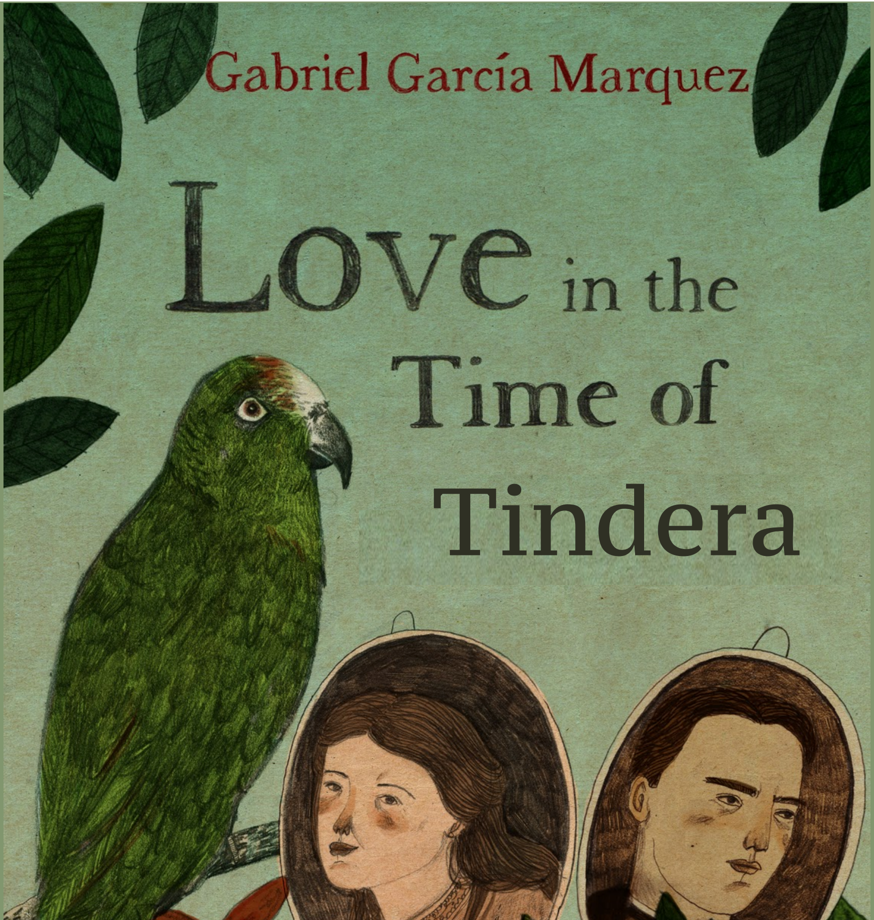
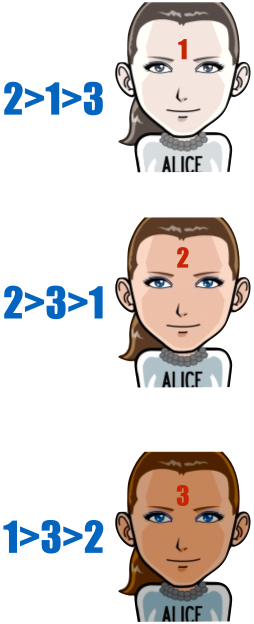
5800 Stablematch

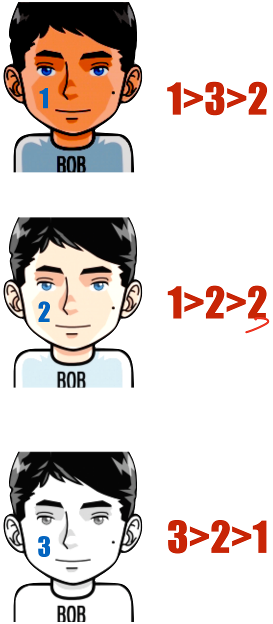
apr1/apr4 2022
shelat


Gabriel García Marquez
Love in the
Time of
Tindera

We have a group of suitors and reviewers



Each has preferences over the other group



2>1>3

2>3>1

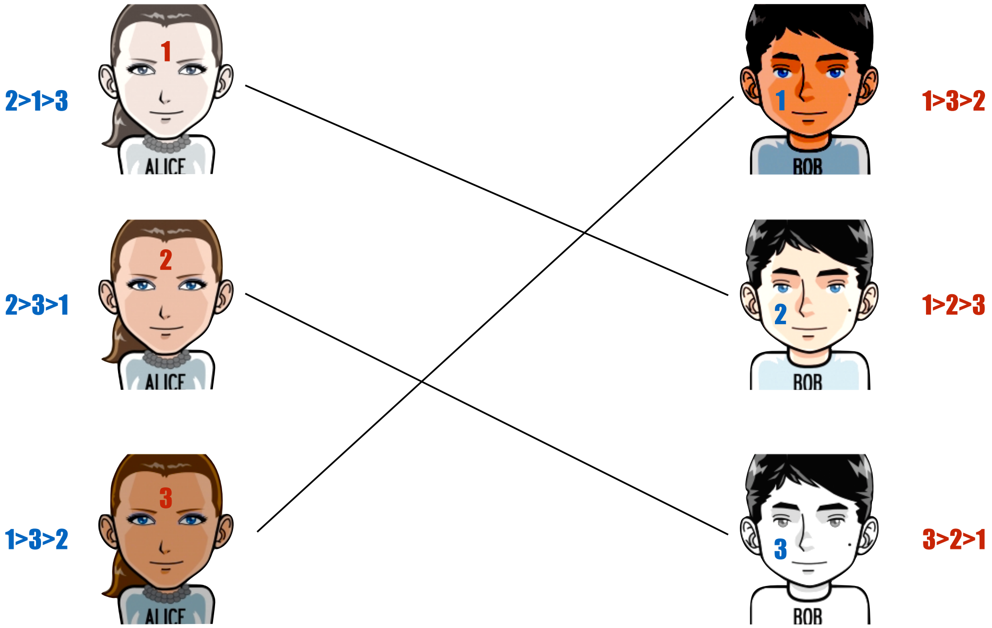
1>3>2

We seek a **stable matching** between the two

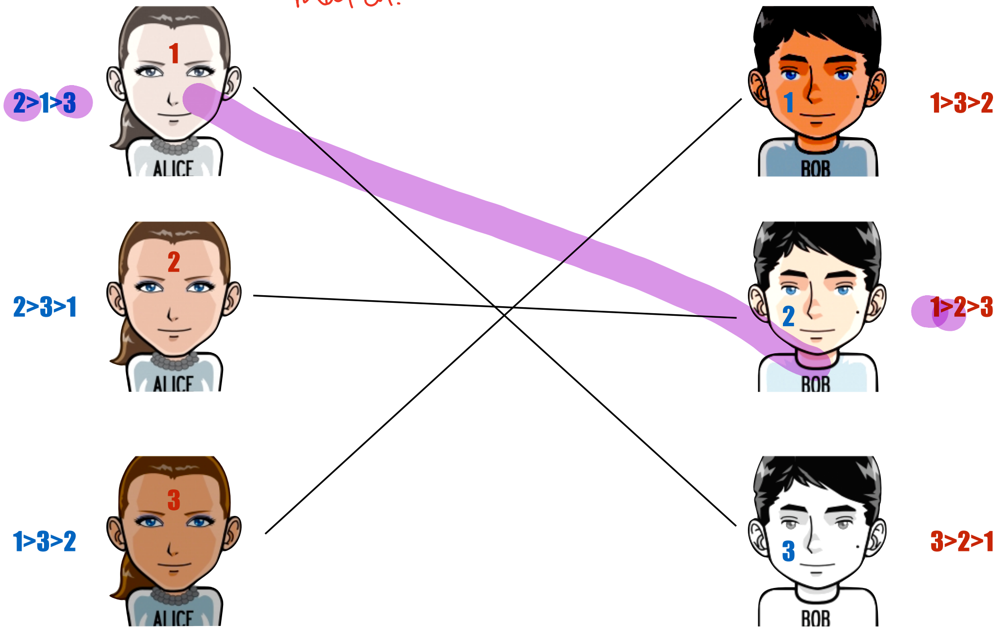
1>3>2

1>2>2

3>2>1

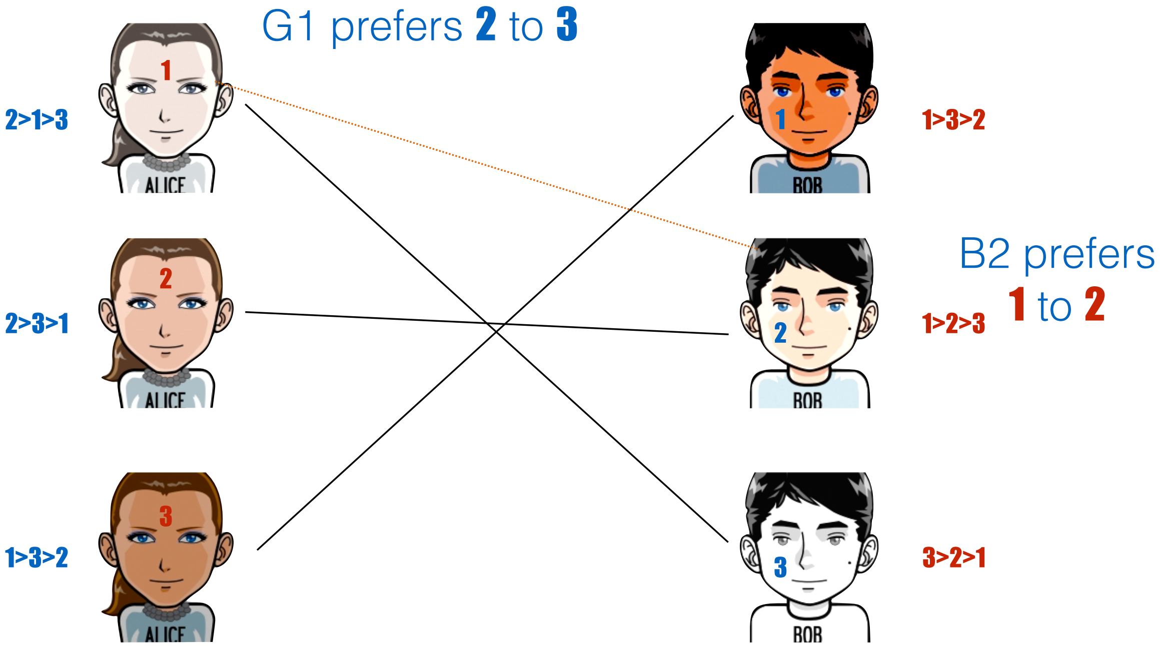

2>1>3
1
ALICE
2>3>1
2
ALICE
1>3>2
3
ALICE
1
BOB
1>3>2
2
BOB
1>2>3
3
BOB
3>2>1

Alice 1 prefers B2 to her current match.

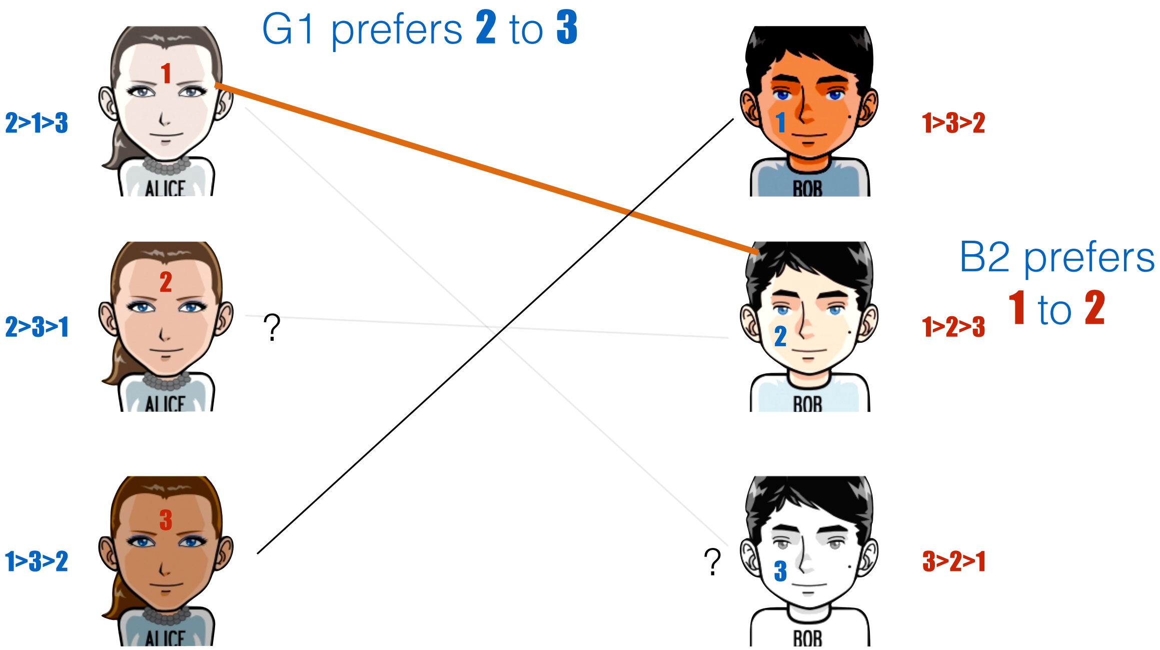


Bob 2 prefers Alice 1 to his current match.

Unstable Matching



Unstable Matching



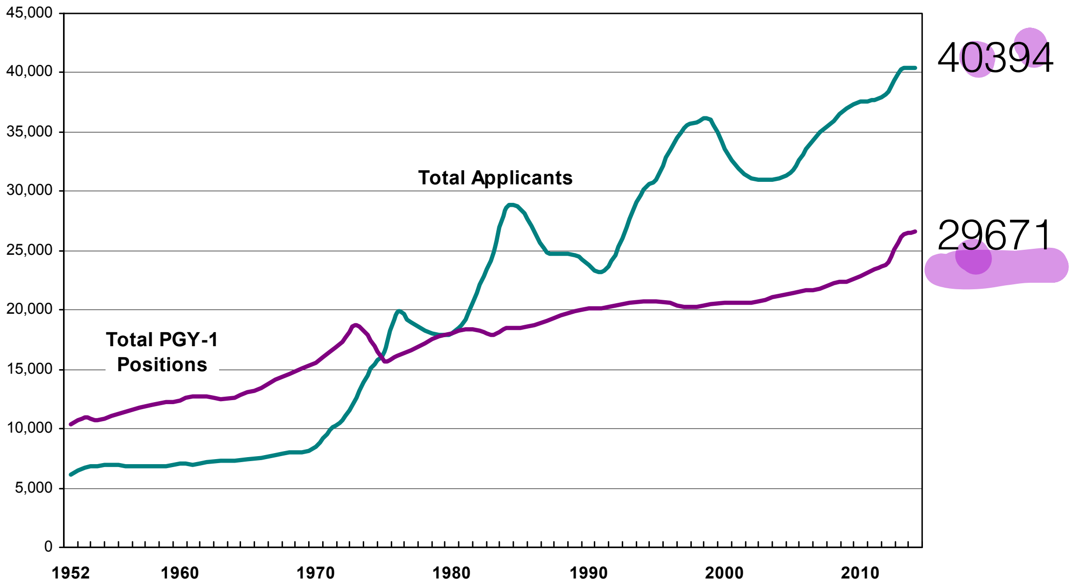
Unstable Matching

Stable Matching

Stable matching has many practical applications

THE MATCH
NATIONAL RESIDENT MATCHING PROGRAM®

Figure 1 Applicants and 1st Year Positions in The Match, 1952 - 2014



Applicant Type	Matched		
	2013 Graduates	Prior Year Graduates[1]	Total
CMG	2571	74	2645
IMG	146	353	499
USMG	23	2	25
TOTAL	**2740**	**429**	**3169**

UNIVERSITAS OSLOENSIS MDCCCXI

NUS
National University of Singapore

SUTD
SINGAPORE UNIVERSITY OF TECHNOLOGY AND DESIGN
Established in collaboration with MIT

NANYANG TECHNOLOGICAL UNIVERSITY


GreekYearbook
www.GreekYearbook.com
University of Virginia
Chi Omega Bid Day 2012

Definition: matchings

proposers

$P = \{ p_1, p_2, \ldots \; p_n \}$

$R = \{ r_1, r_2, \ldots \; r_n \}$

reviewers

$M = \{ (p_{i_1}, r_{j_1}), (p_{i_2}, r_{j_2}), \ldots (p_{i_n}, r_{j_n}) \}$

goal is to find a matching

pairs such that each p_i and each r_j appears in exactly one pair in M.

Definition: matchings

$$P = \{p_1, p_2, \ldots, p_n\}$$

$$R = \{r_1, r_2, \ldots, r_n\}$$

$$M = \{(p_{i_1}, r_{j_1}), \ldots, (p_{i_n}, r_{j_n})\}$$

perfect

stable

matchings

Each `pi` (`rj`) appears only one in a pairing.

A matching is perfect if every `pi` appears.

Proposer

Image credits: Julia Nikolaeva

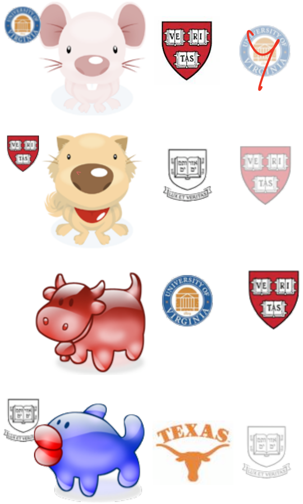
Definition: preferences

$$P = \{p_1, p_2, \ldots, p_n\}$$

" $r_1 \prec_{p_1} r_2$ " p_1 prefers r_2 over r_1

$V >_{Pr} H > Y > .$

fix:



V

H

Y

T

Image credits: Julia Nikolaeva

Example: preferences

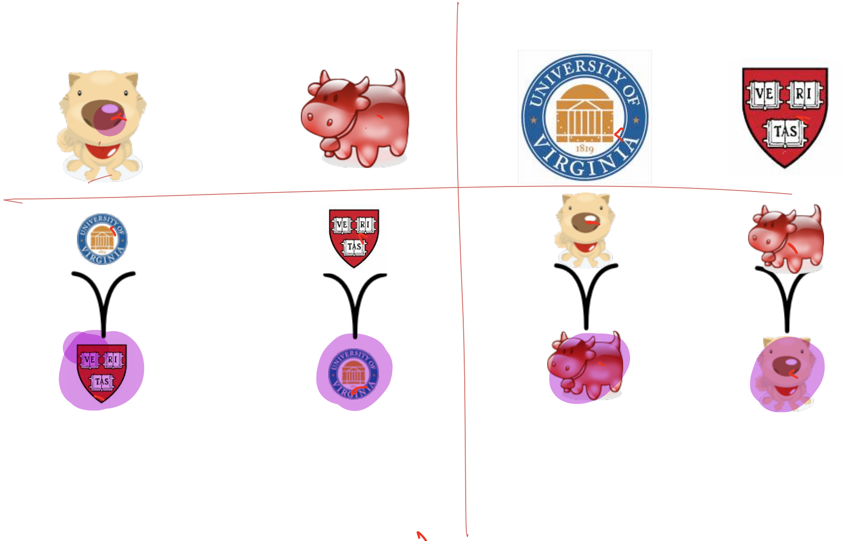
$$P = \{p_1, p_2, \ldots, p_n\}$$

p_i has a preference relation on the set R

$$w_1 \prec_{m_i} w_4 \prec_{m_i} w_2 \prec_{m_i} w_8 \cdots w_n$$

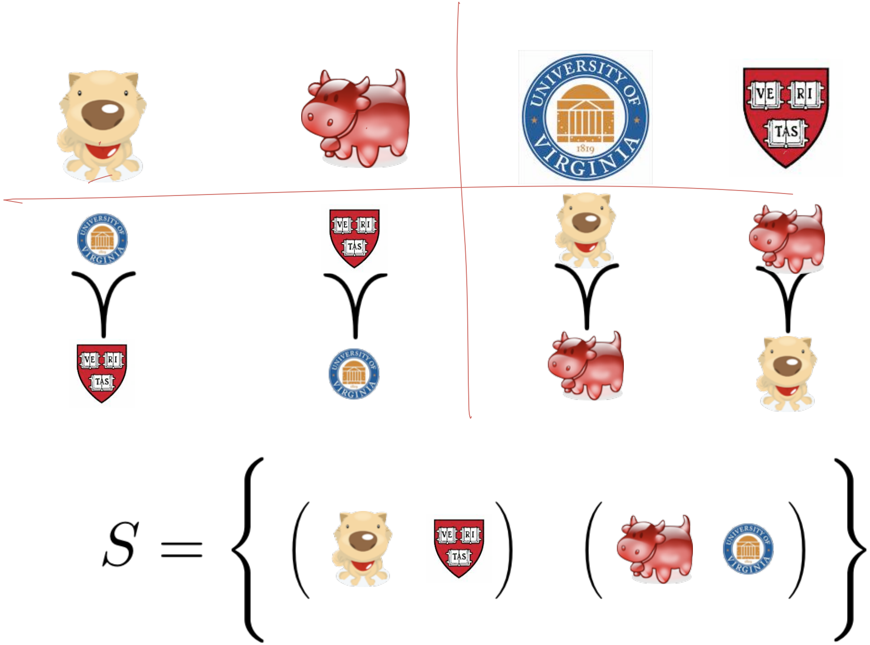
$\prec \quad \prec$

Y H V

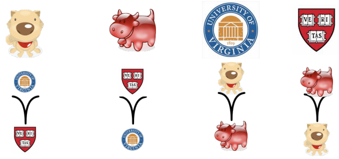


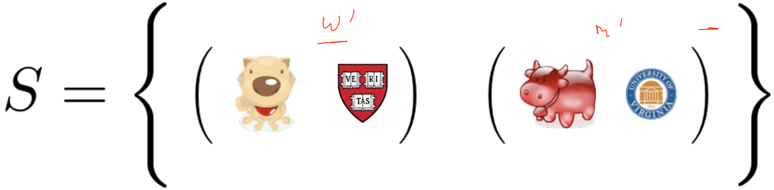
$M = \{ (D,H), (C,V) \}$

this is an example of an unstable matching


UNIVERSITY OF VIRGINIA
1819
VE RI TAS
S =

Def: instability





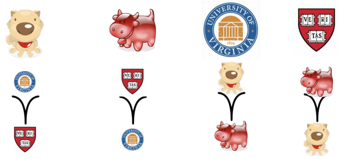
B/c (D, V) is not in S. and
D prefers V to H
V prefers D to C

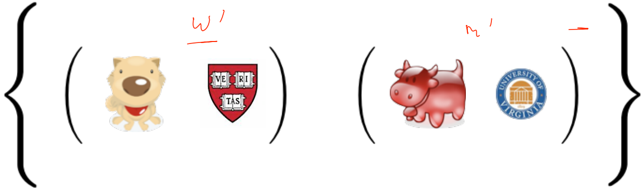
INSTABILITY: it is a pair $(p_i, r_j) \notin S$ that is not in the matching such that

p_i prefers r_j to its match in S and

r_j prefers p_i to its match in S.

Def: instability



$S =$ 

 $(m^*, w^*) \notin S$

$w' \prec_{m^*} w^*$

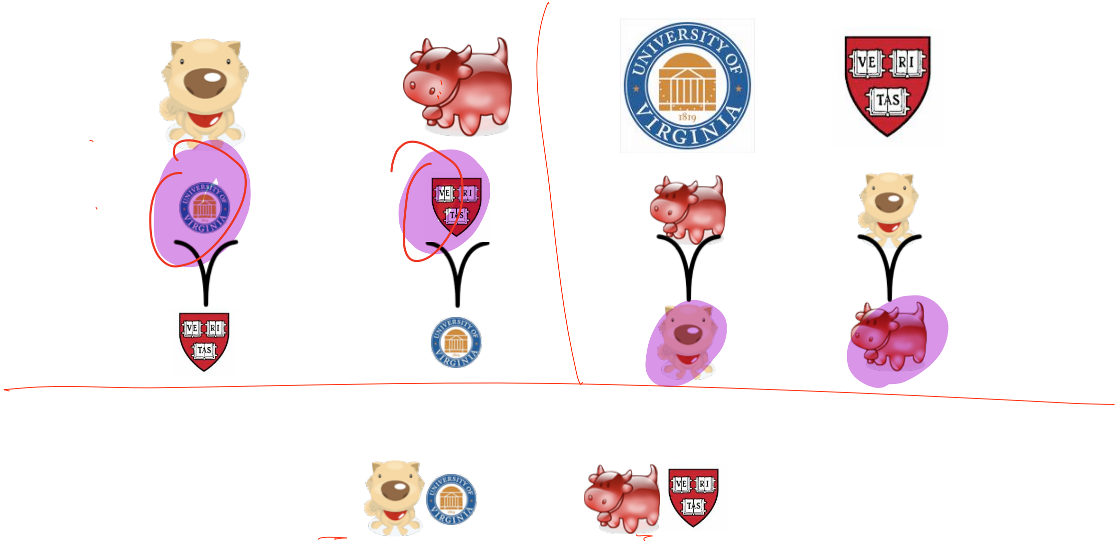
$m' \prec_{w^*} m^*$

M $= \{ (s_1,r_1), (s_2,r_2), \ldots (s_n,r_n) \}$

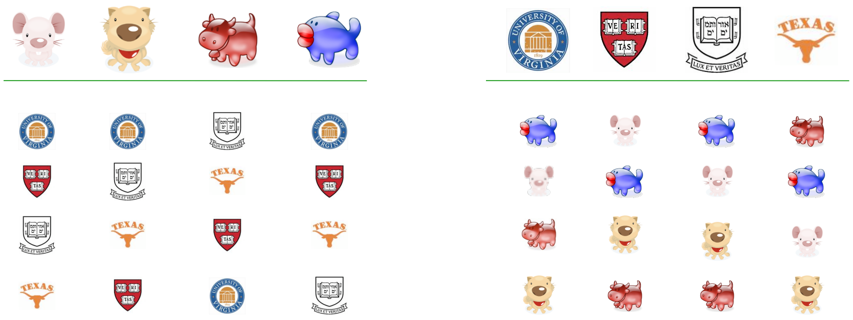
is a stable matching if

No unmatched pair (s^*,r^*) prefer each other to their partners in M

Example 2



Prove: for every input



there exists a stable matching.

proposal algorithm

① Start with everyone unmatched

② while ∃ an unmatched suitor

Let r be the highest ranked reviewer that s hasn't proposed yet

Let s "propose" to r:

if r is unmatched: create pair (s,r)

if r is matched to (s', r) and r prefers $s >_r s'$

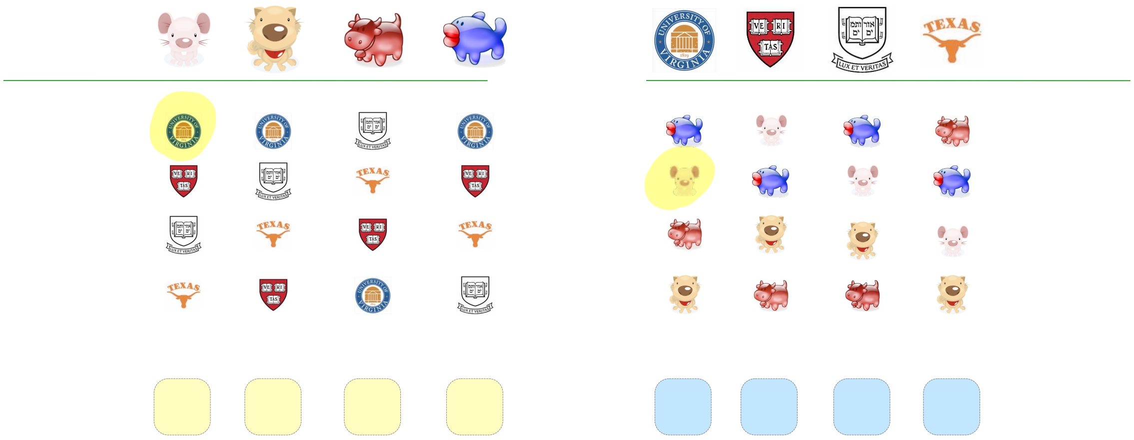
then break (s', r) and create pair (s,r)

otherwise: continue in the loop.

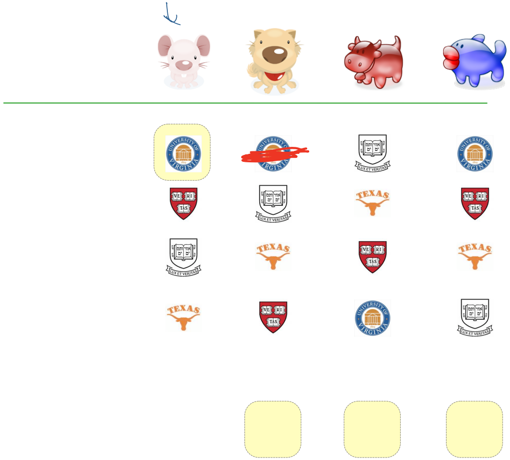
```
StableMatch(M, W, ≺_m, ≺_w)
Initialize all m, w to be FREE
while ∃FREE(m) and hasn't proposed to all W
      do Pick such an m
         Let w ∈ W be highest-ranked to whom m has not yet proposed
         if FREE(w)
            then Make a new pair (m, w)
         elseif (m′, w) is paired and m′ ≺_w m
              do Break pair (m′, w) and make m′ free
                 Make pair (m, w)
return Set of pairs
```

S

R



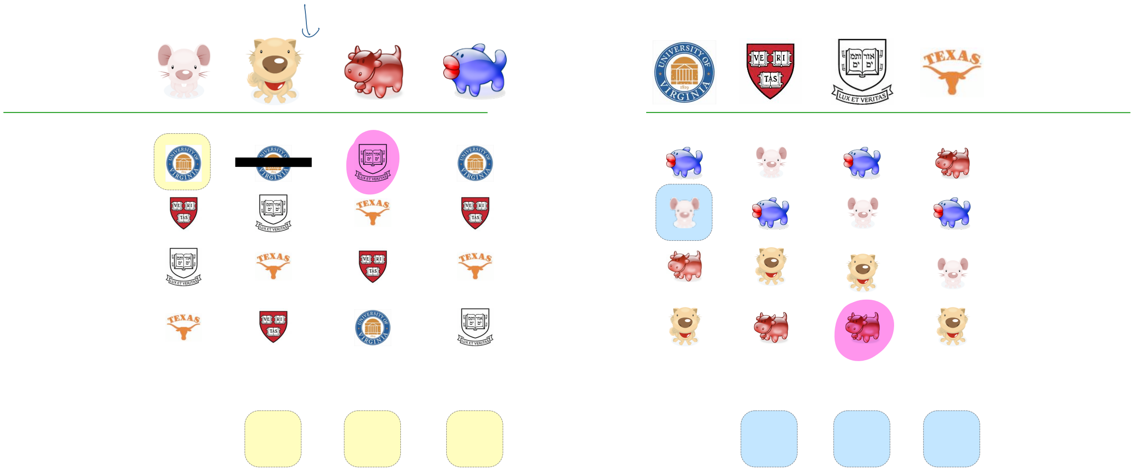
S



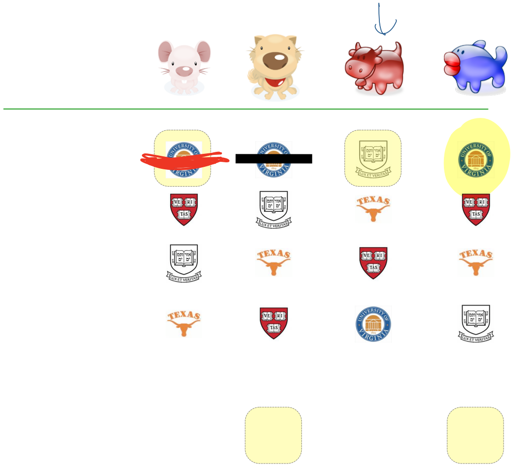
R

S

R



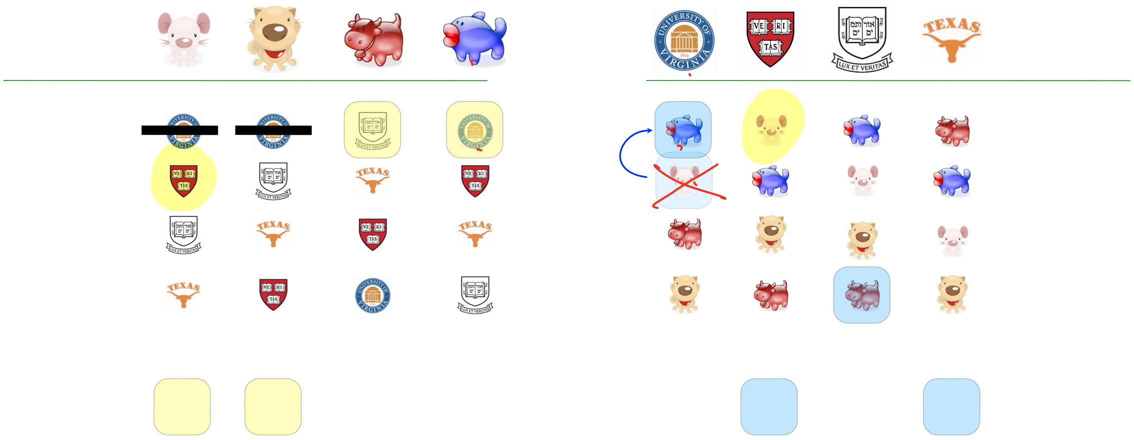
S



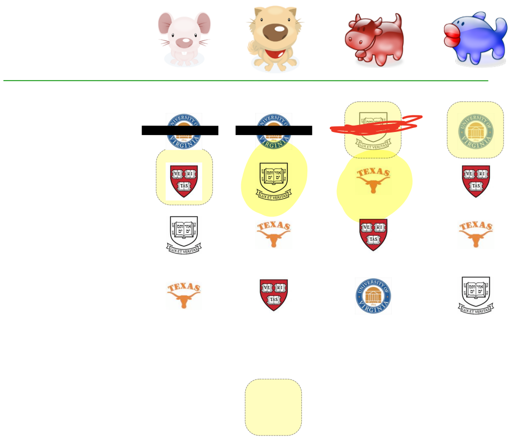
R

S

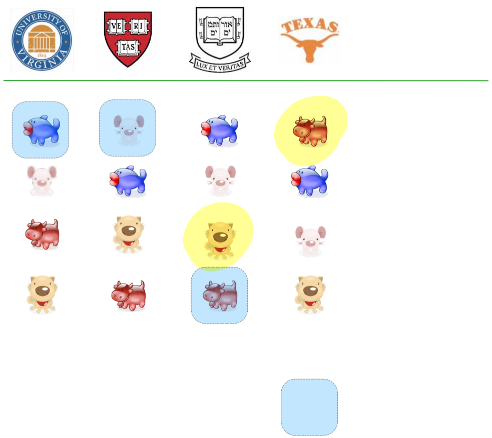
R



S



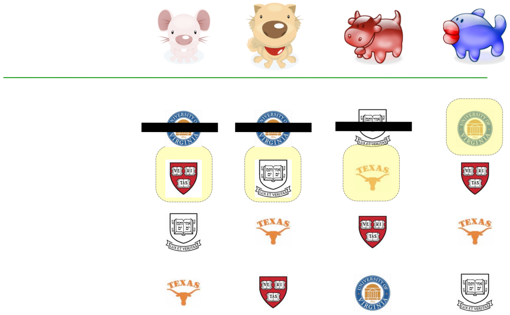
R



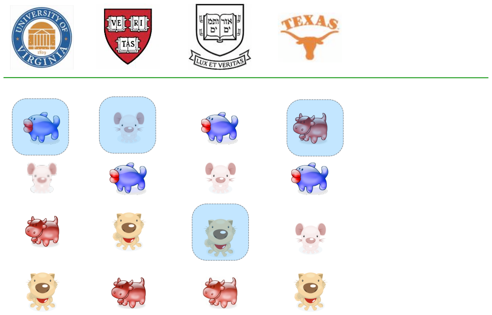
S

R

S



R



Proposal algorithm ends

- EACH suitor proposes at most once to each reviewer
- EACH m proposes $\leq n$ times.

  Since there are n suitors, then $O(n^2)$.

Proposal algorithm ends

$O(n^2)$ steps

each m proposes at most once to each w.

each m proposes at most n times.

size of M is at most n.

output is a matching

① Each suitor appears at most once in the output

This follows because pairs are only created in lines 6, 9, and when a pair is created, both parties are free.

StableMatch($M, W, \prec_m, \prec_w$)

Initialize all m, w to be free
while $\exists$free(m) and hasn't proposed to all W
    do Pick such an m
        Let $w \in W$ be highest-ranked to whom m has not yet proposed
        if free(w)
          then Make a new pair (m, w)
        elseif (m', w) is paired and $m' \prec_w m$
            do Break pair (m', w) and make m' free
                Make pair (m, w)
return Set of pairs

StableMatch($M, W, \prec_m, \prec_w$)

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    do Pick such an m
        Let $w \in W$ be highest-ranked to whom m has not yet proposed
        if FREE(w)
            then Make a new pair (m, w)
        elseif (m', w) is paired and $m' \prec_w m$
            do Break pair (m', w) and make m' free
                Make pair (m, w)
return Set of pairs

output is perfect

$|M| = n$??

$\Rightarrow$ if there is an unmatched suitor,

$\exists$ an unmatched reviewer,

so algorithm cannot have terminated

output is perfect

if $\exists m$ who is free, then

$\exists w$ who has not been asked

output is stable

Proof: By contradiction. Suppose the output M is NOT STABLE.

That means there exists a pair (p^*, r^*) that is NOT in the output M such that $r^* \succ_{p^*} M(p^*)$ and $p^* \succ_{r^*} M(r^*)$

- Consider the moment when r^* is matched with $M(r^*)$ and the moment when p^* is matched with $M(p^*)$

(1) p^* must have proposed to $M(p^*)$ last.

But we know $r^* \succ_{p^*} M(p^*)$

$\Rightarrow$ p^* must have proposed to r^* earlier in the algorithm.

($M(r^*)$ denotes the match of r^* in the output M.)

output is stable

spse not. $\exists (m^*, w), (m, w^*) \in S \quad w \prec_{m^*} w^* \quad m \prec_{w^*} m^*$

What happened when p^* proposed to r^*:

ⓐ (p^*, r^*) pair was created ⇒ but then, another proposa p' proposed to r^*, and r^* preferred p' to p^*.

⇒ this contradicts the assumption that r^* prefers p^* to its current match, because matches are only broken when the reviewer's preference improve.

This contradicts $p^* \succ_{r^*} M(r^*)$.

output is stable

spse not. $\exists (m^*, w), (m, w^*) \in S \quad w \prec_{m^*} w^* \quad m \prec_{w^*} m^*$

(b) 2nd case: r* was already matched to a p' at the time p* proposed, and the match was not broken.

⇒ again this contradicts our assumption that r* prefers p* to its current match.

output is stable

spse not. $\exists (m^*, w), (m, w^*) \in S$ $w \prec_{m^*} w^*$ $m \prec_{w^*} m^*$

m* last proposal was to w

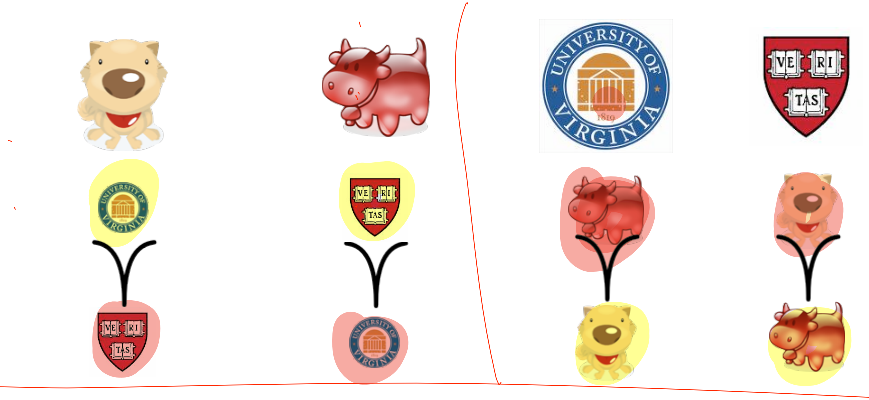
but $w \prec_{m^*} w^*$ and so m* must have already asked w*

and must have been rejected by $m^* \prec_{w^*} m'$

then either $m' \prec_{w^*} m$ or m'=m

which contradicts assumption $m \prec_{w^*} m^*$

Proposer wins



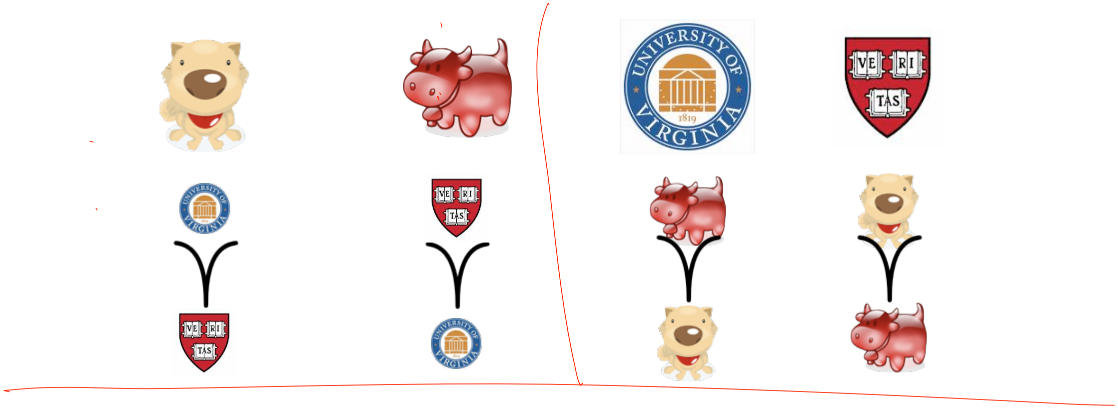
there are 2 stable matchings in this case.

(DV)(CH)

(DH)(CV)

both are stable.

Proposer wins



Remarkable theorem

w is valid for m:

best(m):

GS is Suitor-optimal.

GS matching vs R-opt

S1	S2	S3	S4

R1	R2	R3	R4
S1	S1	S1	S1
S2	S2	S2	S2
S3	S3	S3	S3
S4	S4	S4	S4

S1	S2	S3	S4
R1	R1	R1	R1
R2	R2	R2	R2
R3	R3	R3	R3
R4	R4	R4	R4

R1	R2	R3	R4
S1	S1	S1	S1
S2	S2	S2	S2
S3	S3	S3	S3
S4	S4	S4	S4

S1	S2	S3	S4
R1	R1	R1	R1
R2	R2	R2	R2
R3	R3	R3	R3
R4	R4	R4	R4

R1	R2	R3	R4
S1	S1	S1	S1
S2	S2	S2	S2
S3	S3	S3	S3
S4	S4	S4	S4

Not honest

S1	S2	S3
R2	R1	R1
R1	R2	R3
R3	R3	R2

R1	R2	R3
S1	S2	S2
S2	S1	S3
S3	S3	S1

Not honest

S1	S2	S3
R2	R1	R1
R1	R2	R3
R3	R3	R2

R1	R2	R3
S1	S2	S2
S2	S1	S3
S3	S3	S1

R2	R1	R1
R1	R2	R3
R3	R3	R2

S1	S2	S2
S3	S1	S3
S2	S3	S1

Not honest

S1	S2	S3
R2	R1	R1
R1	R2	R3
R3	R3	R2

R1	R2	R3
S1	S2	S2
S2	S1	S3
S3	S3	S1

R2	R1	R1
R1	R2	R3
R3	R3	R2

S1	S2	S2
S3	S1	S3
S2	S3	S1

THE MATCH℠
NATIONAL RESIDENT MATCHING PROGRAM®

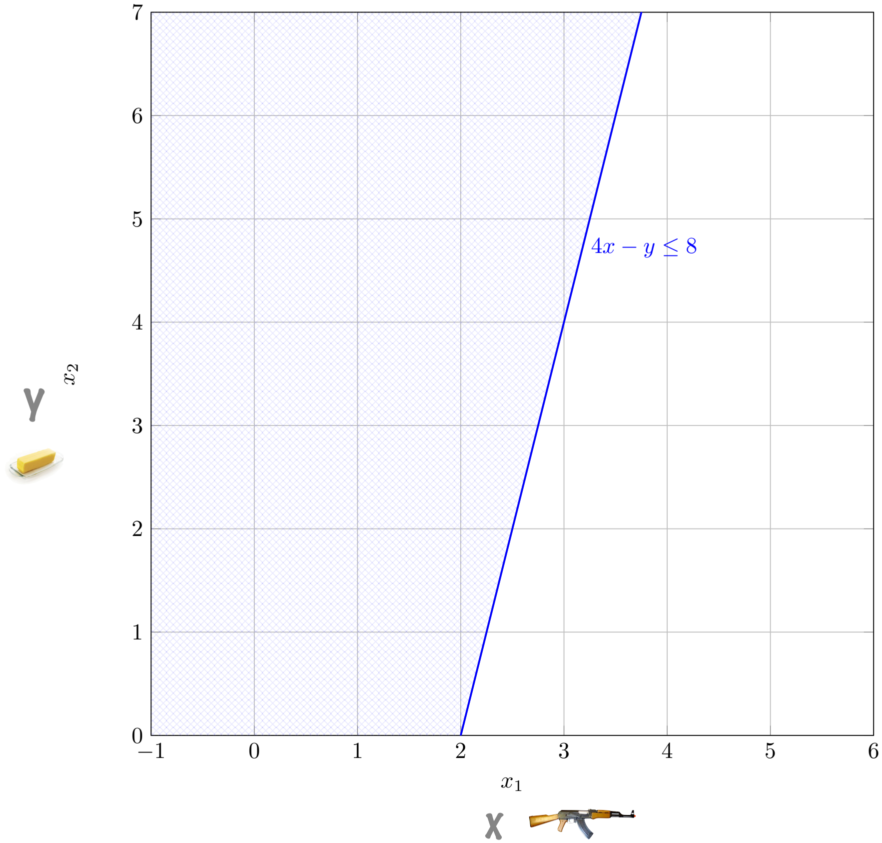
Guns and butter

$$\max x + y$$

$$\begin{aligned} 4x - y &\leq 8 \\ 2x + y &\leq 10 \\ 5x - 2y &\geq -2 \\ x, y &\geq 0 \end{aligned}$$

http://i16.photobucket.com/albums/b20/safebuy/ak47/ak47-electric_lg.jpg

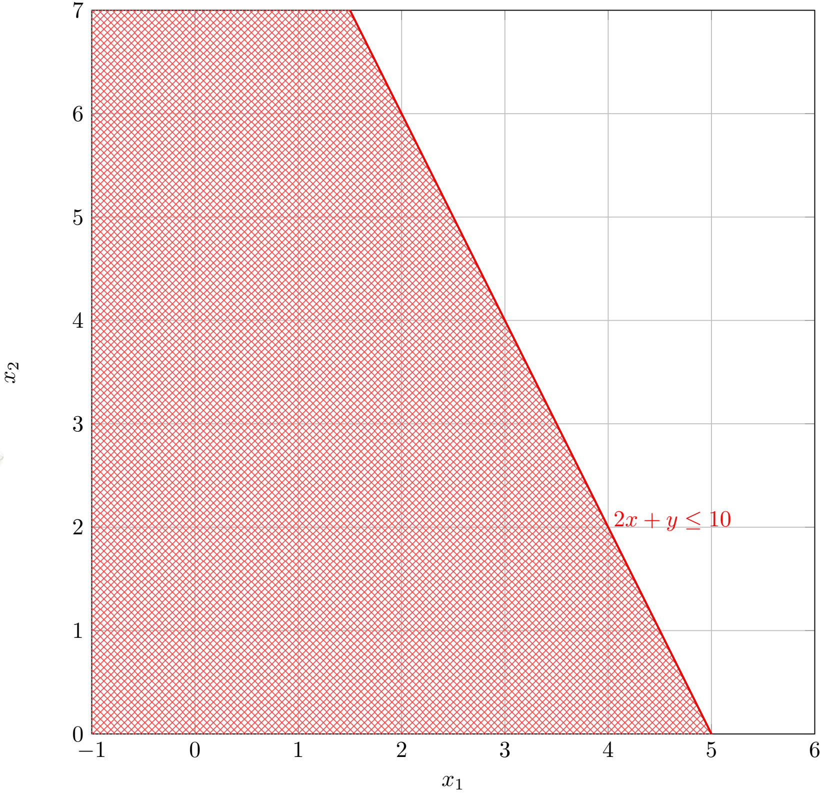
http://2.bp.blogspot.com/_NX4zcMNX4VE/Sb8MQfffllI/AAAAAAAAAL0/eu4J0dfFhJE/s400/gourmet-butter.jpg



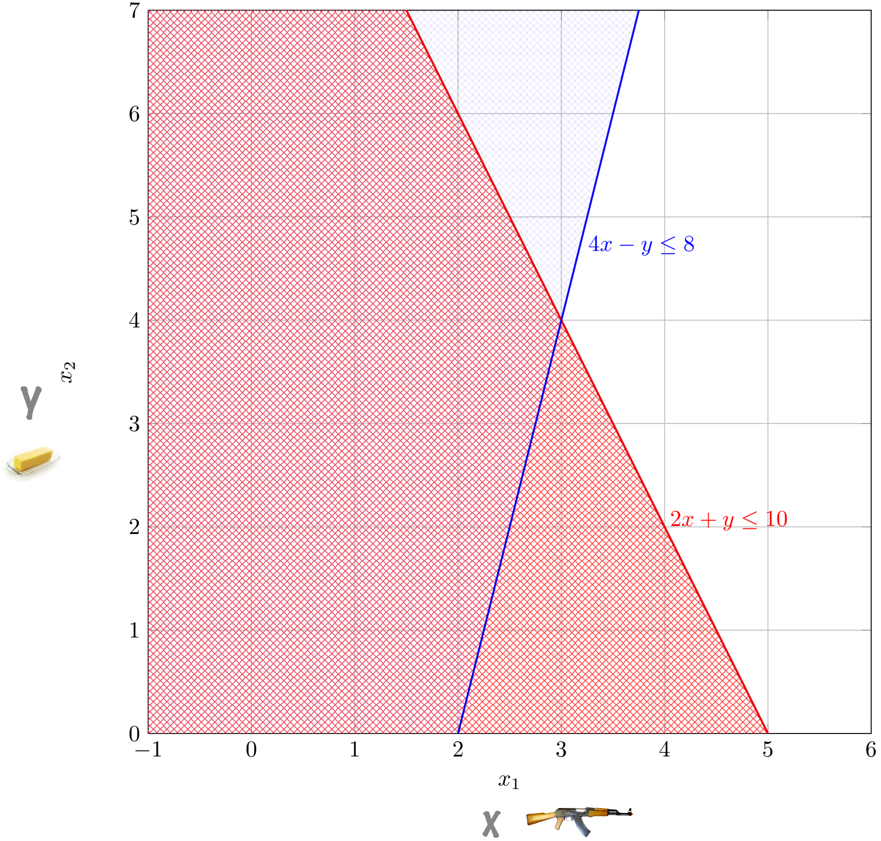
$$
\begin{aligned}
4x - y &\le 8 \\
2x + y &\le 10 \\
5x - 2y &\ge -2 \\
x, y &\ge 0
\end{aligned}
$$

$$\begin{aligned} 4x - y &\leq 8 \\ 2x + y &\leq 10 \\ 5x - 2y &\geq -2 \\ x, y &\geq 0 \end{aligned}$$

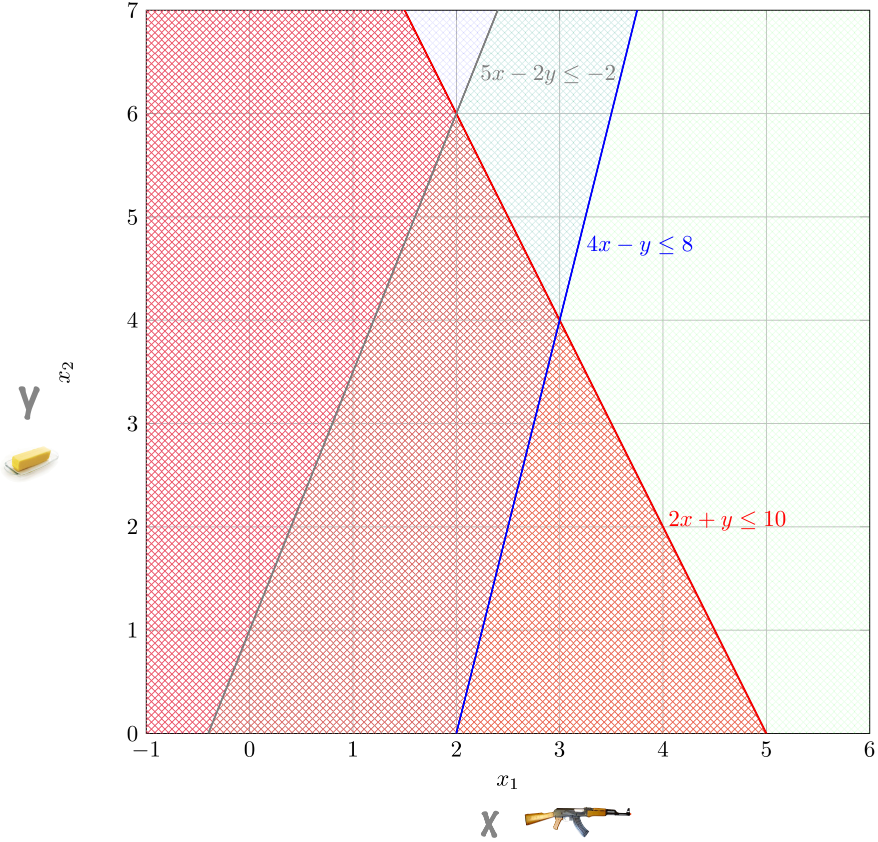
y



x



$$
\begin{aligned}
4x - y &\leq 8 \\
2x + y &\leq 10 \\
5x - 2y &\geq -2 \\
x, y &\geq 0
\end{aligned}
$$



$$
\begin{aligned}
4x - y &\leq 8 \\
2x + y &\leq 10 \\
5x - 2y &\geq -2 \\
x, y &\geq 0
\end{aligned}
$$

Certificate of optimality

$$\max x + y$$

$$\begin{aligned} 4x - y &\leq 8 \\ 2x + y &\leq 10 \\ 5x - 2y &\geq -2 \\ x, y &\geq 0 \end{aligned}$$

Certificate of optimality

$$\max x + y$$

$$\begin{aligned} 4x - y &\leq 8 \\ 2x + y &\leq 10 \\ 5x - 2y &\geq -2 \\ x, y &\geq 0 \end{aligned}$$

7   14x +7y ≤ 70

-1   -5x + 2y ≤ 2

9x+ 9y ≤ 72

BERLIN

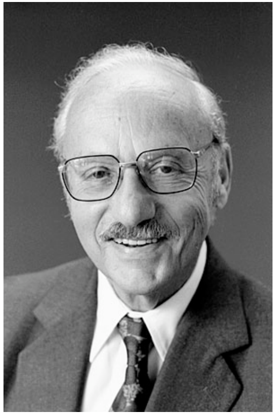


IMAGE:STAMFORD

IMAGE:HISTORY OF AIR CARGO

linear programming
saved Berlin

Stigler diet

CALORIES	3000
PROTEIN	70g
CALCIUM	.8g
IRON	19mg
VITAMIN A	5000iu
THIAMINE	1.8mg
RIBOFLAVIN	2.7mg
NIACIN	18mg
ASCORBIC ACID	75mg

TABLE A. NUTRITIVE VALUES OF COMMON FOODS PER DOLLAR OF EXPENDITURE, AUGUST 15, 1939

Commodity	Unit	Price Aug. 15, 1939 (cents)	Edible Weight per $1.00 (grams)	Calories (1,000)	Protein (grams)	Calcium (grams)	Iron (mg.)	Vitamin A (1,000 I.U.)	Thiamine (mg.)	Ribo-flavin (mg.)	Niacin (mg.)	Ascorbic Acid (mg.)
**1. Wheat Flour (Enriched)	10 lb.	36.0	12,600	44.7	1,411	2.0	365		55.4	33.3	441	
2. Macaroni	1 lb.	14.1	3,217	11.6	418	.7	54		3.2	1.9	68	
3. Wheat Cereal (Enriched)	28 oz.	24.2	3,280	11.8	377	14.4	175		14.4	8.8	114	
4. Corn Flakes	8 oz.	7.1	3,194	11.4	252	.1	56		13.5	2.3	68	
5. Corn Meal	1 lb.	4.6	9,861	36.0	897	1.7	99	30.9	17.4	7.9	106	
6. Hominy Grits	24 oz.	8.5	8,005	28.6	680	.8	80		10.6	1.6	110	
7. Rice	1 lb.	7.5	6,048	21.2	460	.6	41		2.0	4.8	60	
8. Rolled Oats	1 lb.	7.1	6,389	25.3	907	5.1	341		37.1	8.9	64	
9. White Bread (Enriched)	1 lb.	7.9	5,742	15.0	488	2.5	115		13.8	8.5	126	
10. Whole Wheat Bread	1 lb.	9.1	4,985	12.2	484	2.7	125		13.9	6.4	160	
11. Rye Bread	1 lb.	9.2	4,930	12.4	439	1.1	82		9.9	3.0	66	
12. Pound Cake	1 lb.	24.8	1,829	8.0	130	.4	31	18.9	2.8	3.0	17	
13. Soda Crackers	1 lb.	15.1	3,004	12.5	288	.5	50					
14. Milk	1 qt.	11.0	8,867	6.1	310	10.5	18	16.8	4.0	16.0	7	177
**15. Evaporated Milk (can)	14½ oz.	6.7	6,035	8.4	422	15.1	9	26.0	3.0	23.5	11	60
16. Butter	1 lb.	30.8	1,473	10.8	9	.2	3	44.2		.2	2	
*17. Oleomargarine	1 lb.	16.1	2,817	20.6	17	.6	6	55.8	.2			
18. Eggs	1 doz.	32.6	1,857	2.9	238	1.0	52	18.6	2.8	6.5	1	
**19. Cheese (Cheddar)	1 lb.	24.2	1,874	7.4	448	16.4	19	28.1	.8	10.3	4	
20. Cream	½ pt.	14.1	1,689	3.5	49	1.7	3	16.9	.6	2.5		17
21. Peanut Butter	1 lb.	17.9	2,534	15.7	661	1.0	48		9.6	8.1	471	
22. Mayonnaise	½ pt.	16.7	1,198	8.6	18	.2	8	2.7	.4	.5		
23. Crisco	1 lb.	20.3	2,234	20.1								
24. Lard	1 lb.	9.8	4,628	41.7				.2		.5	5	
25. Sirloin Steak	1 lb.	39.6	1,145*	2.9	166	.1	34	.2	2.1	2.9	69	
26. Round Steak	1 lb.	36.4	1,246*	2.2	214	.1	32	.4	2.5	2.4	87	
27. Rib Roast	1 lb.	29.2	1,553*	3.4	213	.1	33			2.0		
28. Chuck Roast	1 lb.	22.6	2,007*	3.6	309	.2	46	.4	1.0	4.0	120	
29. Plate	1 lb.	14.6	3,107*	8.5	404	.2	62		.9			
**30. Liver (Beef)	1 lb.	26.8	1,692*	2.2	333	.2	139	169.2	6.4	50.8	316	525
31. Leg of Lamb	1 lb.	27.6	1,643*	3.1	245	.1	20		2.8	3.9	86	
32. Lamb Chops (Rib)	1 lb.	36.6	1,239*	3.3	140	.1	15		1.7	2.7	54	
33. Pork Chops	1 lb.	30.7	1,477*	3.5	196	.2	30		17.4	2.7	60	
34. Pork Loin Roast	1 lb.	24.2	1,874*	4.4	249	.3	37		18.2	3.6	79	
35. Bacon	1 lb.	25.6	1,772*	10.4	152	.2	23		1.8	1.8	71	
36. Ham—smoked	1 lb.	27.4	1,655*	6.7	212	.2	31		9.9	3.3	50	
37. Salt Pork	1 lb.	16.0	2,835*	18.8	164	.1	26		1.4	1.8		
38. Roasting Chicken	1 lb.	30.3	1,497*	1.8	184	.1	30	.1	.9	1.8	68	46
39. Veal Cutlets	1 lb.	42.3	1,072*	1.7	156	.1	24		1.4	2.4	57	
40. Salmon, Pink (can)	16 oz.	13.0	3,489	5.8	705	6.8	45	3.5	1.0	4.9	209	
41. Apples	1 lb.	4.4	9,072	5.8	27	.5	36	7.3	3.6	2.7	5	544
42. Bananas	1 lb.	6.1	4,982	4.9	60	.4	30	17.4	2.5	3.5	28	498
43. Lemons	1 doz.	26.0	2,380	1.0	21	.5	14		.5		4	952
44. Oranges	1 doz.	30.9	4,439	2.2	40	1.1	18	11.1	3.6	1.3	10	1,998
*45. Green Beans	1 lb.	7.1	5,750	2.4	138	3.7	80	69.0	4.3	5.8	37	862
**46. Cabbage	1 lb.	3.7	8,949	2.6	125	4.0	36	7.2	9.0	4.5	26	5,369
47. Carrots	1 bunch	4.7	6,080	2.7	73	2.8	43	188.5	6.1	4.3	89	608
48. Celery	1 stalk	7.3	3,915	.9	51	3.0	23	.9	1.4	1.4	9	313
49. Lettuce	1 head	8.2	2,247	.4	27	1.1	22	112.4	1.8	3.4	11	449
*50. Onions	1 lb.	3.6	11,844	5.8	166	3.8	59	16.6	4.7	5.9	21	1,184

*51. Potatoes	15 lb.	34.0	18,810	14.5	386	1.8	118	6.7	29.4	7.1	198	2,522
**52. Spinach	1 lb.	8.1	4,592	1.1	106	—	138	918.4	5.7	13.8	33	2,755
**53. Sweet Potatoes	1 lb.	5.1	7,649	9.6	138	2.7	54	290.7	8.4	5.4	83	1,912
54. Peaches (can)	No. 2½	16.8	4,894	3.7	20	.4	10	21.5	.5	1.0	31	196
55. Pears (can)	No. 2½	20.4	4,030	3.0	8	.3	8	.8	.8	.8	5	81
56. Pineapple (can)	No. 2½	21.3	3,993	2.4	16	.4	8	2.0	2.8	.8	7	399
57. Asparagus (can)	No. 2	27.7	1,945	.4	33	.3	12	16.3	1.4	2.1	17	272
58. Green Beans (can)	No. 2	10.0	5,386	1.0	54	2.0	65	53.9	1.6	4.3	32	431
59. Pork and Beans (can)	16 oz.	7.1	6,389	7.5	364	4.0	134	3.5	8.3	7.7	56	
60. Corn (can)	No. 2	10.4	5,452	5.2	136	.2	16	12.0	1.6	2.7	42	218
61. Peas (can)	No. 2	13.8	4,109	2.3	136	.6	45	34.9	4.9	2.5	37	370
62. Tomatoes (can)	No. 2	8.6	6,263	1.3	63	.7	38	53.2	3.4	2.5	36	1,253
63. Tomato Soup (can)	10½ oz.	7.6	3,917	1.6	71	.6	43	57.9	3.5	2.4	67	862
*64. Peaches, Dried	1 lb.	15.7	2,889	8.5	87	1.7	173	86.8	1.2	4.3	55	57
*65. Prunes, Dried	1 lb.	9.0	4,284	12.8	99	2.5	154	85.7	3.9	4.3	65	257
66. Raisins, Dried	15 oz.	9.4	4,524	13.5	104	2.5	136	4.5	6.3	1.4	24	136
67. Peas, Dried	1 lb.	7.9	5,742	20.0	1,367	4.2	345	2.9	28.7	18.4	162	
**68. Lima Beans, Dried	1 lb.	8.9	5,007	17.4	1,055	3.7	459	5.1	26.9	38.2	93	
**69. Navy Beans, Dried	1 lb.	5.9	7,688	26.9	1,691	11.4	792		38.4	24.6	217	
70. Coffee	1 lb.	22.4	2,025	—	—	—	—		4.0	5.1	50	
71. Tea	¼ lb.	17.4	652	—	—	—	—			2.3	42	
72. Cocoa	8 oz.	8.6	2,637	8.7	237	3.0	72		2.0	11.9	40	
73. Chocolate	8 oz.	16.2	1,400	8.0	77	1.3	39		.9	3.4	14	
74. Sugar	10 lb.	51.7	8,773	34.9	—	—	—					
75. Corn Sirup	24 oz.	13.7	4,966	14.7	—	.5	74				5	
76. Molasses	18 oz.	13.6	3,752	9.0	—	10.3	244		1.9	7.5	146	
77. Strawberry Preserves	1 lb.	20.5	2,213	6.4	11	.4	7	.2	.2	.4	3	

* Quantities including inedible portions.

TABLE B. NUTRITIVE VALUES OF COMMON FOODS PER DOLLAR OF EXPENDITURE, AUGUST 15, 1944

Commodity	Price Aug. 15, 1944 (cents)	Calories (1,000)	Protein (grams)	Calcium (grams)	Iron (mg.)	Vitamin A (1,000 I.U.)	Thiamine (mg.)	Riboflavin (mg.)	Niacin (mg.)	Ascorbic Acid (mg.)
1. Wheat Flour	64.6	24.9	786	1.1	203		30.9	18.6	246	
3. Wheat Cereal	23.2	12.3	398	15.0	183		15.0	9.2	119	
5. Corn Meal	6.3	26.3	655	1.2	72	22.6	12.7	5.8	77	
8. Rolled Oats	9.9	18.1	651	3.7	245		26.6	6.4	46	
15. Evaporated Milk	10.0	5.6	283	10.1	6	17.4	2.0	15.7	7	40
46. Cabbage	4.9	2.0	94	3.0	27	5.4	6.8	3.4	20	4,054
51. Potatoes	60.1	6.1	143	.8	50	2.8	12.5	3.0	84	1,071
52. Spinach	11.6	.8	74	—	96	641.3	4.0	9.6	23	1,924
53. Sweet Potatoes	12.3	4.0	57	1.1	22	120.5	3.5	2.2	34	793
69. Navy Beans	10.8	14.7	924	6.2	433		21.0	13.4	119	
74. Sugar	67.0	26.9	—	—	—					
78. Pancake Flour[1]	12.2	16.0	479	18.1	46		3.7	1.9	41	
79. Beets[2]	7.3	2.2	85	1.1	70	152.3	2.9	6.3	29	395
80. Liver (Pork)[3]	21.9	2.7	408	.2	518	145.0	10.4	51.8	472	580

[1] Unit: 20 oz.; edible weight: 4,647 g. [2] Unit: 1 bunch; edible weight: 4,971 g. [3] Unit: 1 lb.; edible weight: 2,071 g.

	Brownie	Dumpling	Espresso	Amelia
cost	5	2	3	8
cals	400	200	150	500
choc	3	2	0	0
sugar	2	2	4	4
fat	2	4	0	5

requirements: 500 calories, 6 oz choc, 10 oz sugar, 8 oz fat

	Brownie	Dumpling	Espresso	Amelia
cost	5	2	3	8
cals	400	200	150	500
choc	3	2	0	0
sugar	2	2	4	4
fat	2	4	0	5

requirements: 500 calories, 6 oz choc, 10 oz sugar, 8 oz fat

	Brownie	Dumpling	Espresso	Amelia
cost	5	2	3	8
cals	400	200	150	500
choc	3	2	0	0
sugar	2	2	4	4
fat	2	4	0	5

requirements: 500 calories, 6 oz choc, 10 oz sugar, 8 oz fat

$$\min 5x_1 + 2x_2 + 3x_3 + 8x_4$$

$$\begin{bmatrix} 400 & 200 & 150 & 500 \\ 3 & 2 & 0 & 0 \\ 2 & 2 & 4 & 4 \\ 2 & 4 & 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \geq \begin{bmatrix} 500 \\ 6 \\ 10 \\ 8 \end{bmatrix}$$

$$\min 5x_1 + 2x_2 + 3x_3 + 8x_4$$

$$\begin{bmatrix} 400 & 200 & 150 & 500 \\ 3 & 2 & 0 & 0 \\ 2 & 2 & 4 & 4 \\ 2 & 4 & 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \geq \begin{bmatrix} 500 \\ 6 \\ 10 \\ 8 \end{bmatrix}$$

$$\min 5x_1 + 2x_2 + 3x_3 + 8x_4$$

$$\begin{bmatrix} 400 & 200 & 150 & 500 \\ 3 & 2 & 0 & 0 \\ 2 & 2 & 4 & 4 \\ 2 & 4 & 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \geq \begin{bmatrix} 500 \\ 6 \\ 10 \\ 8 \end{bmatrix}$$

```
H-representation
begin
8 4 rational
-500 400 200 150 500
-6   3   2   0    0
-10  2   2   4    4
-6   2   4   0    5
0    1   0   0    0
0    0   1   0    0
0    0   0   1    0
0    0   0   0    1
end
minimize
0 5 2 3 8
```

$$\min 5x_1 + 2x_2 + 3x_3 + 8x_4$$

$$\begin{bmatrix} 400 & 200 & 150 & 500 \\ 3 & 2 & 0 & 0 \\ 2 & 2 & 4 & 4 \\ 2 & 4 & 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \geq \begin{bmatrix} 500 \\ 6 \\ 10 \\ 8 \end{bmatrix}$$

$$\min 5x_1 + 2x_2 + 3x_3 + 8x_4$$

$$\begin{bmatrix} 400 & 200 & 150 & 500 \\ 3 & 2 & 0 & 0 \\ 2 & 2 & 4 & 4 \\ 2 & 4 & 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \geq \begin{bmatrix} 500 \\ 6 \\ 10 \\ 8 \end{bmatrix}$$

```
H-representation
begin
8 4 rational
-500 400 200 150 500
-6   3   2   0    0
-10  2   2   4    4
-6   2   4   0    5
0    1   0   0    0
0    0   1   0    0
0    0   0   1    0
0    0   0   0    1
end
minimize
0 5 2 3 8
```

$$\min 5x_1 + 2x_2 + 3x_3 + 8x_4$$

$$\begin{bmatrix} 400 & 200 & 150 & 500 \\ 3 & 2 & 0 & 0 \\ 2 & 2 & 4 & 4 \\ 2 & 4 & 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \geq \begin{bmatrix} 500 \\ 6 \\ 10 \\ 8 \end{bmatrix}$$

```
H-representation
begin
8 4 rational
-500 400 200 150 500
-6   3   2   0    0
-10  2   2   4    4
-6   2   4   0    5
0    1   0   0    0
0    0   1   0    0
0    0   0   1    0
0    0   0   0    1
end
minimize
0 5 2 3 8
```

```
*Objective function is
 0 + 5 X[1] + 2 X[2] + 3 X[3] + 8 X[4]
*LP status: a dual pair (x, y) of optimal solutions found.
begin
  primal_solution
  1 :  0
  2 :  3
  3 :  1
  4 :  0
  dual_solution
  2 :  -1/4
  5 :  -11/4
  3 :  -3/4
  8 :  -5
  optimal_value : 9
end
*number of pivot operations = 4
```

shortest paths as LP

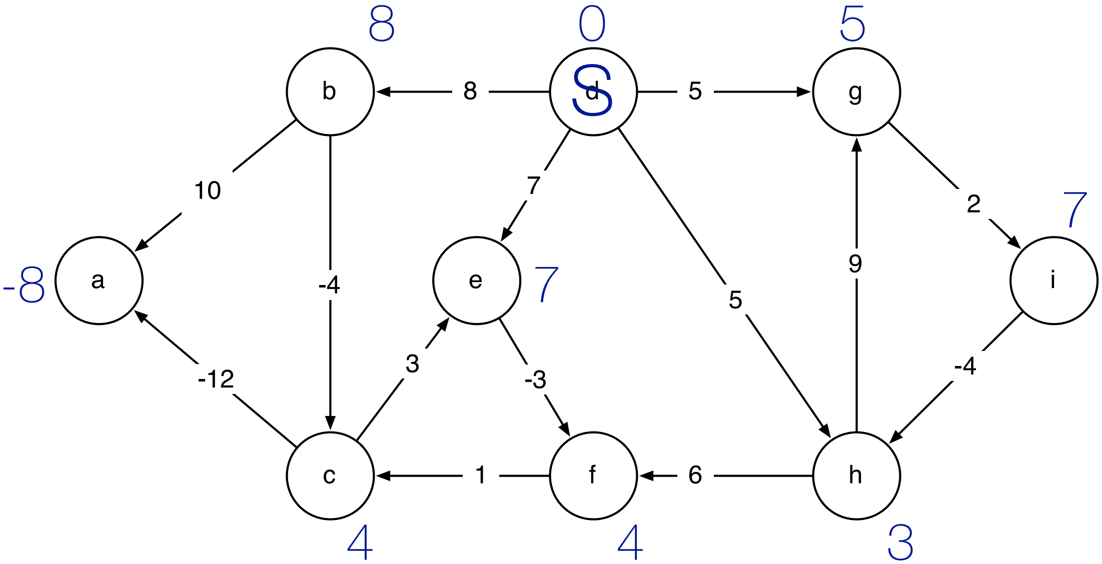
inputs:

shortest paths as LP

$$\max d_t$$

$$d_y - d_x \leq l(x,y) \qquad \forall e = (x,y) \in E$$

$$d_s = 0$$



$$\max d_t$$

$$d_y - d_x \leq l(x,y) \qquad \forall e = (x,y) \in E$$

$$d_s = 0$$

dt = 30

max flow as lp

INPUT: $(G, c, s, t) \quad G = (V, E) \quad c : E \to \mathbb{Z}_+$

max flow as lp

$$\max \sum_v f(s,v) - \sum_v f(v,s)$$

$$f(u,v) \le c(u,v) \qquad \text{for } (u,v) \text{ in } E$$

$$\sum_u f(u,v) = \sum_w f(v,w) \quad \forall v$$

$$f(u,v) \ge 0 \qquad \text{for } (u,v) \text{ in } E$$

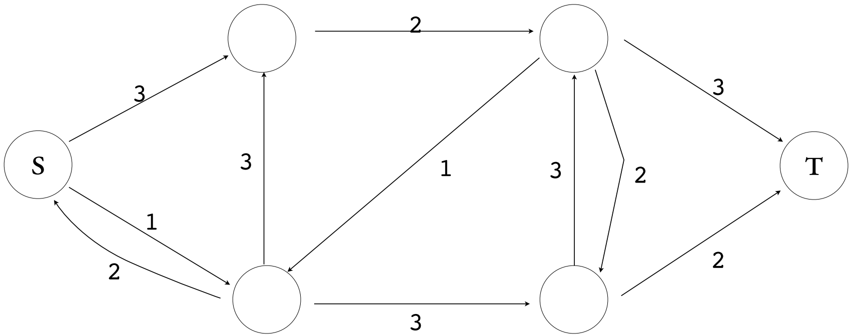
max flow as lp

$$\max \sum_v f(s,v) - \sum_v f(v,s)$$

$$f(u,v) \leq c(u,v) \qquad \text{FOR } (u,v) \text{ IN } E$$

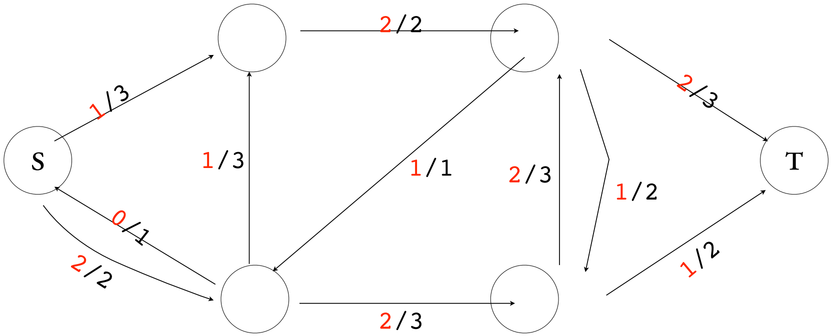
$$\sum_u f(u,v) = \sum_w f(v,w) \quad \forall v$$

$$f(u,v) \geq 0 \qquad \text{FOR } (u,v) \text{ IN } E$$



min-cost flow as lp

INPUT: (G, c, s, t) $G = (V, E)$ $c : E \to \mathbb{Z}_+$ $x : E \to \mathbb{Z}_+$ d



min-cost flow as lp

min-cost flow as lp

$$\min_e x_e \cdot f(e)$$

$$f(e) \leq c(e)$$

$$f(e) \geq 0$$

$$\sum_u f(u,v) = \sum_w f(v,w)$$

$$\sum_v f(s,v) - \sum_v f(v,s) = d$$