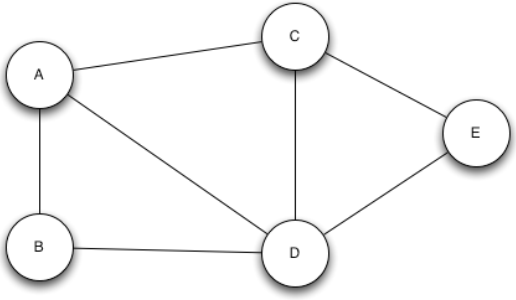
5800

Shortest paths

mar 8/10 2022
shelat

simple graph questions



WHAT IS THE LENGTH OF THE PATH FROM A TO E?

E
S

shortest path property

DEFINITION:

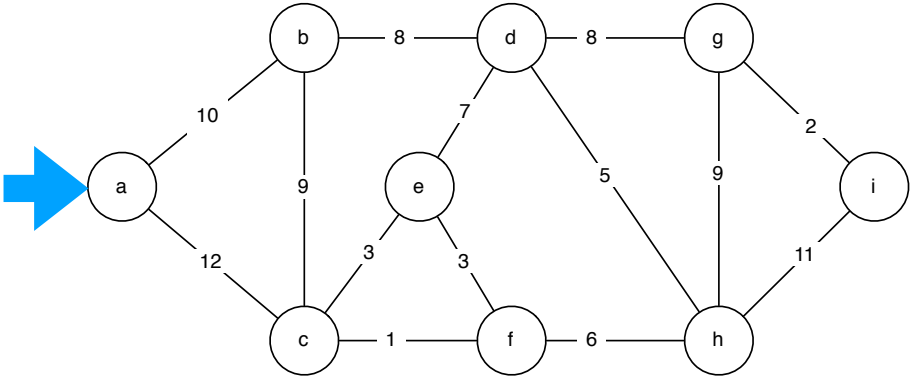
$$\delta(s, v)$$

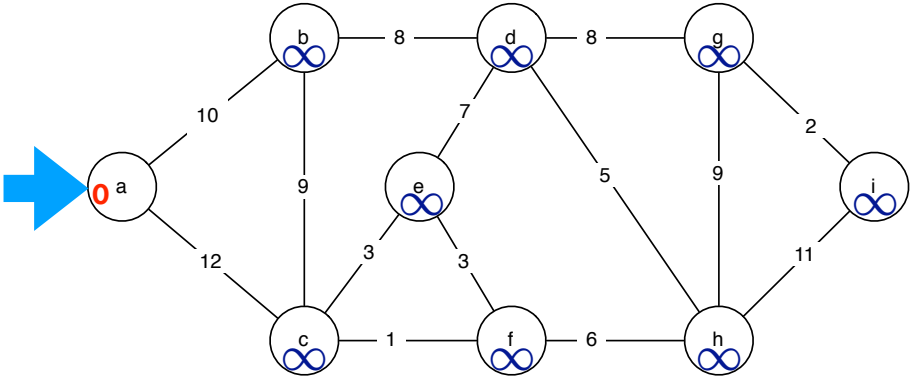
shortest path property

DEFINITION:

$\delta(s, v)$ Length of the shortest path from s to v, set to ∞ if there is no path from s to v

shortest paths from a

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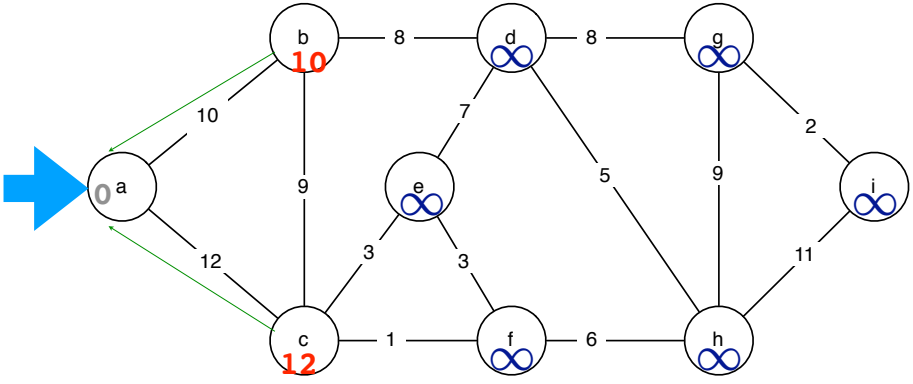
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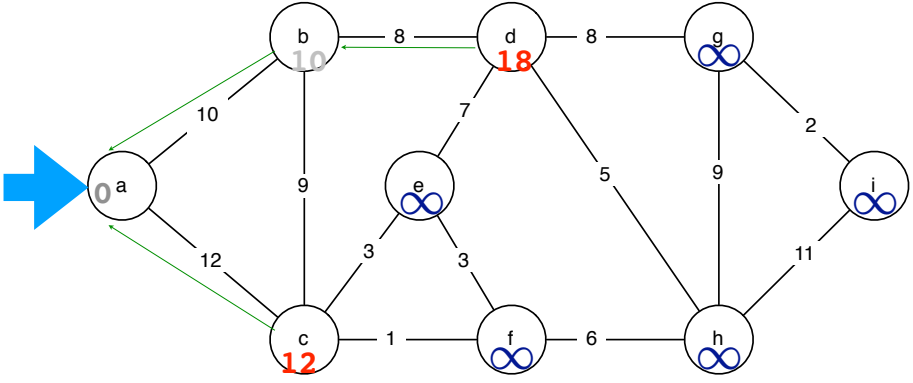
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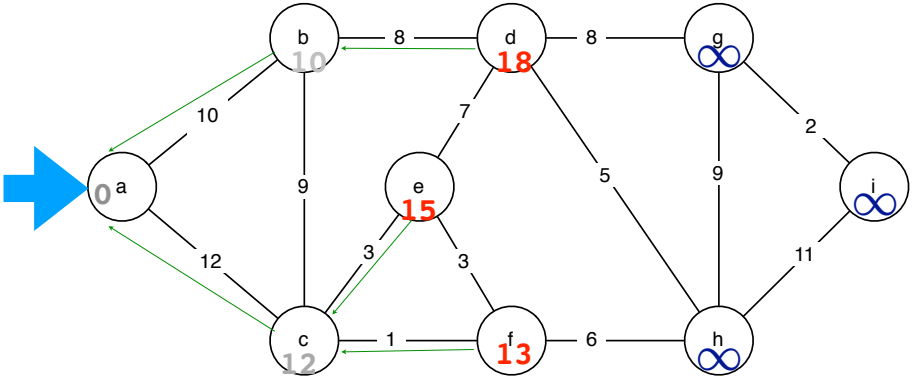
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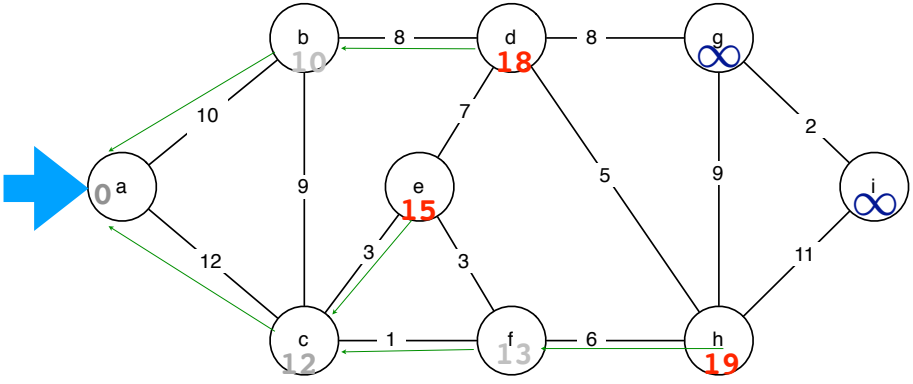
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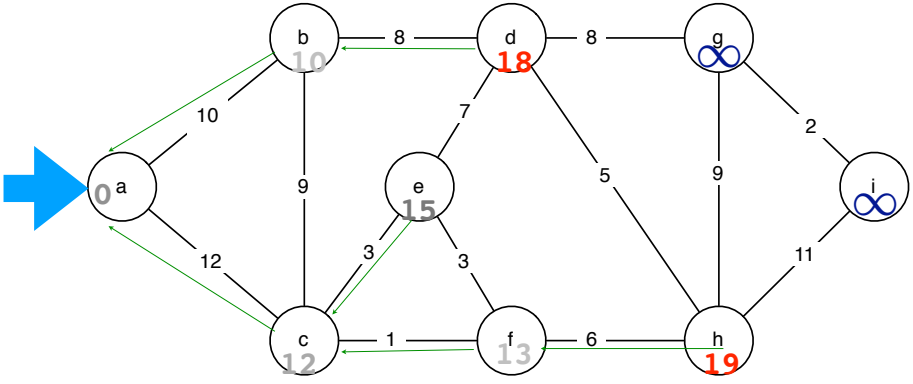
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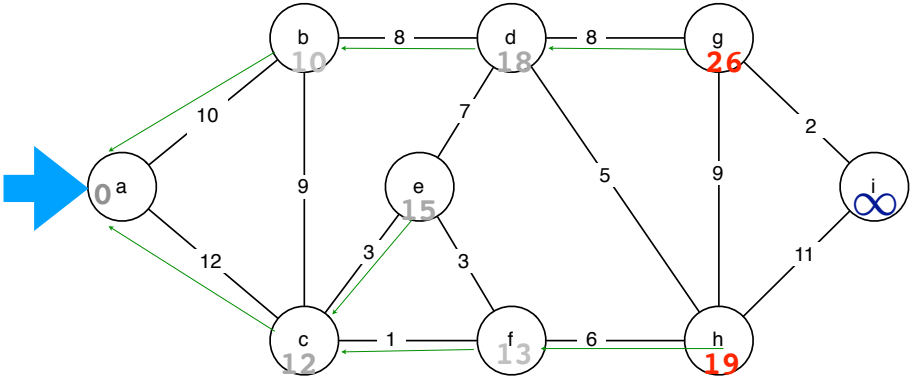
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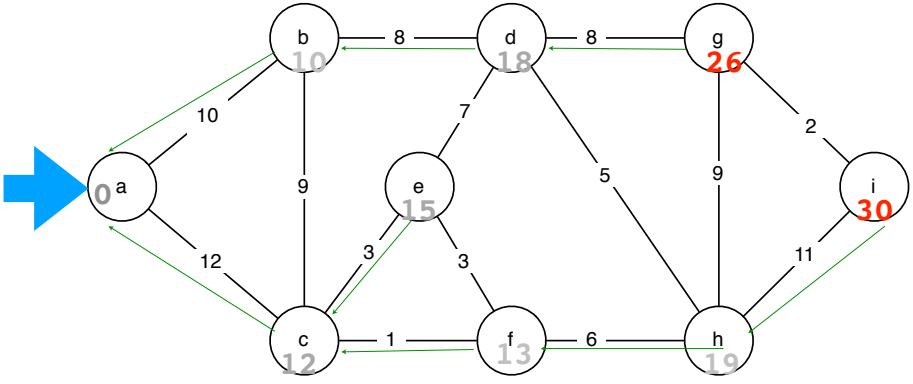
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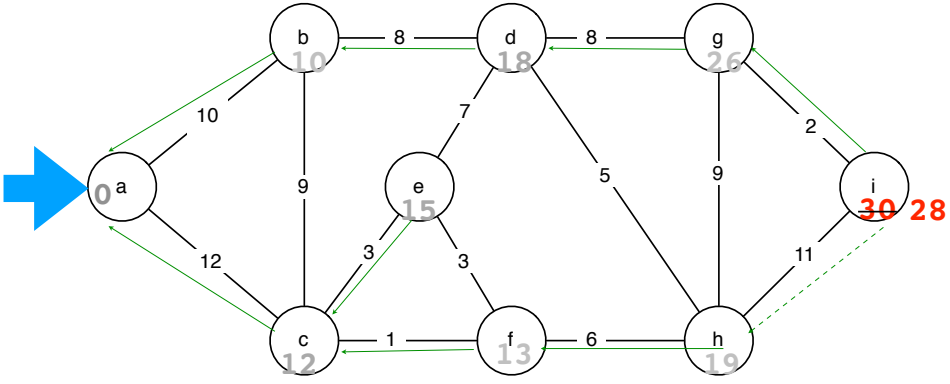
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Algorithm

```
DIJKSTRA(G = (V, E), s)
for all v ∈ V
        do d_u ← ∞
           π_u ← NIL
d_s ← 0
Q ← MAKEQUEUE(V)    ▷ use d_u as key
while Q ≠ ∅
        do u ← EXTRACTMIN(Q)
           for each v ∈ Adj(u)
                 do if d_v > d_u + w(u, v)
                       then d_v ← d_u + w(u, v)
                            π_v ← u
                            DECREASEKEY(Q, v)
```

Very similar structure

```
DIJKSTRA(G = (V, E), s)
for all v ∈ V
      do d_u ← ∞
         π_u ← NIL
d_s ← 0
Q ← MAKEQUEUE(V)    ▷ use d_u as key
while Q ≠ ∅
      do u ← EXTRACTMIN(Q)
         for each v ∈ Adj(u)
               do if d_v > d_u + w(u, v)
                     then d_v ← d_u + w(u, v)
                          π_v ← u
                          DECREASEKEY(Q, v)
```

```
PRIM(G = (V, E))
Q ← ∅    ▷  Q is a Priority Queue
Initialize each v ∈ V with key k_v ← ∞, π_v ← NIL
Pick a starting node r and set k_r ← 0
Insert all nodes into Q with key k_v.
while Q ≠ ∅
      do u ← EXTRACT-MIN(Q)
         for each v ∈ Adj(u)
               do if v ∈ Q and w(u, v) < k_v
                     then π_v ← u
                          DECREASE-KEY(Q, v, w(u, v
```

running time

```
DIJKSTRA(G = (V, E), s)
for all v ∈ V
        do d_u ← ∞
           π_u ← NIL
d_s ← 0
Q ← MAKEQUEUE(V)      ▷ use d_u as key
while Q ≠ ∅
        do u ← EXTRACTMIN(Q)
           for each v ∈ Adj(u)
                do if d_v > d_u + w(u, v)
                      then d_v ← d_u + w(u, v)
                           π_v ← u
                           DECREASEKEY(Q, v)
```

The running time is $\Theta(E \log V)$ because each DecreaseKey operation is called at most once on each edge.

why does Dijkstra work?

TRIANGLE INEQUALITY:

$$\forall (u, v) \in E, \ \delta(s, v) \leq \delta(s, u) + w(u, v)$$

UPPER BOUND: $d_v \geq \delta(s, v)$

Theorem

Given any weighted directed graph $G = (V, E)$ with non-negative weights, and a source s, Dijkstra(G, s) terminates with $d_v = \delta(s, v)$ for all $v \in V$.

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Let S be the set of elements not in Q.
At the beginning, this set is empty.

Theorem

Given any weighted directed graph $G = (V, E)$ with non-negative weights, and a source s, Dijkstra(G, s) terminates with $d_v = \delta(s, v)$ for all $v \in V$.

Let S be the set of elements not in Q.
At the beginning, this set is empty.

Property 1: For all $v \in S, d_v = \delta(s, v)$.

Proof (cont)

Suppose this property holds for the first i iterations of the loop.

$$w(p) = w(s, x) + w(x, y) + w(y, u)$$

Proof (cont)

Suppose this property holds for the first i iterations of the loop.

Proof (cont) Suppose this property holds for the first i iterations of the loop.

Let u be the node extracted on line 7. By lines, 9,10,11, it follows that $d_u = d_z + w(z, u)$ for some node $z \in S$. By the hypothesis, $d_u = \delta(s, z) + w(z, u)$.

Consider any path p from s to u. Let $e = (x, y)$ be the first edge on path p that crosses cut $(S, V - S)$. By lines 9,10,11 and the inductive hypothesis, $d_y \leq \delta(s, x) + w(x, y)$. We now analyze the weight of path p:

$$\begin{aligned} w(p) &= w(s \rightsquigarrow x) + w(x, y) + w(y \rightsquigarrow u) \\ &\geq \delta(s, x) + w(x, y) + \delta(y, u) \end{aligned}$$

Substituting from above, we have that

$$w(p) \geq d_y + \delta(y, u)$$

However, by line 6, $d_u \leq d_y$, and so

$$w(p) \geq d_u + \delta(y, u)$$

Since all edges have non-negative weight, $\delta(y, u) \geq 0$ and so

$$w(p) \geq d_u$$

which implies that $d_u = \delta(s, u)$.

breadth first search

INPUT: $G = (V, E), s$ (All edge weights are 1)

OUTPUT:

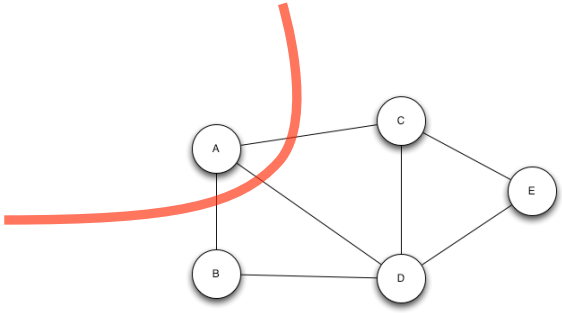
breadth first search

INPUT: $G = (V, E), s$ (All edge weights are 1)

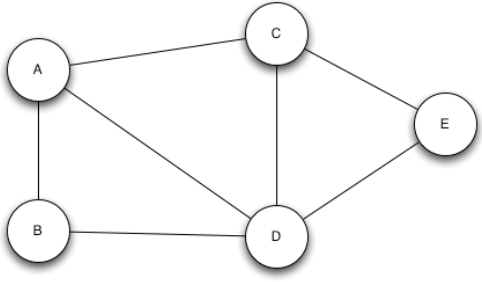
OUTPUT: $\forall v \in V \quad d_v = \delta(s, v)$

SMALLEST # OF EDGES FROM S TO V

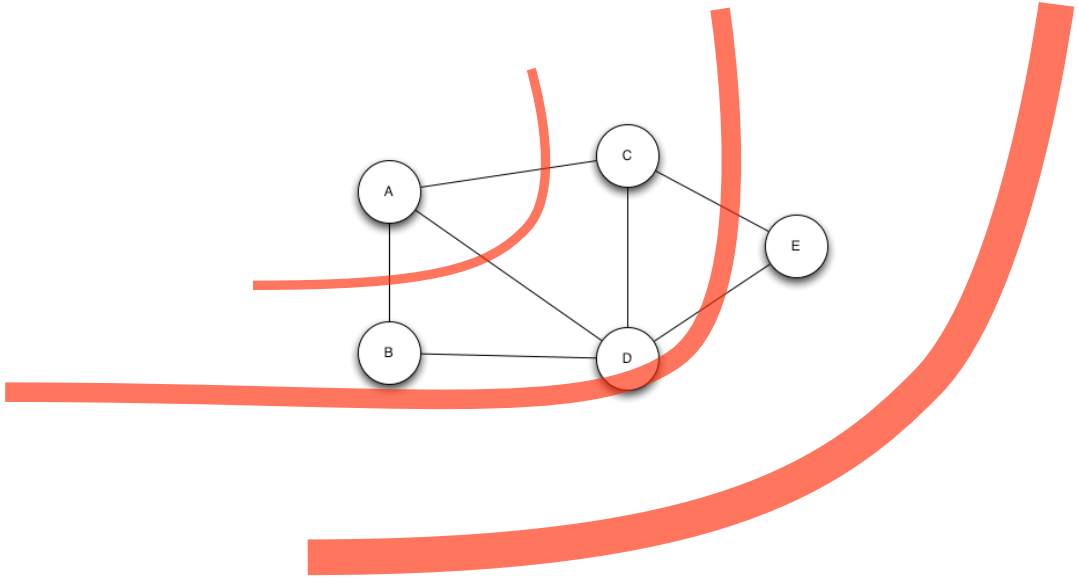
breadth-first search



breadth-first search



breadth-first search



breadth first search

INPUT: $G = (V, E), s$

OUTPUT: $d_v \quad \forall v \in V$

SMALLEST # OF EDGES FROM S TO V

breadth first search

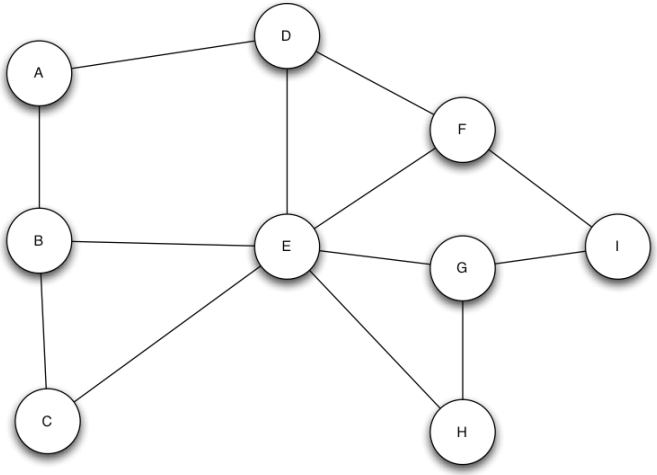
INPUT: $G = (V, E), s$

OUTPUT: $d_v \quad \forall v \in V$

SMALLEST # OF EDGES FROM S TO V

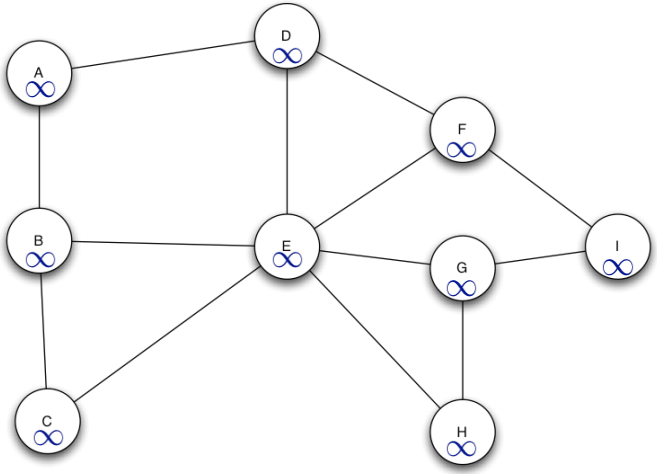
Big Idea: like Dijkstra, but use a simpler data structure: Queue.

bfs(G, a)



Q

bfs(G, a)



Q

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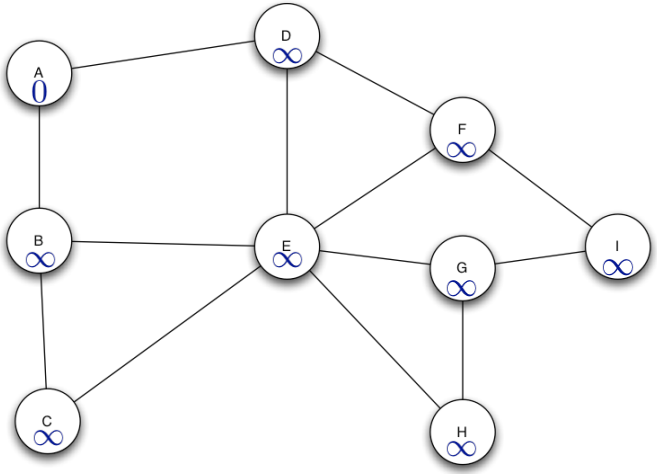
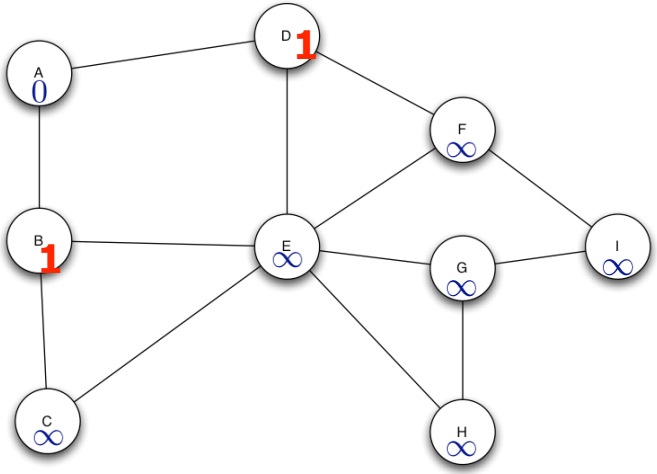


Q

A

bfs(G, a)



Q

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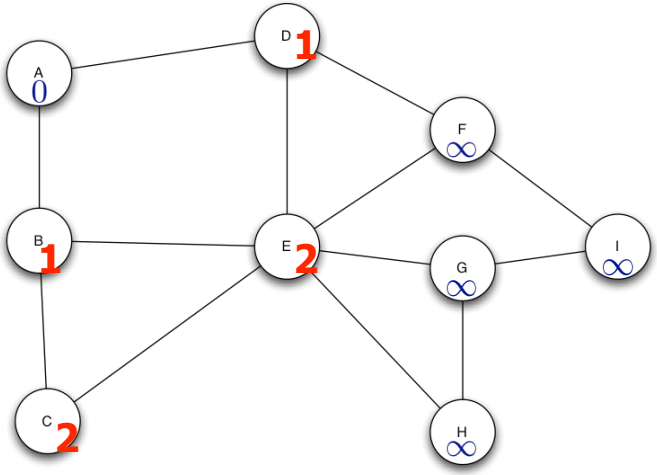
bfs(G, a)



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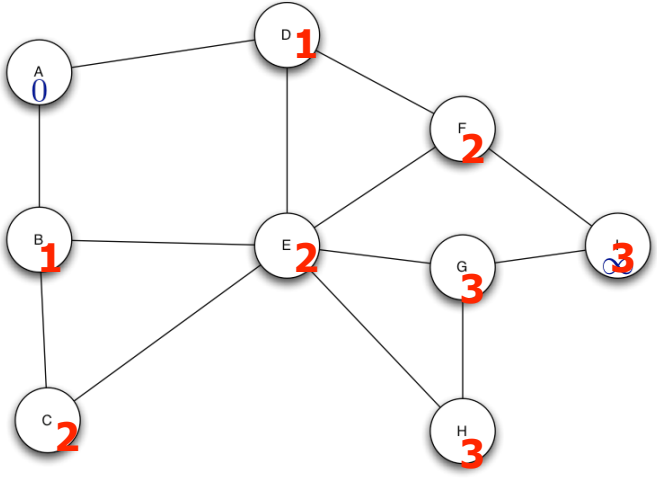
bfs(G, a)



Q

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breadth first search

```
BFS(V, E, s)
for each u ∈ V − {s}
    do d[u] ← ∞
d[s] ← 0
Q ← ∅
ENQUEUE(Q, s)
while Q ≠ ∅
    do u ← DEQUEUE(Q)
        for each v ∈ Adj[u]
            do if d[v] = ∞
                then d[v] ← d[u] + 1
                     ENQUEUE(Q, v)
```

BFS theorem

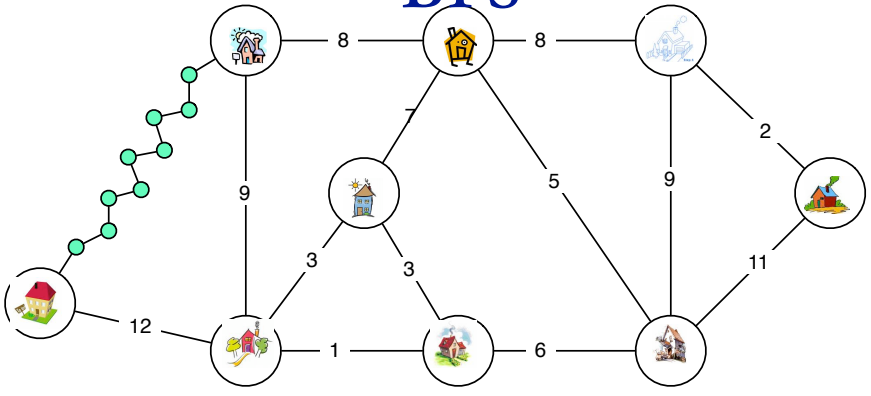
BFS theorem

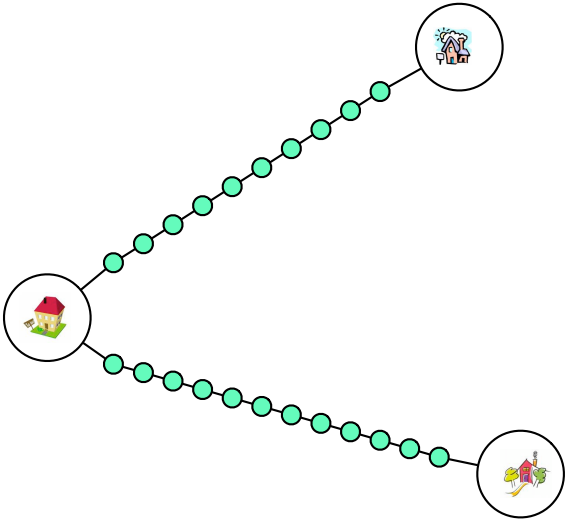
Thm: Let G be a graph and s a vertex. After $BFS(G, s)$ runs, then $d_v = \delta(s, v)$ for all nodes in the graph.

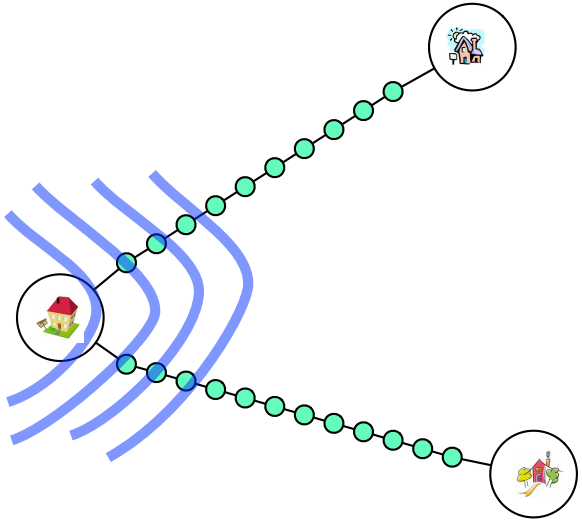
```
BFS(V, E, s)
for each u    V - {s}
    do d[u]    ∞
d[s]    0
Q
ENQUEUE(Q, s)
while Q =
    do u    DEQUEUE(Q)
        for each v    Adj[u]
            do if d[v] = ∞
                then d[v]    d[u] + 1
                    ENQUEUE(Q, v)
```

```
DIJKSTRA(G = (V, E), s)
for all v ∈ V
        do d_u ← ∞
            π_u ← NIL
d_s ← 0
Q ← MAKEQUEUE(V)    ▷ use d_u as key
while Q ≠ ∅
        do u ← EXTRACTMIN(Q)
            for each v ∈ Adj(u)
                do if d_v > d_u + w(u, v)
                    then d_v ← d_u + w(u, v)
                        π_v ← u
                        DECREASEKEY(Q, v)
```

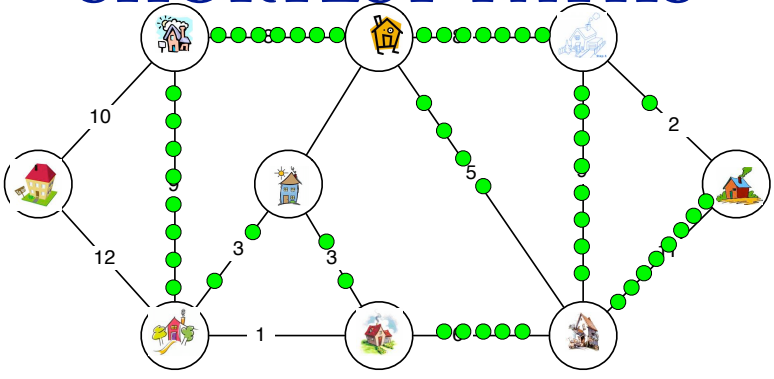
BFS







SHORTEST PATHS



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What about Negative edge weights?

Mid-market rates as of **2016-03-31 17:40 UTC**

Currency code ▲▼	Currency name ▲▼	Units per EUR	EUR per Unit
USD	US Dollar	1.1386632306	0.8782227907
EUR	Euro	1.0000000000	1.0000000000
GBP	British Pound	0.7921136388	1.2624451227
INR	Indian Rupee	75.3658843112	0.0132686030
AUD	Australian Dollar	1.4859561878	0.6729673514
CAD	Canadian Dollar	1.4796754127	0.6758238945
SGD	Singapore Dollar	1.5347639238	0.6515660060
CHF	Swiss Franc	1.0917416715	0.9159676012
MYR	Malaysian Ringgit	4.4140052400	0.2265516114
JPY	Japanese Yen	128.1388820287	0.0078040325
CNY	Chinese Yuan Renminbi	7.3411003512	0.1362193612
NZD	New Zealand Dollar	1.6484648003	0.6066250246
THB	Thai Baht	39.9627318192	0.0250233143
HUF	Hungarian Forint	313.9042436792	0.0031856849
AED	Emirati Dirham	4.1823100458	0.2391023117

XE Currency Table: AED - Emirati Dirham

Mid-market rates as of **2016-03-31 17:39 UTC**

Currency code ▲▼	Currency name ▲▼	Units per AED	AED per Unit
USD	US Dollar	0.2722570106	3.6730000000
EUR	Euro	0.2391289974	4.1818433177
GBP	British Pound	0.1893997890	5.2798369266
INR	Indian Rupee	18.0207422309	0.0554916100
AUD	Australian Dollar	0.3552996418	2.8145257760
CAD	Canadian Dollar	0.3538334124	2.8261887234
SGD	Singapore Dollar	0.3669652245	2.7250538559
CHF	Swiss Franc	0.2610686193	3.8304105746
MYR	Malaysian Ringgit	1.0548325619	0.9480177576
JPY	Japanese Yen	30.6399242607	0.0326371564
CNY	Chinese Yuan Renminbi	1.7555154332	0.5696332719
NZD	New Zealand Dollar	0.3941937299	2.5368237088
THB	Thai Baht	9.5553789460	0.1046530970
HUF	Hungarian Forint	75.0637936939	0.0133220019
AED	Emirati Dirham	1.0000000000	1.0000000000

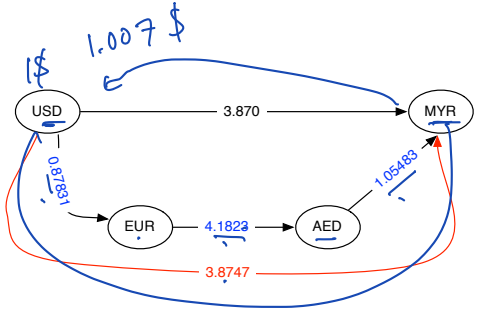
Mid-market rates as of **2016-03-31 17:40 UTC**

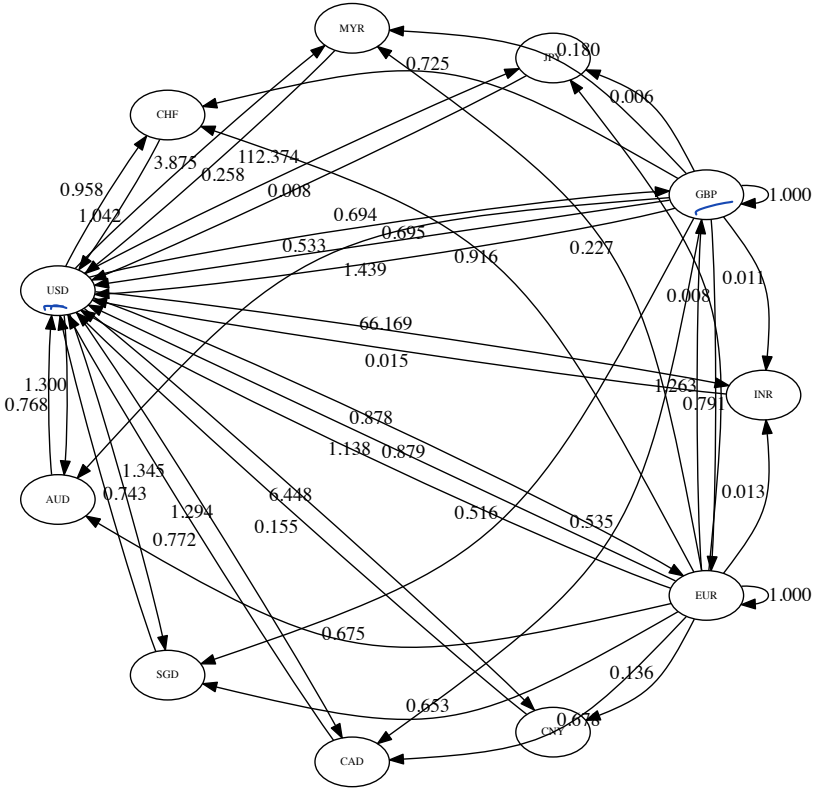
Currency code ▲▼	Currency name ▲▼	Units per EUR	EUR per Unit
USD	US Dollar	1.1386632306	0.8782227907
EUR	Euro	1.0000000000	1.0000000000
GBP	British Pound	0.7921136388	1.2624451227
INR	Indian Rupee	75.3658843112	0.0132686030
AUD	Australian Dollar	1.4859561878	0.6729673514
CAD	Canadian Dollar	1.4796754127	0.6758238945
SGD	Singapore Dollar	1.5347639238	0.6515660060
CHF	Swiss Franc	1.0917416715	0.9159676012
MYR	Malaysian Ringgit	4.4140052400	0.2265516114
JPY	Japanese Yen	128.1388820287	0.0078040325
CNY	Chinese Yuan Renminbi	7.3411003512	0.1362193612
NZD	New Zealand Dollar	1.6484648003	0.6066250246
THB	Thai Baht	39.9627318192	0.0250233143
HUF	Hungarian Forint	313.9042436792	0.0031856849
AED	Emirati Dirham	4.1823100458	0.2391023117

XE Currency Table: AED - Emirati Dirham

Mid-market rates as of **2016-03-31 17:39 UTC**

Currency code ▲▼	Currency name ▲▼	Units per AED	AED per Unit
USD	US Dollar	0.2722570106	3.6730000000
EUR	Euro	0.2391289974	4.1818433177
GBP	British Pound	0.1893997890	5.2798369266
INR	Indian Rupee	18.0207422309	0.0554916100
AUD	Australian Dollar	0.3552996418	2.8145257760
CAD	Canadian Dollar	0.3538334124	2.8261887234
SGD	Singapore Dollar	0.3669652245	2.7250538559
CHF	Swiss Franc	0.2610686193	3.8304105746
MYR	Malaysian Ringgit	1.0548325619	0.9480177576
JPY	Japanese Yen	30.6399242607	0.0326371564
CNY	Chinese Yuan Renminbi	1.7555154332	0.5696332719
NZD	New Zealand Dollar	0.3941937299	2.5368237088
THB	Thai Baht	9.5553789460	0.1046530970
HUF	Hungarian Forint	75.0637936939	0.0133220019
AED	Emirati Dirham	1.0000000000	1.0000000000

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1.439
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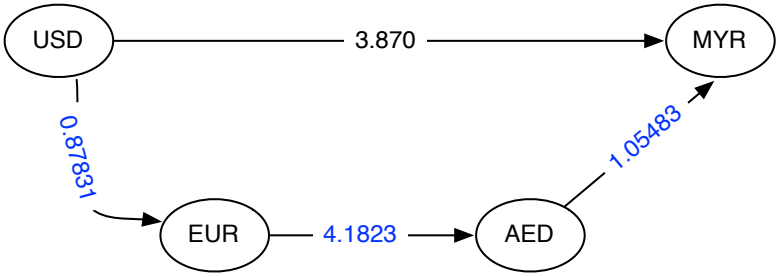
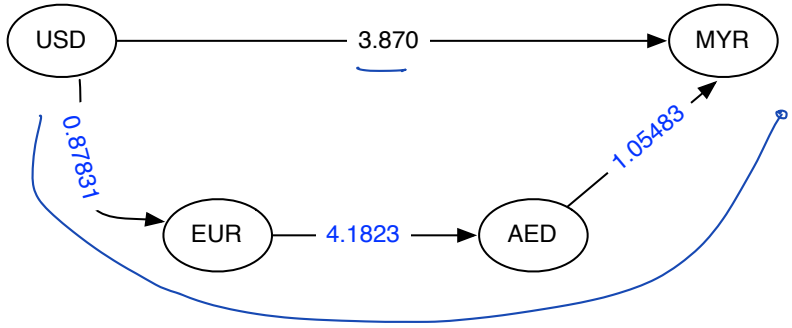
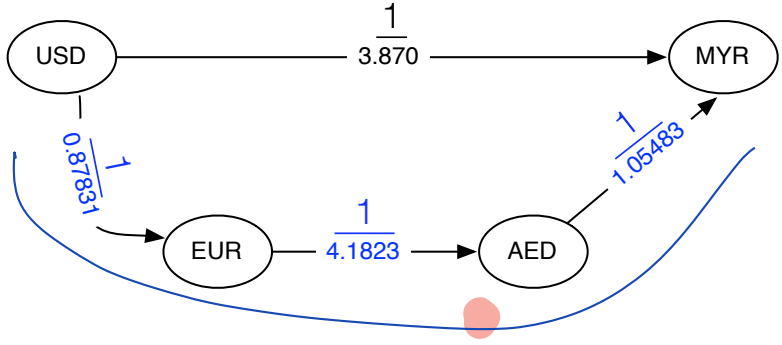


Trying to find Max weight (mult) Paths

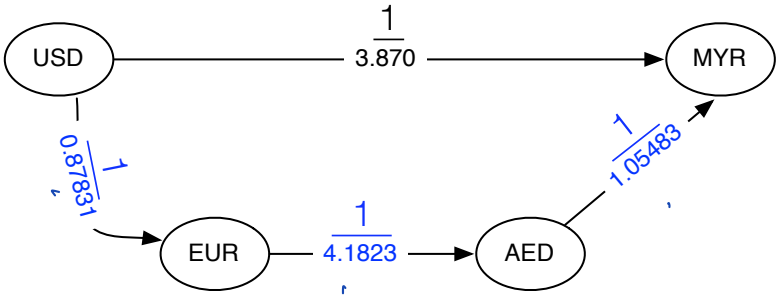


Trying to find Max weight (mult) Paths

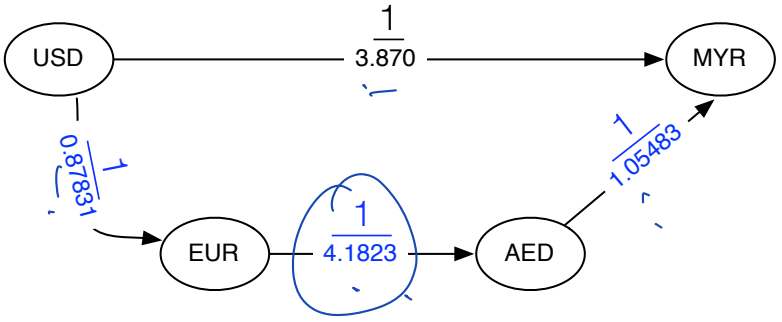
take inverses of each edge weight.



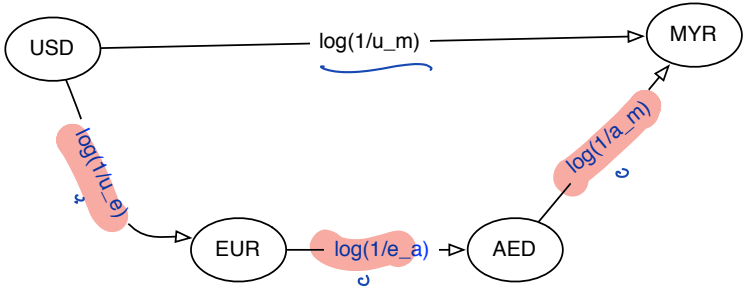
Trying to find MIN weight (mult) Paths



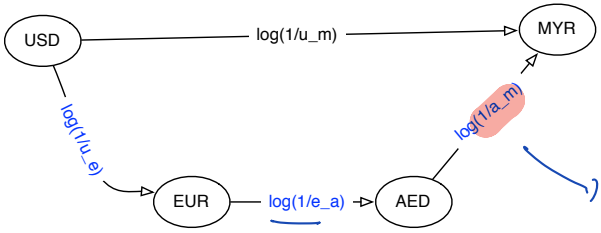
Trying to find MIN weight (mult) Paths



Trying to find MIN weight (mult) Paths



MIN weight paths with additive weights



could be

$$\frac{1}{x} < 1$$

$$\log\left(\frac{1}{4.1823}\right)$$

$$= \log_{10}(0.2391\ldots)$$

$$= -0.62\ldots$$

where does old argument break down

Recall this line in our proof of Dijkstra:

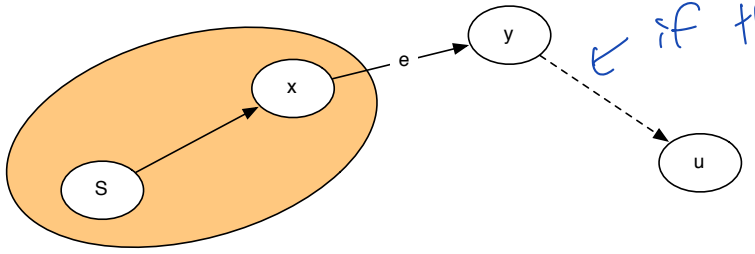
$$w(p) \geq d_y + w(y, u)$$

in previous graphs, we concluded that

$$\underline{w(y,u) \geq 0}$$

$$\Rightarrow w(p) \geq d_u$$

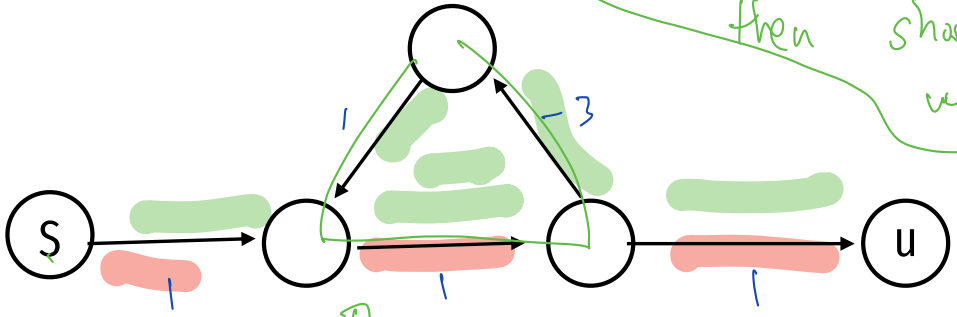
where does old argument break down



if this weight was negative, then we cannot be certain that some other future path does not have a shorter route to u.

2nd problem: cycles

if G has negative weight cycles, then shortest paths are not well defined.



cycle if weight = -1

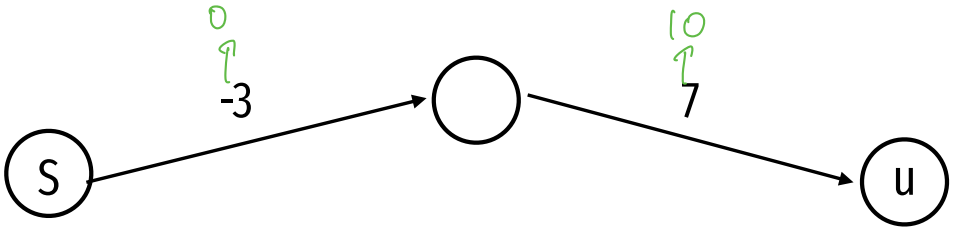
wt = 3

wt = 1+1+-3+1+1+1

= 2

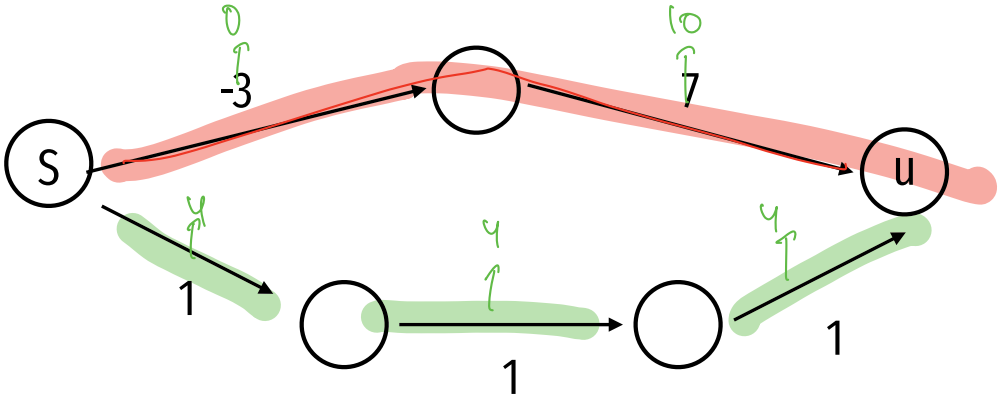
first ideas: Add to each edge

Idea: add a constant to each edge so that all weights are non-negative



first ideas: Add to each edge

Idea: add a constant to each edge so that all weights are non-negative

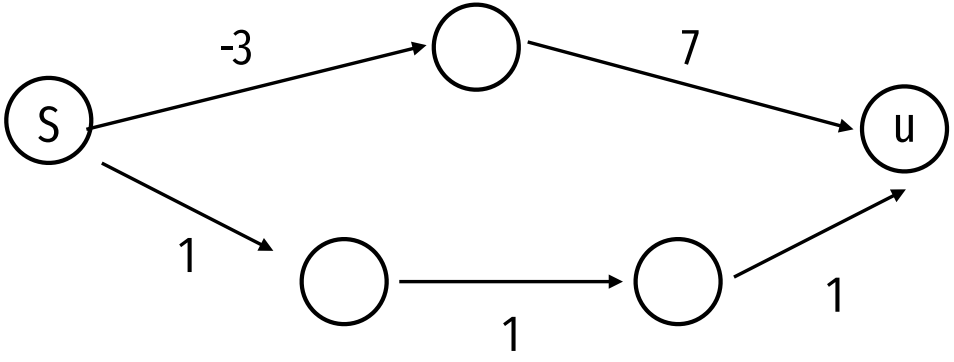


In the new graph, the shortest path changes to become the upper path.

So this idea does not preserve shortest paths in G.

first ideas: Add to each edge

Idea: add a constant to each edge so that all weights are non-negative



Problem: this can change the shortest paths of the graph.

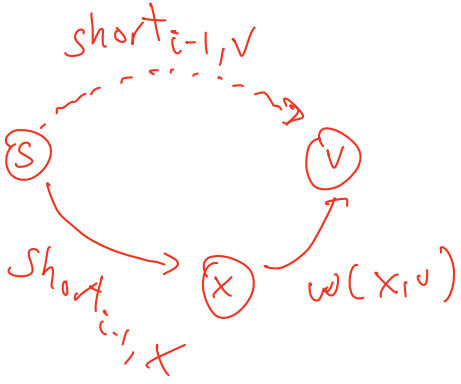
sssp(G,s) = single source (s) shortest paths (with neg edge weights)

$\mathrm{SHORT}_{i,v}$ = length of the shortest path from s to v that uses at most i edges

$$\mathrm{short}_{0,v} = \begin{cases} 0 & \text{if } v = s \\ \text{else } \infty \end{cases}$$

sssp(G,s)

$\text{SHORT}_{i,v} =$ Length of the shortest path from s to v that uses at most i edges.



Two possibilities:

1. use i-1 edges to get from s to v
2. use i-1 edges to get to some intermediate node x and then use the edge (x,v)

sssp(G,s)

$$\text{SHORT}_{i,v} = \begin{cases} \infty & i = 0 \\ 0 & v = s \\ \min_{x \in V} & \left\{ \begin{array}{l} \text{SHORT}_{i-1,v} \\ \text{SHORT}_{i-1,x} + w(x,v) \end{array} \right\} \end{cases}$$

Either the shortest path from s to v

uses at most $i - 1$ edges or

it uses at most $i - 1$ edges to x and then uses edge $e = (x, v)$

max len of a simple path:

is $V-1$.

and so we only need to consider

$\text{short}_{|V|-1,u}$ for every $u \in V$.

max len of a simple path:

The max length of a path from s to v is $|V| - 1$.
Otherwise, the path contains a cycle.

BELLMAN-FORD(G,s)

1. $short_{0,v} = \infty$ for all $v \in V$
2. $short_{0,s} = 0$.
3. for $i = 1$ to $|V| - 1$
4. for each node $v \in V - \{s\}$
5. $$short_{i,v} = \min_{x \in Adj(v)} \begin{cases} short_{i-1,v} \\ short_{i-1,x} + w(x,v) \end{cases}$$

BELLMAN-FORD(G, s)

$\text{SHORT}_{0,s} \leftarrow 0$

$\forall v \in V - \{s\}$, $\text{SHORT}_{0,v} \leftarrow \infty$

for $i = 1, \ldots, V - 1$

  do for each $v \in V - \{s\}$

    do $\text{SHORT}_{i,v} = \min_{x \in Adj(v)} \left\{ \begin{array}{l} \text{SHORT}_{i-1,v} \\ w(x,v) + \text{SHORT}_{i-1,x} \end{array} \right\}$

equivalent loops

- we can rewrite this loop to instead loop over ~~every~~ edge of the graph: for each edge $e = (x, y)$

$$\text{short}_{i,y} = \min \left\{ \begin{array}{l} \text{short}_{i-1,y} \\ \text{short}_{i-1,x} + w(x,y) \\ \text{short}_{i,y} \end{array} \right.$$

- what is the run time of this algorithm??

$\Theta(V \cdot E)$

BELLMAN-FORD(G, s)

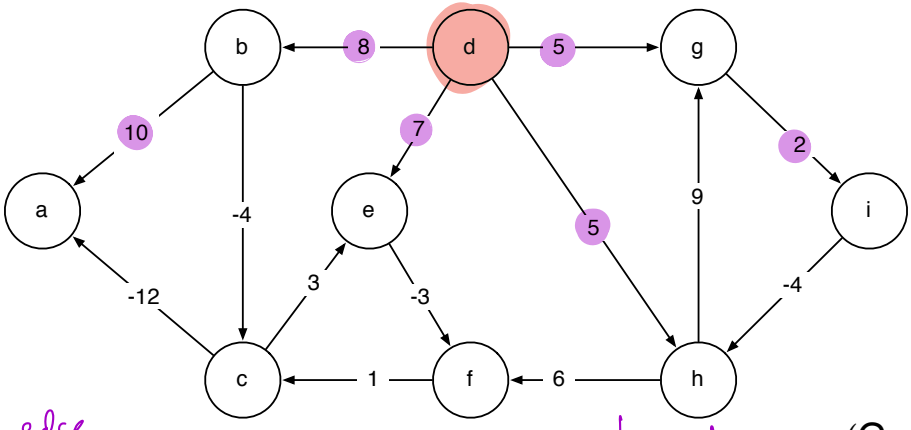
$\text{SHORT}_{0,s} \leftarrow 0$

$\forall v \in V - \{s\}$, $\text{SHORT}_{0,v} \leftarrow \infty$

for $i = 1, \ldots, V - 1$ V

  do for each $e = (x, y) \in E$

    do $\text{SHORT}_{i,y} = \min \left\{ \begin{array}{l} \text{SHORT}_{i-1,y} \\ \text{SHORT}_{i,y} \\ w(x,y) + \text{SHORT}_{i-1,x} \end{array} \right\}$



loop over each edge,

$$\text{SHORT}_{i,v} = \begin{cases} \infty & i = 0 \\ 0 & v = s \\ \min_{x \in V} & \left\{ \begin{array}{l} \text{SHORT}_{i-1,v} \\ \text{SHORT}_{i-1,x} + w(x,v) \end{array} \right\} \end{cases}$$

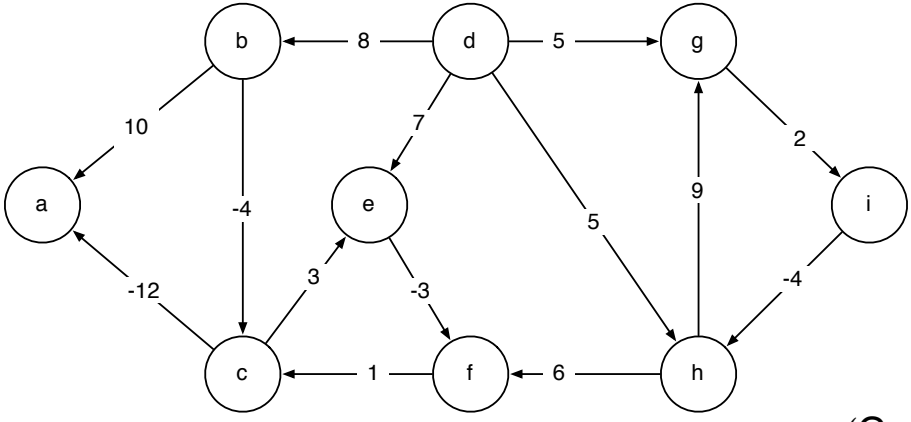
(d,b)

$\text{short}_{1,b} = \min \begin{cases} \text{short}_{0,b} \\ \text{short}_{0,d} + 8 \end{cases}$

short$_{i,v}$

BF(G,d)

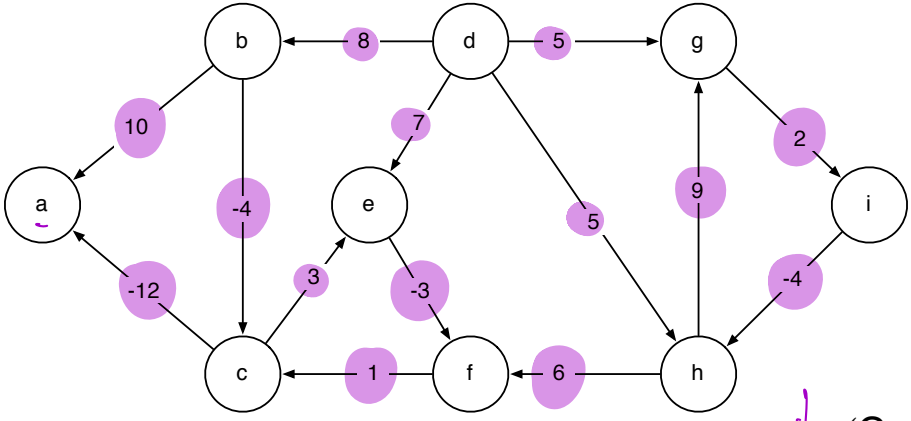
	0	1	2	3	4	5	6	7
A	∞	∞						
B	∞	8						
C	∞	-						
D	0	-						
E	∞	7						
F	∞	-						
G	∞	5						
H	∞	5						
I	∞	-						



$$\text{SHORT}_{i,v} = \begin{cases} \infty & i = 0 \\ 0 & v = s \\ \min_{x \in V} & \left\{ \begin{array}{l} \text{SHORT}_{i-1,v} \\ \text{SHORT}_{i-1,x} + w(x,v) \end{array} \right\} \end{cases}$$

BF(G,d)

	0	1	2	3	4	5	6	7
A	∞							
B	∞							
C	∞							
D	0							
E	∞							
F	∞							
G	∞							
H	∞							
I	∞							

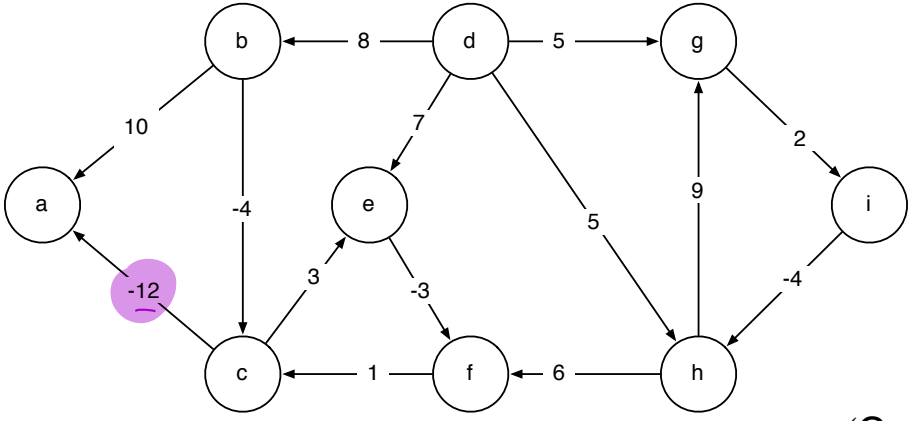


Short$_{2,v}$

$$\text{SHORT}_{i,v} = \begin{cases} \infty & i = 0 \\ 0 & v = s \\ \min_{x \in V} & \left\{ \begin{array}{l} \text{SHORT}_{i-1,v} \\ \text{SHORT}_{i-1,x} + w(x,v) \end{array} \right\} \end{cases}$$

BF(G,d)

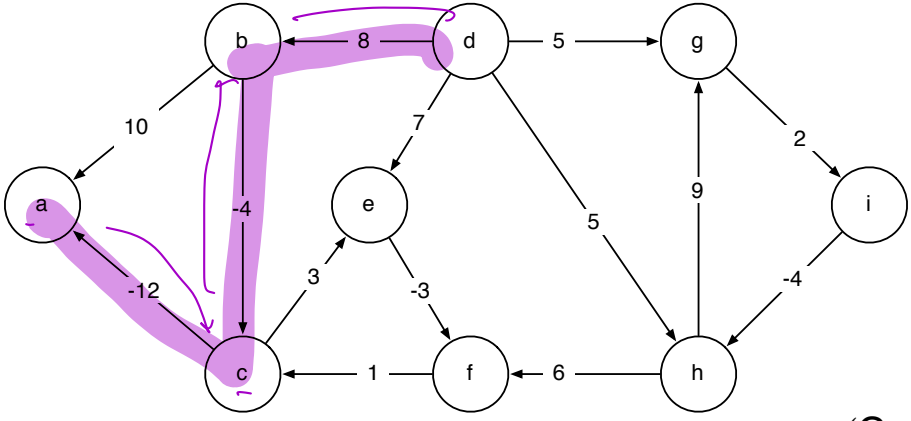
	0	1	2	3	4	5	6	7
A	∞		18					
B	∞	8	8					
C	∞		4					
D	0	0	0					
E	∞	7	7					
F	∞		4					
G	∞	5	5					
H	∞	5	5					
I	∞		7					



$$\text{SHORT}_{i,v} = \begin{cases} \infty & i = 0 \\ 0 & v = s \\ \min_{x \in V} & \left\{ \begin{array}{l} \text{SHORT}_{i-1,v} \\ \text{SHORT}_{i-1,x} + w(x,v) \end{array} \right\} \end{cases}$$

BF(G,d)

	0	1	2	3	4	5	6	7
A	∞		18	-8				
B	∞	8	8					
C	∞		4					
D	0	0	0					
E	∞	7	7					
F	∞		4					
G	∞	5	5					
H	∞	5	5					
I	∞		7					



V^2

$$\text{SHORT}_{i,v} = \begin{cases} \infty & i = 0 \\ 0 & v = s \\ \min_{x \in V} & \left\{ \begin{array}{l} \text{SHORT}_{i-1,v} \\ \text{SHORT}_{i-1,x} + w(x,v) \end{array} \right\} \end{cases}$$

BF(G,d)

	0	1	2	3	4	5	6	7
A	∞		18	-8				
B	∞	8	8	8				
C	∞		4	4				
D	0	0	0	0				
E	∞	7	7	7				
F	∞		4	4				
G	∞	5	5	5				
H	∞	5	5	3				
I	∞		7	7				

optimization

BELLMAN-FORD(G, s)

$\text{SHORT}_{0,s} \leftarrow 0$
$\forall v \in V - \{s\}$, $\text{SHORT}_{0,v} \leftarrow \infty$
for $i = 1, \ldots, V - 1$
  do for each $e = (x, y) \in E$
    do $\text{SHORT}_{i,y} = \min \left\{ \begin{array}{l} \text{SHORT}_{i-1,y} \\ \text{SHORT}_{i,y} \\ w(x,y) + \text{SHORT}_{i-1,x} \end{array} \right\}$

BELLMAN-FORD(G, s)

$d_s \leftarrow 0$
$\forall v \in V - \{s\}$, $d_v \leftarrow \infty$
for $i = 1, \ldots, V - 1$
  do for each $e = (x, y) \in E$
    do $d_y \leftarrow \min \{ d_y, w(x, y) + d_x \}$

$\Theta(E \cdot V)$ time and $\Theta(V)$ space.

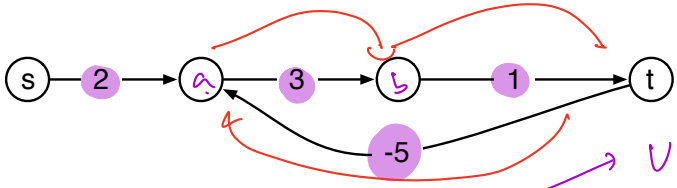
running time

```
BELLMAN-FORD(G, s)
d_s ← 0
∀v ∈ V − {s}, d_v ← ∞
for i = 1, ..., V − 1
        do for each e = (x, y) ∈ E
                do d_y ← min { d_y, w(x, y) + d_x }
```

negative cycles?

if G contains a negative cycle, then some distance value will decrease on the Vth iteration of the BF algorithm..

$3+1+(-5) = (-1)$ negative cycle.



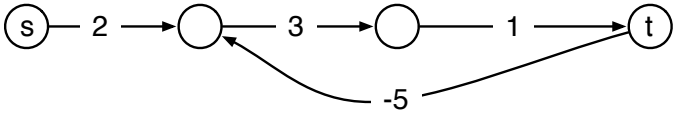
V th step

	1	2	3	4
S	0	0	0	0
A	2	2	2	2
B	.	5	5	5
T	.	.	6	6

$\min\left(2,\ 6-5=1\right)$

∞ if there is no value listed.

negative cycles?



S	0	0	0	0
A	2	2	2	1
B		5	5	5
T			6	6

if some value decreases on the Vth step then $\exists$ a negative cycle.

To show: if $\exists$ a negative cycle, some value decreases.

applications of BF

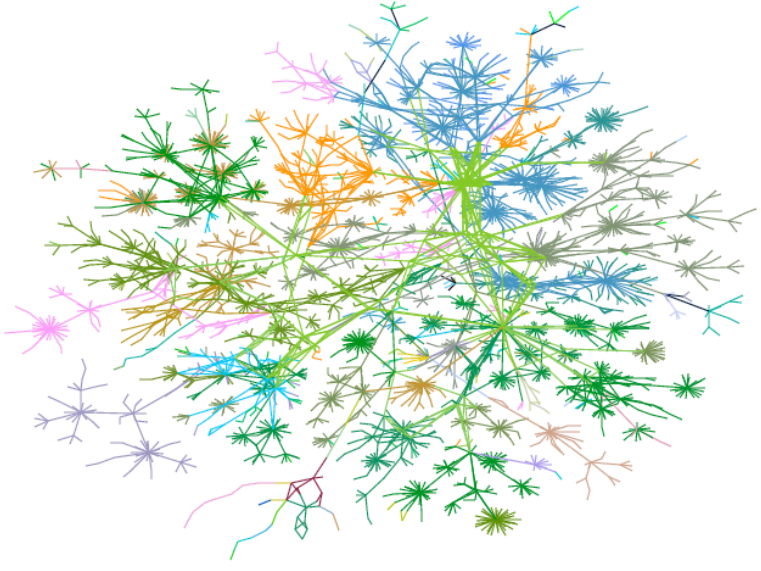
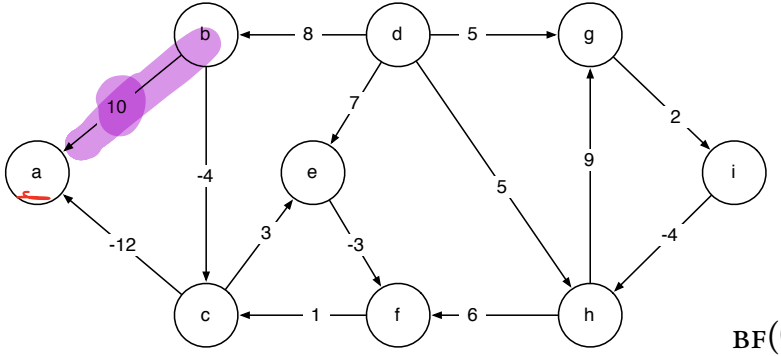


image: cheswick et al

Figure 3: Lucent's intranet as of 1 October 1999.



WHAT HAPPENS WHEN
B CHANGES...

BF(G,d)

	0	1	2	3	4	5	6	7
A	∞							
B	∞	8						
C	∞							
D	0	0						
E	∞	7						
F	∞							
G	∞	5						
H	∞	5						
I	∞							

DISTANCE VECTOR

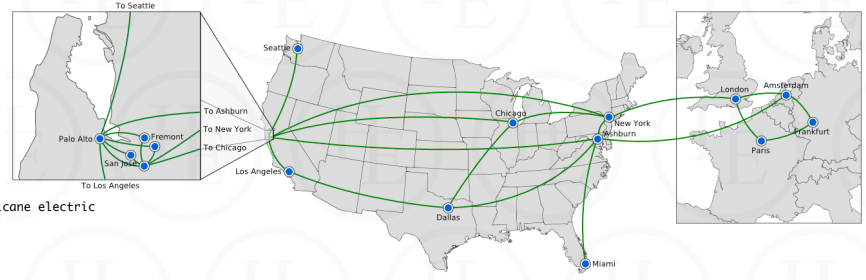
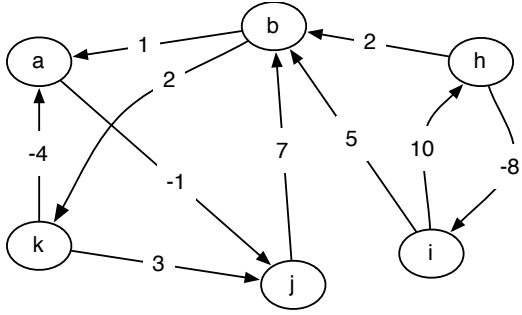


image: hurricane electric

⇒ BGP uses a distance vector (ie, BF inspired) algorithm to maintain shortest paths.

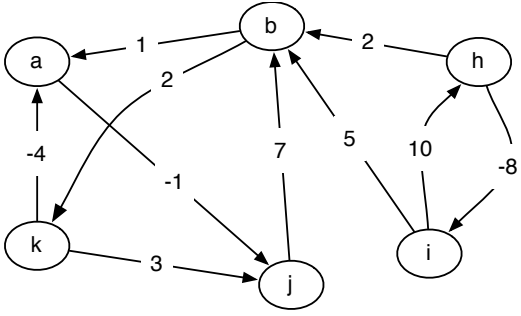
ALL-PAIRS SHORTEST PATH



First approach: run BF from each node.
What is the running time? $V \cdot \underline{EV} = \Theta(EV^2)$ $O(V^4)$

$O(V^2)$

ALL-PAIRS SHORTEST PATH



First approach: run BF from each node.
What is the running time?

$$O(EV^2)$$

NEW APPROACH TO ALL-PAIRS

$\text{ASHORT}_{i,j,k}$ = length of the shortest path between nodes i,j that only use intermediate nodes 1....k.

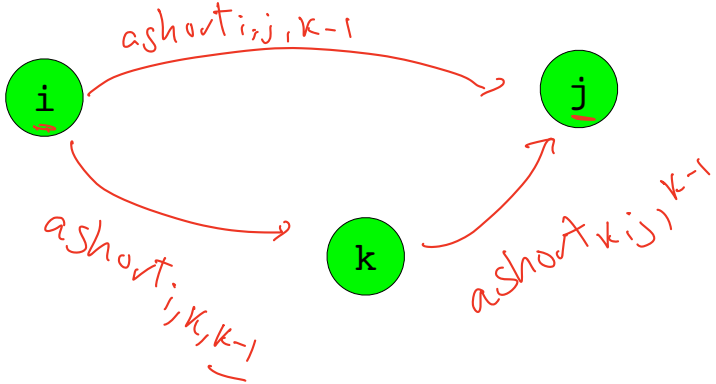
NEW APPROACH TO ALL-PAIRS

$\text{ASHORT}_{\text{i,j,k}} =$ Length of the shortest path from i to j that only traverses nodes $1, \ldots, k$.

$1 \ldots k-1$

$$\text{ASHORT}_{i,j,k} = \min \begin{cases} w(i,j) & \text{if } k = 0 \quad \text{(base case)} \\ \text{ashort}_{i,j,k-1} \\ \text{ashort}_{i,k,k-1} + \text{ashort}_{k,j,k-1} \end{cases}$$

Q: how does node k help in getting from i to j?



using the first $k-1$ intermediate nodes

$\text{ashort}_{i,j,|V|}$

$$\mathrm{ASHORT}_{\mathtt{i},\mathtt{j},\mathtt{k}} =$$

$$\mathrm{ASHORT}_{i,j,k} = \begin{cases} w_{i,j} & k = 0 \\ \min \begin{cases} \mathrm{ASHORT}_{i,j,k-1} \\ \mathrm{ASHORT}_{i,k,k-1} + \mathrm{ASHORT}_{k,j,k-1} \end{cases} & k \geq 1 \end{cases}$$

FLOYD-WARSHALL(G, W)

INITIALIZE $ashort_{i,j,0}$

for $k = 1$ to $|V|$ $\Theta(V^3)$

for $i = 1$ to $|V|$

for $j = 1$ to $|V|$

$ashort_{i,j,k} = \min$ { equation from previous slide