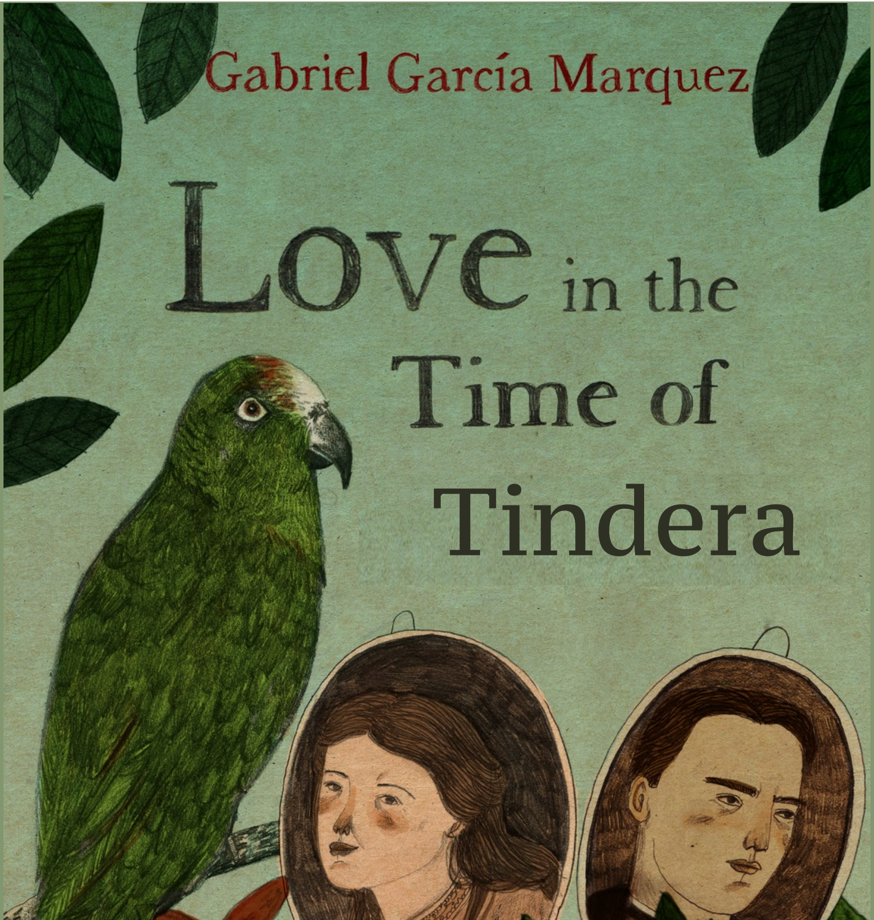
5800 Stablematch

apr1/apr4 2022
shelat


Gabriel García Marquez
Love in the
Time of
Tindera

We have a group of suitors and reviewers

2>1>3

2>3>1

1>3>2

Each has preferences over the other group

1>3>2

1>2>2

3>2>1

2>1>3

2>3>1

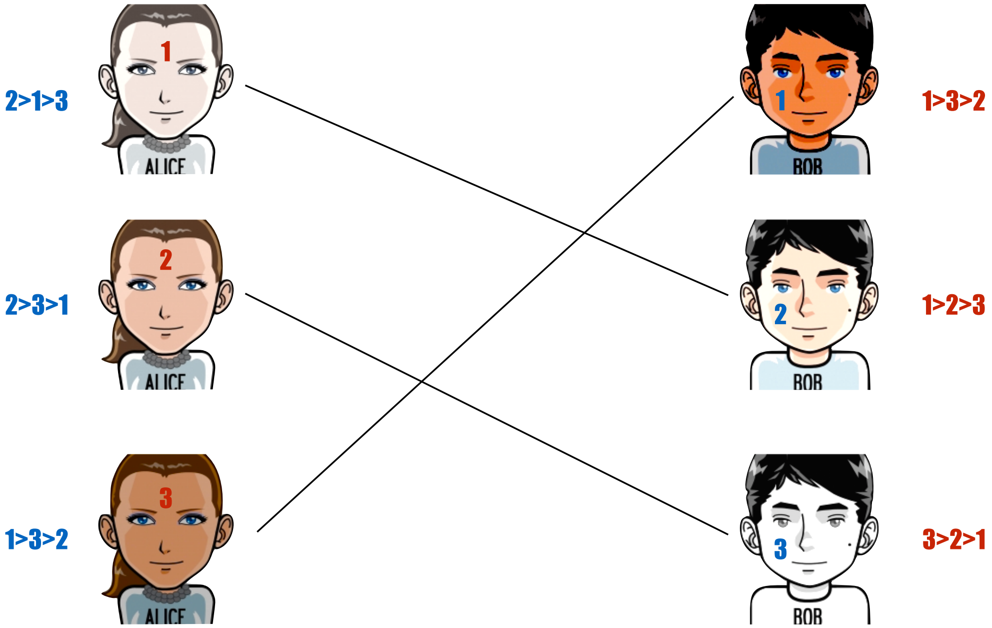
1>3>2

We seek a **stable matching** between the two

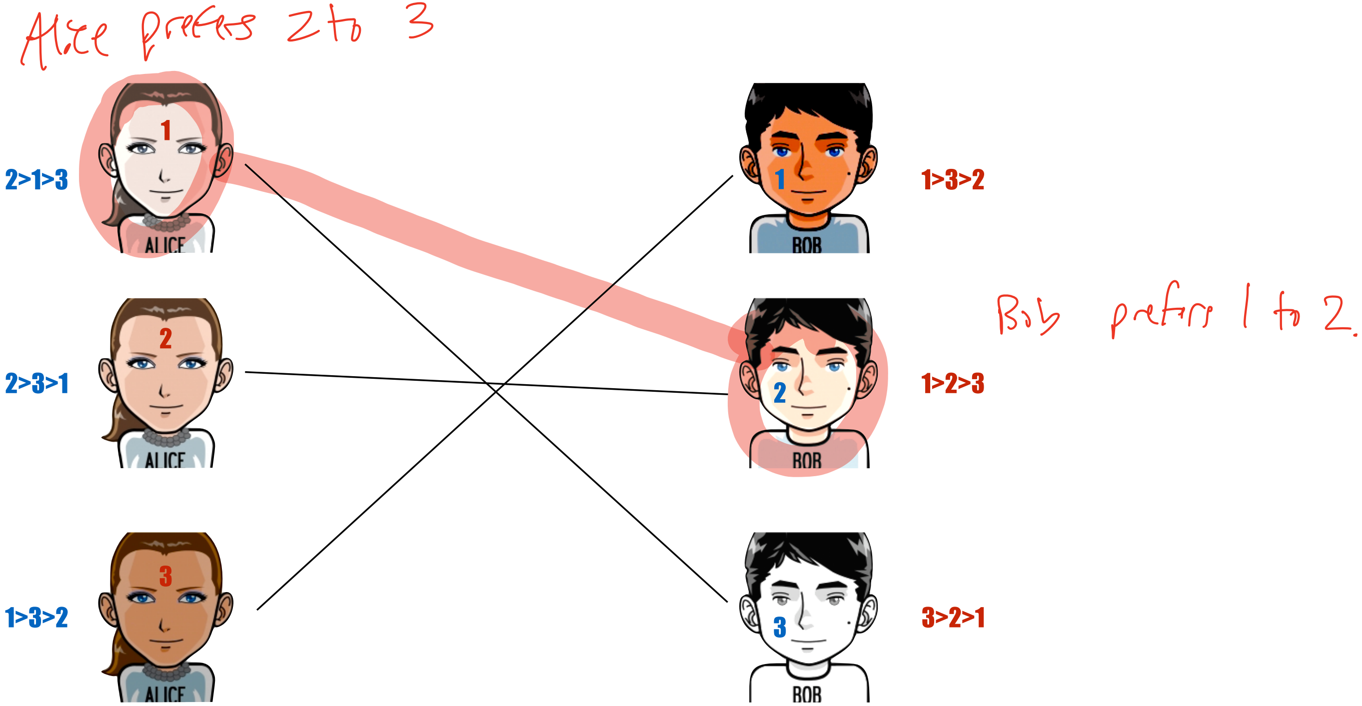
1>3>2

1>2>2

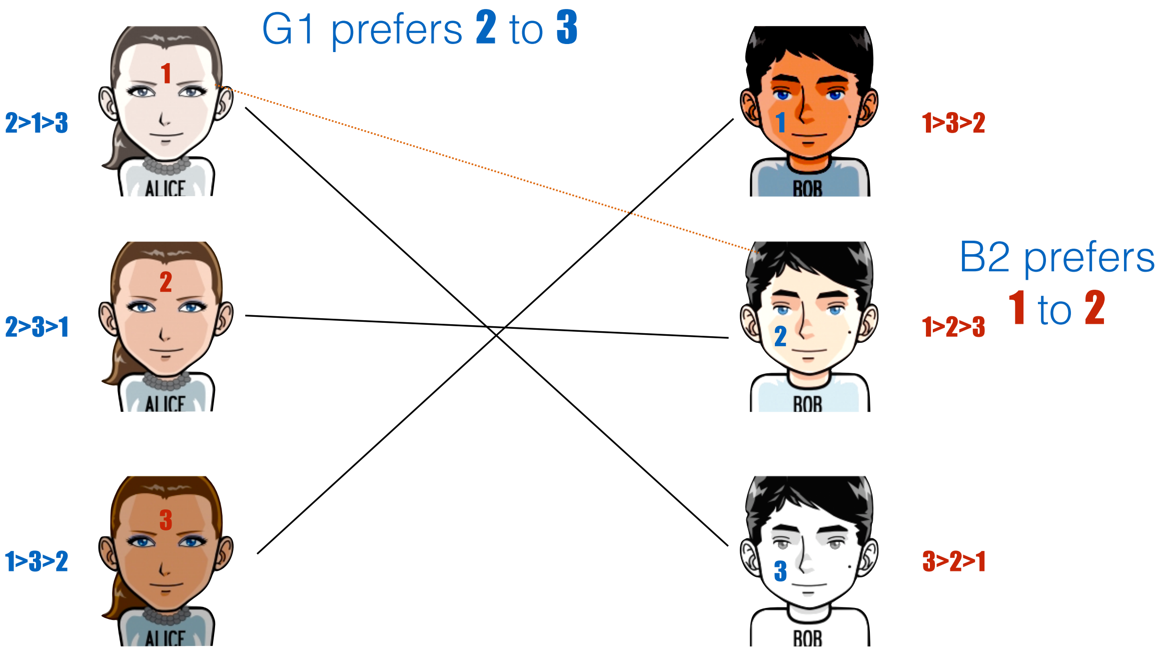
3>2>1



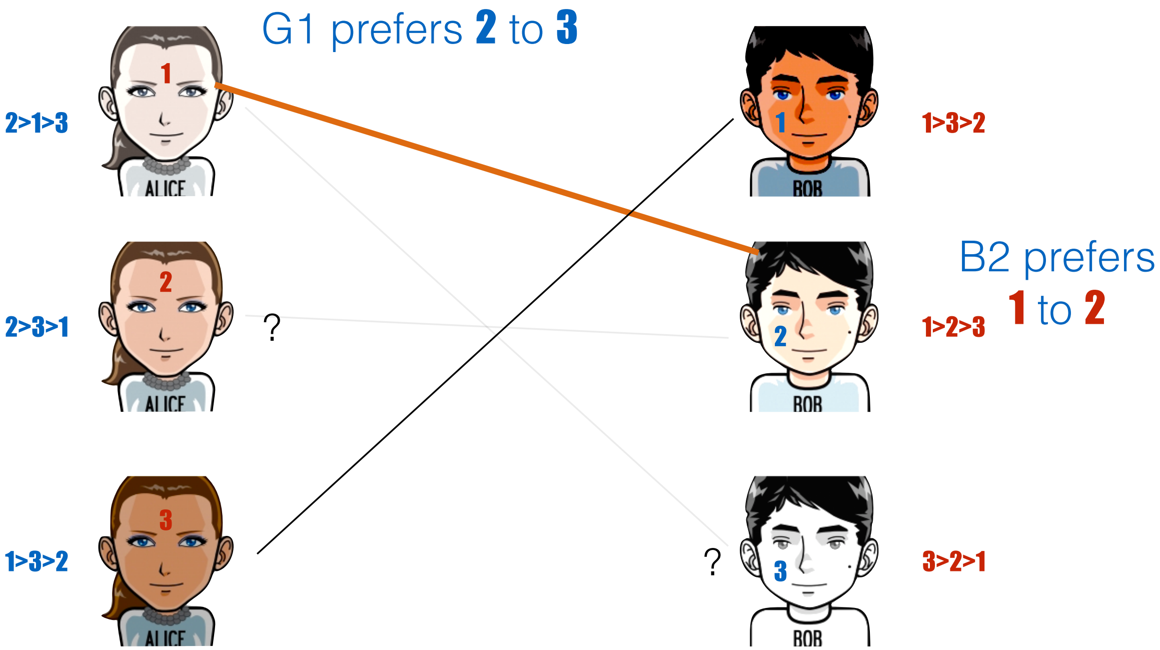
2>1>3
1
ALICE
2>3>1
2
ALICE
1>3>2
3
ALICE
1
BOB
1>3>2
2
BOB
1>2>3
3
BOB
3>2>1



Unstable Matching



Unstable Matching



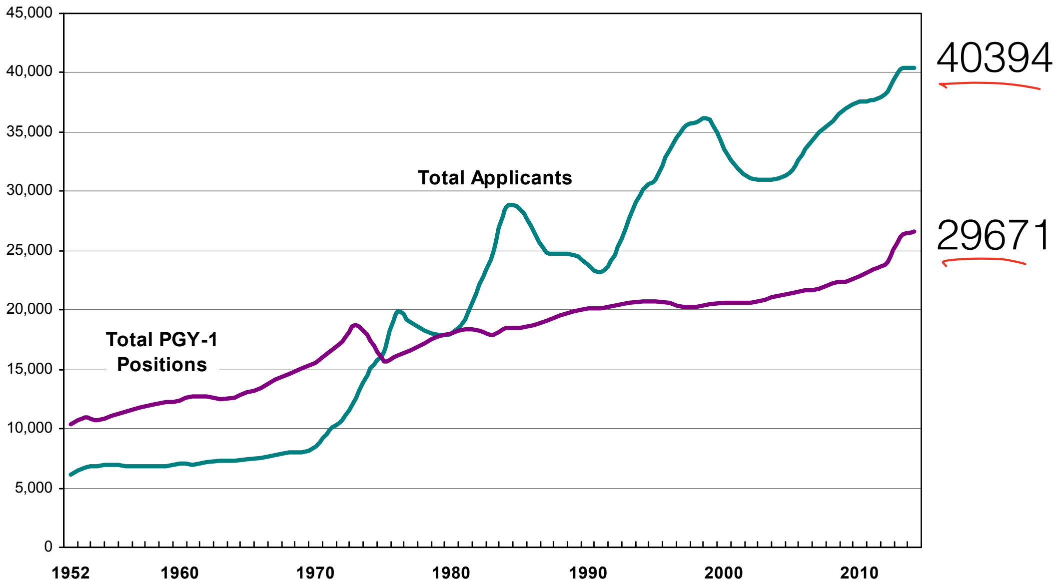
Unstable Matching

Stable Matching

Stable matching has many practical applications

THE MATCH
NATIONAL RESIDENT MATCHING PROGRAM®

Figure 1 Applicants and 1st Year Positions in The Match, 1952 - 2014



Applicant Type	Matched		
	2013 Graduates	Prior Year Graduates[1]	Total
CMG	2571	74	2645
IMG	146	353	499
USMG	23	2	25
TOTAL	**2740**	**429**	**3169**

UNIVERSITAS OSLOENSIS
MDCCCXI

NUS
National University
of Singapore

SUTD
SINGAPORE UNIVERSITY OF
TECHNOLOGY AND DESIGN
Established in collaboration with MIT

NANYANG
TECHNOLOGICAL
UNIVERSITY


GreekYearbook
www.GreekYearbook.com
University of Virginia
Chi Omega Bid Day 2012

Definition: matchings

proposers

$P = \{p_1, p_2, \ldots p_n\}$

reviewers

$R = \{r_1, r_2 \ldots r_n\}$

set of reviewers, both have n elements

$M = \{(p_i, r_j)\}$ set of pairs such that each p_i and each r_j occur in at most one pair.

A matching is perfect if $|M| = n$ and each p_i, r_j occurs in exactly one pair.

Definition: matchings

$$P = \{p_1, p_2, \ldots, p_n\}$$
$$R = \{r_1, r_2, \ldots, r_n\}$$
$$M = \{(p_{i_1}, r_{j_1}), \ldots, (p_{i_n}, r_{j_n})\}$$

Each `pi` (`rj`) appears only one in a pairing.
A matching is perfect if every `pi` appears.

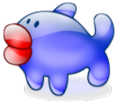


Image credits: Julia Nikolaeva

Definition: preferences

$$P = \{p_1, p_2, \ldots, p_n\}$$

proposal p_i may prefer reviewer r_2 to r_1

$r_1 \prec_{p_i} r_2$

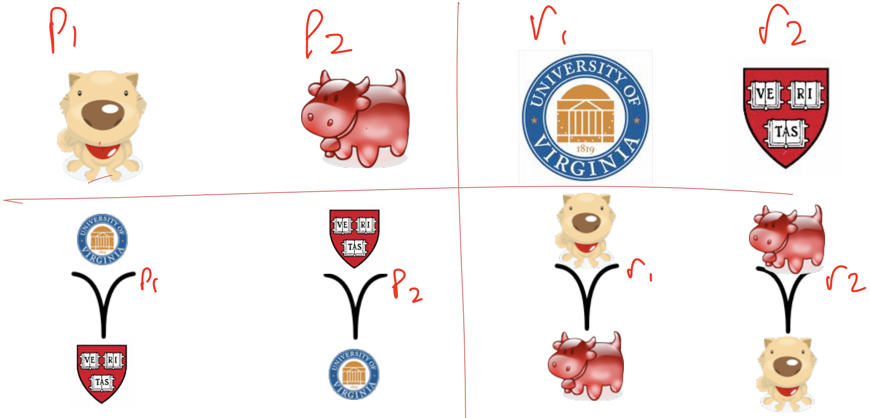
Example: preferences

$$P = \{p_1, p_2, \ldots, p_n\}$$

p_i has a preference relation on the set R

$$r_1 \prec_{p_1} r_4 \prec_{p_1} r_3 \prec_{p_1} r_2$$

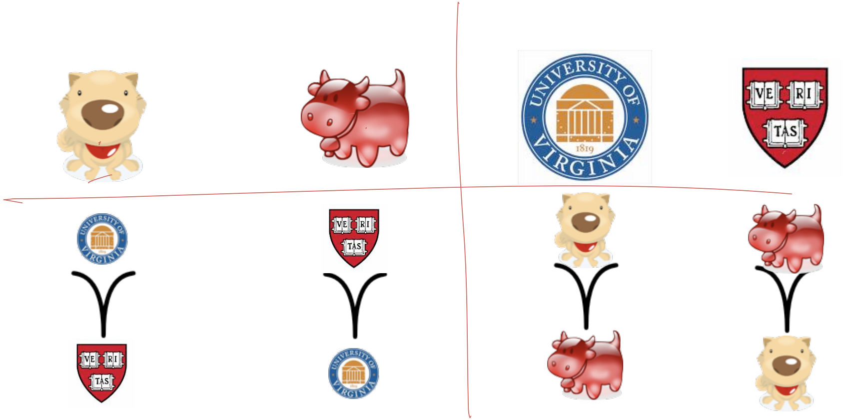
Consider one matching

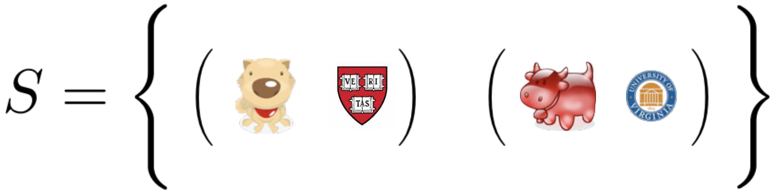


$$S = \{ (p_1, r_2) \ (p_2, r_1) \}$$

Is this a stable matching??

Is this a stable match?





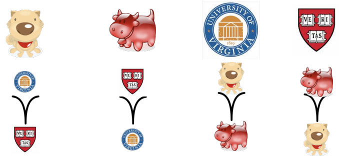
NOT STABLE.

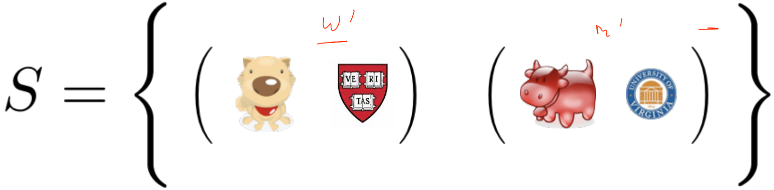
(D, V)

D prefers V to H

V prefers D to C.

Def: instability





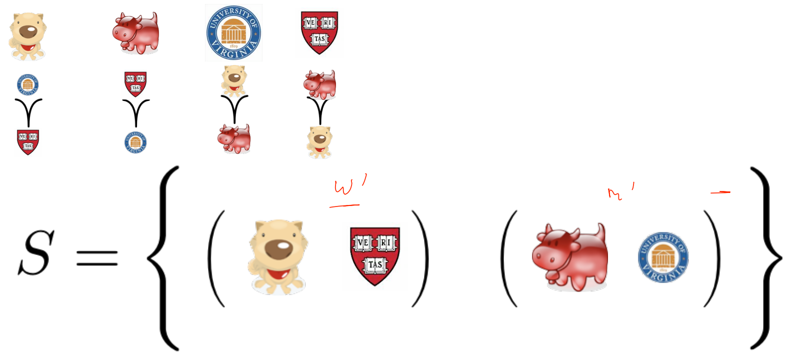
is an unmatched pair $(p^*, r^*) \notin S$ such that

p^* prefers r^* to its current match $S(p^*)$

r^* prefers p^* to its current match $S(r^*)$

$S(r^*)$ denotes the match of r^* in S.

Def: instability



$$(m^*, w^*) \notin S$$

$$w' \prec_{m^*} w^*$$
$$m' \prec_{w^*} m^*$$

There is a pair, not in the matching S, in which each party prefers each other to their current matches in S.

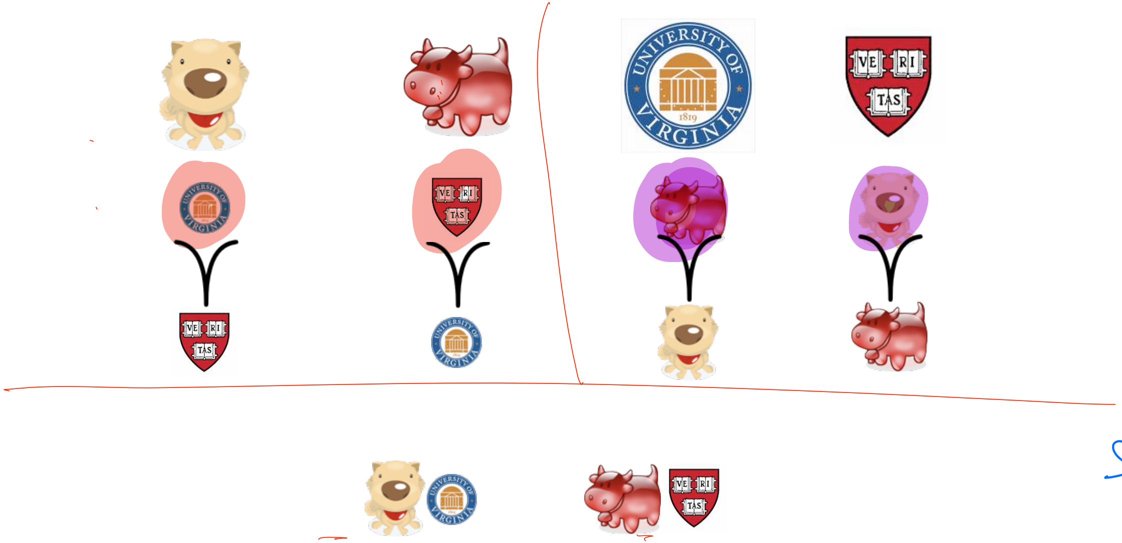
M $= \{ (p_1,r_1), (p_2,r_2), \ldots (p_n,r_n) \}$

is a stable matching if

No unmatched pair (p^*,r^*) prefer each other to their partners in M

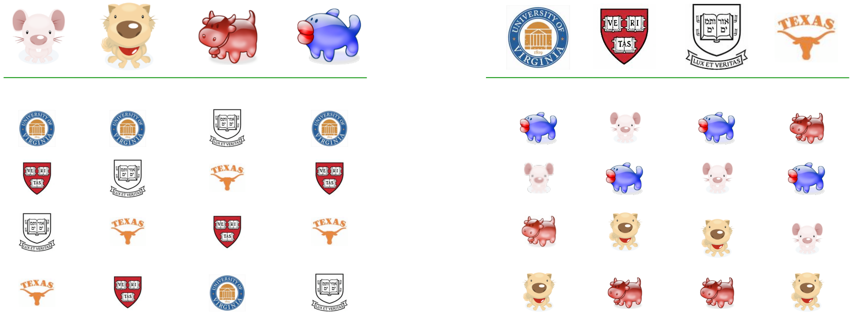
(ie, a matching with no instable pairs.)

Example 2



STABLE ✓
matching

Prove: for every input



there exists a stable matching.

proposal algorithm

① Start with all participants unmatched

② while there exists an unmatched proposer p who has not exhausted preferences:

- Let r be the highest ranked reviewer that p has not already proposed to.
- Let p propose to r.

If r is unmatched : Add the pair (p, r) to S.

If r is matched in (p', r) but r prefers p to p'

then: break the pair (p', r)

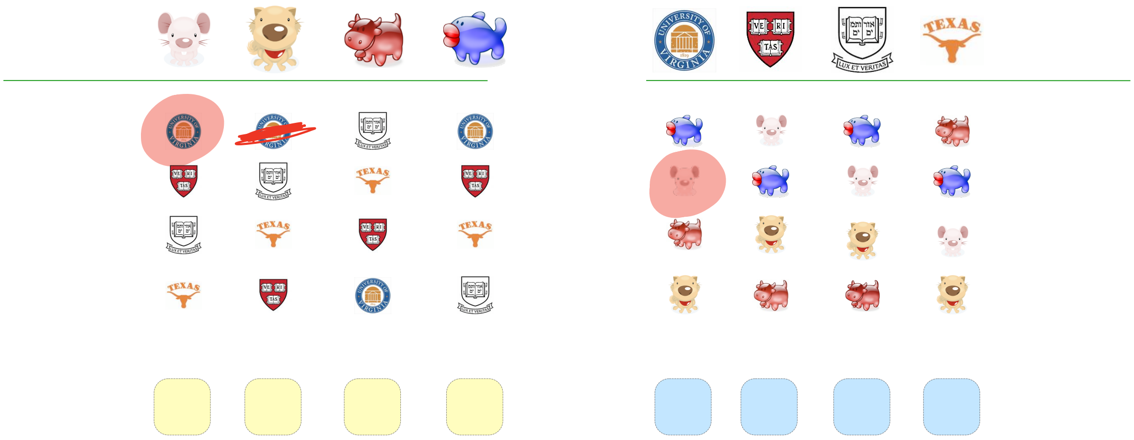
add the pair (p, r)

Else continue.

STABLEMATCH($M, W, \prec_m, \prec_w$)

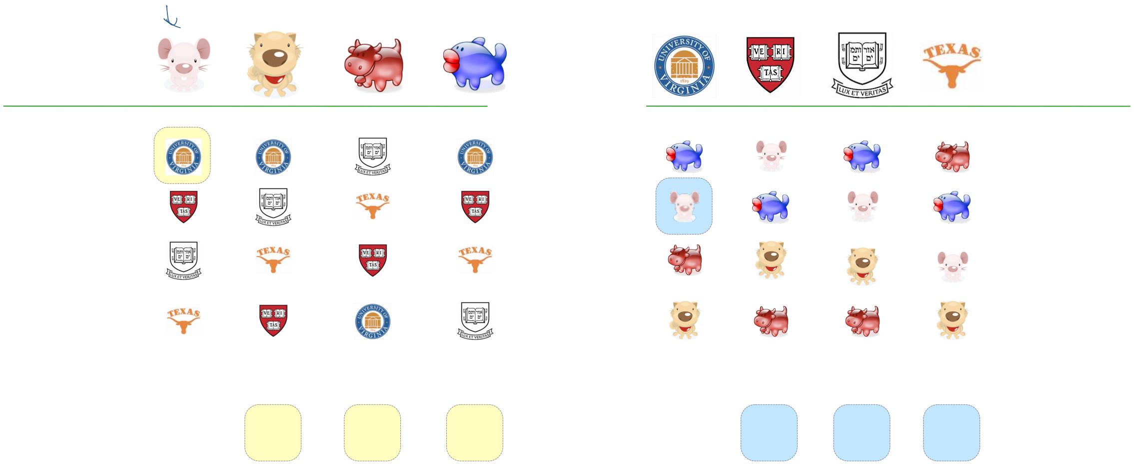
```
Initialize all m, w to be FREE
while ∃FREE(m) and hasn't proposed to all W
      do Pick such an m
         Let w ∈ W be highest-ranked to whom m has not yet proposed
         if FREE(w)
            then Make a new pair (m, w)
         elseif (m′, w) is paired and m′ ≺_w m
              do Break pair (m′, w) and make m′ free
                 Make pair (m, w)
return Set of pairs
```

S R



S

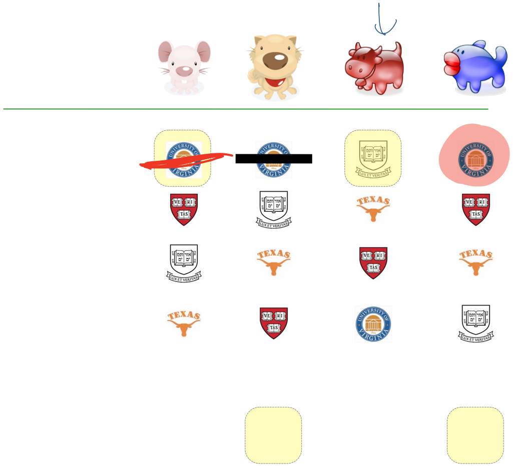
R



S

R

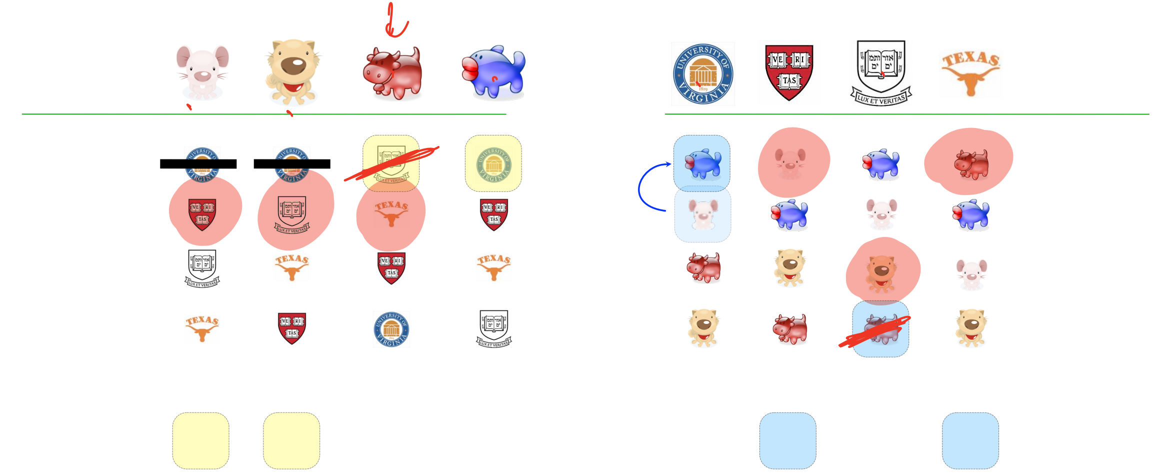
S



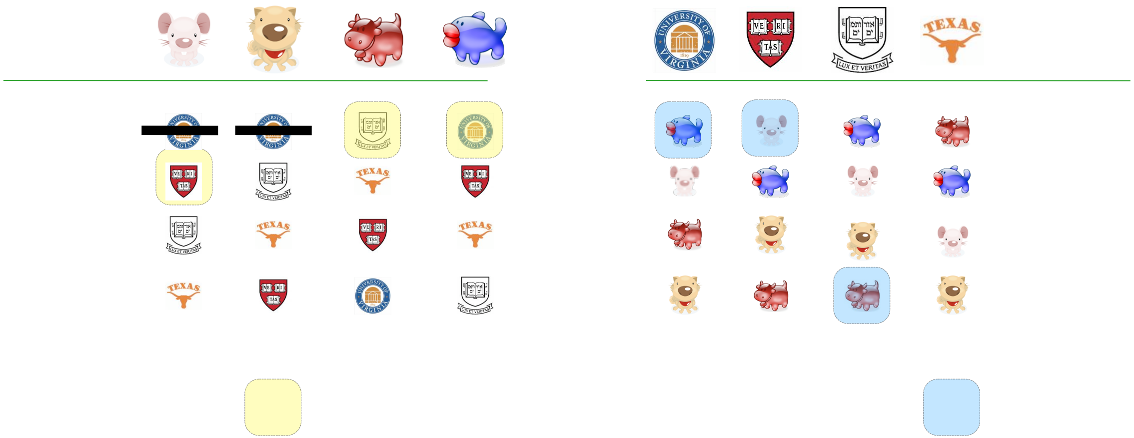
R

S

R



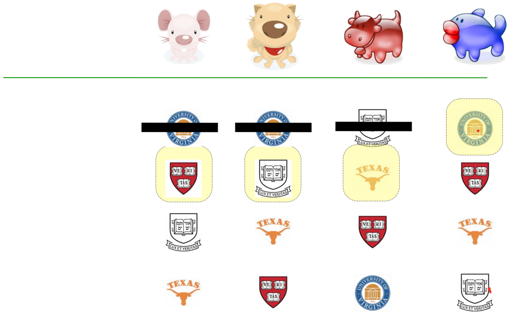
S R



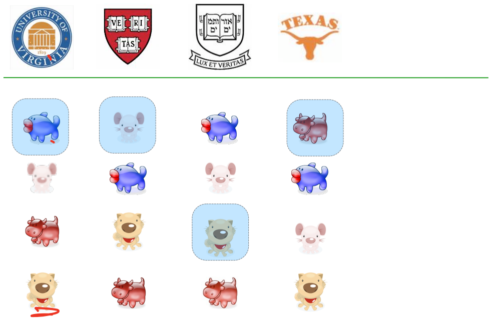
S

R

S



R



Proposal algorithm ends

① EACH proposer proposes at most once to each reviewer.

EACH proposer proposes $O(n)$

② There are only n proposers $\Rightarrow$ $O(n^2)$ proposals

Proposal algorithm ends

$O(n^2)$ steps

each p_i proposes at most once to each r_j.

each $p \in P$ proposes at most n times.

size of M is at most n.

The output is a matching

① Each proposer p appears in at most one pair.
reviewer r

→ By lines 6, 9 in the algorithm, a pair is created only between 2 elements that are unmatched at the time.

The output is a matching

Each $p \in P$ appears at most once in the output.
Each $r \in R$ appears at most once in the output.

This follows directly from the pair lines in the code.
When a pair is made, the two participants are unmatched in each case.

StableMatch$(M, W, \prec_m, \prec_w)$

Initialize all m, w to be free
while $\exists$free(m) and hasn't proposed to all W
    do Pick such an m
        Let $w \in W$ be highest-ranked to whom m has not yet proposed
        if free(w)
            then Make a new pair (m, w)
        elseif (m', w) is paired and $m' \prec_w m$
            do Break pair (m', w) and make m' free
                Make pair (m, w)
return Set of pairs

StableMatch$(M, W, \prec_m, \prec_w)$

```
Initialize all m, w to be FREE
while ∃FREE(m) and hasn't proposed to all W
      do Pick such an m
         Let w ∈ W be highest-ranked to whom m has not yet proposed
         if FREE(w)
            then Make a new pair (m, w)
         elseif (m′, w) is paired and m′ ≺_w m
              do Break pair (m′, w) and make m′ free
                 Make pair (m, w)
return Set of pairs
```

Reviewers' matches improve

Once a reviewer has been matched, they remain matched for the rest of the execution. Their match either remains the same or improves.

Follows again from lines 7, 8 and 6.

The output is perfect

This means that the size of the output pair is n.

If there is an unmatched p, then, there also exists an unmatched reviewer. because $|R| = n$.

$\Rightarrow$ GS will continue until all n proposers are matched.

output is perfect

if $\exists m$ who is free, then

$\exists w$ who has not been asked

The output is stable

Proof by contradiction.

Proof by contradiction: Suppose the output S of GS is NOT STABLE.

This means there exists $(p^*, r^*) \notin S$ such that p^* prefers r^* to $S(p^*)$ and r^* prefers p^* to $S(r^*)$.

$S(p^*) = r$, $S(r^*) = p$

In other words, S contains (p^*, r) and (p, r^*).

Consider the moment when r^* is matched with p, and when p^* is matched with r.

The output is stable

Proof by contradiction.

Suppose the output M is not stable. That means there exists an unmatched pair $(p^*, r^*) \notin M$ such that p^* prefers r^* to their current match $M(p^*)$ and r^* prefers p^* to their current match $M(r^*)$.

output is stable

Spse not. $(p^*, r), (p, r^*) \in M$ But ② $r \prec_{p^*} r^*$ and ① $p \prec_{r^*} p^*$ LAST

① Since p^* is matched to r, p^* must have proposed to r after proposing to r^* because p^* prefers r^* to r. By Assumption 2.

② At the time p^* proposed to r^*:

① either (r^*, p') was already in M and r^* preferred p' to p^*

No match was broken and either $p' = p$ or r^* prefers p to p'.

In either case $p^* <_{r^*} p$ which contradicts the Assumption ①

② (p^*, r^*) was created at this step. But then later r^* accepts proposal from p. $\Rightarrow$ $p^* <_{r^*} p$ which contradicts ①

output is stable

Spse not. $(p^*, r), (p, r^*) \in M$ But $r \prec_{p^*} r^*$ and $p \prec_{r^*} p^*$

Consider the moment when r* is matched with p.

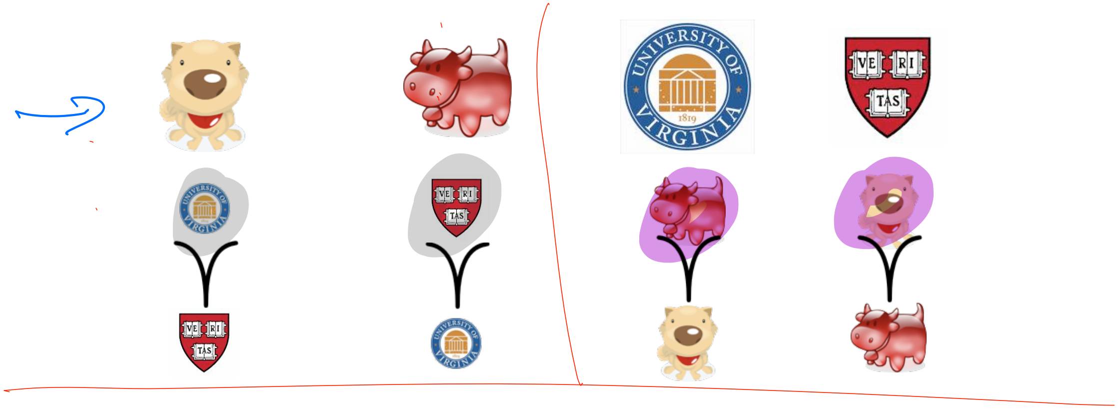
Since p* prefer r*, then p* must have already proposed to r* at the time of the proposal to r.

At that proposal to r*, either:

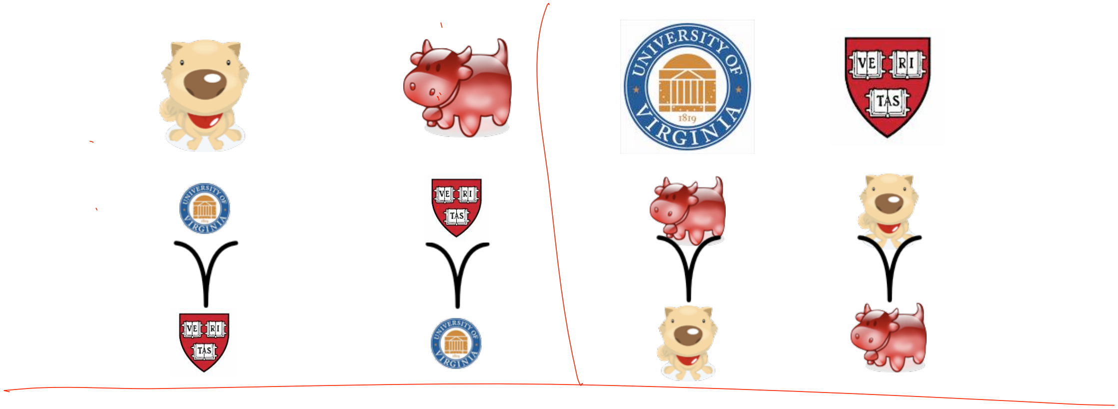
r* was matched to p', and r* preferred p' to p*. But since reviewer matches only improve, this contradicts the assumption that r* prefers p* to p.

r* was not matched and paired to p*, but then broken and paired with p. This contradicts the assumption that r* prefers p*.

Proposer wins



Proposer wins



Remarkable theorem

r is valid for p: There exists a stable matching S in which $(p, r) \in S$

best(p): best(p) is valid for p and there is no valid r^* such that $best(p) \prec_p r^*$

a reviewer

Thm: GS produces the matching $M = \{(p, best(p))\}\ \forall p \in P$.
no matter what order the proposals arrive.

[GS produces the proposer-optimal stable matching]

GS is Proposer-optimal.

Proof: Suppose that GS doesn't proposer optimal match.

Consider the first time that some proposer p is not paired with best(p).

r^*

proposer- optimal match

S^*

$r \neq r^*$.

Output of GS

GS is Proposer-optimal.

Suppose that GS did not return a proposer-optimal matching S^*.
Consider the first moment in GS when a proposer p is rejected by a valid match r.
This must also be $r = best(p)$ since p proposes in decreasing order.

S^*

(p, r)

(p', r')

Lets consider who p' matches with in S^*, the optimal match.

Output of GS

(p', r)

p' ≠ p because we are assuming that S was not optimal.

r prefers p' to p.
p' prefers r to r'.

This implies that S^* is unstable by (p', r), which contradicts our assumption that S^* is an optimal matching.

GS is Proposer-optimal.

Suppose that GS did not return a proposer-optimal matching S^*.
Consider the first moment in GS when a proposer p is rejected by a valid match r.
This must also be $r = best(p)$ since p proposes in decreasing order.

$$S^*$$

(p, r)

(p', r'')

In this matching, p' is paired with r''.

Output of GS

(p', r)

p is matched with another reviewer, And r is matched to another proposer

GS is Proposer-optimal.

Suppose that GS did not return a proposer-optimal matching S^*.

Consider the first moment in GS when a proposer p is rejected by a valid match r.

This must also be $r = best(p)$ since p proposes in decreasing order.

S^*

(p, r)

(p', r'')

In this matching, p' is paired with r''.

Output of GS

(p', r)

p is matched with another reviewer, And r is matched to another proposer

Since (p,r) is not in the output of GS, either r is already matched with a higher ranked p', or r breaks for a higher p'.

Since (p,r) is valid, i.e. $(p, r) \in S^*$, who is p' paired with ? Let it be (p', r'').

GS matching vs P-opt

S^*

(p, r)

(p', r'')

In this matching, p' is paired with r''.

Output of GS

(p', r)

p is matched with another reviewer, And r is matched to another proposer

In GS, p' could not have been rejected yet. Must be that p' prefers r to r''. This means (p',r'') is an instability in S*. This contradicts the assumption that S* is a matching (recall, it was defined as the best matching for p).

S1	S2	S3	S4
R1	R1	R1	R1
R2	R2	R2	R2
R3	R3	R3	R3
R4	R4	R4	R4

R1	R2	R3	R4
S1	S1	S1	S1
S2	S2	S2	S2
S3	S3	S3	S3
S4	S4	S4	S4

S1	S2	S3	S4
R1	R1	R1	R1
R2	**R2**	R2	R2
R3	R3	**R3**	R3
R4	R4	R4	**R4**

R1	R2	R3	R4
S1	S1	S1	S1
S2	**S2**	S2	S2
S3	S3	**S3**	S3
S4	S4	S4	**S4**

Not honest

S1	S2	S3
R2	R1	R1
R1	R2	R3
R3	R3	R2

R1	R2	R3
S1	S2	S2
S2	S1	S3
S3	S3	S1

Not honest

S1	S2	S3
R2	R1	R1
R1	R2	R3
R3	R3	R2

R1	R2	R3
S1	S2	S2
S2	S1	S3
S3	S3	S1

R2	R1	R1
R1	R2	R3
R3	R3	R2

S1	S2	S2
S3	S1	S3
S2	S3	S1

Not honest

S1	S2	S3
R2	R1	R1
R1	R2	R3
R3	R3	R2

R1	R2	R3
S1	S2	S2
S2	S1	S3
S3	S3	S1

R2	R1	R1
R1	R2	R3
R3	R3	R2

S1	S2	S2
S3	S1	S3
S2	S3	S1

THE MATCH
NATIONAL RESIDENT MATCHING PROGRAM®

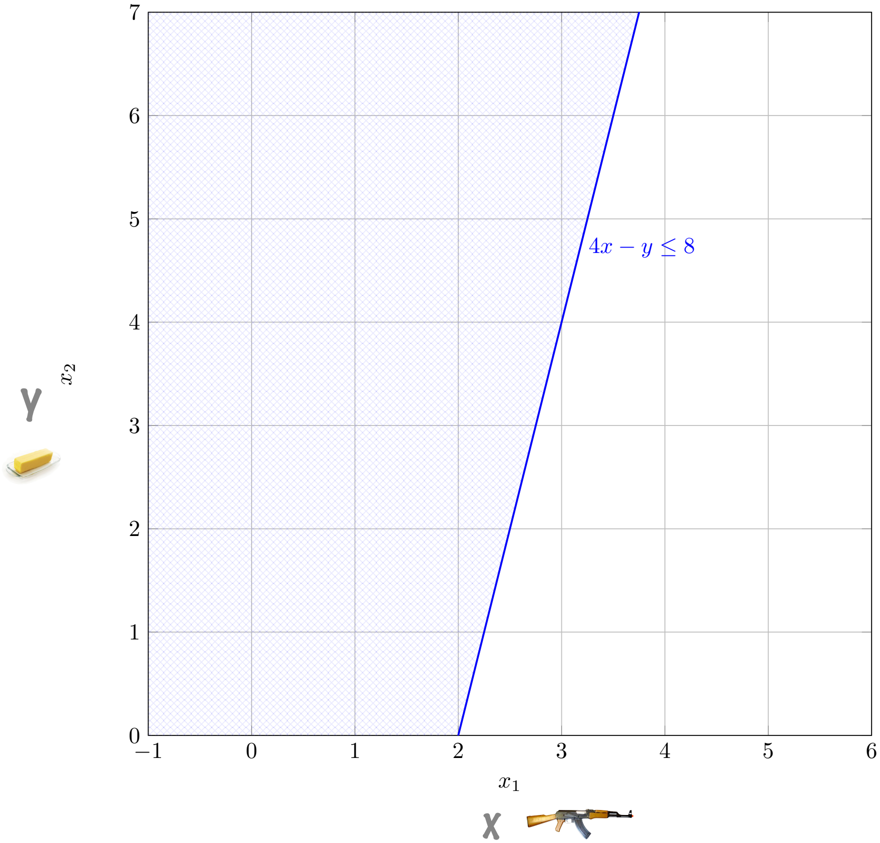
Guns and butter

$$\max x + y$$

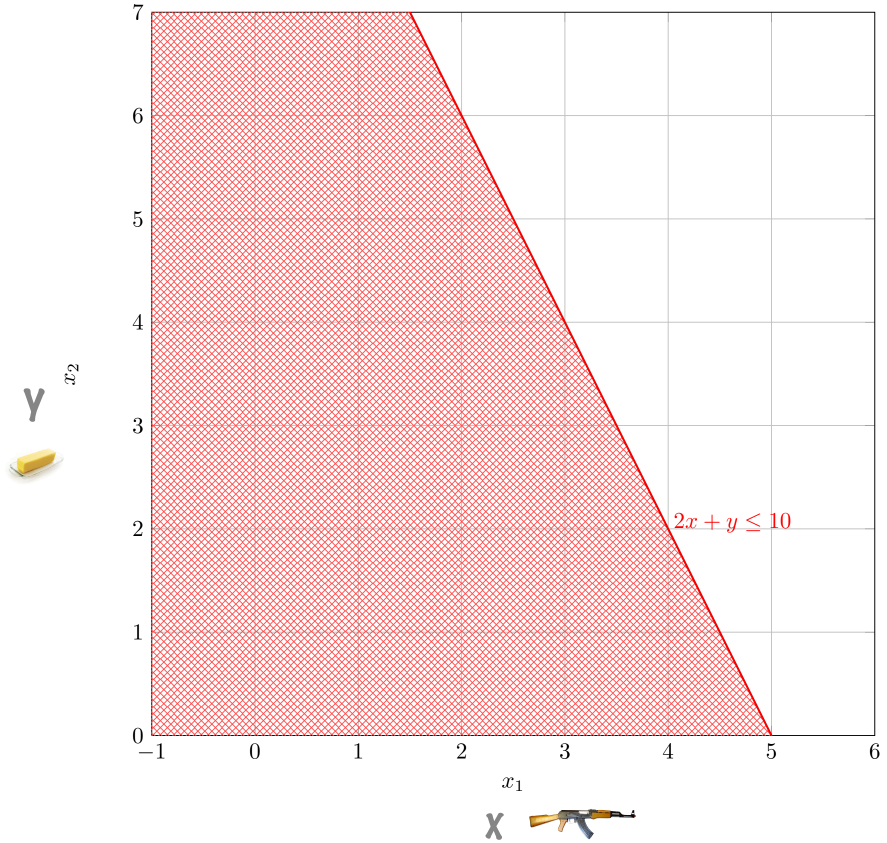
$$\begin{aligned} 4x - y &\leq 8 \\ 2x + y &\leq 10 \\ 5x - 2y &\geq -2 \\ x, y &\geq 0 \end{aligned}$$

http://i16.photobucket.com/albums/b20/safebuy/ak47/ak47-electric_lg.jpg

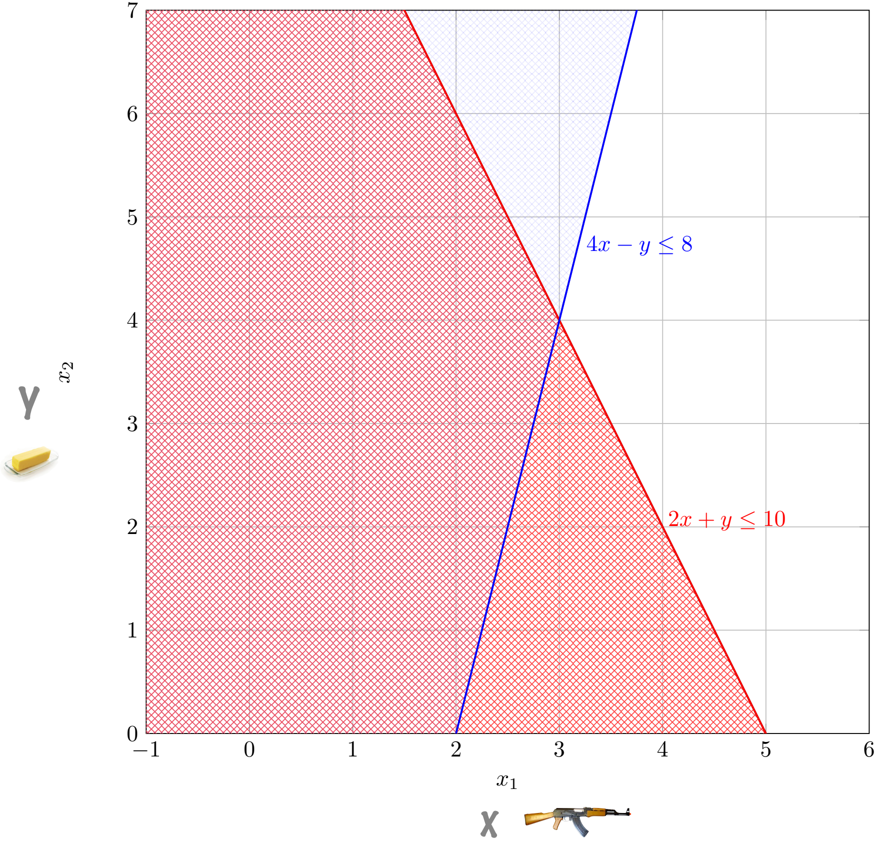
http://2.bp.blogspot.com/_NX4zcMNX4VE/Sb8MQfffllI/AAAAAAAAAL0/eu4J0dfFhJE/s400/gourmet-butter.jpg



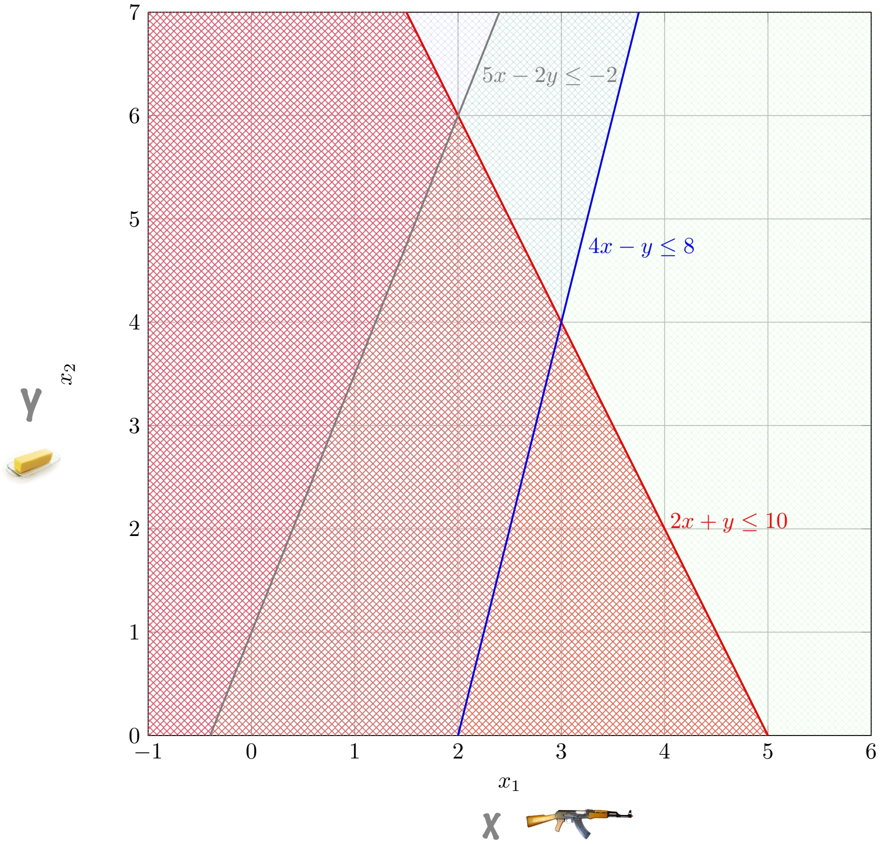
$$
\begin{aligned}
4x - y &\leq 8 \\
2x + y &\leq 10 \\
5x - 2y &\geq -2 \\
x, y &\geq 0
\end{aligned}
$$



$$
\begin{aligned}
4x - y &\leq 8 \\
2x + y &\leq 10 \\
5x - 2y &\geq -2 \\
x, y &\geq 0
\end{aligned}
$$



$$
\begin{aligned}
4x - y &\leq 8 \\
2x + y &\leq 10 \\
5x - 2y &\geq -2 \\
x, y &\geq 0
\end{aligned}
$$



$$
\begin{aligned}
4x - y &\leq 8\\
2x + y &\leq 10\\
5x - 2y &\geq -2\\
x, y &\geq 0
\end{aligned}
$$

Certificate of optimality

$$\max x + y$$

$$\begin{aligned} 4x - y &\leq 8 \\ 2x + y &\leq 10 \\ 5x - 2y &\geq -2 \\ x, y &\geq 0 \end{aligned}$$

Certificate of optimality

$$\max x + y$$

$$\begin{aligned} 4x - y &\leq 8 \\ 2x + y &\leq 10 \\ 5x - 2y &\geq -2 \\ x, y &\geq 0 \end{aligned}$$

7 $14x + 7y \leq 70$

-1 $-5x + 2y \leq 2$

$9x + 9y \leq 72$

Stigler diet

CALORIES	3000
PROTEIN	70g
CALCIUM	.8g
IRON	19mg
VITAMIN A	5000iu
THIAMINE	1.8mg
RIBOFLAVIN	2.7mg
NIACIN	18mg
ASCORBIC ACID	75mg

TABLE A. NUTRITIVE VALUES OF COMMON FOODS PER DOLLAR OF EXPENDITURE, AUGUST 15, 1939

Commodity	Unit	Price Aug. 15, 1939 (cents)	Edible Weight per $1.00 (grams)	Calories (1,000)	Protein (grams)	Calcium (grams)	Iron (mg.)	Vitamin A (1,000 I.U.)	Thiamine (mg.)	Ribo-flavin (mg.)	Niacin (mg.)	Ascorbic Acid (mg.)
**1. Wheat Flour (Enriched)	10 lb.	36.0	12,600	44.7	1,411	2.0	365		55.4	33.3	441	
2. Macaroni	1 lb.	14.1	3,217	11.6	418	.7	54		3.2	1.9	68	
3. Wheat Cereal (Enriched)	28 oz.	24.2	3,280	11.8	377	14.4	175		14.4	8.8	114	
4. Corn Flakes	8 oz.	7.1	3,194	11.4	252	.1	56		13.5	2.3	68	
5. Corn Meal	1 lb.	4.6	9,861	36.0	897	1.7	99	30.9	17.4	7.9	106	
6. Hominy Grits	24 oz.	8.5	8,005	28.6	680	.8	80		10.6	1.6	110	
7. Rice	1 lb.	7.5	6,048	21.2	460	.6	41		2.0	4.8	60	
8. Rolled Oats	1 lb.	7.1	6,389	25.3	907	5.1	341		37.1	8.9	64	
9. White Bread (Enriched)	1 lb.	7.9	5,742	15.6	488	2.5	115		13.8	8.5	126	
10. Whole Wheat Bread	1 lb.	9.1	4,985	12.2	484	2.7	125		13.9	6.4	160	
11. Rye Bread	1 lb.	9.2	4,930	12.4	439	1.1	82		9.9	3.0	66	
12. Pound Cake	1 lb.	24.8	1,829	8.0	130	.4	31	18.9	2.8	3.0	17	
13. Soda Crackers	1 lb.	15.1	3,004	12.5	288	.5	50					
14. Milk	1 qt.	11.0	8,867	6.1	310	10.5	18	16.8	4.0	16.0	7	177
**15. Evaporated Milk (can)	14½ oz.	6.7	6,035	8.4	422	15.1	9	26.0	3.0	23.5	11	60
16. Butter	1 lb.	30.8	1,473	10.8	9	.2	3	44.2		.2	2	
*17. Oleomargarine	1 lb.	16.1	2,817	20.6	17	.6	6	55.8	.2			
18. Eggs	1 doz.	32.6	1,857	2.9	238	1.0	52	18.6	2.8	6.5	1	
**19. Cheese (Cheddar)	1 lb.	24.2	1,874	7.4	448	16.4	19	28.1	.8	10.3	4	
20. Cream	½ pt.	14.1	1,689	3.5	49	1.7	3	16.9	.6	2.5		17
21. Peanut Butter	1 lb.	17.9	2,534	15.7	661	1.0	48		9.6	8.1	471	
22. Mayonnaise	½ pt.	16.7	1,198	8.6	18	.2	8	2.7	.4	.5		
23. Crisco	1 lb.	20.3	2,234	20.1								
24. Lard	1 lb.	9.8	4,628	41.7				.2		.5	5	
25. Sirloin Steak	1 lb.	39.6	1,145*	2.9	166	.1	34	.2	2.1	2.9	69	
26. Round Steak	1 lb.	36.4	1,246*	2.2	214	.1	32	.4	2.5	2.4	87	
27. Rib Roast	1 lb.	29.2	1,553*	3.4	213	.1	33			2.0		
28. Chuck Roast	1 lb.	22.6	2,007*	3.6	309	.2	46	.4	1.0	4.0	120	
29. Plate	1 lb.	14.6	3,107*	8.5	404	.2	62		.9			
**30. Liver (Beef)	1 lb.	26.8	1,692*	2.2	333	.2	139	169.2	6.4	50.8	316	525
31. Leg of Lamb	1 lb.	27.6	1,643*	3.1	245	.1	20		2.8	3.9	86	
32. Lamb Chops (Rib)	1 lb.	36.6	1,239*	3.3	140	.1	15		1.7	2.7	54	
33. Pork Chops	1 lb.	30.7	1,477*	3.5	196	.2	30		17.4	2.7	60	
34. Pork Loin Roast	1 lb.	24.2	1,874*	4.4	249	.3	37		18.2	3.6	79	
35. Bacon	1 lb.	25.6	1,772*	10.4	152	.2	23		1.8	1.8	71	
36. Ham—smoked	1 lb.	27.4	1,655*	6.7	212	.2	31		9.9	3.3	50	
37. Salt Pork	1 lb.	16.0	2,835*	18.8	164	.1	26		1.4	1.8		
38. Roasting Chicken	1 lb.	30.3	1,497*	1.8	184	.1	30	.1	.9	1.8	68	46
39. Veal Cutlets	1 lb.	42.3	1,072*	1.7	156	.1	24		1.4	2.4	57	
40. Salmon, Pink (can)	16 oz.	13.0	3,489	5.8	705	6.8	45	3.5	1.0	4.9	209	
41. Apples	1 lb.	4.4	9,072	5.8	27	.5	36	7.3	3.6	2.7	5	544
42. Bananas	1 lb.	6.1	4,982	4.9	60	.4	30	17.4	2.5	3.5	28	498
43. Lemons	1 doz.	26.0	2,380	1.0	21	.5	14		.5		4	952
44. Oranges	1 doz.	30.9	4,439	2.2	40	1.1	18	11.1	3.6	1.3	10	1,998
*45. Green Beans	1 lb.	7.1	5,750	2.4	138	3.7	80	69.0	4.3	5.8	37	862
**46. Cabbage	1 lb.	3.7	8,949	2.6	125	4.0	36	7.2	9.0	4.5	26	5,369
47. Carrots	1 bunch	4.7	6,080	2.7	73	2.8	43	188.5	6.1	4.3	89	608
48. Celery	1 stalk	7.3	3,915	.9	51	3.0	23	.9	1.4	1.4	9	313
49. Lettuce	1 head	8.2	2,247	.4	27	1.1	22	112.4	1.8	3.4	11	449
*50. Onions	1 lb.	3.6	11,844	5.8	166	3.8	59	16.6	4.7	5.9	21	1,184

*51. Potatoes	15 lb.	34.0	18,810	14.3	336	1.8	118	6.7	29.4	7.1	198	2,522
**52. Spinach	1 lb.	8.1	4,592	1.1	106	—	138	918.4	5.7	13.8	33	2,755
**53. Sweet Potatoes	1 lb.	5.1	7,649	9.6	138	2.7	54	290.7	8.4	5.4	83	1,912
54. Peaches (can)	No. 2½	16.8	4,894	3.7	20	.4	10	21.5	.5	1.0	31	196
55. Pears (can)	No. 2½	20.4	4,030	3.0	8	.3	8	.8	.8	.8	5	81
56. Pineapple (can)	No. 2½	21.3	3,993	2.4	16	.4	8	2.0	2.8	.8	7	399
57. Asparagus (can)	No. 2	27.7	1,945	.4	33	.3	12	16.3	1.4	2.1	17	272
58. Green Beans (can)	No. 2	10.0	5,386	1.0	54	2.0	65	53.9	1.6	4.3	32	431
59. Pork and Beans (can)	16 oz.	7.1	6,389	7.5	364	4.0	134	3.5	8.3	7.7	56	
60. Corn (can)	No. 2	10.4	5,452	5.2	136	.2	16	12.0	1.6	2.7	42	218
61. Peas (can)	No. 2	13.8	4,109	2.3	136	.6	45	34.9	4.9	2.5	37	370
62. Tomatoes (can)	No. 2	8.6	6,263	1.3	63	.7	38	53.2	3.4	2.5	36	1,253
63. Tomato Soup (can)	10½ oz.	7.6	3,917	1.6	71	.6	43	57.9	3.5	2.4	67	862
*64. Peaches, Dried	1 lb.	15.7	2,889	8.5	87	1.7	173	86.8	1.2	4.3	55	57
*65. Prunes, Dried	1 lb.	9.0	4,284	12.8	99	2.5	154	85.7	3.9	4.3	65	257
66. Raisins, Dried	15 oz.	9.4	4,524	13.5	104	2.5	136	4.5	6.3	1.4	24	136
67. Peas, Dried	1 lb.	7.9	5,742	20.0	1,367	4.2	345	2.9	28.7	18.4	162	
**68. Lima Beans, Dried	1 lb.	8.9	5,097	17.4	1,055	3.7	459	5.1	26.9	38.2	93	
**69. Navy Beans, Dried	1 lb.	5.9	7,688	26.9	1,691	11.4	792		38.4	24.6	217	
70. Coffee	1 lb.	22.4	2,025	—	—	—	—		4.0	5.1	50	
71. Tea	¼ lb.	17.4	652	—	—	—	—			2.3	42	
72. Cocoa	8 oz.	8.6	2,637	8.7	237	3.0	72		2.0	11.9	40	
73. Chocolate	8 oz.	16.2	1,400	8.0	77	1.3	39		.9	3.4	14	
74. Sugar	10 lb.	51.7	8,773	34.9	—	—	—					
75. Corn Sirup	24 oz.	13.7	4,966	14.7	—	.5	74				5	
76. Molasses	18 oz.	13.6	3,752	9.0	—	10.3	244		1.9	7.5	146	
77. Strawberry Preserves	1 lb.	20.5	2,213	6.4	11	.4	7	.2	.2	.4	3	

* Quantities including inedible portions.

TABLE B. NUTRITIVE VALUES OF COMMON FOODS PER DOLLAR OF EXPENDITURE, AUGUST 15, 1944

Commodity	Price Aug. 15, 1944 (cents)	Calories (1,000)	Protein (grams)	Calcium (grams)	Iron (mg.)	Vitamin A (1,000 I.U.)	Thiamine (mg.)	Riboflavin (mg.)	Niacin (mg.)	Ascorbic Acid (mg.)
1. Wheat Flour	64.6	24.9	786	1.1	203		30.9	18.6	246	
3. Wheat Cereal	23.2	12.3	398	15.0	183		15.0	9.2	119	
5. Corn Meal	6.3	26.3	655	1.2	72	22.6	12.7	5.8	77	
8. Rolled Oats	9.9	18.1	651	3.7	245		26.6	6.4	46	
15. Evaporated Milk	10.0	5.6	283	10.1	6	17.4	2.0	15.7	7	40
46. Cabbage	4.0	2.0	94	3.0	27	5.4	6.8	3.4	20	4,034
51. Potatoes	60.1	6.1	143	.8	50	2.8	12.5	3.0	84	1,071
52. Spinach	11.6	.8	74	—	96	641.3	4.0	9.6	23	1,924
53. Sweet Potatoes	12.3	4.0	57	1.1	22	120.5	3.5	2.2	34	793
69. Navy Beans	10.8	14.7	924	6.2	433		21.0	13.4	119	
74. Sugar	67.0	26.9	—	—	—					
78. Pancake Flour[1]	12.2	16.0	479	18.1	46		3.7	1.9	41	
79. Beets[2]	7.3	2.2	85	1.1	70	132.3	2.9	6.3	29	395
80. Liver (Pork)[3]	21.9	2.7	408	.2	518	145.0	10.4	51.8	472	580

[1] Unit: 20 oz.; edible weight: 4,647 g.
[2] Unit: 1 bunch; edible weight: 4,071 g.
[3] Unit: 1 lb.; edible weight: 2,071 g.

	Brownie	Dumpling	Espresso	Amelia
cost	5	2	3	8
cals	400	200	150	500
choc	3	2	0	0
sugar	2	2	4	4
fat	2	4	0	5

requirements: 500 calories, 6 oz choc, 10 oz sugar, 8 oz fat

	Brownie	Dumpling	Espresso	Amelia
cost	5	2	3	8
cals	400	200	150	500
choc	3	2	0	0
sugar	2	2	4	4
fat	2	4	0	5

requirements: 500 calories, 6 oz choc, 10 oz sugar, 8 oz fat

	Brownie	Dumpling	Espresso	Amelia
cost	5	2	3	8
cals	400	200	150	500
choc	3	2	0	0
sugar	2	2	4	4
fat	2	4	0	5

requirements: 500 calories, 6 oz choc, 10 oz sugar, 8 oz fat

$$\min 5x_1 + 2x_2 + 3x_3 + 8x_4$$

$$\begin{bmatrix} 400 & 200 & 150 & 500 \\ 3 & 2 & 0 & 0 \\ 2 & 2 & 4 & 4 \\ 2 & 4 & 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \geq \begin{bmatrix} 500 \\ 6 \\ 10 \\ 8 \end{bmatrix}$$

$$\min 5x_1 + 2x_2 + 3x_3 + 8x_4$$

$$\begin{bmatrix} 400 & 200 & 150 & 500 \\ 3 & 2 & 0 & 0 \\ 2 & 2 & 4 & 4 \\ 2 & 4 & 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \geq \begin{bmatrix} 500 \\ 6 \\ 10 \\ 8 \end{bmatrix}$$

$$\min 5x_1 + 2x_2 + 3x_3 + 8x_4$$

$$\begin{bmatrix} 400 & 200 & 150 & 500 \\ 3 & 2 & 0 & 0 \\ 2 & 2 & 4 & 4 \\ 2 & 4 & 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \geq \begin{bmatrix} 500 \\ 6 \\ 10 \\ 8 \end{bmatrix}$$

```
H-representation
begin
8 4 rational
-500 400 200 150 500
-6   3   2   0    0
-10  2   2   4    4
-6   2   4   0    5
0    1   0   0    0
0    0   1   0    0
0    0   0   1    0
0    0   0   0    1
end
minimize
0 5 2 3 8
```

$$\min 5x_1 + 2x_2 + 3x_3 + 8x_4$$

$$\begin{bmatrix} 400 & 200 & 150 & 500 \\ 3 & 2 & 0 & 0 \\ 2 & 2 & 4 & 4 \\ 2 & 4 & 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \geq \begin{bmatrix} 500 \\ 6 \\ 10 \\ 8 \end{bmatrix}$$

$$\min 5x_1 + 2x_2 + 3x_3 + 8x_4$$

$$\begin{bmatrix} 400 & 200 & 150 & 500 \\ 3 & 2 & 0 & 0 \\ 2 & 2 & 4 & 4 \\ 2 & 4 & 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \geq \begin{bmatrix} 500 \\ 6 \\ 10 \\ 8 \end{bmatrix}$$

```
H-representation
begin
8 4 rational
-500 400 200 150 500
-6   3   2   0    0
-10  2   2   4    4
-6   2   4   0    5
0    1   0   0    0
0    0   1   0    0
0    0   0   1    0
0    0   0   0    1
end
minimize
0 5 2 3 8
```

$$\min 5x_1 + 2x_2 + 3x_3 + 8x_4$$

$$\begin{bmatrix} 400 & 200 & 150 & 500 \\ 3 & 2 & 0 & 0 \\ 2 & 2 & 4 & 4 \\ 2 & 4 & 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \geq \begin{bmatrix} 500 \\ 6 \\ 10 \\ 8 \end{bmatrix}$$

```
H-representation
begin
8 4 rational
-500 400 200 150 500
-6   3   2   0    0
-10  2   2   4    4
-6   2   4   0    5
0    1   0   0    0
0    0   1   0    0
0    0   0   1    0
0    0   0   0    1
end
minimize
0 5 2 3 8
```

```
*Objective function is
 0 + 5 X[1] + 2 X[2] + 3 X[3] + 8 X[4]
*LP status: a dual pair (x, y) of optimal solutions found.
begin
  primal_solution
  1 :  0
  2 :  3
  3 :  1
  4 :  0
  dual_solution
  2 :  -1/4
  5 :  -11/4
  3 :  -3/4
  8 :  -5
  optimal_value : 9
end
*number of pivot operations = 4
```

BERLIN

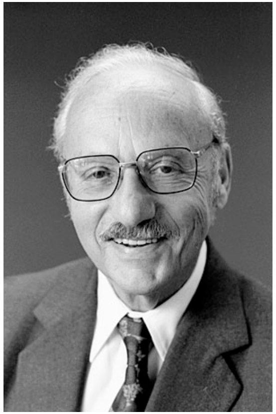


IMAGE:STAMFORD

IMAGE:HISTORY OF AIR CARGO

linear programming
saved Berlin

shortest paths as LP

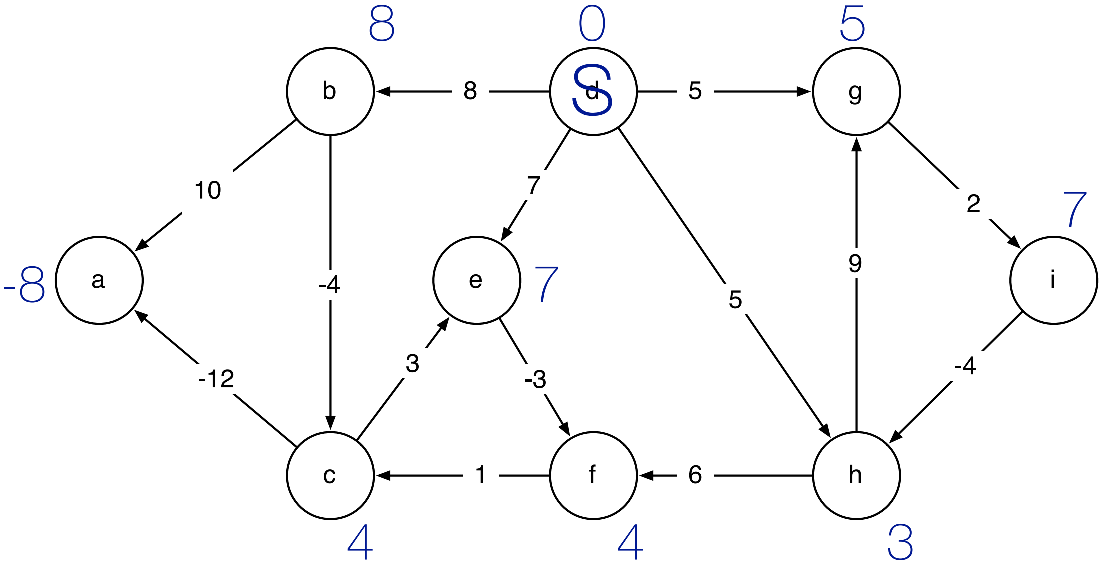
inputs:

shortest paths as LP

$$\max d_t$$

$$d_y - d_x \leq l(x,y) \qquad \forall e = (x,y) \in E$$

$$d_s = 0$$



$$\max d_t$$

$$d_y - d_x \le l(x,y) \qquad \forall e = (x,y) \in E$$

$$d_s = 0$$

dt = 30

max flow as lp

INPUT: (G, c, s, t) $G = (V, E)$ $c : E \to \mathbb{Z}_+$

max flow as lp

$$\max \sum_{v} f(s,v) - \sum_{v} f(v,s)$$

$f(u,v) \leq c(u,v)$ FOR (U,V) IN E

$\sum_{u} f(u,v) = \sum_{w} f(v,w) \quad \forall v$

$f(u,v) \geq 0$ FOR (U,V) IN E

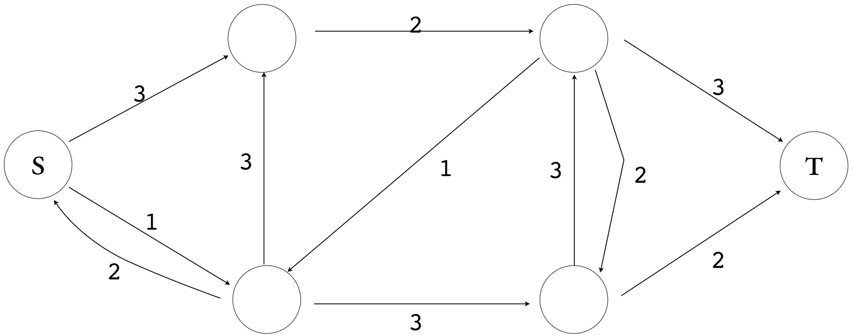
max flow as lp

$$\max \sum_v f(s,v) - \sum_v f(v,s)$$

$$f(u,v) \le c(u,v) \qquad \text{for } (u,v) \text{ in } E$$

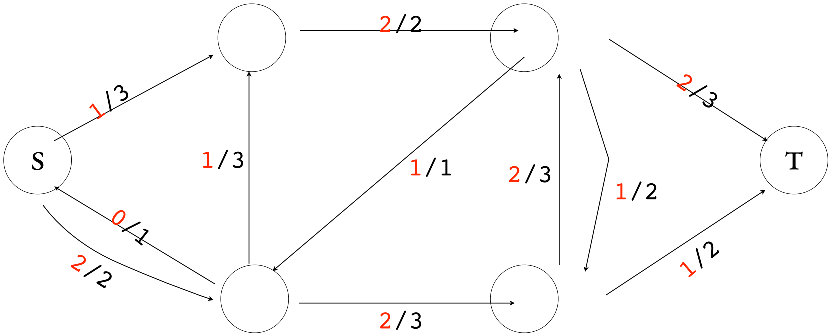
$$\sum_u f(u,v) = \sum_w f(v,w) \qquad \forall v$$

$$f(u,v) \ge 0 \qquad \text{for } (u,v) \text{ in } E$$



min-cost flow as lp

INPUT: (G, c, s, t) $G = (V, E)$ $c : E \to \mathbb{Z}_+$ $x : E \to \mathbb{Z}_+$ d



min-cost flow as lp

min-cost flow as lp

$$\min_e x_e \cdot f(e)$$

$$f(e) \leq c(e)$$

$$f(e) \geq 0$$

$$\sum_u f(u,v) = \sum_w f(v,w)$$

$$\sum_v f(s,v) - \sum_v f(v,s) = d$$