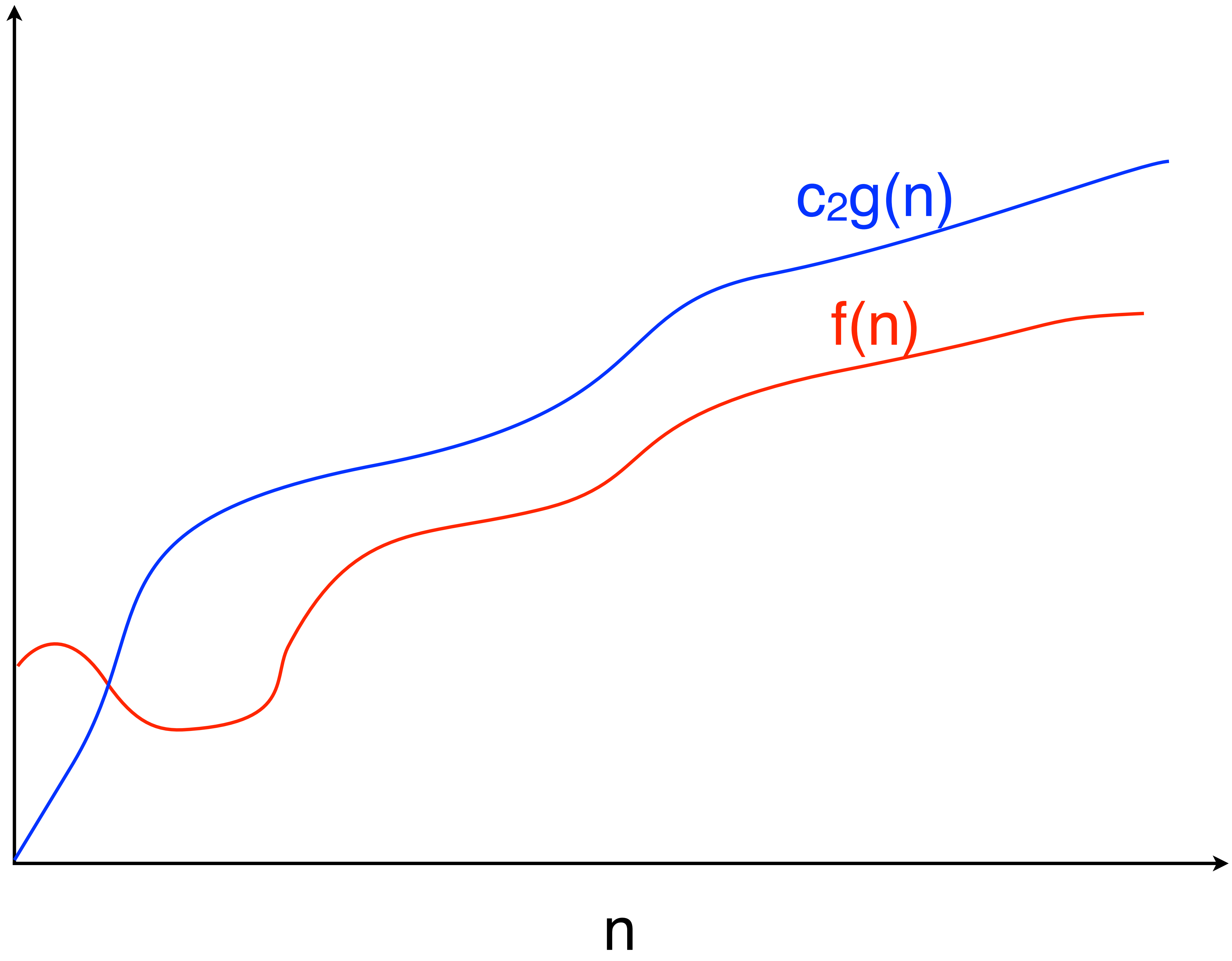
Asymptotic notation

$O(g)$	at most within const of g	for large n
$\Omega(g)$	at least within const of g	for large n
$\Theta(g)$	within a const of g	for large n

Omega sandwich

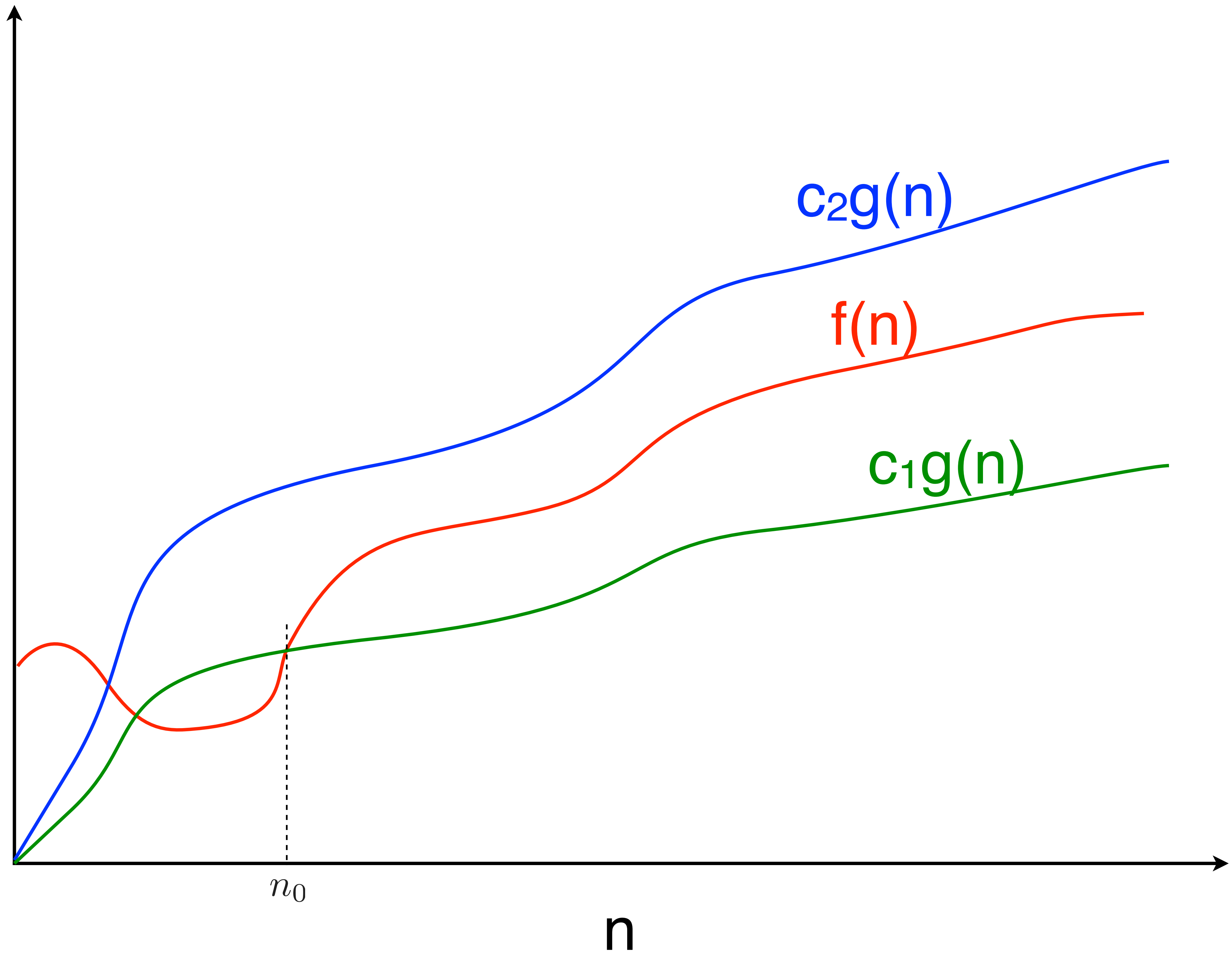


Omega sandwich



$$f(n) = O(g(n))$$

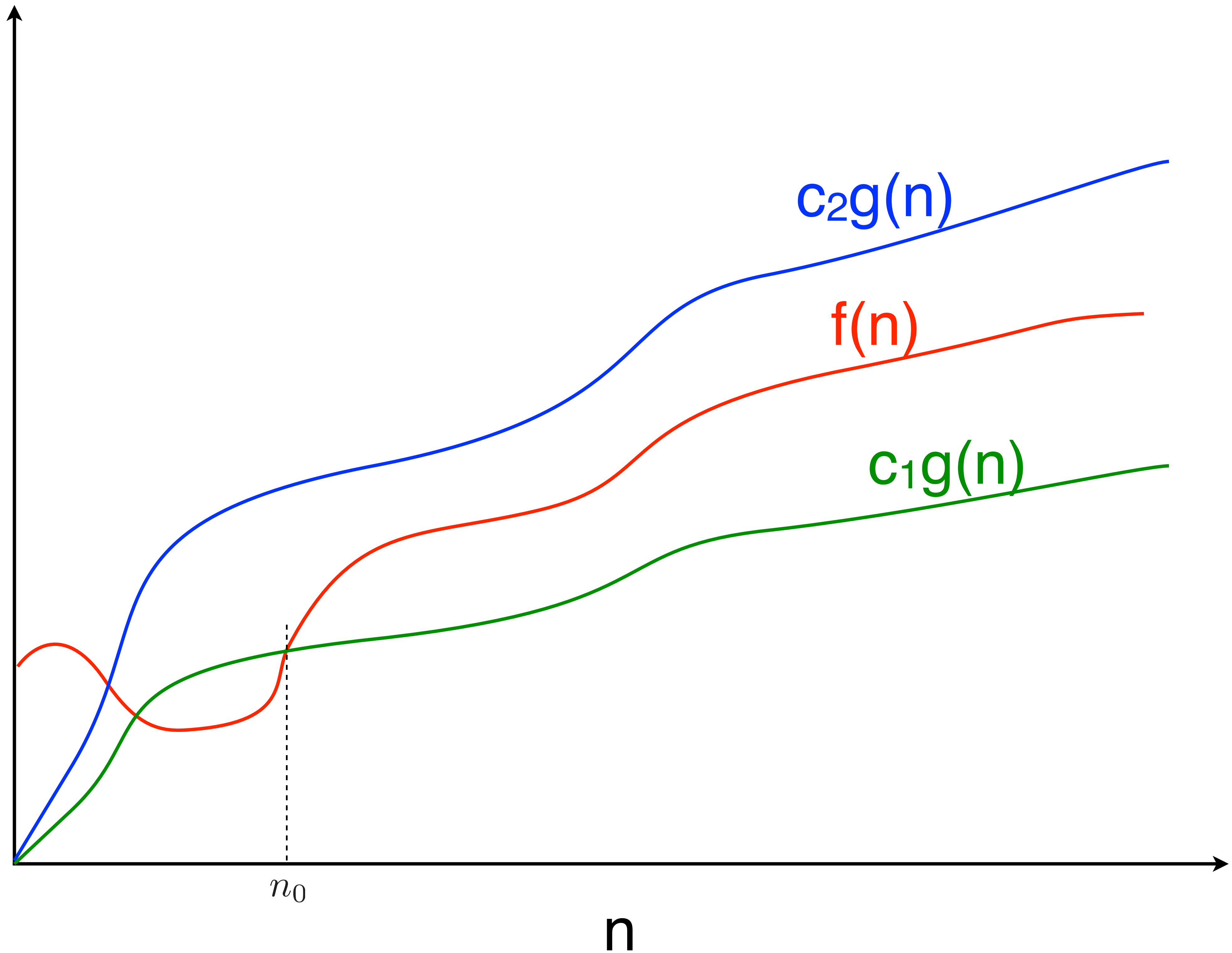
Omega sandwich



$$f(n) = O(g(n))$$

$$f(n) = \Omega(g(n))$$

Omega sandwich



$$f(n) = O(g(n))$$

$$f(n) = \Theta(g(n))$$

$$f(n) = \Omega(g(n))$$

“intuition here”

$$T(2^k) = 2 + T(2^{k-1})$$

$$= 2 + 2 + T(2^{k-2})$$

$$= \overbrace{2 + 2 + \cdots + 2}^{k} + T(2^0)$$

$$= 2k + T(1) \ = O(\log(2^k))$$

“intuition here”

$$T(2^k) = 2 + T(2^{k-1})$$

$$= 2 + 2 + T(2^{k-2})$$

$$= \overbrace{2 + 2 + \cdots + 2}^{k} + T(2^0)$$

$$= 2k + T(1) \ = O(\log(2^k))$$

$$\forall 0 < n < m, T(n) \leq T(m)$$

$$T(m) \leq T(2^{\lceil \log(m) \rceil}) = 2\lceil \log(m) \rceil + 2$$

“intuition here”

$$T(2^k) = 2 + T(2^{k-1})$$

$$= 2 + 2 + T(2^{k-2})$$

$$= \overbrace{2 + 2 + \cdots + 2}^{k} + T(2^0)$$

$$= 2k + T(1) \;= O(\log(2^k))$$

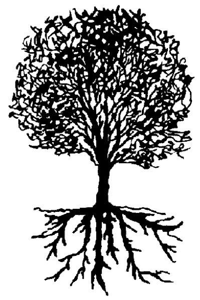
$$\forall 0 < n < m, T(n) \leq T(m)$$

$$T(m) \leq T(2^{\lceil \log(m) \rceil}) = 2\lceil \log(m) \rceil + 2$$

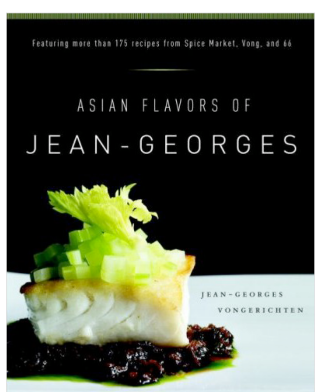
$$T(m) = \Omega(\log(m))$$

$$= \Theta(\log(m))$$

How to solve recurrence relations



?-✓



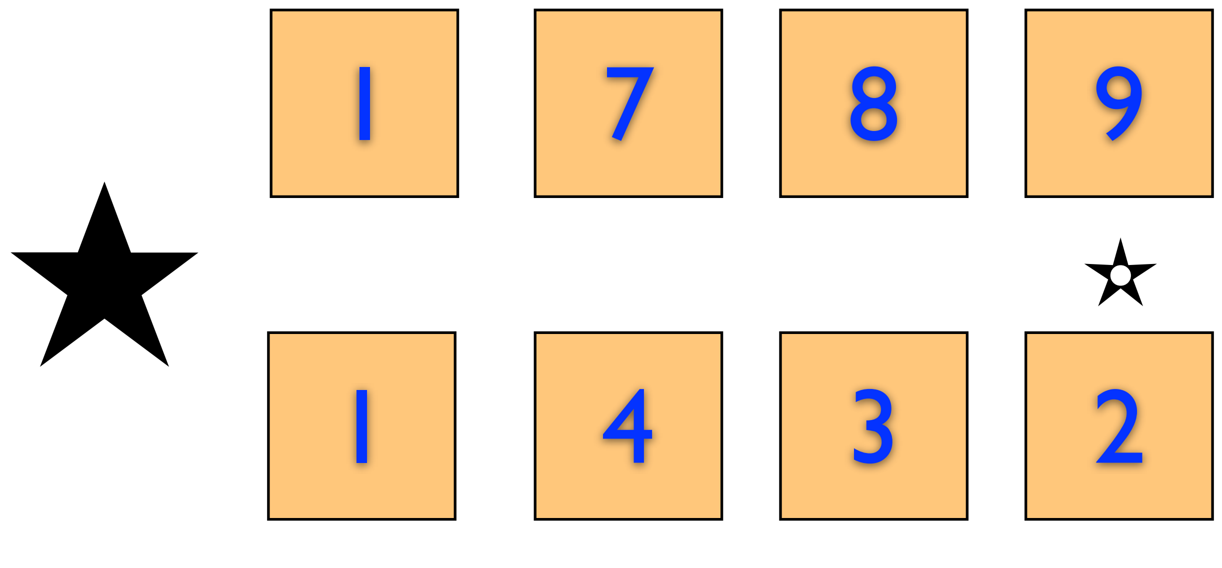
http://www.drblank.com/law301.jpg

Multiplication

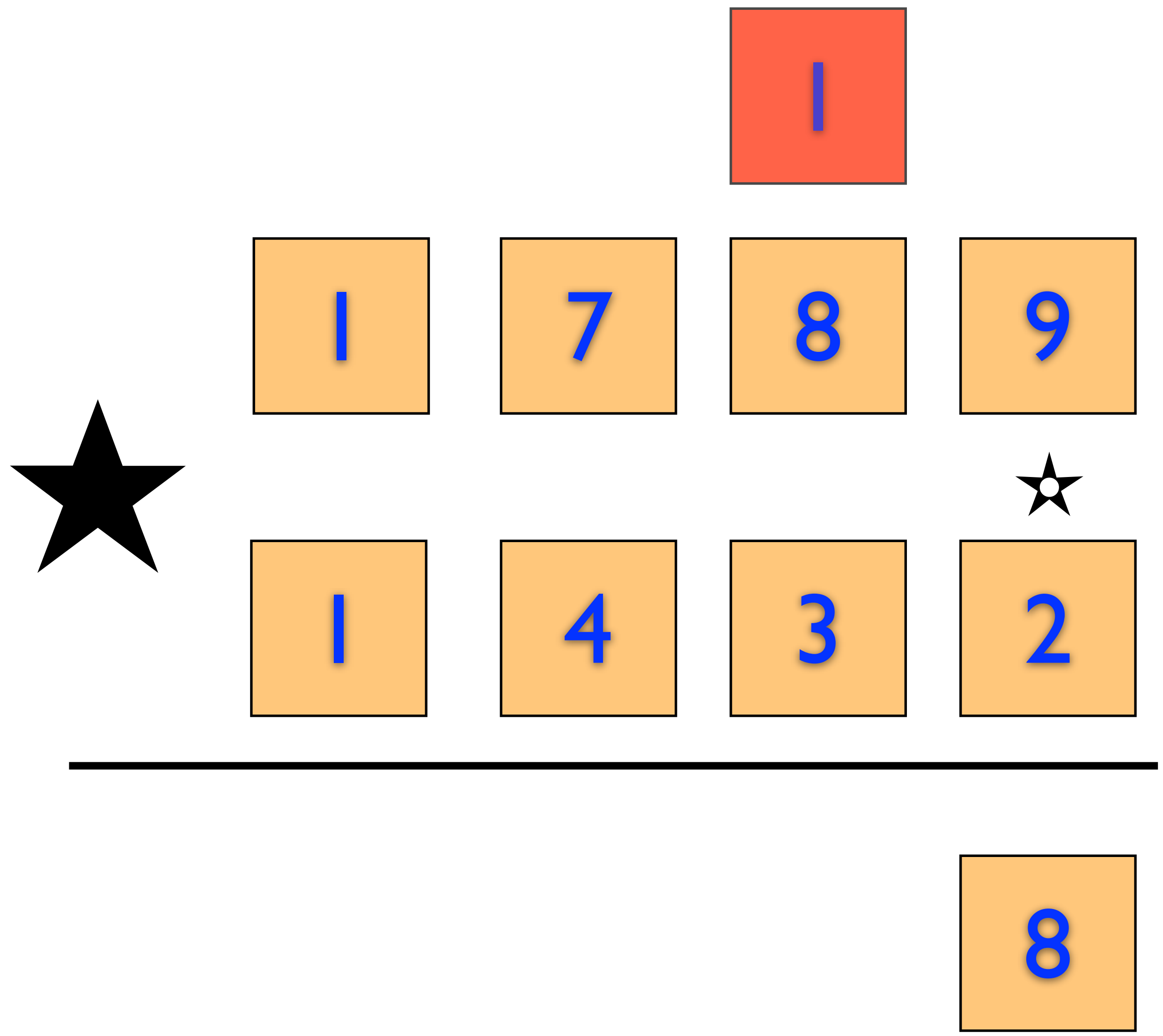
How much work does it take?

	1	7	8	9
★	1	4	3	2

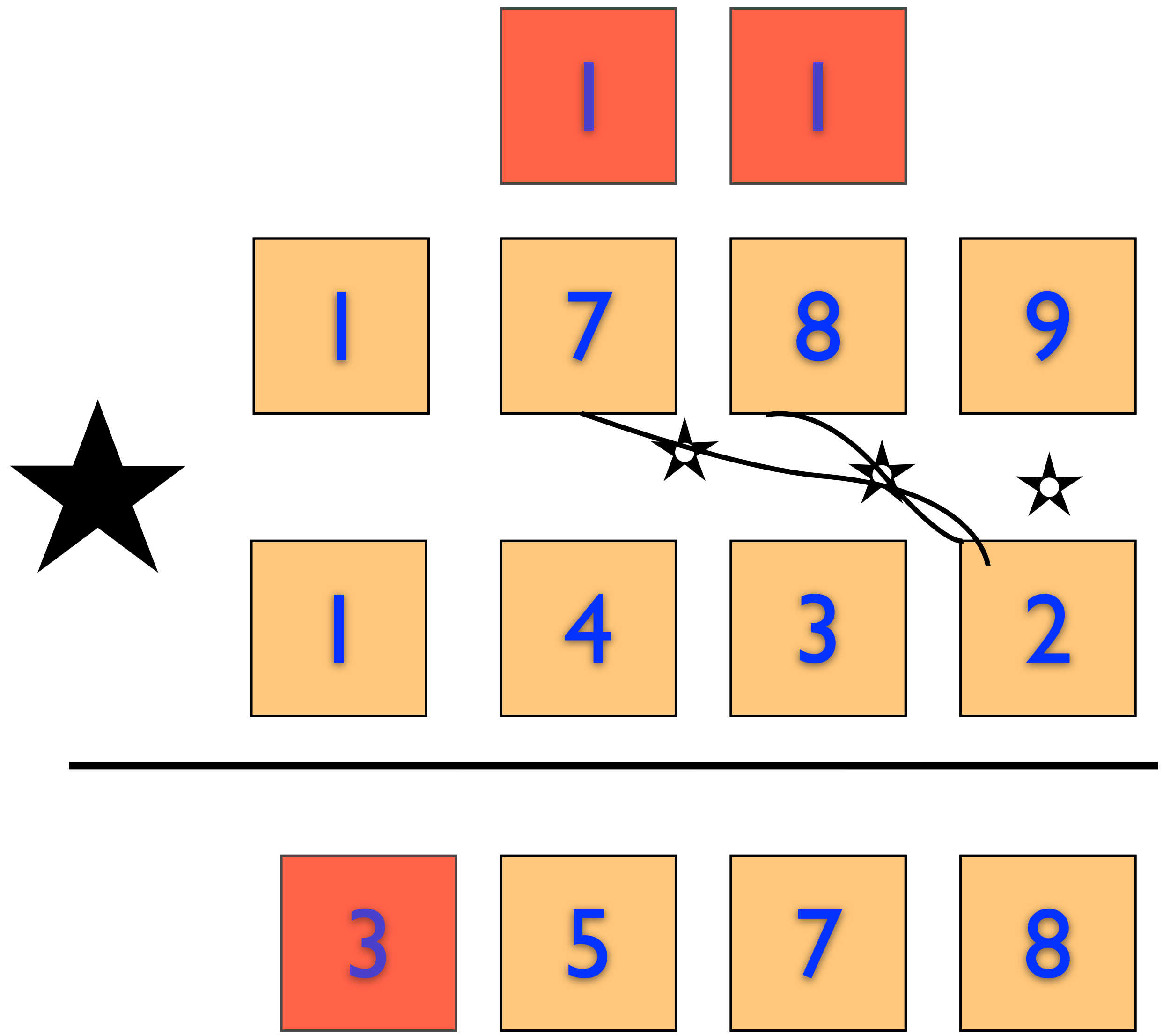
(n-1)(n+1) ✛



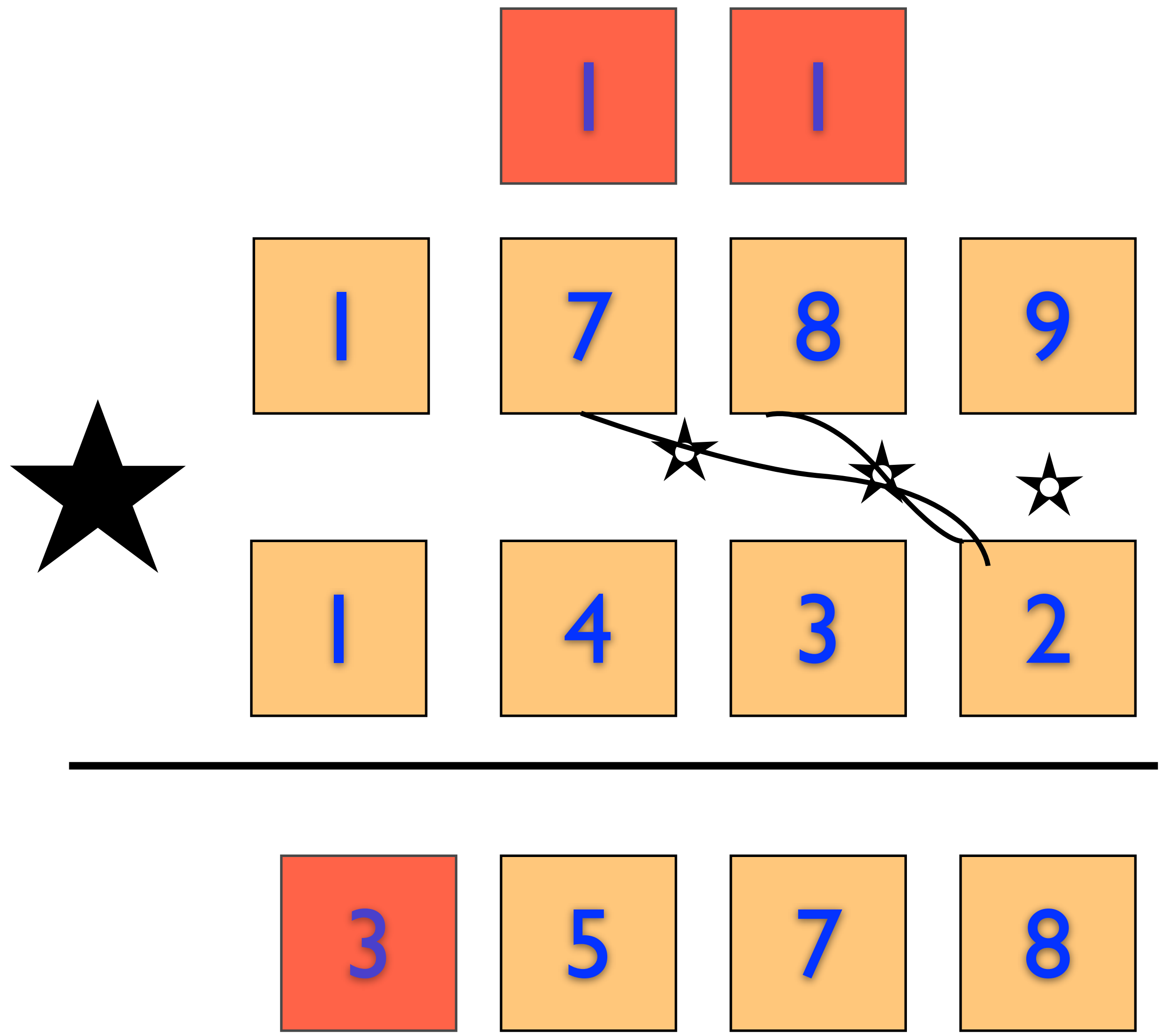
(n-1)(n+1)



(n-1)(n+1) ✢

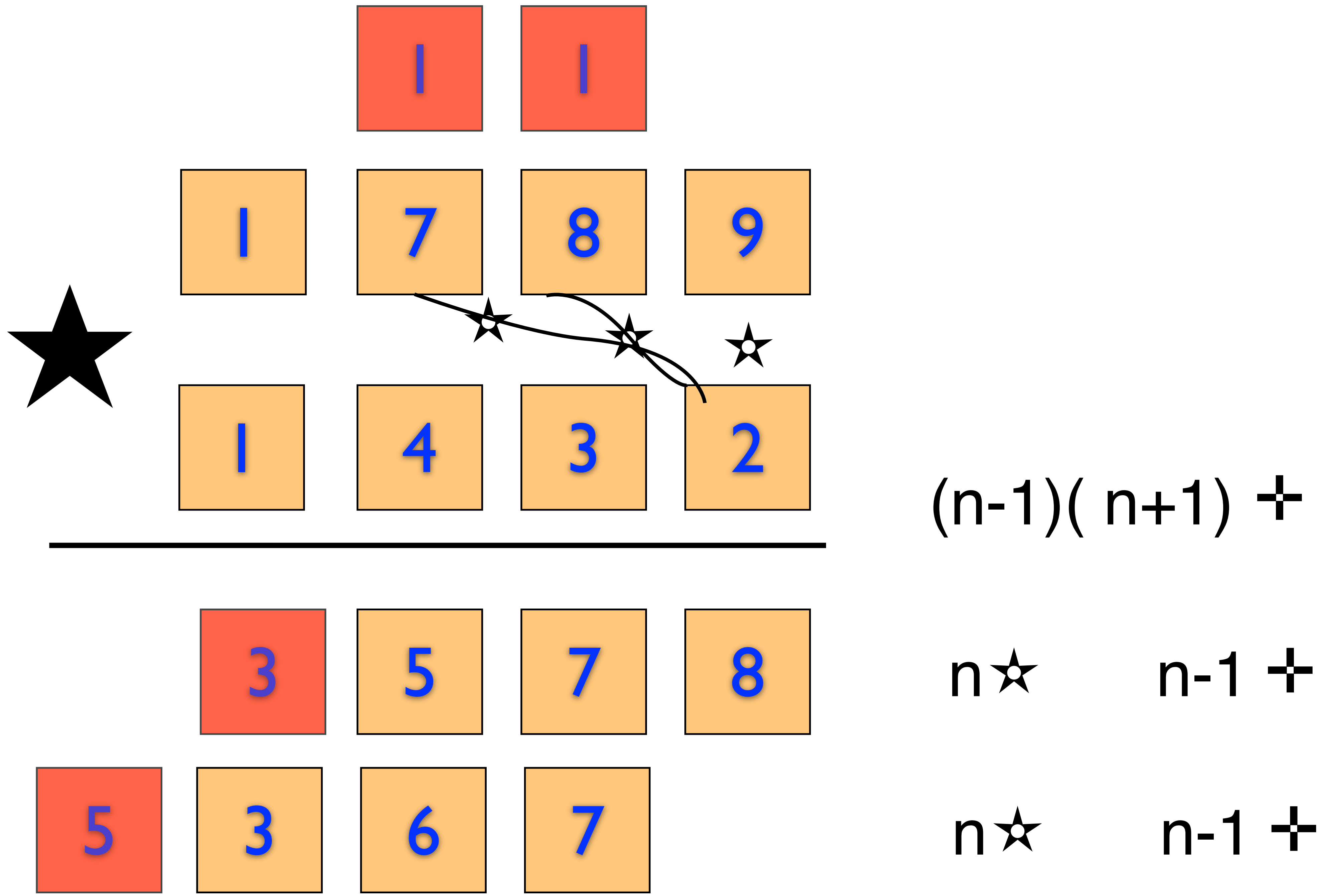


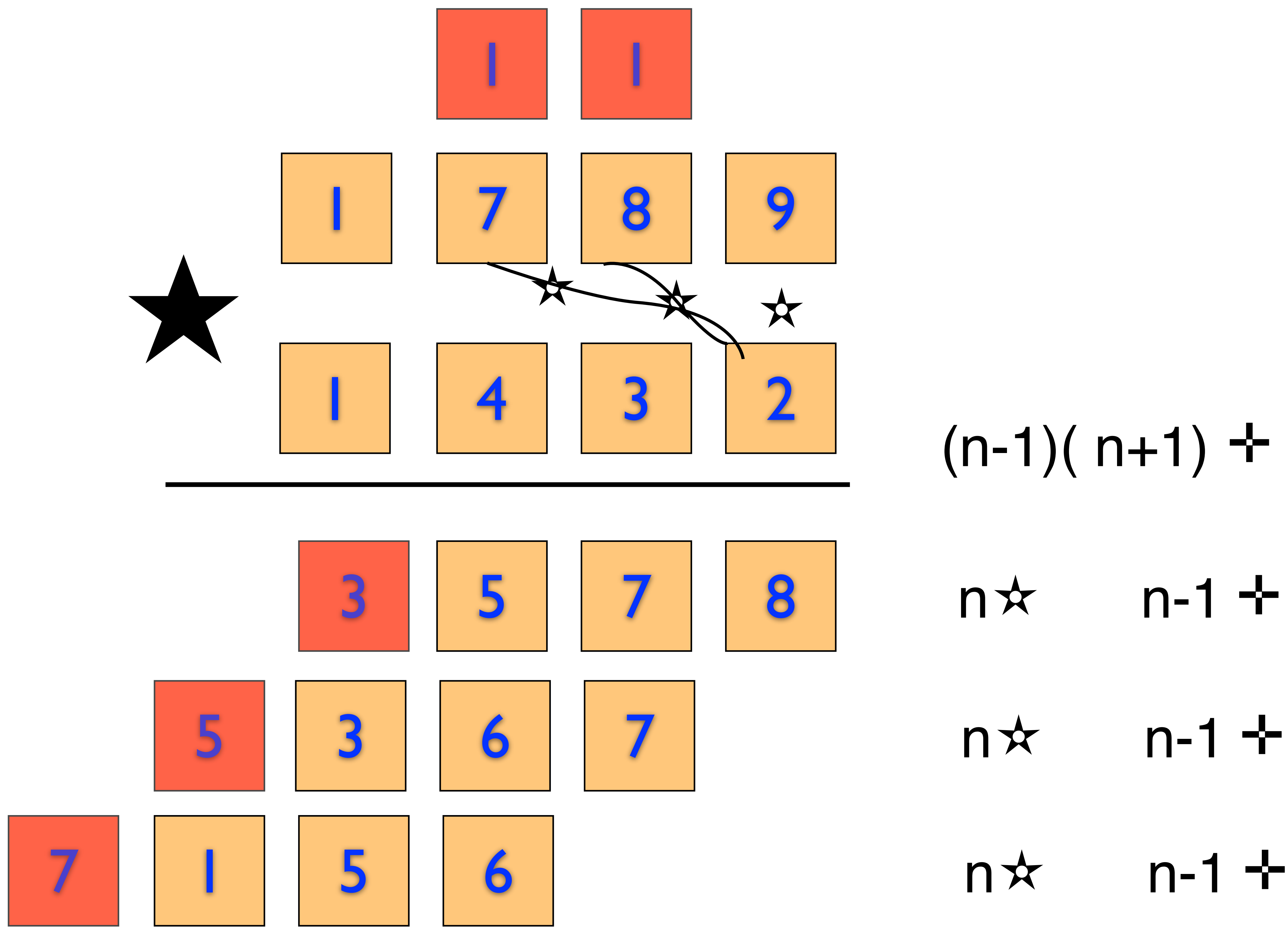
(n-1)(n+1)

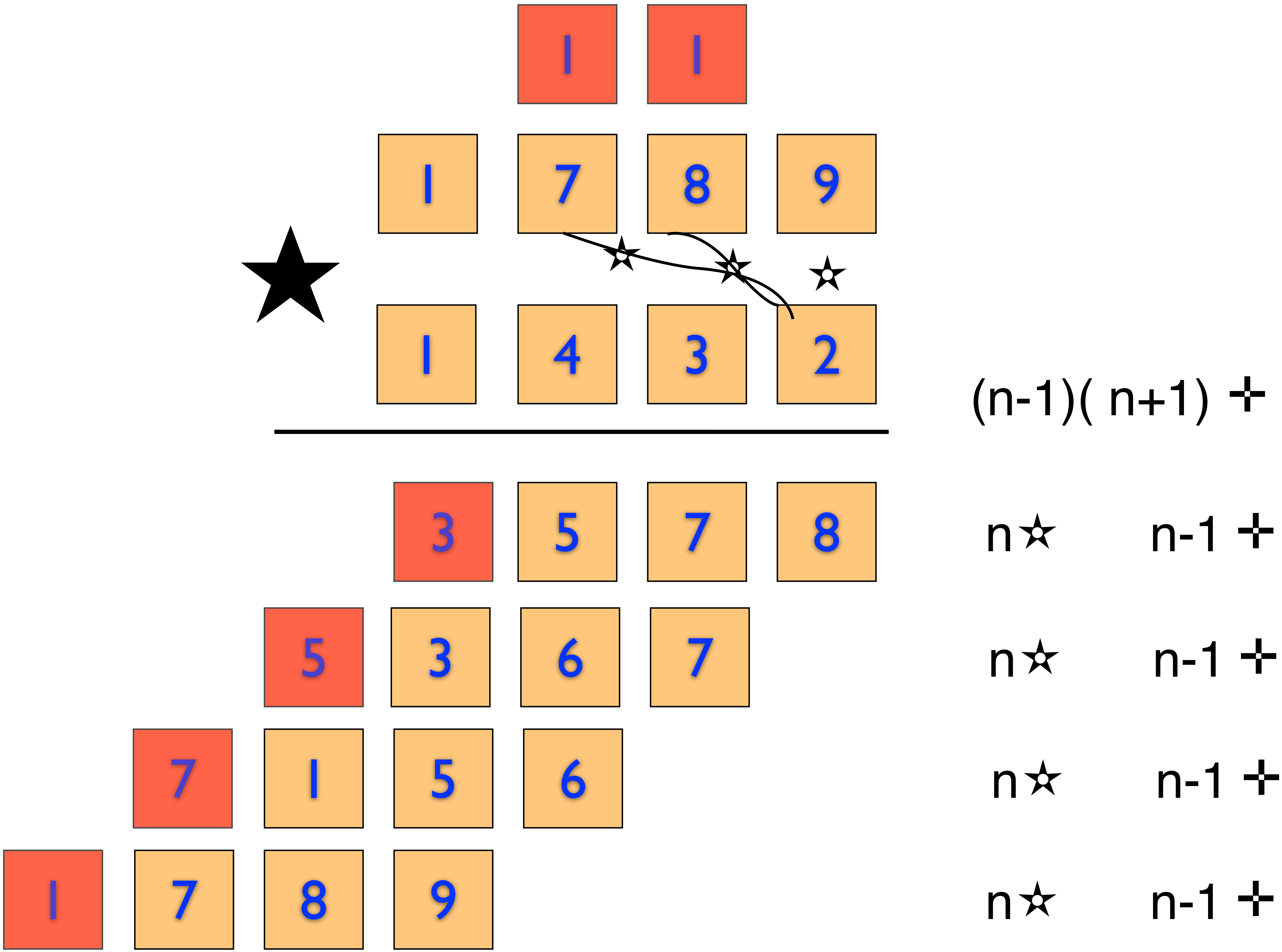


(n-1)(n+1) ✢

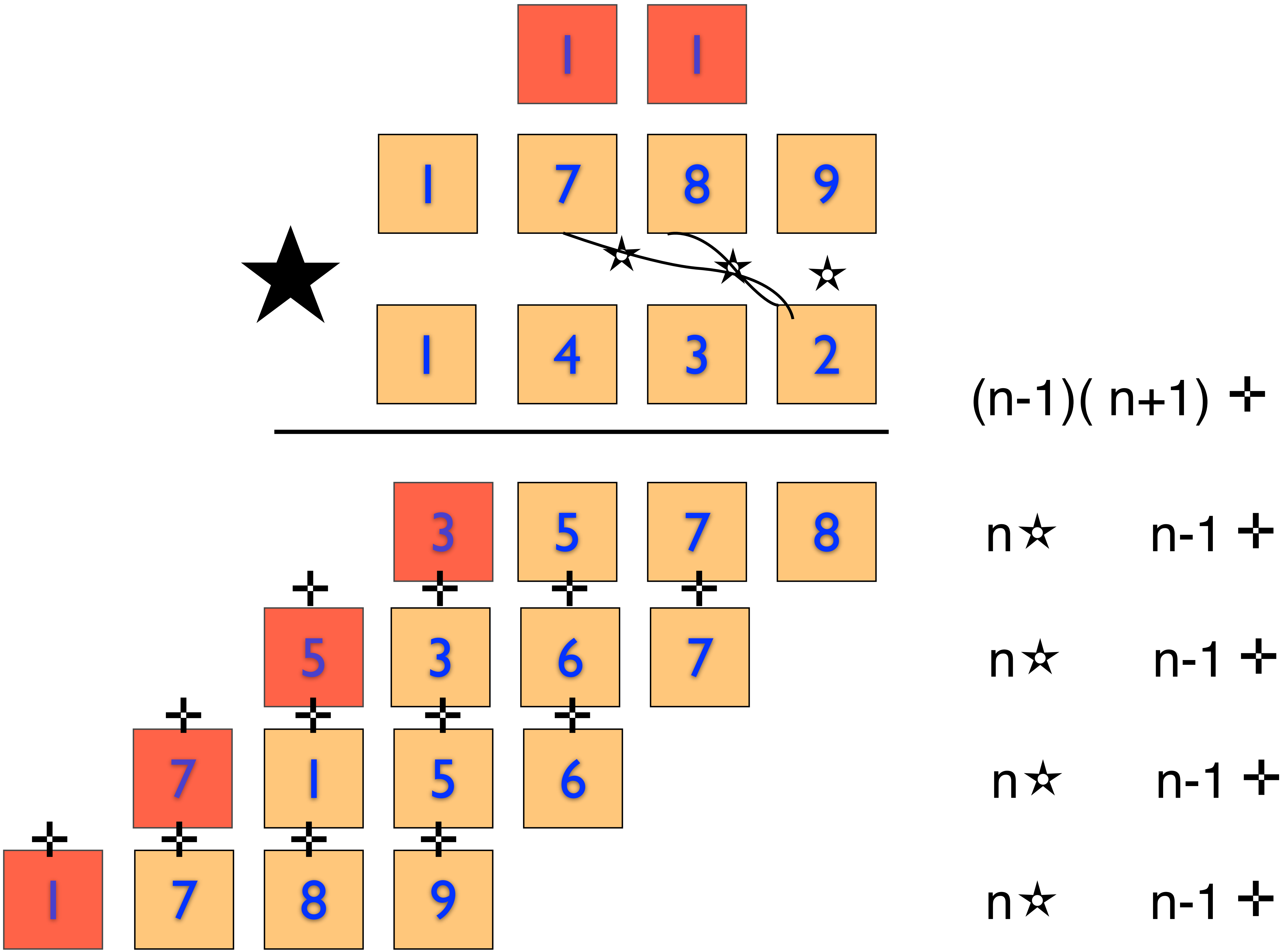
n☆ n-1 ✢


1 1
1 7 8 9
1 4 3 2
3 5 7 8
5 3 6 7
(n-1)(n+1) ✢
n☆ n-1 ✢
n☆ n-1 ✢


1 1
1 7 8 9
1 4 3 2
(n-1)(n+1) ✢
3 5 7 8
n☆ n-1 ✢
5 3 6 7
n☆ n-1 ✢
7 1 5 6
n☆ n-1 ✢

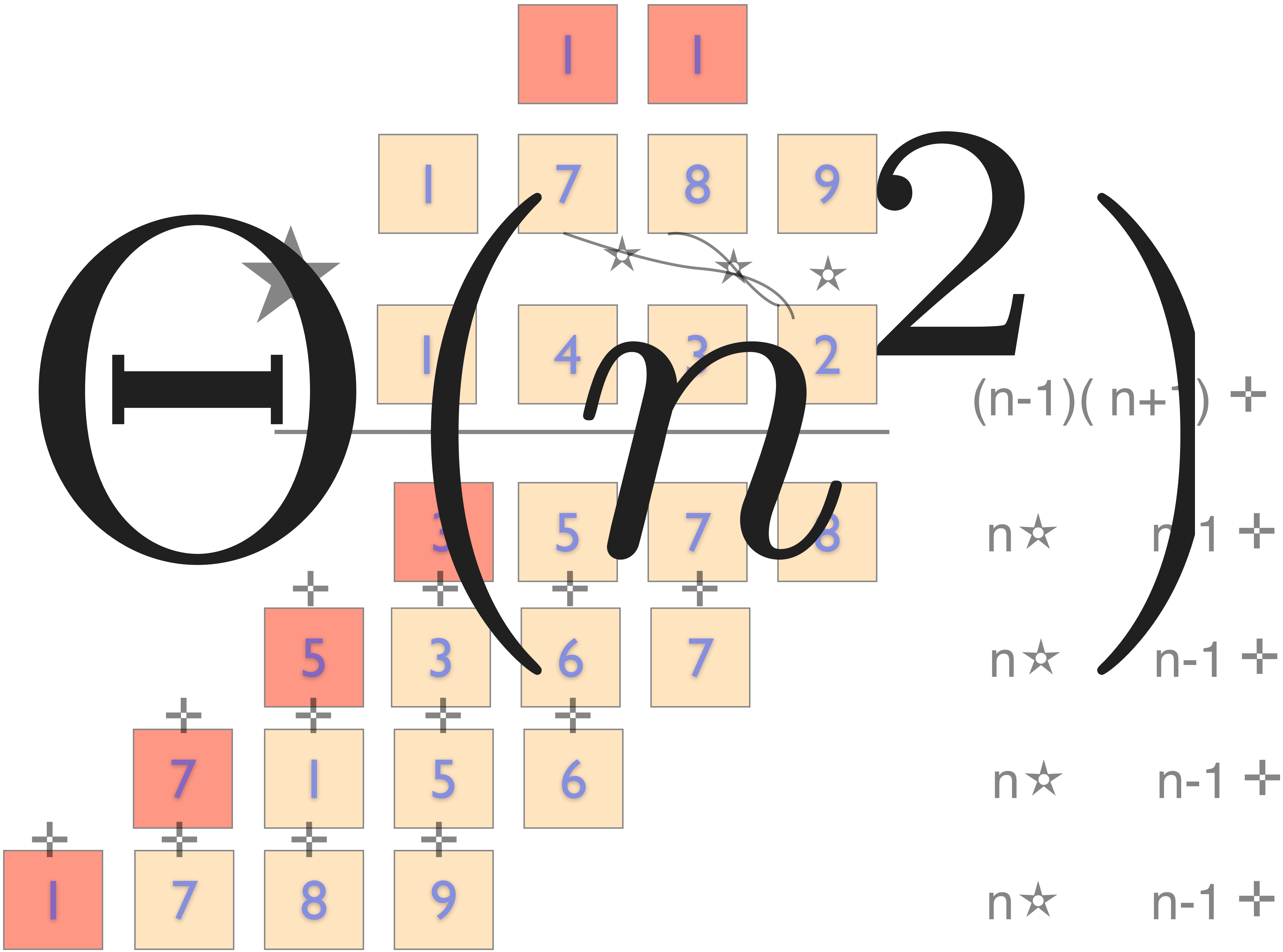


1 1
1 7 8 9
1 4 3 2
(n-1)(n+1) ✢
3 5 7 8
n☆ n-1 ✢
5 3 6 7
n☆ n-1 ✢
7 1 5 6
n☆ n-1 ✢
1 7 8 9
n☆ n-1 ✢



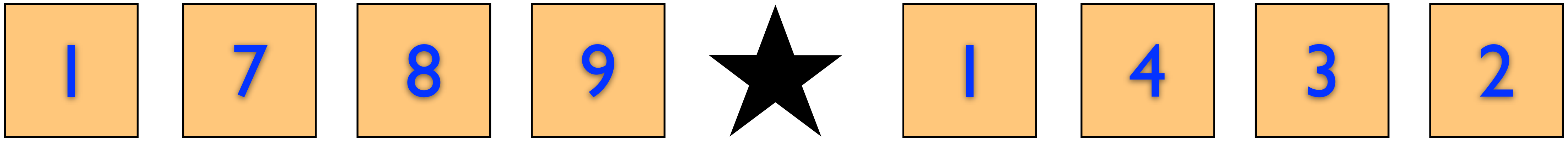
1 1
1 7 8 9
1 4 3 2
(n-1)(n+1) ✢
3 5 7 8
n☆ n-1 ✢
5 3 6 7
n☆ n-1 ✢
7 1 5 6
n☆ n-1 ✢
1 7 8 9
n☆ n-1 ✢

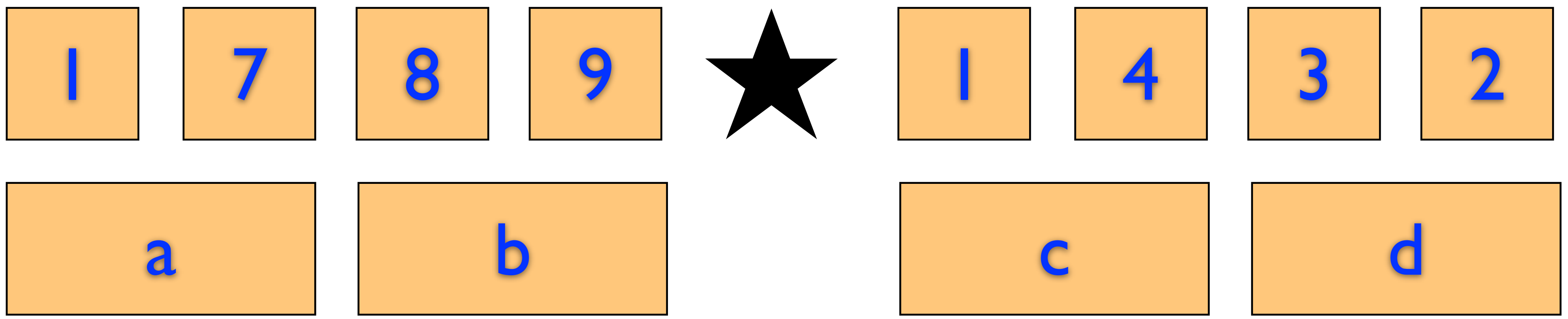
$$\Theta(n^2)$$

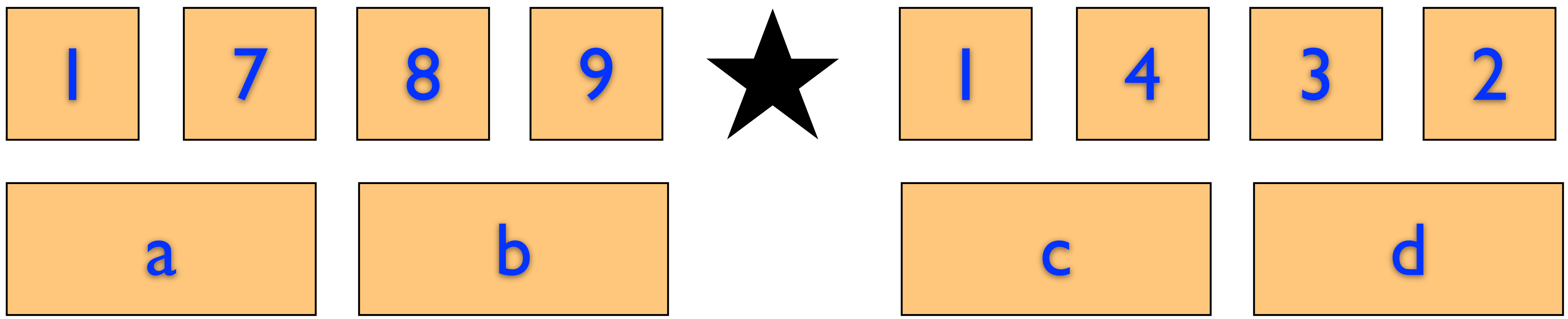


Theme 1

A first attempt…



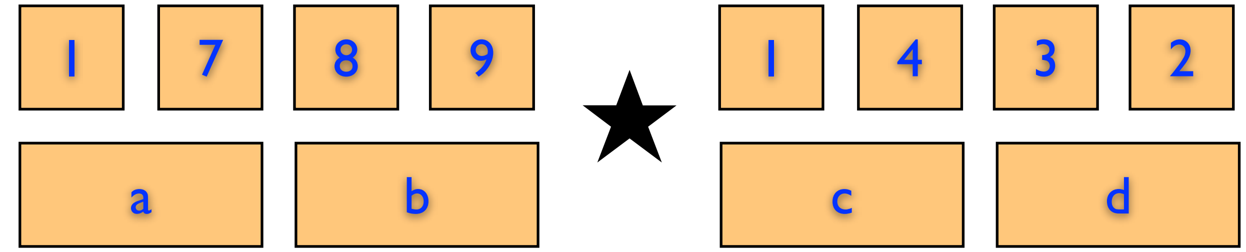

1
7
8
9
1
4
3
2
a
b
c
d



$$ac100^2 + (ad + bc)100 + bd$$

n-digit inputs

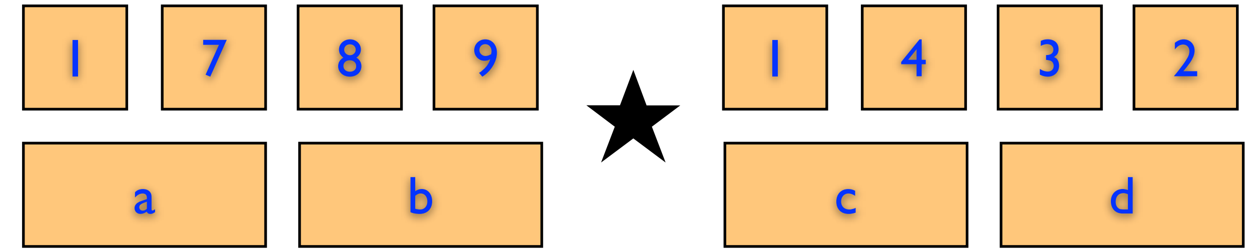
Mult(ab, cd)



$$ac100^2 + (ad + bc)100 + bd$$

Base case: return b*d if inputs are 1-digit

Mult(ab, cd)



$$ac100^2 + (ad + bc)100 + bd$$

Base case: return b*d if inputs are 1-digit

Compute x = Mult(a,c)

Compute y = Mult(a,d)

Compute z = Mult(b,c)

Compute w = Mult(b,d)

Return r = $x*100^2 + (y+z)100 + w$

$$T(n) = 4T(n/2) + 3O(n)$$

calculations: