

5800

Stablematch

apr1/apr4 2022

shelat

Gabriel García Márquez

Love in the
Time of
Tindera





We have a
group of
suitors and
reviewers



2>1>3



2>3>1



1>3>2



Each has preferences over the other group



1>3>2



1>2>2



3>2>1

2>1>3



2>3>1



1>3>2



We seek a
stable matching
between
the two



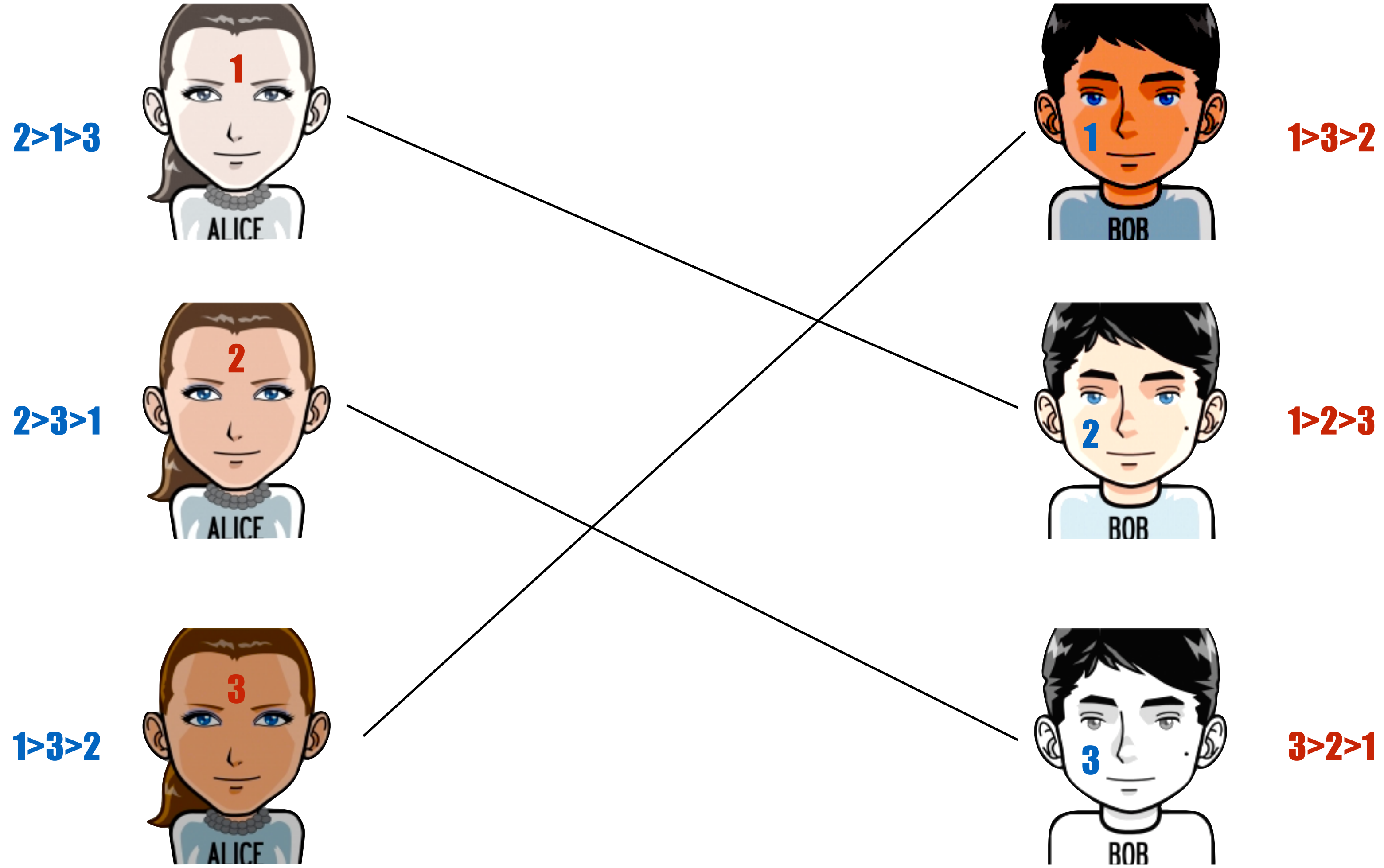
1>3>2

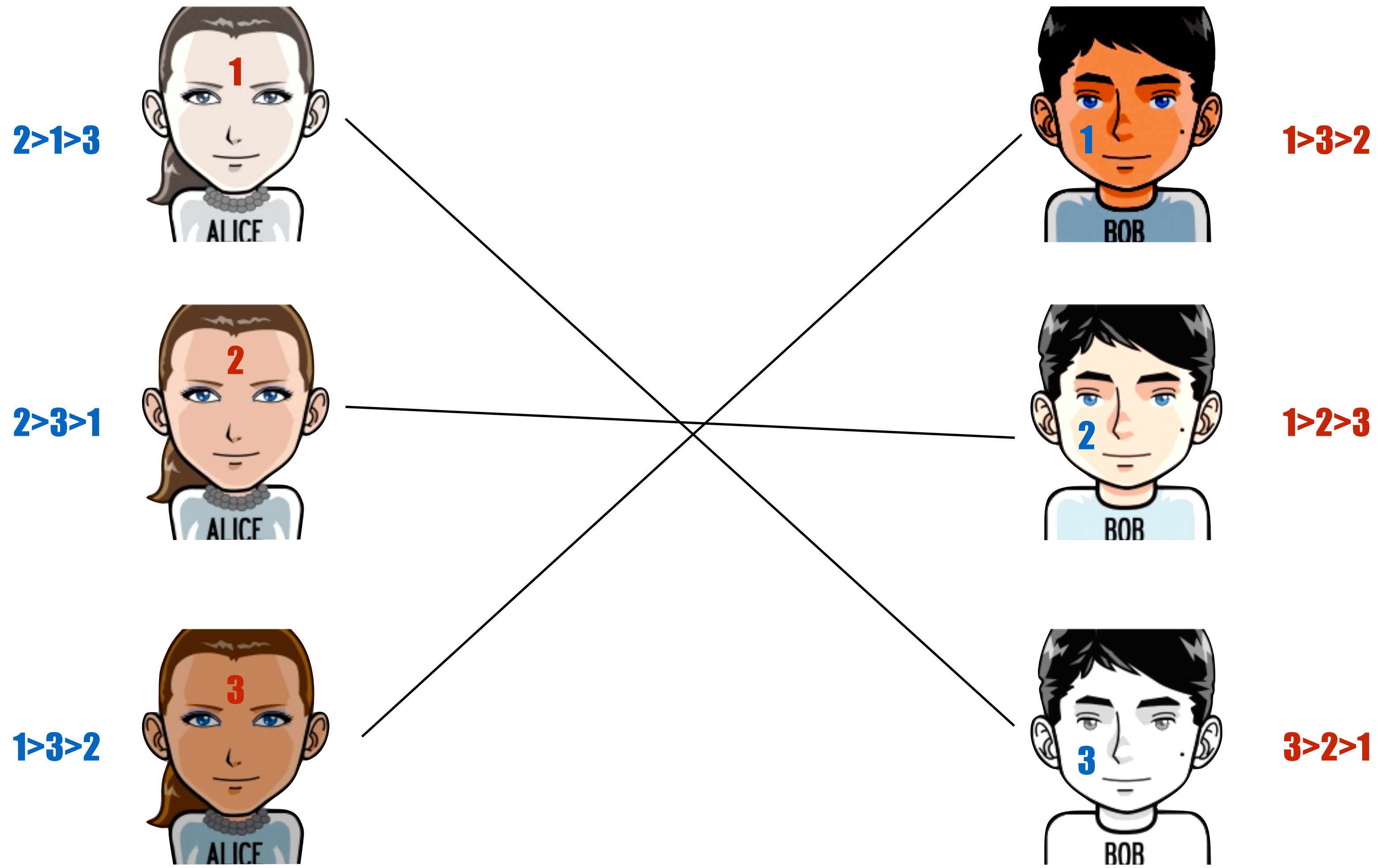


1>2>2

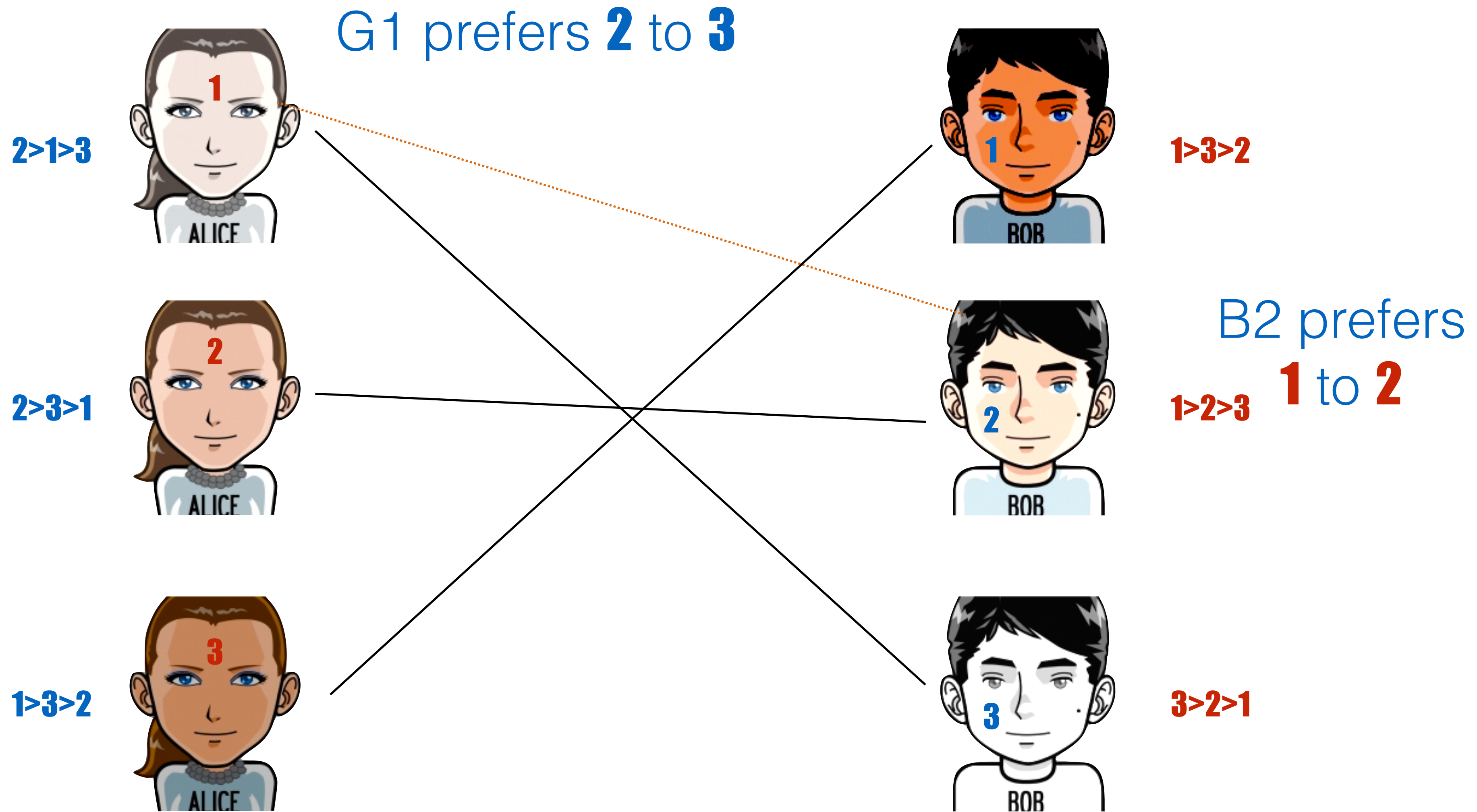


3>2>1





Unstable Matching

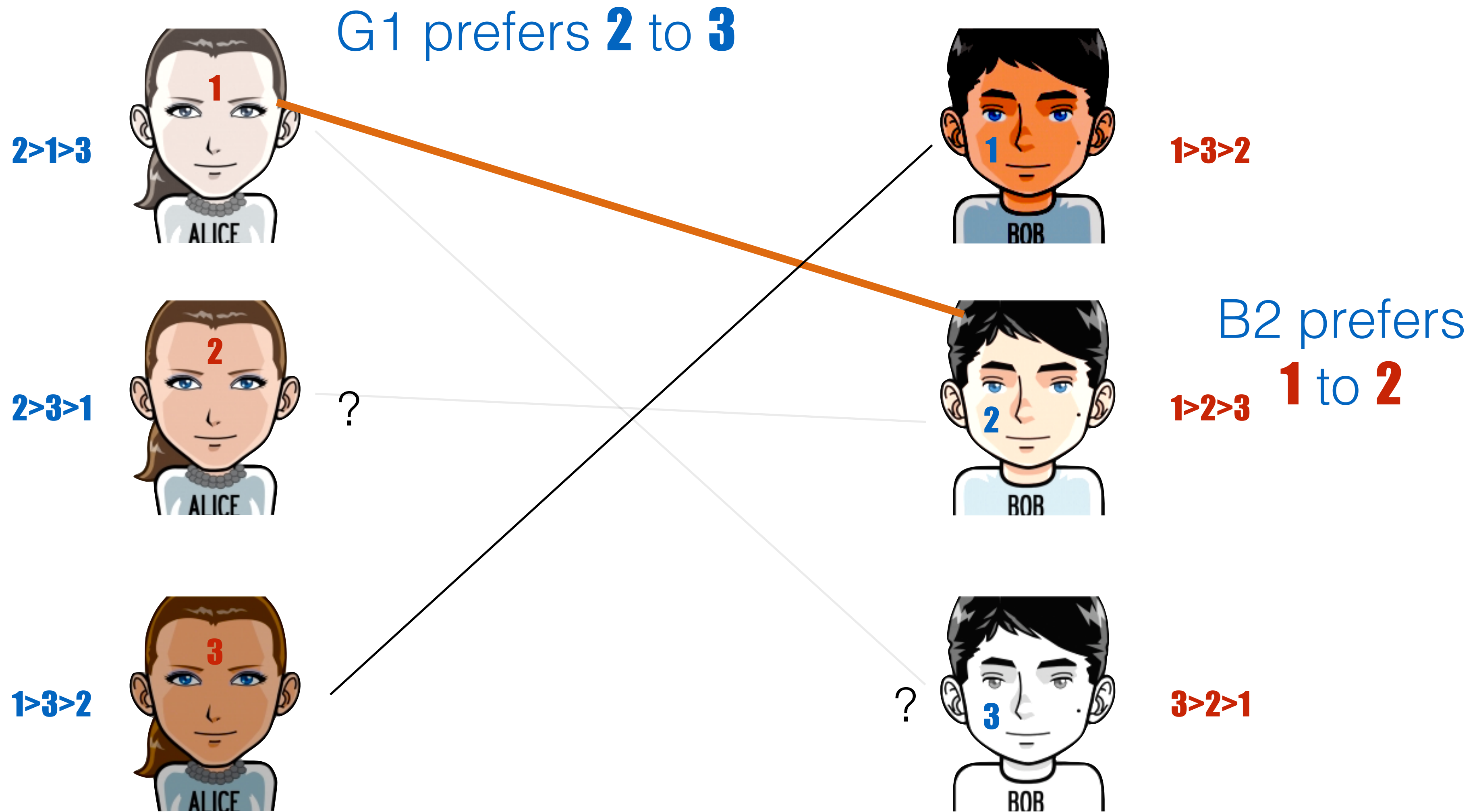


G1 prefers **2** to **3**

1 > **3** > **2**

B2 prefers **1** to **2**

Unstable Matching



Unstable Matching

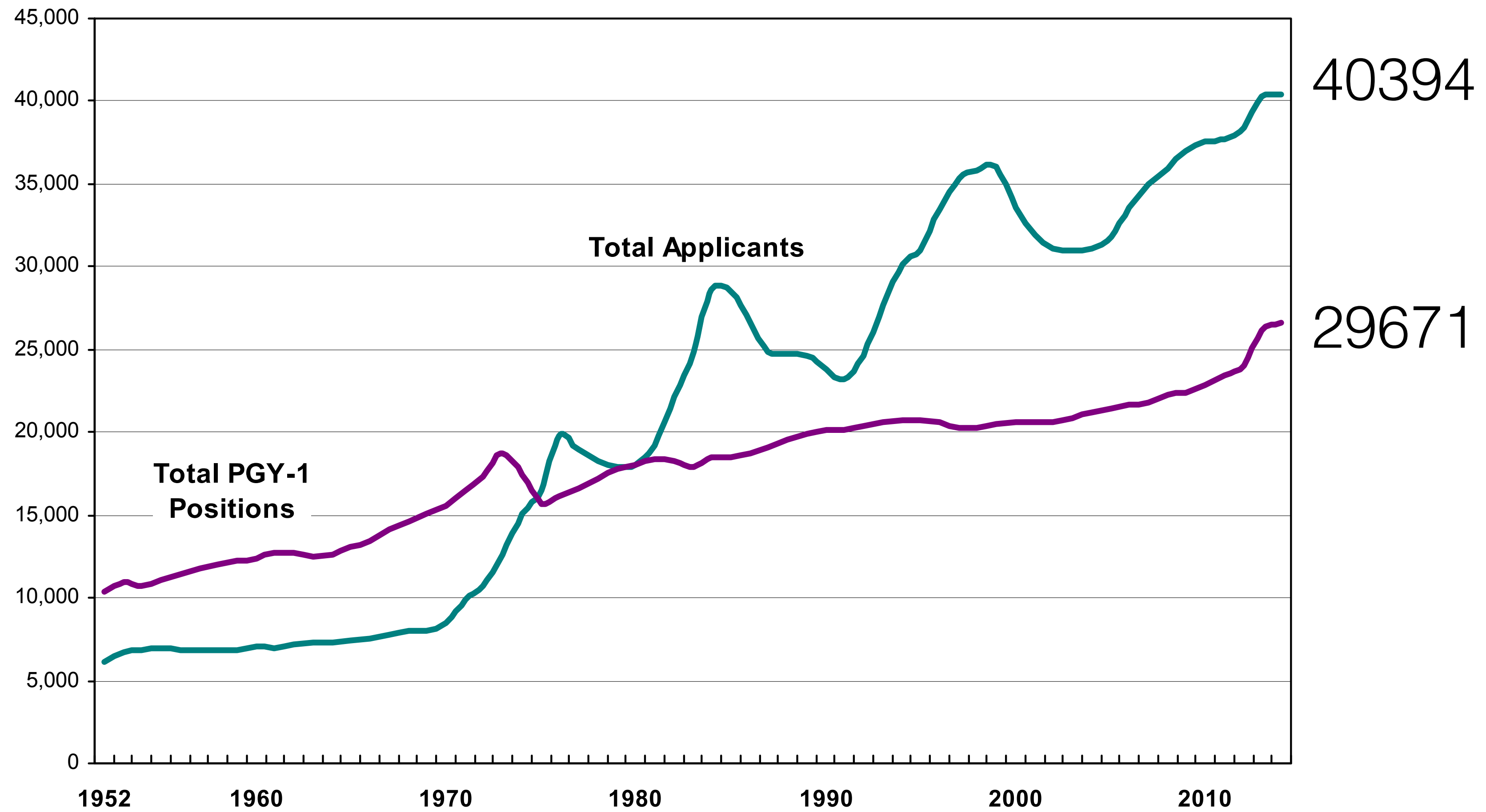
Stable Matching

Stable
matching has
many practical
applications

THE MATCHSM

NATIONAL RESIDENT MATCHING PROGRAM[®]

Figure 1 Applicants and 1st Year Positions in The Match, 1952 - 2014





Applicant Type	Matched		
	2013 Graduates	Prior Year Graduates ¹	Total
CMG	2571	74	2645
IMG	146	353	499
USMG	23	2	25
TOTAL	2740	429	3169





University of Virginia
Chi Omega Bid Day 2012

Definition: matchings

$$P =$$

$$R =$$

$$M =$$

Definition: matchings

$$P = \{p_1, p_2, \dots, p_n\}$$

$$R = \{r_1, r_2, \dots, r_n\}$$

$$M = \{(p_{i_1}, r_{j_1}), \dots, (p_{i_n}, r_{j_n})\}$$

Each p_i (r_j) appears only one in a pairing.

A matching is perfect if every p_i appears.



Image credits: Julia Nikolaeva

Definition: preferences

$$P = \{p_1, p_2, \dots, p_n\}$$

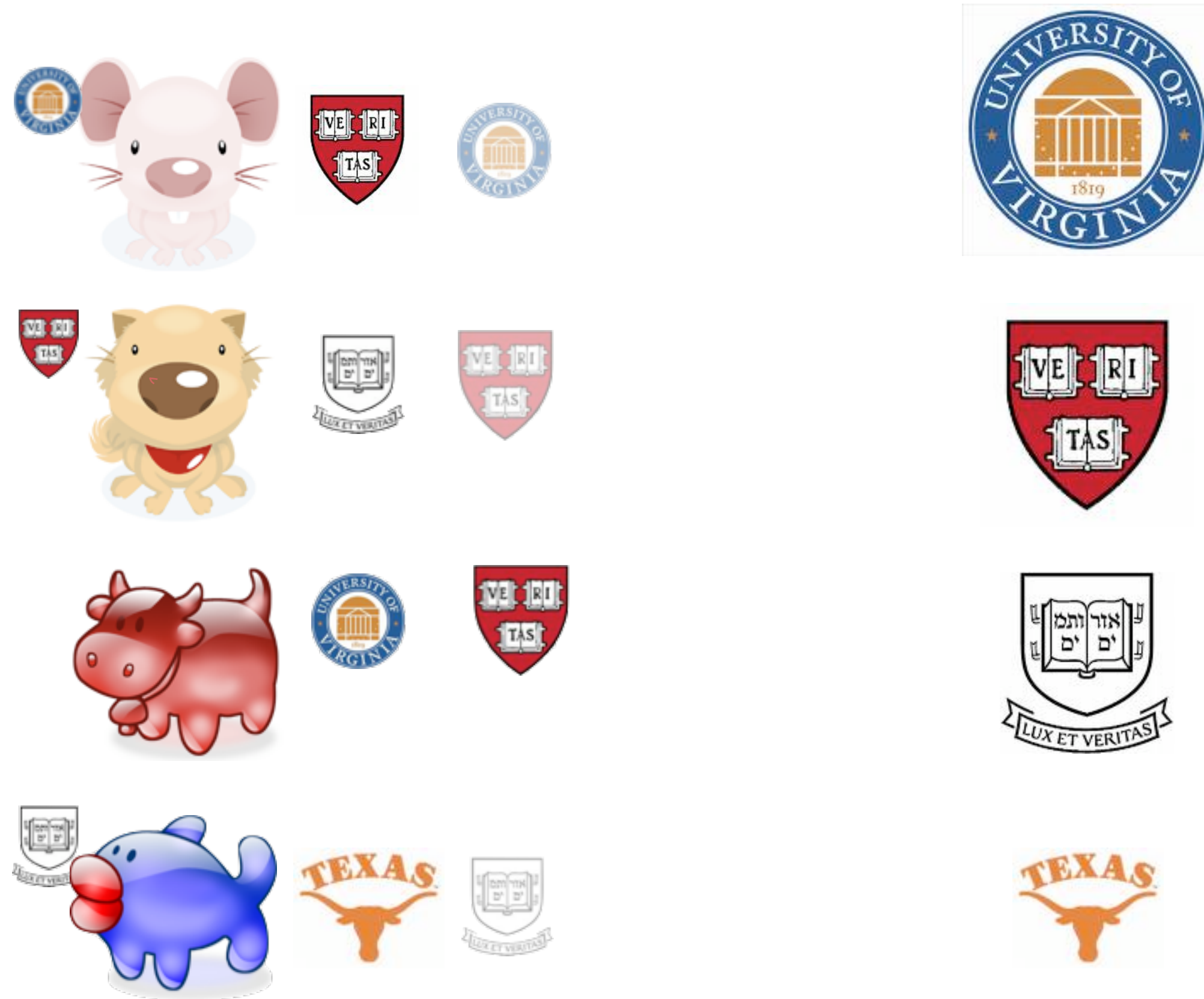


Image credits: Julia Nikolaeva

Example: preferences

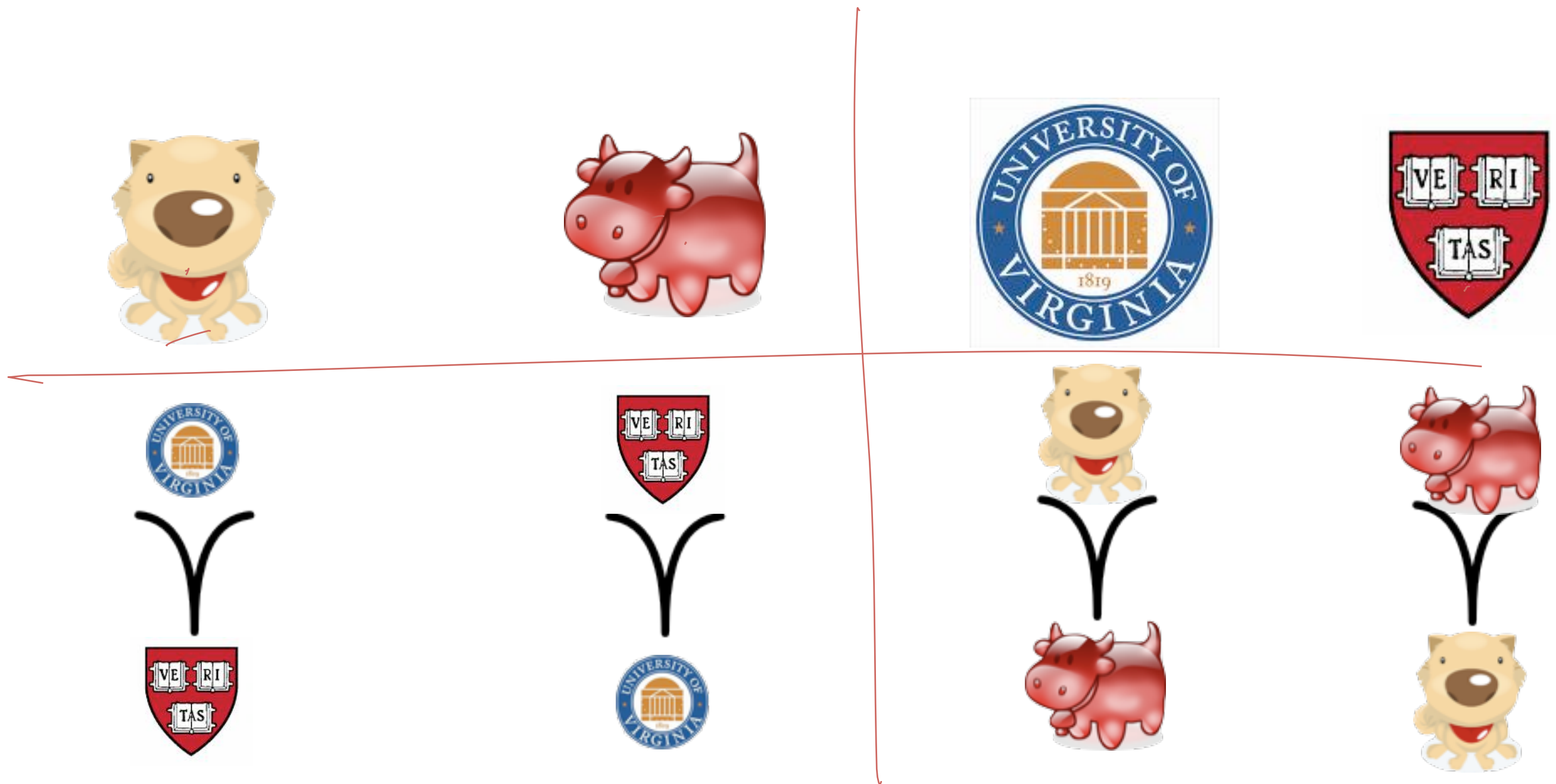
$$P = \{p_1, p_2, \dots, p_n\}$$

p_i has a preference relation
on the set R

$$w_1 \prec_{m_i} w_4 \prec_{m_i} w_2 \prec_{m_i} w_8 \dots w_n$$

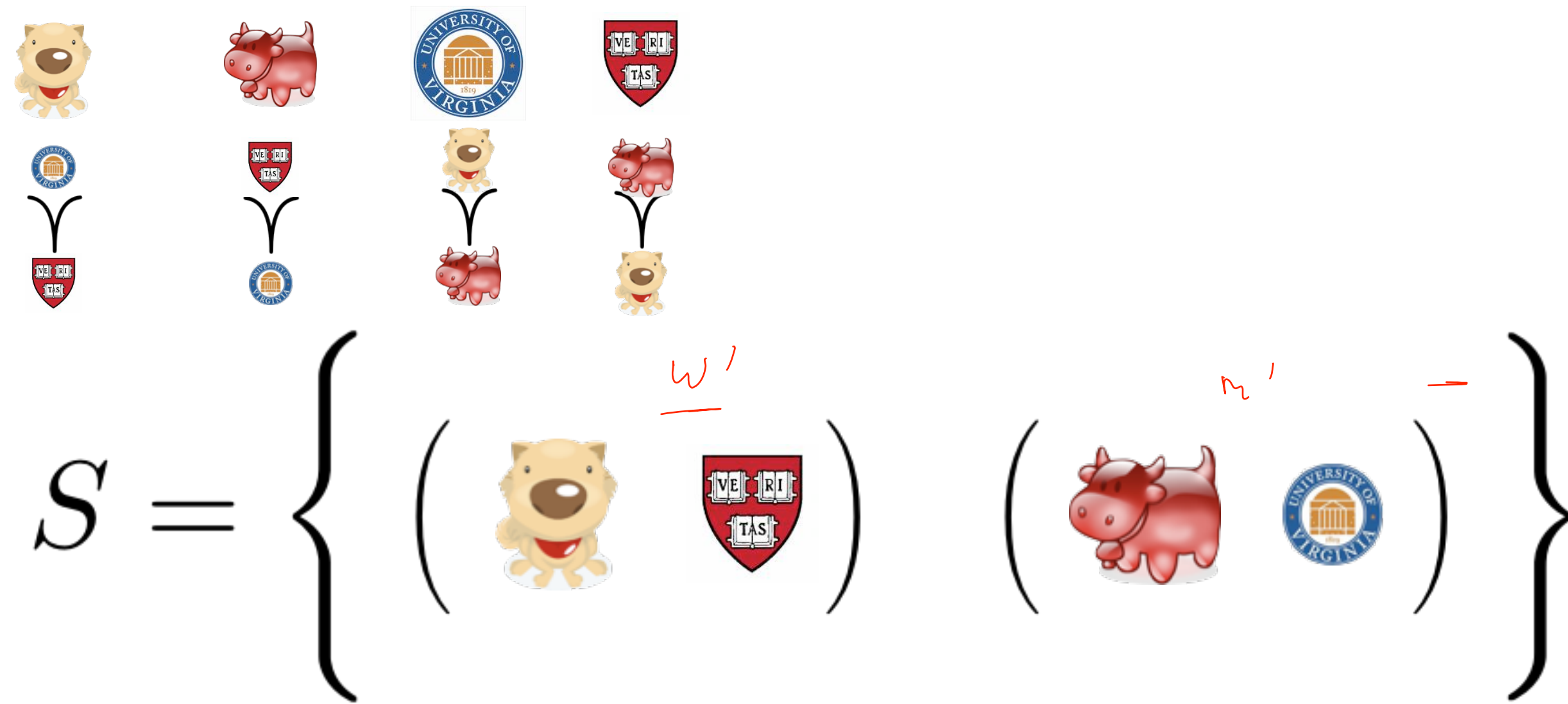




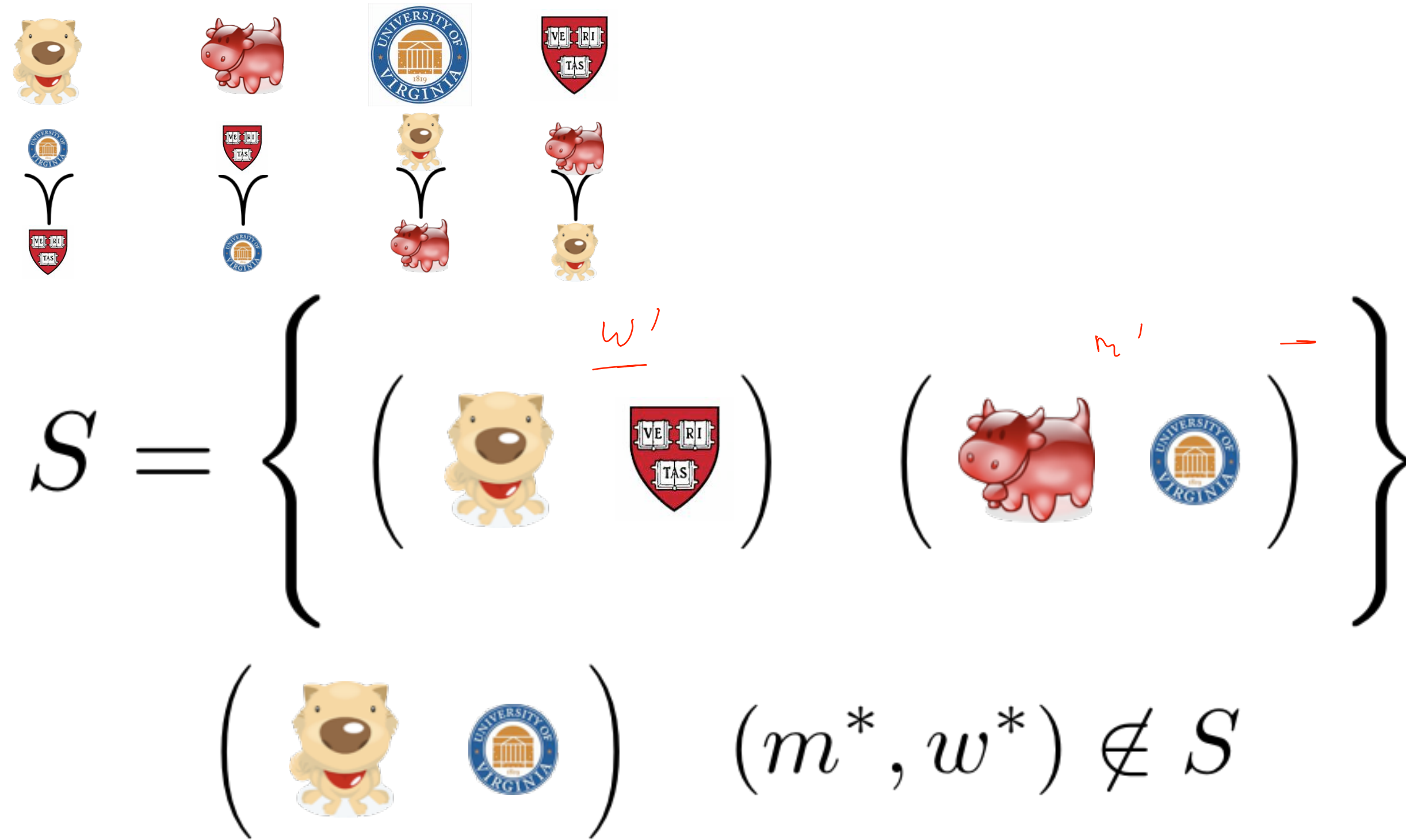


$$S = \left\{ \left(\text{dog} \quad \text{shield} \right) \quad \left(\text{cow} \quad \text{seal} \right) \right\}$$

Def: instability



Def: instability



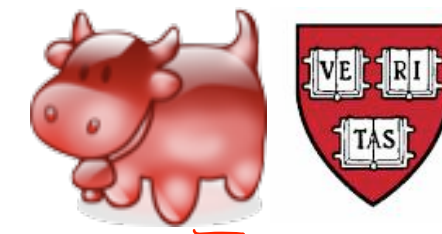
$$w' \prec_{m^*} w^*$$

$$m' \prec_{w^*} m^*$$

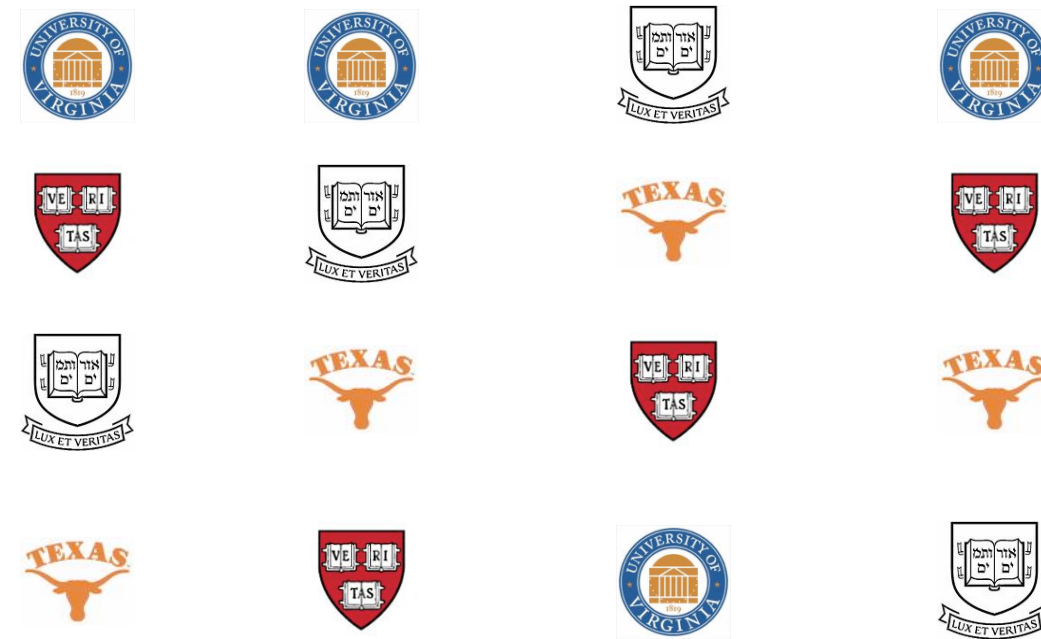
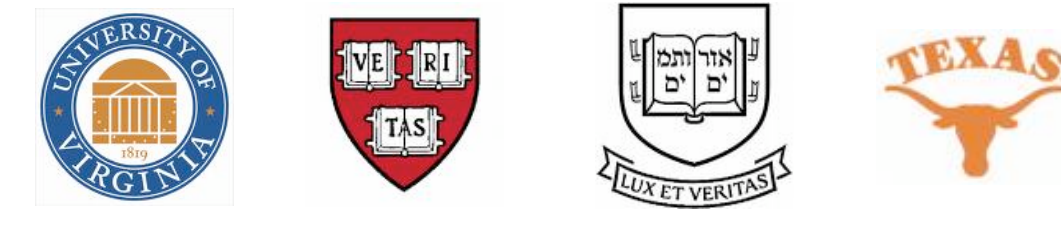
M = $\{ (s_1, r_1), (s_2, r_2), \dots (s_n, r_n) \}$
is a stable matching if

No unmatched pair (s^*, r^*) prefer each other to their partners in M

Example 2



Prove: for every input



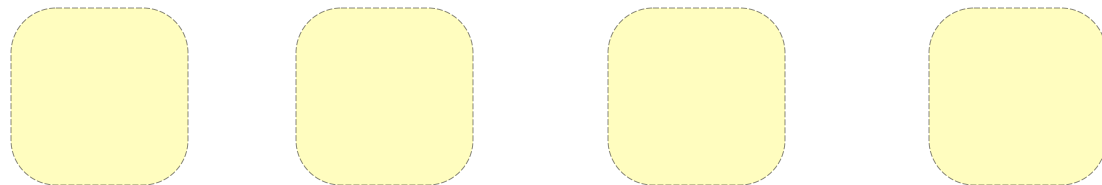
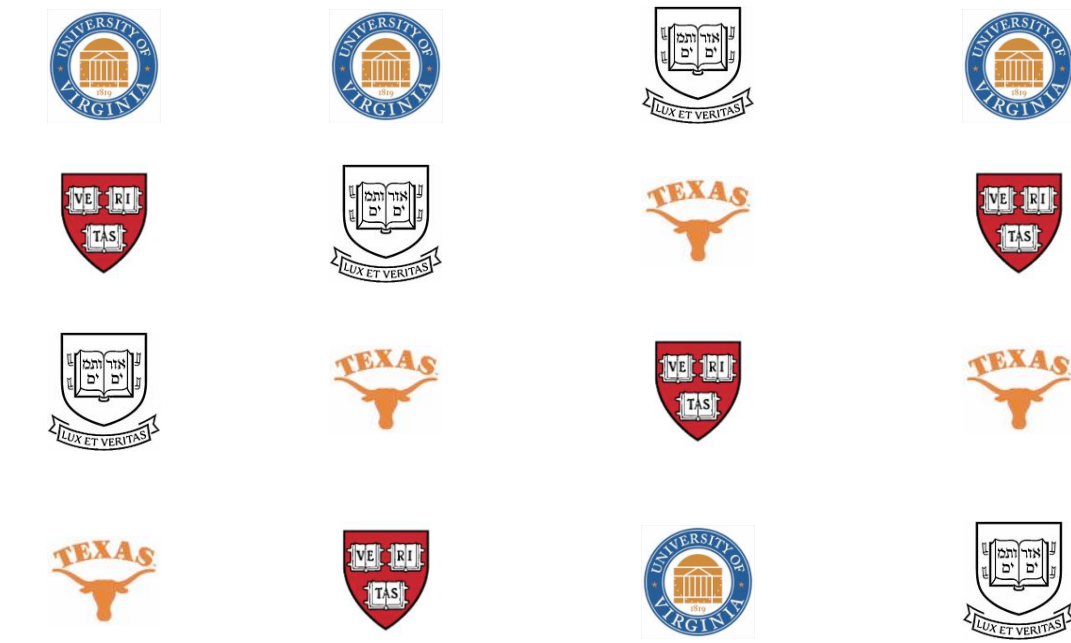
there exists a stable matching.

proposal algorithm

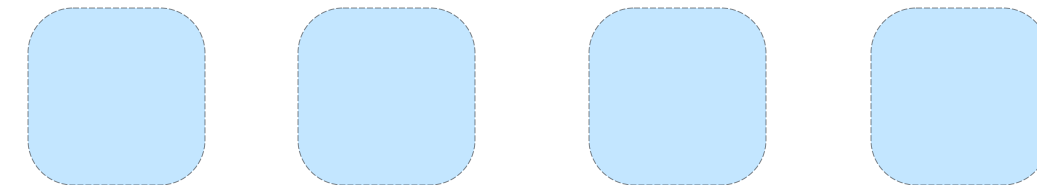
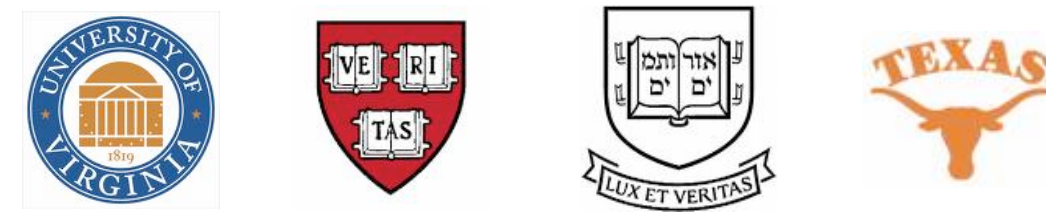
STABLEMATCH(M, W, \prec_m, \prec_w)

```
1  Initialize all  $m, w$  to be FREE
2  while  $\exists$ FREE( $m$ ) and hasn't proposed to all  $W$ 
3      do Pick such an  $m$ 
4          Let  $w \in W$  be highest-ranked to whom  $m$  has not yet proposed
5          if FREE( $w$ )
6              then Make a new pair  $(m, w)$ 
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8              do Break pair  $(m', w)$  and make  $m'$  free
9                  Make pair  $(m, w)$ 
10 return Set of pairs
```

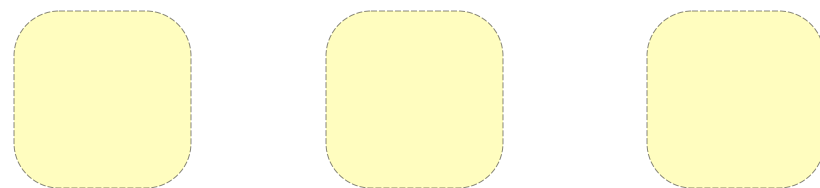
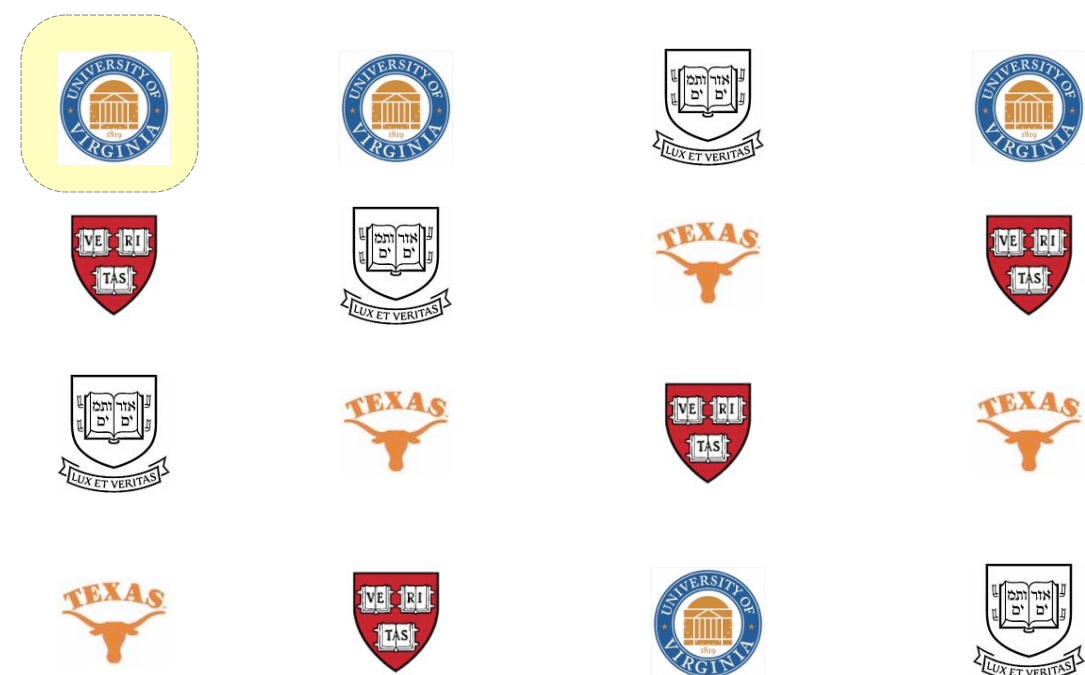
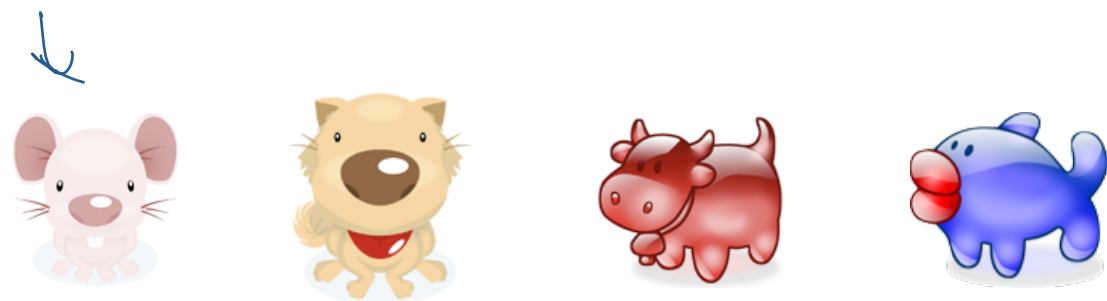

S



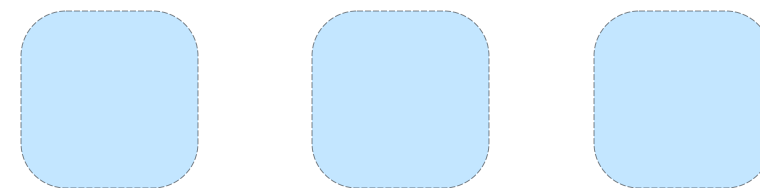
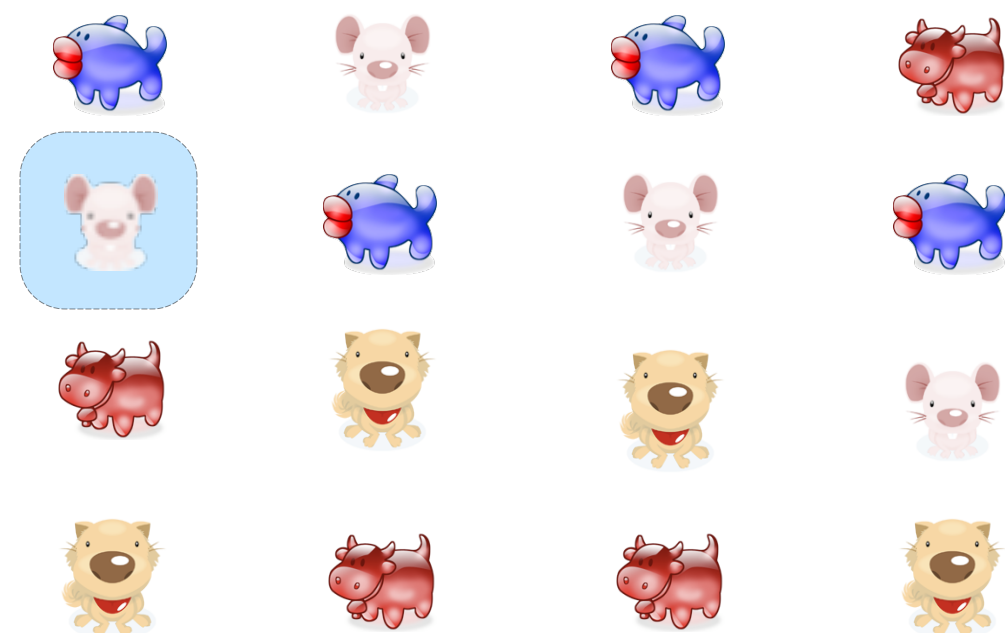
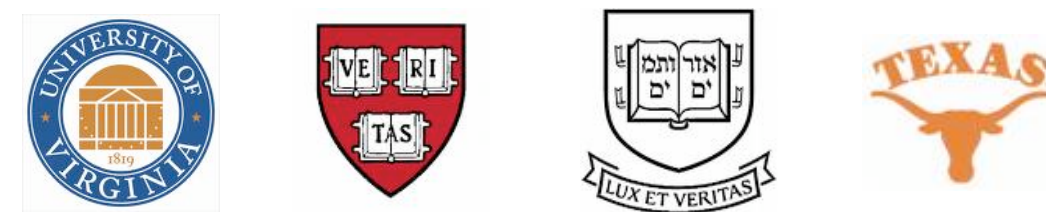
R



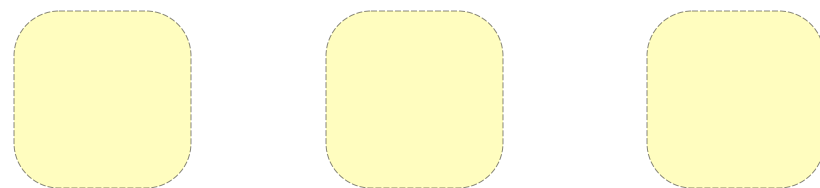
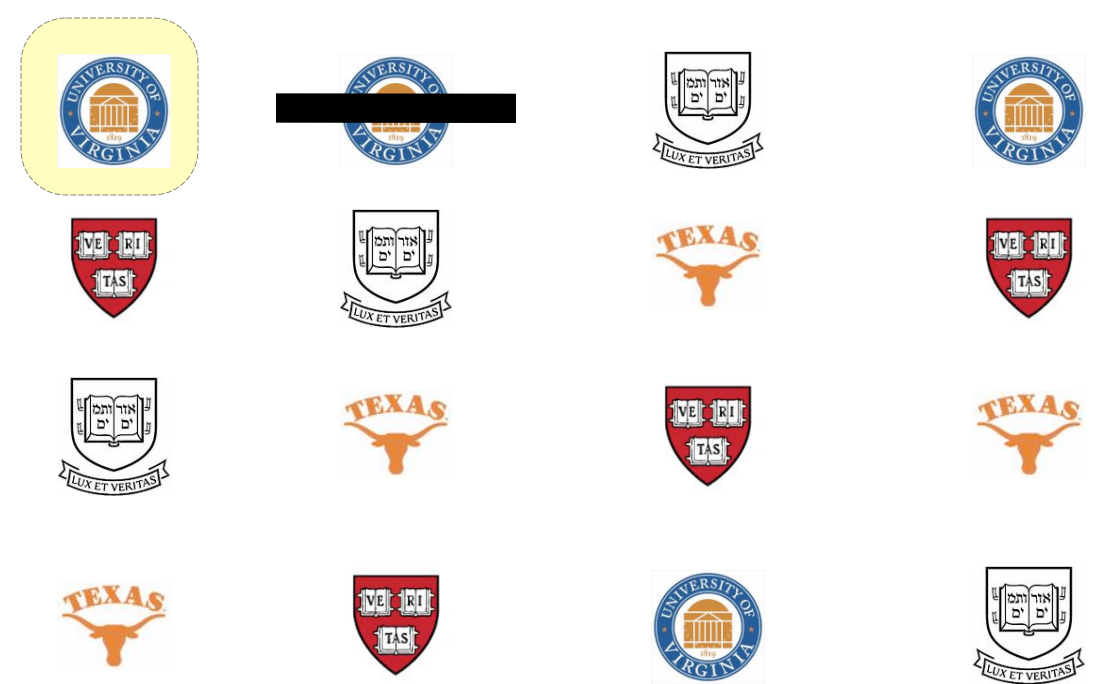
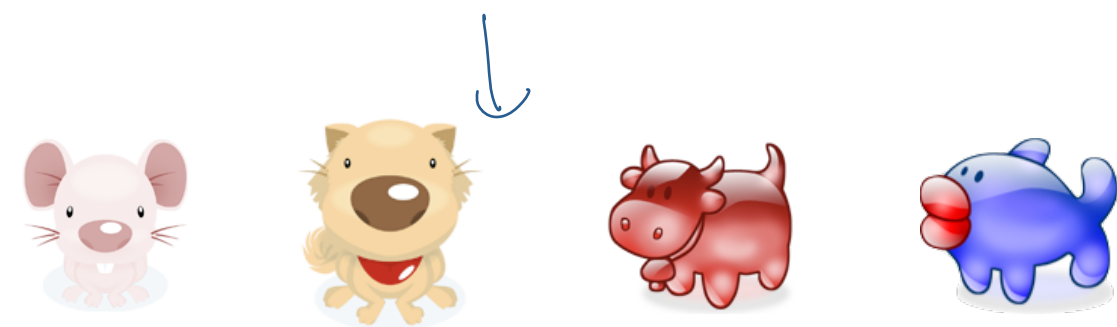
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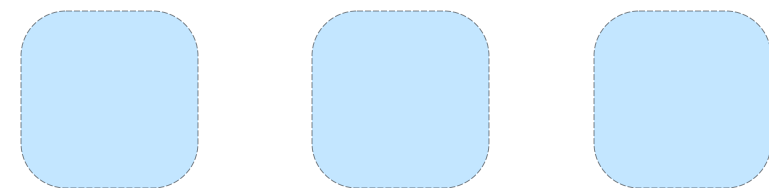
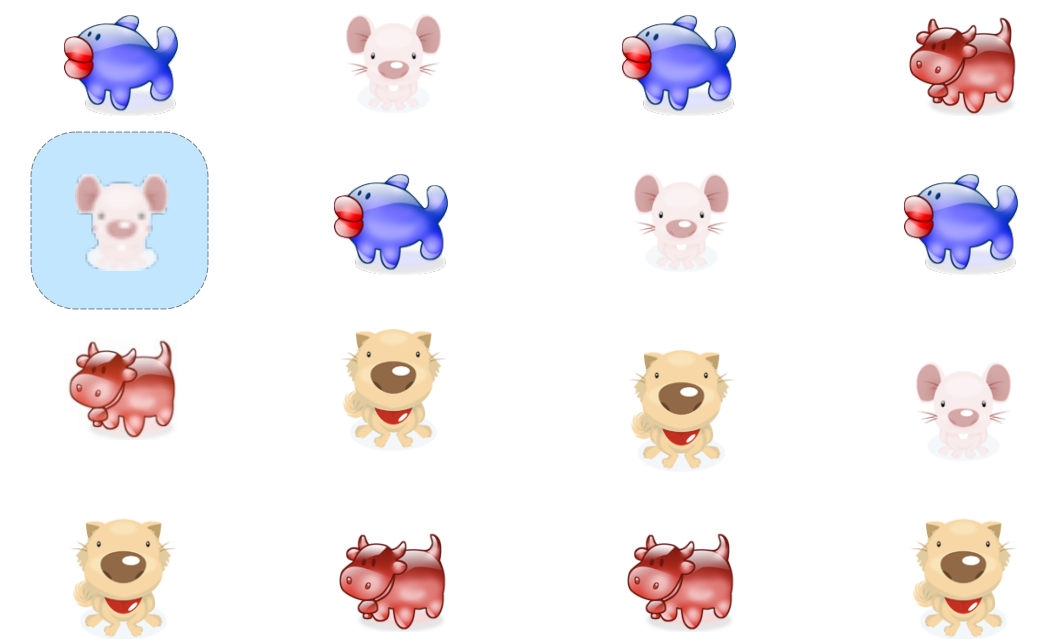
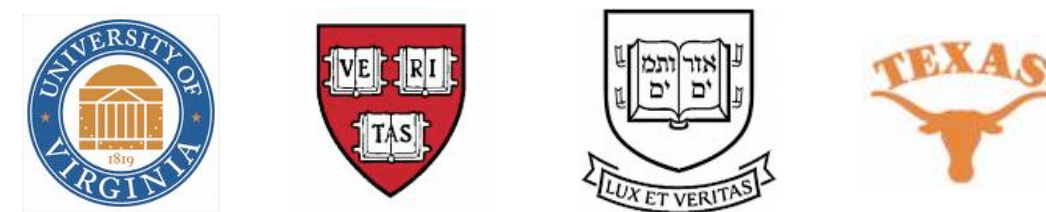
R



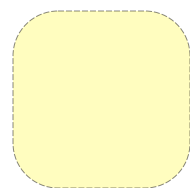
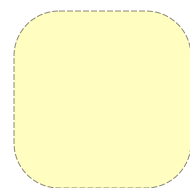
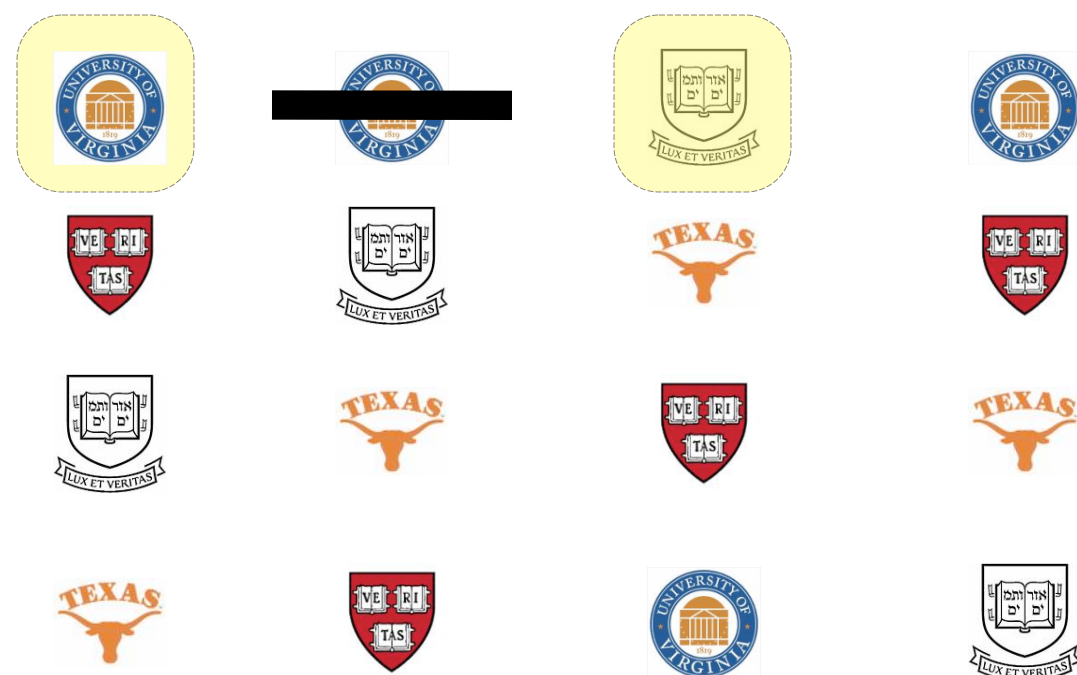
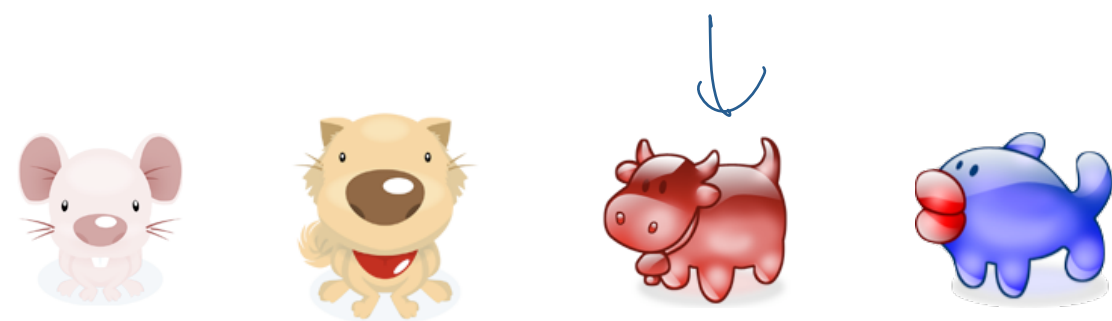
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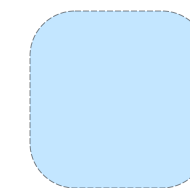
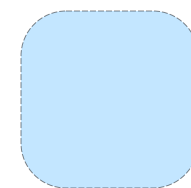
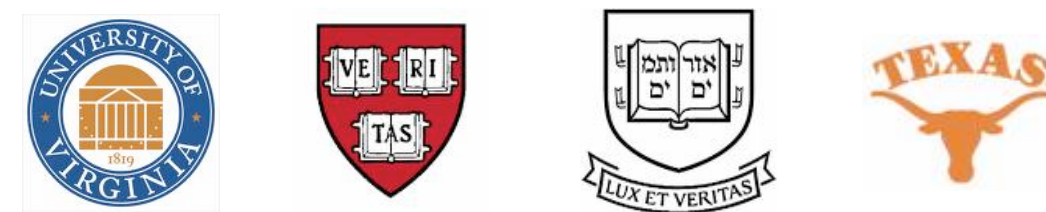
R



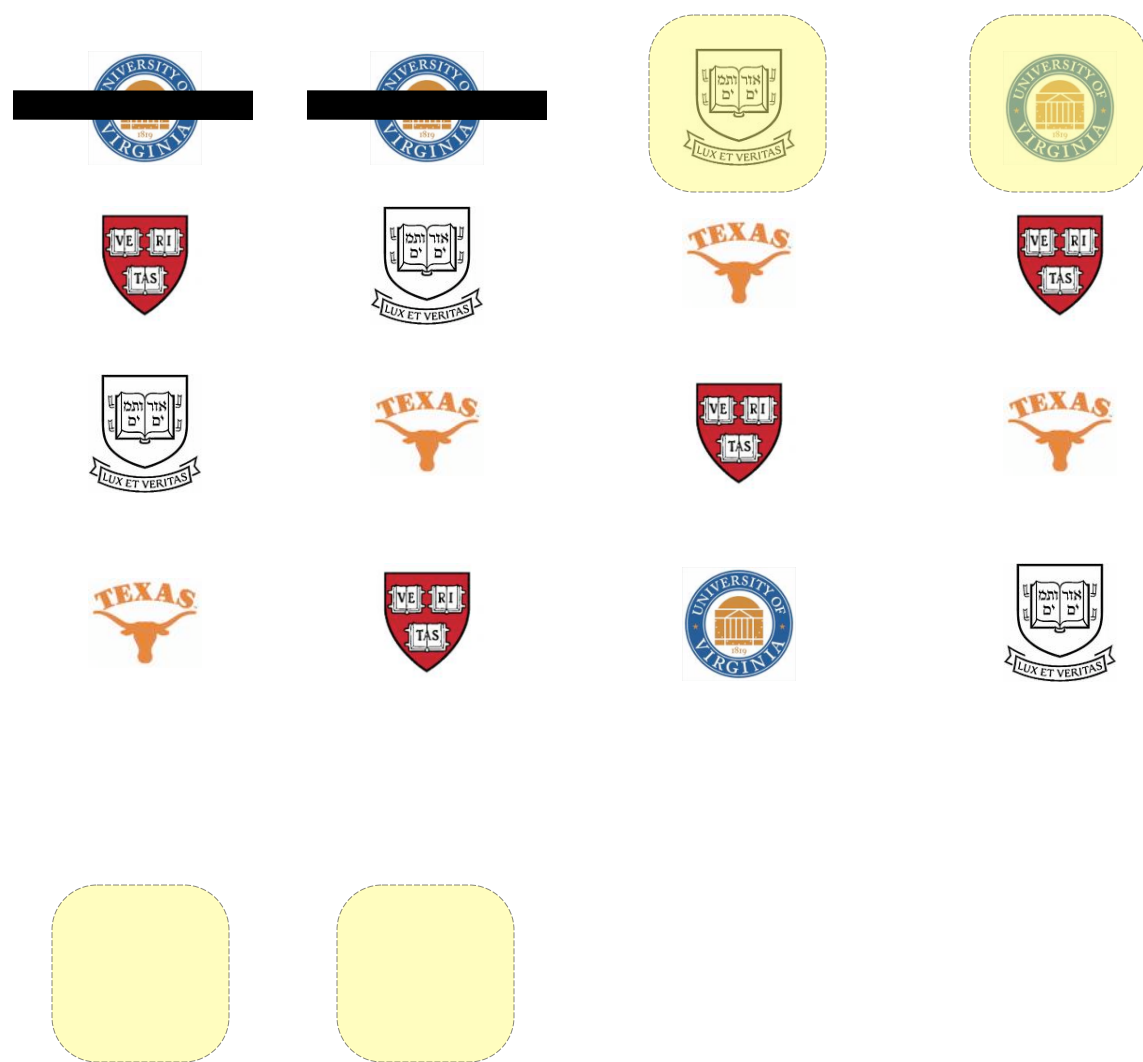
S



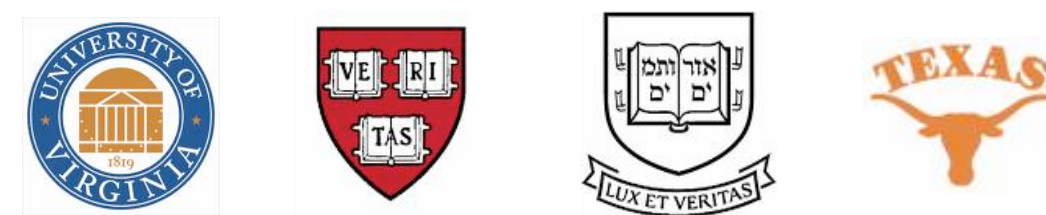
R



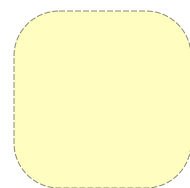
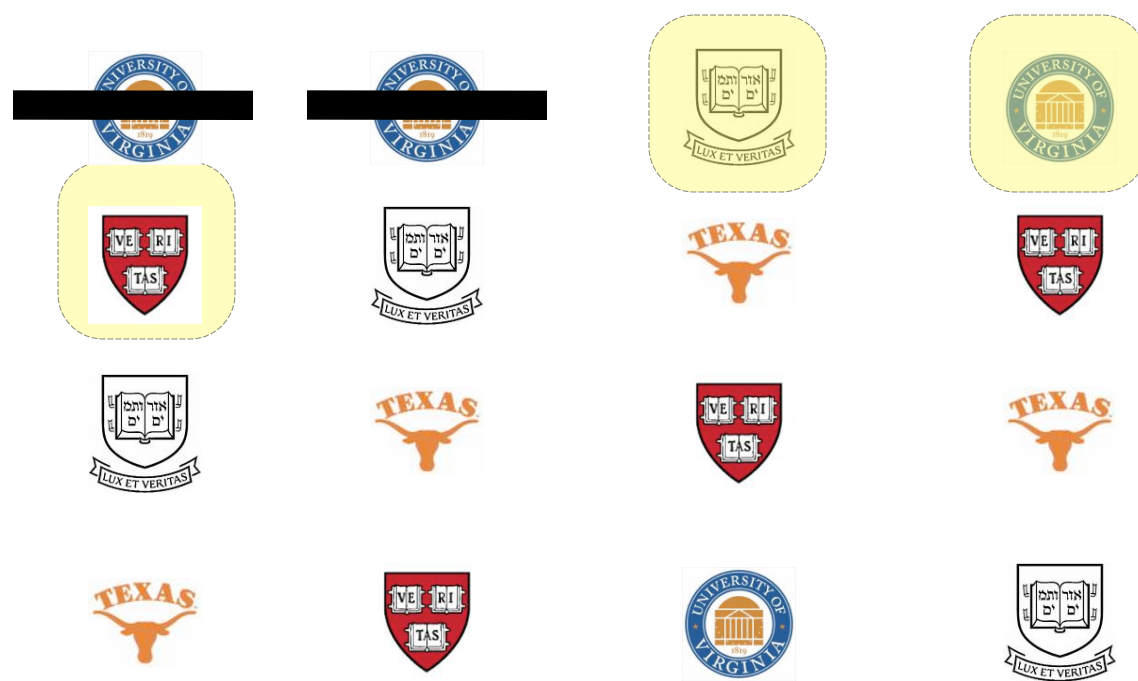
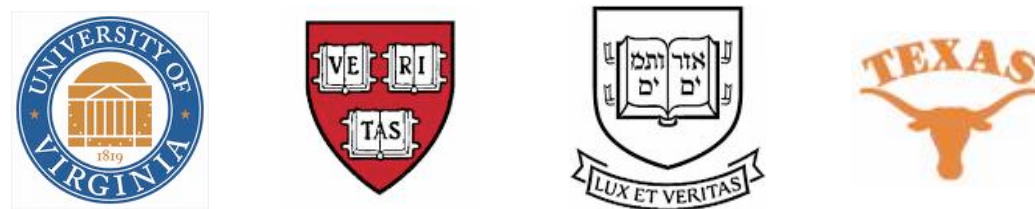
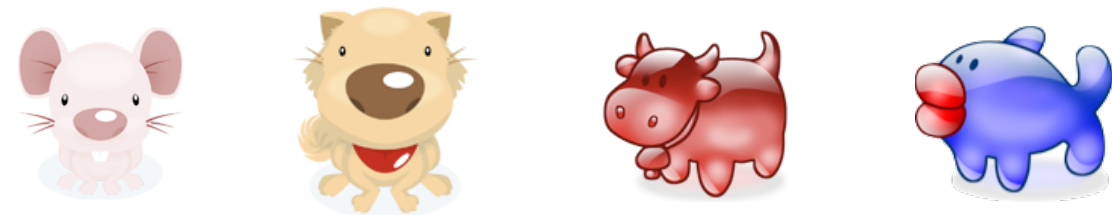
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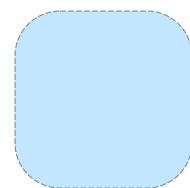
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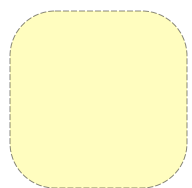
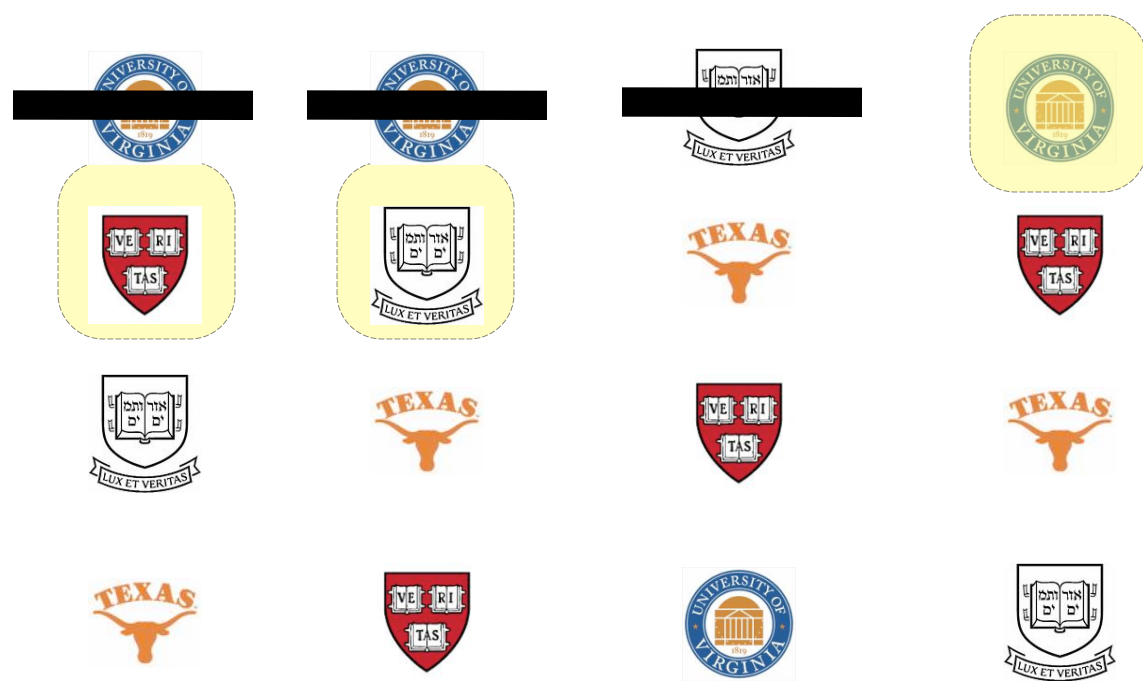
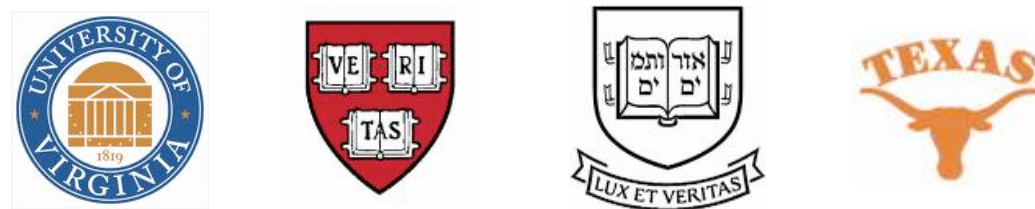
S



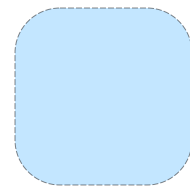
R



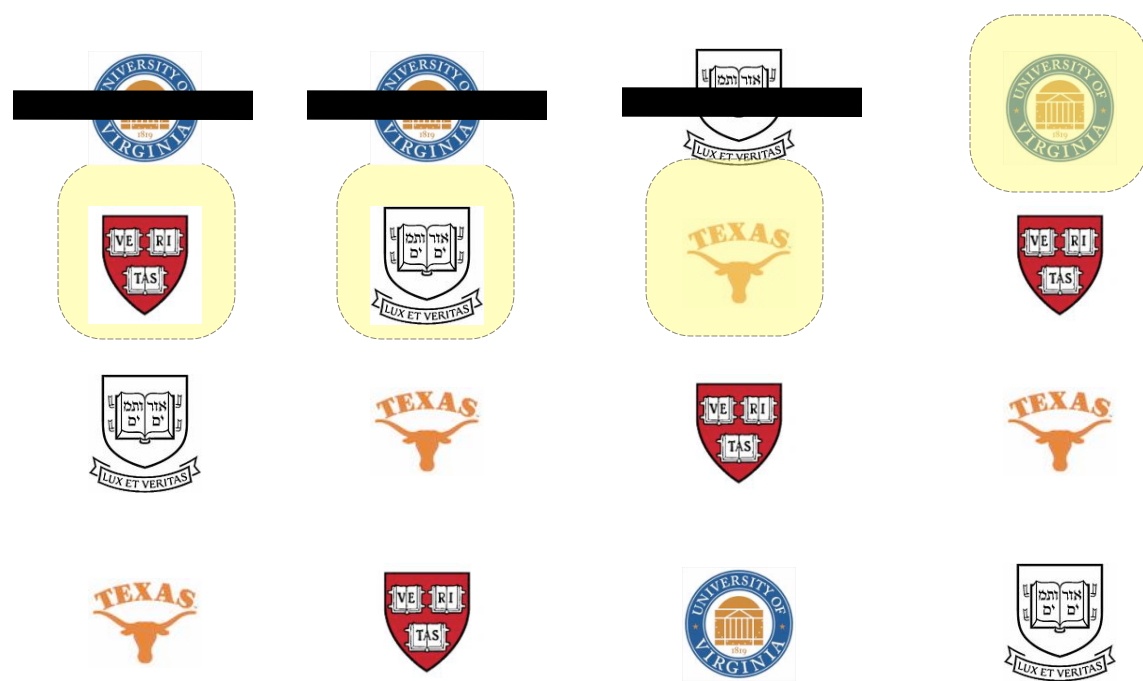
S



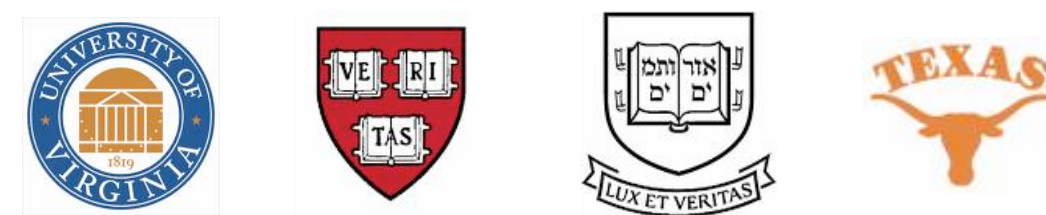
R



S



R



Proposal algorithm ends

Proposal algorithm ends

$O(n^2)$ steps

each m proposes at most once to each w .


each m proposes at most n times.

size of M is at most n .

output is a matching

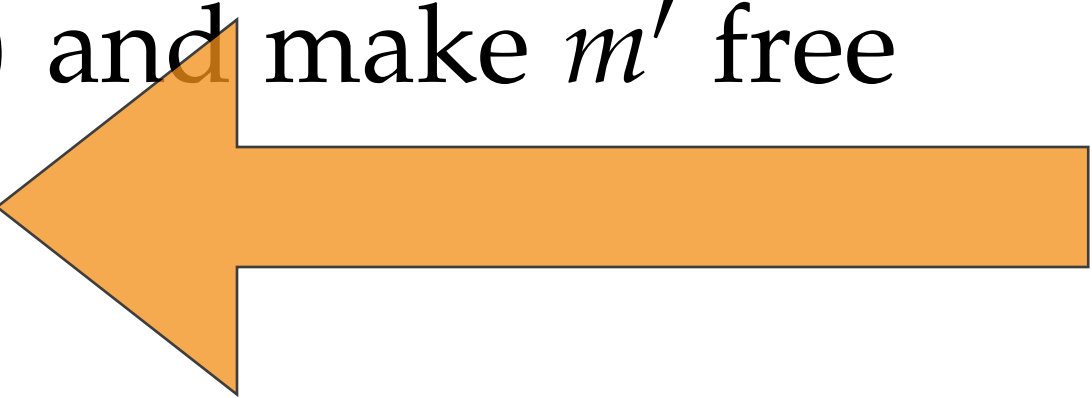
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STABLEMATCH(M, W, \prec_m, \prec_w)

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```



output is perfect

output is perfect

if $\exists m$ who is free, then

$\exists w$ who has not been
asked

output is stable

output is stable

spse not. $\exists (m^*, w), (m, w^*) \in S \quad w \prec_{m^*} w^* \quad m \prec_{w^*} m^*$

output is stable

spse not. $\exists(m^*, w), (m, w^*) \in S \quad w \prec_{m^*} w^* \quad m \prec_{w^*} m^*$

m^* last proposal was to w

but $w \prec_{m^*} w^*$ and so m^* must have already asked w^*

and must have been rejected by $m^* \prec_{w^*} m'$

then either $m' \prec_{w^*} m$ or $m'=m$

which contradicts assumption $m \prec_{w^*} m^*$

Proposer wins



Proposer wins



Remarkable theorem

w is valid for m :

$\text{best}(m)$:

GS is Suitor-optimal.

GS matching vs R-opt

S1



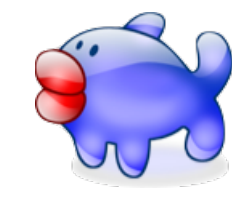
S2



S3



S4



R1



R2



R3



R4



S1

S1

S1

S1

S2

S2

S2

S2

S3

S3

S3

S3

S4

S4

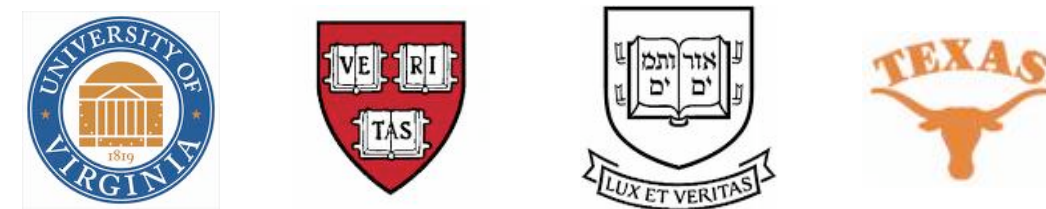
S4

S4

S1 S2 S3 S4



R1 R2 R3 R4



R1 R1 R1 R1

R2 R2 R2 R2

R3 R3 R3 R3

R4 R4 R4 R4

S1 S1 S1 S1

S2 S2 S2 S2

S3 S3 S3 S3

S4 S4 S4 S4

S1



S2



S3



S4



R1



R2



R3



R4



R1

R1

R1

R1

S1

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S1

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R3

R3

R3

R3

S3

S3

S3

S3

R4

R4

R4

R4

S4

S4

S4

S4

Not honest

S1



S2



S3



R1



R2



R3



R2

R1

R1

R1

R2

R3

R3

R3

R2

S1

S2

S2

S2

S1

S3

S3

S3

S1

Not honest

S1 S2 S3



R1 R2 R3



R2 R1 R1

R1 R2 R3

R3 R3 R2

S1 S2 S2

S2 S1 S3

S3 S3 S1

R2 R1 R1

R1 R2 R3

R3 R3 R2

S1 S2 S2

S3 S1 S3

S2 S3 S1

Not honest

S1 S2 S3



R1 R2 R3



R2 R1 R1

R1 R2 R3

R3 R3 R2

S1 S2 S2

S2 S1 S3

S3 S3 S1

R2 R1 R1

R1 R2 R3

R3 R3 R2

S1 S2 S2

S3 S1 S3

S2 S3 S1

THE MATCH[™]
NATIONAL RESIDENT MATCHING PROGRAM®

Guns and butter



$$\max x + y$$

$$4x - y \leq 8$$

$$2x + y \leq 10$$

$$5x - 2y \geq -2$$

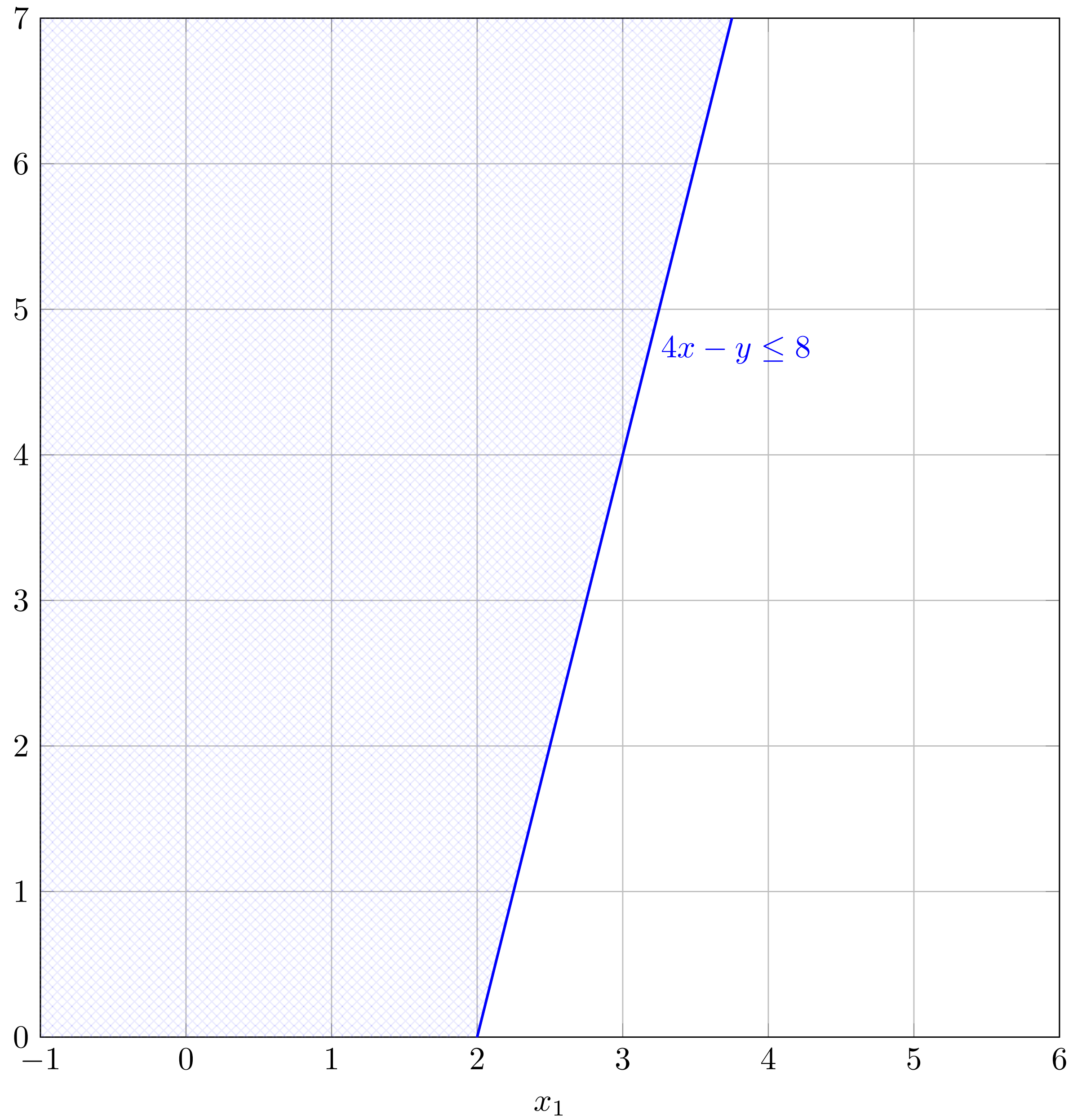
$$x, y \geq 0$$

http://i16.photobucket.com/albums/b20/safebuy/ak47/ak47-electric_lg.jpg

http://2.bp.blogspot.com/_NX4zcMNx4VE/Sb8MQffl1I/AAAAAAAAAL0/eu4J0dfFhJE/s400/gourmet-butter.jpg



y x_2



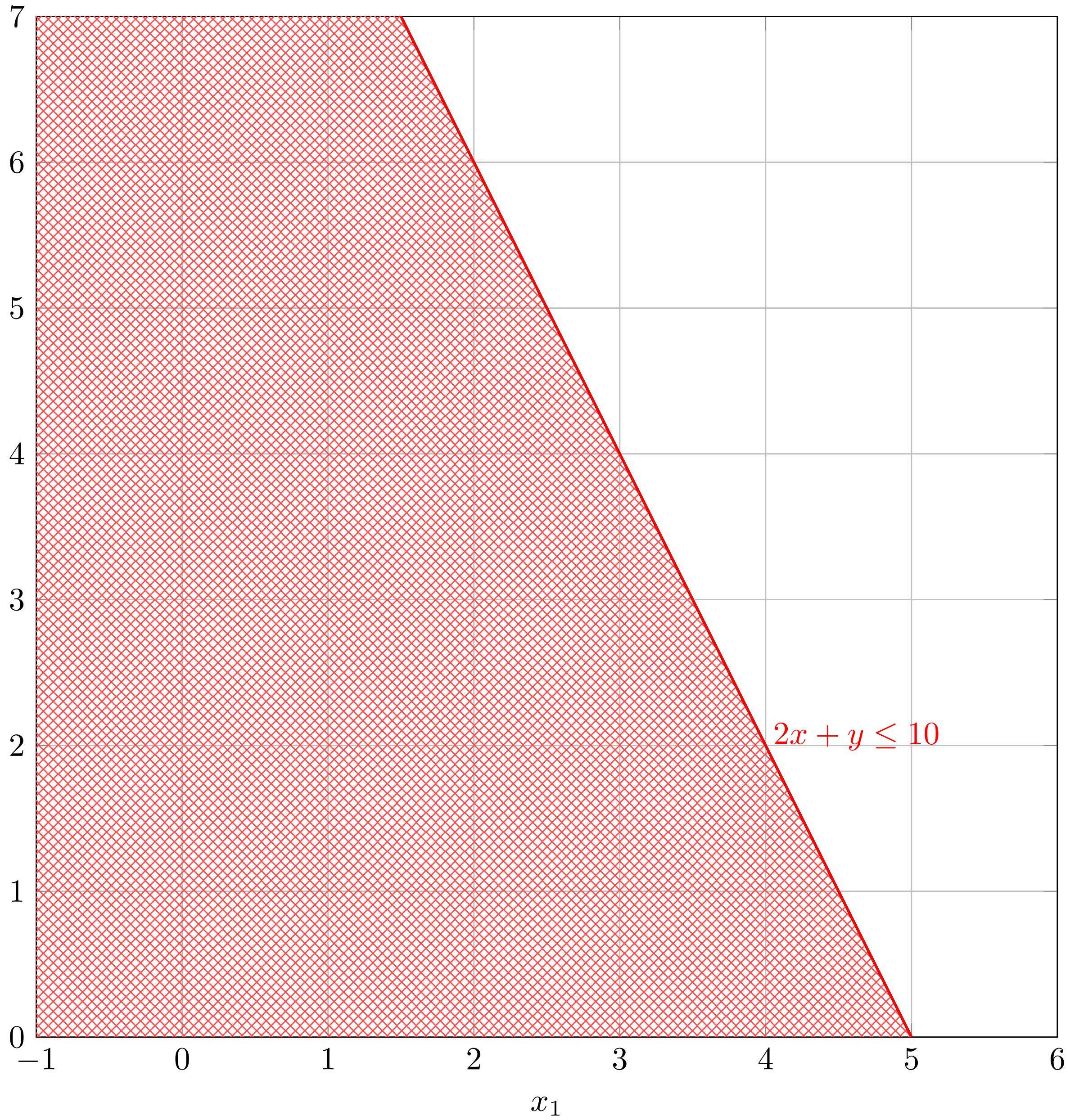
$$\begin{aligned} 4x - y &\leq 8 \\ 2x + y &\leq 10 \\ 5x - 2y &\geq -2 \\ x, y &\geq 0 \end{aligned}$$



x



y
 x_2



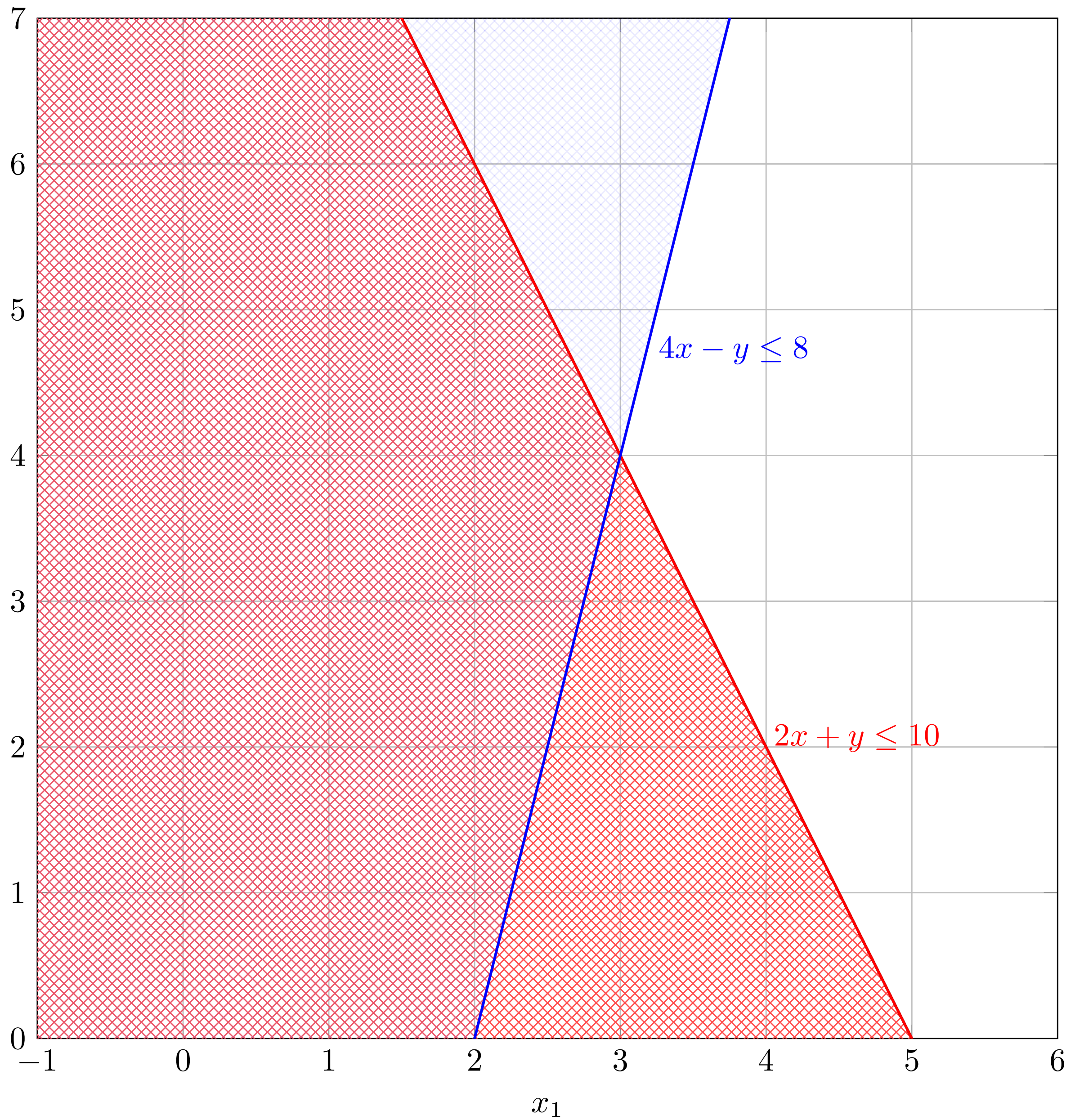
$$\begin{aligned} 4x - y &\leq 8 \\ 2x + y &\leq 10 \\ 5x - 2y &\geq -2 \\ x, y &\geq 0 \end{aligned}$$



x
 x_1



y
 x_2



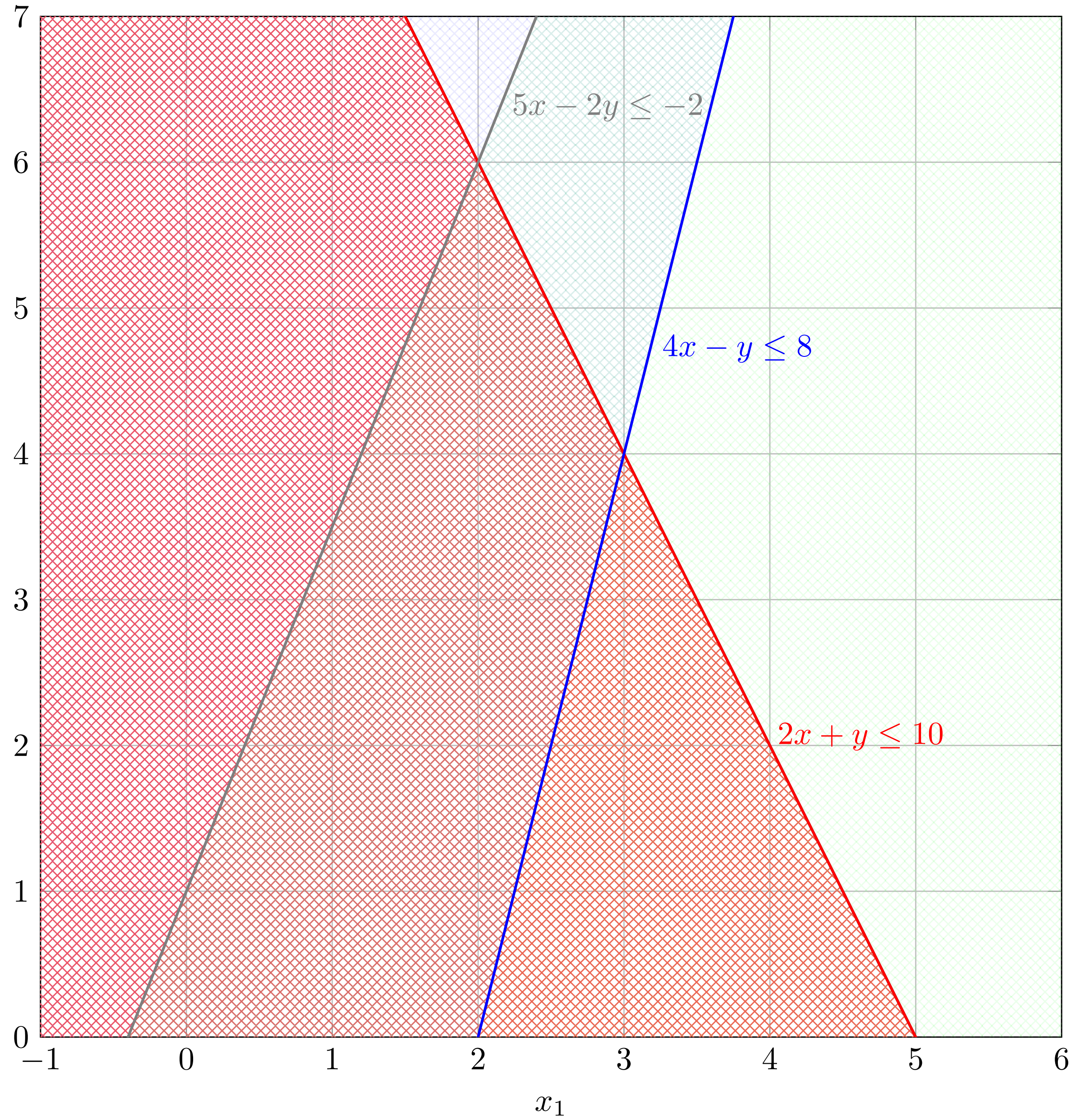
$$\begin{aligned} 4x - y &\leq 8 \\ 2x + y &\leq 10 \\ 5x - 2y &\geq -2 \\ x, y &\geq 0 \end{aligned}$$



x
 x_1



y
 x_2



x
 x_1

$$\begin{aligned} 4x - y &\leq 8 \\ 2x + y &\leq 10 \\ 5x - 2y &\geq -2 \\ x, y &\geq 0 \end{aligned}$$

Certificate of optimality

$$\max x + y$$

$$4x - y \leq 8$$

$$2x + y \leq 10$$

$$5x - 2y \geq -2$$

$$x, y \geq 0$$

Certificate of optimality

$$\max x + y$$

$$4x - y \leq 8$$

$$2x + y \leq 10$$

$$5x - 2y \geq -2$$

$$x, y \geq 0$$

7

-1

$$14x + 7y \leq 70$$

$$-5x + 2y \leq 2$$

$$9x + 9y \leq 72$$



BERLIN

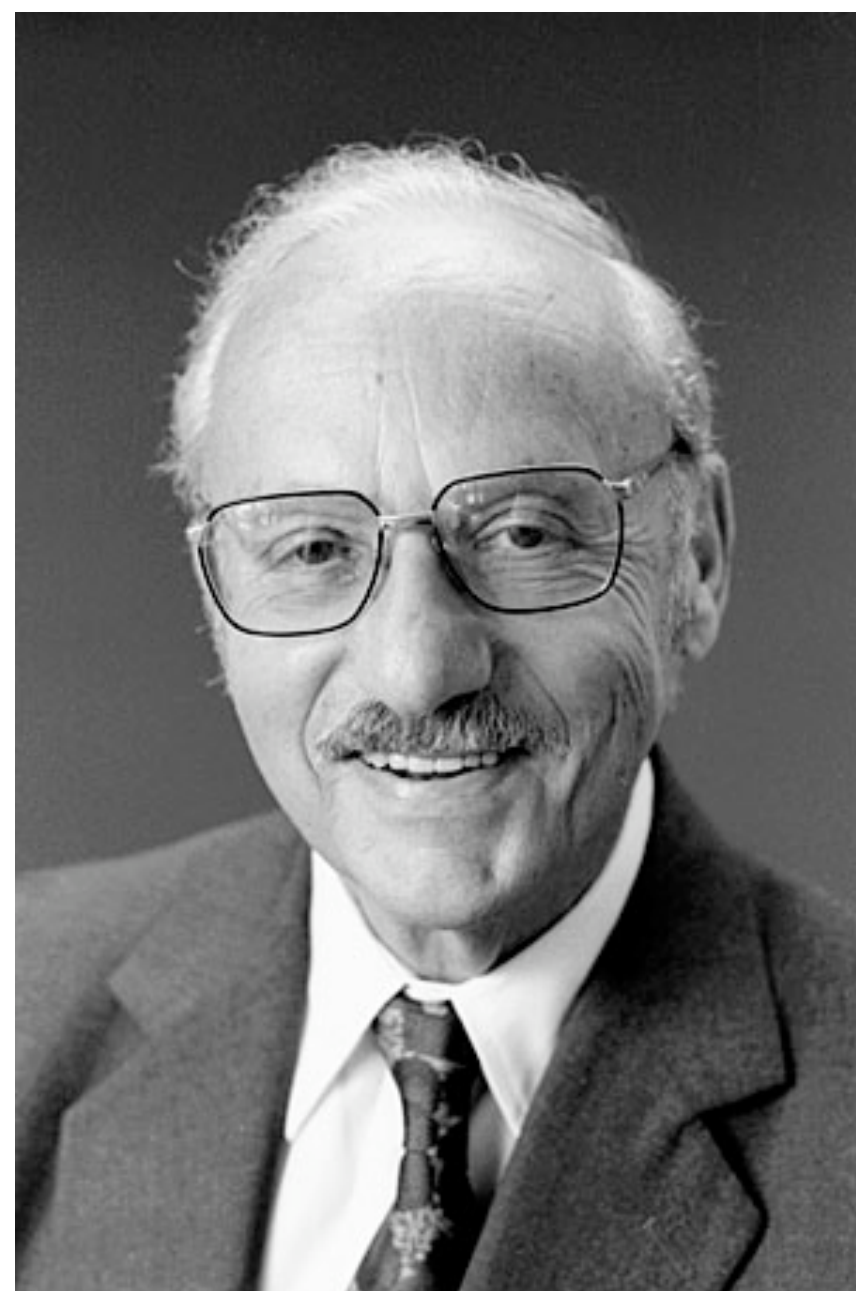


IMAGE:STAMFORD



IMAGE:HISTORY OF AIR CARGO

linear programming
saved Berlin

Stigler diet

CALORIES	3000
PROTEIN	70g
CALCIUM	.8g
IRON	19mg
VITAMIN A	5000iu
THIAMINE	1.8mg
RIBOFLAVIN	2.7mg
NIACIN	18mg
ASCORBIC ACID	75mg

TABLE A. NUTRITIVE VALUES OF COMMON FOODS PER DOLLAR OF EXPENDITURE, AUGUST 15, 1939

Commodity	Unit	Price Aug. 15, 1939 (cents)	Edible Weight per \$1.00 (grams)	Calories (1,000)	Protein (grams)	Calcium (grams)	Iron (mg.)	Vitamin A (1,000 I.U.)	Thiamine (mg.)	Ribo- flavin (mg.)	Niacin (mg.)	Ascorbic Acid (mg.)
**1. Wheat Flour (Enriched)	10 lb.	36.0	12,600	44.7	1,411	2.0	365		55.4	33.3	441	
2. Macaroni	1 lb.	14.1	3,217	11.6	418	.7	54		3.2	1.9	68	
3. Wheat Cereal (Enriched)	28 oz.	24.2	3,280	11.8	377	14.4	175		14.4	8.8	114	
4. Corn Flakes	8 oz.	7.1	3,104	11.4	252	.1	56		15.5	2.3	68	
5. Corn Meal	1 lb.	4.6	9,861	36.0	897	1.7	99	30.0	17.4	7.9	106	
6. Hominy Grits	24 oz.	8.5	8,065	28.6	680	.8	90		10.6	1.6	110	
7. Rice	1 lb.	7.5	6,048	21.2	400	.6	41		2.0	4.8	60	
8. Rolled Oats	1 lb.	7.1	6,389	25.3	907	5.1	341		37.1	8.9	64	
9. White Bread (Enriched)	1 lb.	7.9	5,742	15.6	488	2.5	115		15.8	8.5	126	
10. Whole Wheat Bread	1 lb.	9.1	4,985	12.2	464	2.7	125		15.9	6.4	100	
11. Rye Bread	1 lb.	9.2	4,930	12.4	439	1.1	82		9.9	3.0	66	
12. Pound Cake	1 lb.	24.8	1,829	8.0	130	.4	31	18.9	2.8	3.0	17	
13. Soda Crackers	1 lb.	15.1	3,004	12.5	288	.5	50					
14. Milk	1 qt.	11.0	8,807	6.1	310	10.5	18	16.8	4.0	16.0	7	177
**15. Evaporated Milk (can)	14½ oz.	6.7	6,085	8.4	422	15.1	9	20.0	3.0	23.5	11	60
16. Butter	1 lb.	30.8	1,478	10.8	9	.2	3	44.2		.2	2	
*17. Oleomargarine	1 lb.	16.1	2,817	20.6	17	.6	6	55.8	.2			
18. Eggs	1 doz.	32.6	1,857	2.9	238	1.0	52	18.6	2.8	6.6	1	
**19. Cheese (Cheddar)	1 lb.	24.2	1,874	7.4	448	16.4	19	28.1	.8	10.3	4	
20. Cream	½ pt.	14.1	1,689	3.5	49	1.7	3	15.9	.6	2.5		17
21. Peanut Butter	1 lb.	17.9	2,534	15.7	661	1.0	48		9.8	3.1	471	
22. Mayonnaise	½ pt.	16.7	1,198	8.6	18	.2	8	2.7	.4	.5		
23. Crisco	1 lb.	20.3	2,254	20.1								
24. Lard	1 lb.	9.8	4,628	41.7				.2		.5	5	
25. Sirloin Steak	1 lb.	39.6	1,145*	2.9	166	.1	54	.2	2.1	2.9	69	
26. Round Steak	1 lb.	36.4	1,246*	2.2	214	.1	52	.4	2.5	2.4	87	
27. Rib Roast	1 lb.	29.2	1,553*	3.4	213	.1	33			2.0		
28. Chuck Roast	1 lb.	22.6	2,007*	3.6	309	.2	46	.4	1.0	4.0	120	
29. Plate	1 lb.	14.6	3,107*	8.5	494	.2	62		.9			
**30. Liver (Beef)	1 lb.	20.8	1,692*	2.2	333	.2	130	169.2	6.4	50.8	316	525
31. Leg of Lamb	1 lb.	27.6	1,643*	3.1	245	.1	20		2.8	3.9	86	
32. Lamb Chops (Rib)	1 lb.	36.6	1,239*	3.3	140	.1	15		1.7	2.7	54	
33. Pork Chops	1 lb.	30.7	1,477*	3.5	196	.2	30		17.4	2.7	60	
34. Pork Loin Roast	1 lb.	24.2	1,874*	4.4	240	.3	37		18.2	3.0	79	
35. Bacon	1 lb.	25.0	1,772*	10.4	152	.2	23		1.8	1.8	71	
36. Ham—smoked	1 lb.	27.4	1,655*	6.7	212	.2	31		9.9	3.3	50	
37. Salt Pork	1 lb.	16.0	2,835*	18.8	164	.1	20		1.4	1.8		
38. Roasting Chicken	1 lb.	30.3	1,497*	1.8	184	.1	30	.1	.9	1.8	68	46
39. Veal Cutlets	1 lb.	42.3	1,072*	1.7	156	.1	24		1.4	2.4	57	
40. Salmon, Pink (can)	16 oz.	15.0	3,439	5.8	705	6.6	45	3.5	1.0	4.9	209	
41. Apples	1 lb.	4.4	9,072	5.8	27	.5	36	7.3	3.6	2.7	5	544
42. Bananas	1 lb.	6.1	4,982	4.9	60	.4	30	17.4	2.5	3.5	28	438
43. Lemons	1 doz.	26.0	2,389	1.0	21	.6	14		.5		4	952
44. Oranges	1 doz.	30.9	4,439	2.2	40	1.1	18	11.1	3.6	1.3	10	1,998
*45. Green Beans	1 lb.	7.1	5,750	2.4	138	3.7	60	69.0	4.3	5.8	37	862
**46. Cabbage	1 lb.	3.7	8,949	2.6	125	4.0	36	7.2	9.0	4.5	26	5,369
47. Carrots	1 bunch	4.7	6,080	2.7	73	2.8	43	188.5	6.1	4.3	89	608
48. Celery	1 stalk	7.3	3,913	.9	51	3.0	23	.9	1.4	1.4	9	313
49. Lettuce	1 head	8.2	2,247	.4	27	1.1	22	112.4	1.8	3.4	11	449
*50. Onions	1 lb.	3.6	11,844	5.8	166	3.8	59	16.6	4.7	5.9	21	1,184

*51. Potatoes	15 lb.	34.0	16,810	14.8	336	1.8	118	6.7	29.4	7.1	108	2,522
**52. Spinach	1 lb.	8.1	4,592	1.1	100	—	138	918.4	5.7	13.8	53	2,755
**53. Sweet Potatoes	1 lb.	5.1	7,649	9.6	138	2.7	54	200.7	8.4	6.4	85	1,912
54. Peaches (can)	No. 2 $\frac{1}{2}$	16.8	4,894	3.7	20	.4	10	21.5	.5	1.0	31	196
55. Pears (can)	No. 2 $\frac{1}{2}$	20.4	4,030	3.0	8	.8	8	.8	.8	.8	5	81
56. Pineapple (can)	No. 2 $\frac{1}{2}$	21.3	3,993	2.4	16	.4	8	2.0	2.8	.8	7	399
57. Asparagus (can)	No. 2	27.7	1,945	.4	33	.3	12	16.3	1.4	2.1	17	272
58. Green Beans (can)	No. 2	10.0	3,386	1.0	54	2.0	65	53.0	1.6	4.3	32	451
59. Pork and Beans (can)	16 oz.	7.1	6,389	7.5	364	4.0	134	3.5	8.3	7.7	56	—
60. Corn (can)	No. 2	10.4	3,452	5.2	136	.2	16	12.0	1.6	2.7	42	218
61. Peas (can)	No. 2	13.8	4,109	2.3	136	.6	45	34.9	4.9	2.5	37	370
62. Tomatoes (can)	No. 2	8.6	6,263	1.3	63	.7	38	53.2	3.4	2.5	36	1,253
63. Tomato Soup (can)	10 $\frac{1}{2}$ oz.	7.6	3,917	1.6	71	.6	43	37.9	3.5	2.4	67	302
*64. Peaches, Dried	1 lb.	15.7	2,889	8.5	87	1.7	173	36.9	1.2	4.3	55	57
*65. Prunes, Dried	1 lb.	9.0	4,284	12.8	99	2.5	154	35.7	3.9	4.3	65	237
66. Raisins, Dried	15 oz.	9.4	4,524	13.5	104	2.3	136	4.3	6.3	1.4	24	136
67. Peas, Dried	1 lb.	7.9	5,742	20.0	1,367	4.2	345	2.0	28.7	18.4	162	—
**68. Lima Beans, Dried	1 lb.	8.9	5,097	17.4	1,055	3.7	459	3.1	26.9	38.2	93	—
**69. Navy Beans, Dried	1 lb.	8.9	7,688	26.9	1,691	11.4	792	—	33.4	24.6	217	—
70. Coffee	1 lb.	22.4	2,025	—	—	—	—	—	4.0	5.1	50	—
71. Tea	$\frac{1}{2}$ lb.	17.4	632	—	—	—	—	—	—	2.3	42	—
72. Cocoa	8 oz.	8.6	2,657	8.7	237	3.0	72	—	2.0	11.9	40	—
73. Chocolate	8 oz.	10.2	1,400	8.0	77	1.3	30	—	.9	3.4	14	—
74. Sugar	10 lb.	51.7	8,773	34.9	—	—	—	—	—	—	—	—
75. Corn Syrup	24 oz.	13.7	4,968	14.7	—	.5	74	—	—	—	5	—
76. Molasses	18 oz.	13.6	3,732	9.0	—	10.3	244	—	1.9	7.5	146	—
77. Strawberry Preserves	1 lb.	20.5	2,213	6.4	11	.4	7	.2	.2	.4	3	—

* Quantities including inedible portions.

TABLE B. NUTRITIVE VALUES OF COMMON FOODS PER DOLLAR OF EXPENDITURE, AUGUST 15, 1944

Commodity	Price Aug. 15, 1944 (cents)	Calories (1,000)	Protein (grams)	Calcium (grams)	Iron (mg.)	Vitamin A (1,000 I.U.)	Thiamine (mg.)	Riboflavin (mg.)	Niacin (mg.)	Ascorbic Acid (mg.)
1. Wheat Flour	64.6	24.9	756	1.1	203	—	30.9	18.6	246	—
3. Wheat Cereal	23.2	12.3	398	15.0	183	—	15.0	9.2	119	—
5. Corn Meal	6.3	20.3	655	1.2	72	22.6	12.7	5.8	77	—
8. Rolled Oats	9.9	18.1	651	3.7	245	—	28.6	6.4	48	—
15. Evaporated Milk	10.0	5.6	233	10.1	6	17.4	2.9	15.7	7	40
46. Cabbage	4.0	2.0	94	3.0	27	5.4	6.8	3.4	20	4,034
51. Potatoes	60.1	6.1	143	.8	50	2.8	12.5	3.0	84	1,071
52. Spinach	11.6	.8	74	—	96	641.3	4.0	9.6	23	1,924
53. Sweet Potatoes	12.3	4.0	37	1.1	22	120.5	3.5	2.2	34	793
69. Navy Beans	10.8	14.7	924	0.2	433	—	21.0	13.4	119	—
74. Sugar	67.0	26.9	—	—	—	—	—	—	—	—
78. Pancake Flour ¹	12.2	16.0	479	19.1	40	—	3.7	1.9	41	—
79. Beets ²	7.3	2.2	85	1.1	70	132.3	2.9	6.3	29	895
80. Liver (Pork) ³	21.9	2.7	408	.2	518	145.0	10.4	51.8	472	530

¹ Unit: 20 oz.; edible weight: 4,647 g.² Unit: 1 bunch; edible weight: 4,971 g.³ Unit: 1 lb.; edible weight: 2,071 g.

	Brownie	Dumpling	Espresso	Amelia
cost	5	2	3	8
cals	400	200	150	500
choc	3	2	0	0
sugar	2	2	4	4
fat	2	4	0	5

requirements: 500 calories, 6 oz choc, 10 oz sugar, 8 oz fat

	Brownie	Dumpling	Espresso	Amelia
cost	5	2	3	8
cals	400	200	150	500
choc	3	2	0	0
sugar	2	2	4	4
fat	2	4	0	5

requirements: 500 calories, 6 oz choc, 10 oz sugar, 8 oz fat

	Brownie	Dumpling	Espresso	Amelia
cost	5	2	3	8
cals	400	200	150	500
choc	3	2	0	0
sugar	2	2	4	4
fat	2	4	0	5

requirements: 500 calories, 6 oz choc, 10 oz sugar, 8 oz fat

$$\min 5x_1 + 2x_2 + 3x_3 + 8x_4$$

$$\begin{bmatrix} 400 & 200 & 150 & 500 \\ 3 & 2 & 0 & 0 \\ 2 & 2 & 4 & 4 \\ 2 & 4 & 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \geq \begin{bmatrix} 500 \\ 6 \\ 10 \\ 8 \end{bmatrix}$$

$$\min 5x_1 + 2x_2 + 3x_3 + 8x_4$$

$$\begin{bmatrix} 400 & 200 & 150 & 500 \\ 3 & 2 & 0 & 0 \\ 2 & 2 & 4 & 4 \\ 2 & 4 & 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \geq \begin{bmatrix} 500 \\ 6 \\ 10 \\ 8 \end{bmatrix}$$

$$\min 5x_1 + 2x_2 + 3x_3 + 8x_4$$

$$\begin{bmatrix} 400 & 200 & 150 & 500 \\ 3 & 2 & 0 & 0 \\ 2 & 2 & 4 & 4 \\ 2 & 4 & 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \geq \begin{bmatrix} 500 \\ 6 \\ 10 \\ 8 \end{bmatrix}$$

H-representation

begin

8 4 rational

-500 400 200 150 500

-6 3 2 0 0

-10 2 2 4 4

-6 2 4 0 5

0 1 0 0 0

0 0 1 0 0

0 0 0 1 0

0 0 0 0 1

end

minimize

0 5 2 3 8

$$\min 5x_1 + 2x_2 + 3x_3 + 8x_4$$

$$\begin{bmatrix} 400 & 200 & 150 & 500 \\ 3 & 2 & 0 & 0 \\ 2 & 2 & 4 & 4 \\ 2 & 4 & 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \geq \begin{bmatrix} 500 \\ 6 \\ 10 \\ 8 \end{bmatrix}$$

$$\min 5x_1 + 2x_2 + 3x_3 + 8x_4$$

$$\begin{bmatrix} 400 & 200 & 150 & 500 \\ 3 & 2 & 0 & 0 \\ 2 & 2 & 4 & 4 \\ 2 & 4 & 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \geq \begin{bmatrix} 500 \\ 6 \\ 10 \\ 8 \end{bmatrix}$$

H-representation

begin

8 4 rational

-500 400 200 150 500

-6 3 2 0 0

-10 2 2 4 4

-6 2 4 0 5

0 1 0 0 0

0 0 1 0 0

0 0 0 1 0

0 0 0 0 1

end

minimize

0 5 2 3 8

$$\min 5x_1 + 2x_2 + 3x_3 + 8x_4$$

$$\begin{bmatrix} 400 & 200 & 150 & 500 \\ 3 & 2 & 0 & 0 \\ 2 & 2 & 4 & 4 \\ 2 & 4 & 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \geq \begin{bmatrix} 500 \\ 6 \\ 10 \\ 8 \end{bmatrix}$$

*Objective function is

$$0 + 5 X[1] + 2 X[2] + 3 X[3] + 8 X[4]$$

*LP status: a dual pair (x, y) of optimal solutions found.

H-representation

begin

8 4 rational

```
-500 400 200 150 500
-6   3   2   0   0
-10  2   2   4   4
-6   2   4   0   5
0    1   0   0   0
0    0   1   0   0
0    0   0   1   0
0    0   0   0   1
```

end

minimize

```
0 5 2 3 8
```

begin

primal_solution

1 : 0

2 : 3

3 : 1

4 : 0

dual_solution

2 : -1/4

5 : -11/4

3 : -3/4

8 : -5

optimal_value : 9

end

*number of pivot operations = 4

shortest paths as LP

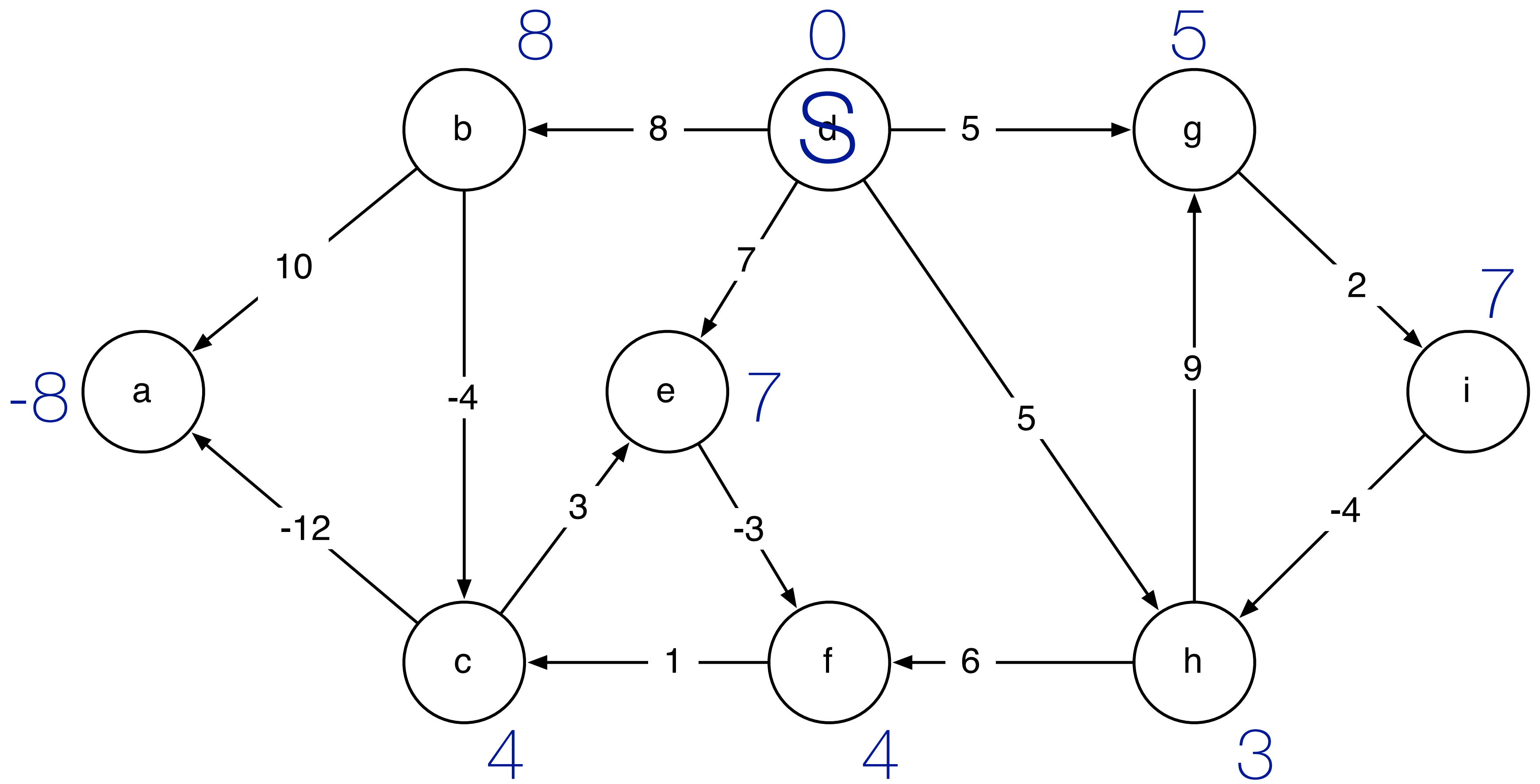
inputs:

shortest paths as LP

$$\max d_t$$

$$d_y - d_x \leq l(x, y) \quad \forall e = (x, y) \in E$$

$$d_s = 0$$



$\max d_t$

$$d_y - d_x \leq l(x, y) \quad \forall e = (x, y) \in E$$

$$d_s = 0$$

$$dt = 30$$

max flow as lp

INPUT: (G, c, s, t) $G = (V, E)$ $c : E \rightarrow \mathbb{Z}_+$

max flow as lp

$$\max \sum_v f(s, v) - \sum_v f(v, s)$$

$$f(u, v) \leq c(u, v) \quad \text{FOR } (u, v) \text{ IN } E$$

$$\sum_u f(u, v) = \sum_w f(v, w) \quad \forall v$$

$$f(u, v) \geq 0 \quad \text{FOR } (u, v) \text{ IN } E$$

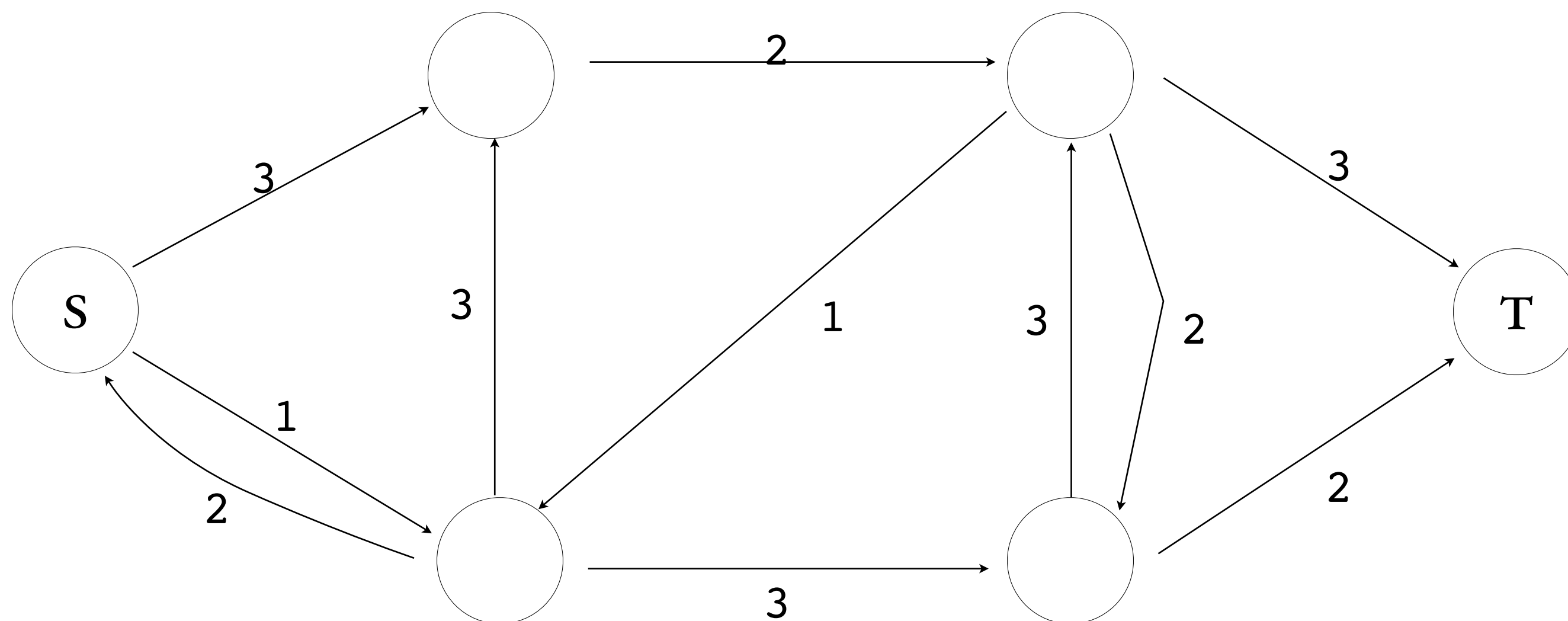
max flow as lp

$$\max \sum_v f(s, v) - \sum_v f(v, s)$$

$$f(u, v) \leq c(u, v) \quad \text{FOR } (u, v) \text{ IN } E$$

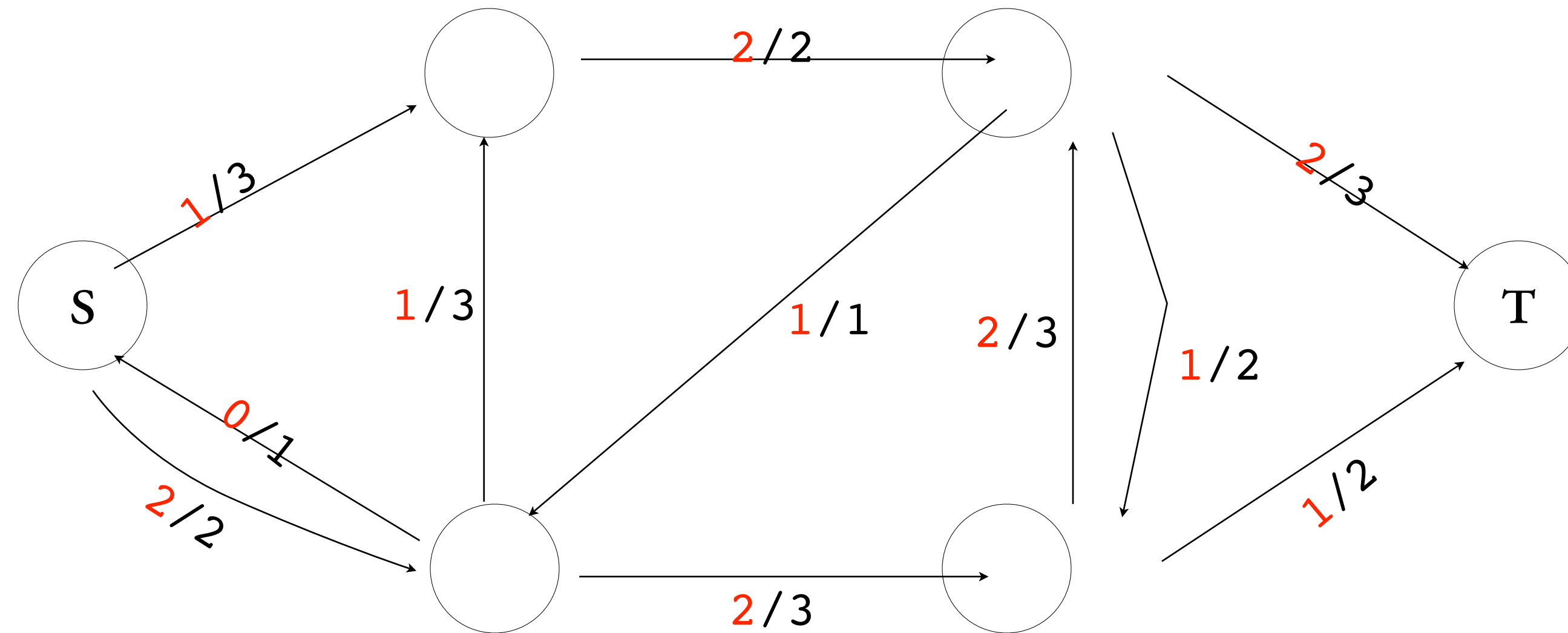
$$\sum_u f(u, v) = \sum_w f(v, w) \quad \forall v$$

$$f(u, v) \geq 0 \quad \text{FOR } (u, v) \text{ IN } E$$



min-cost flow as lp

INPUT: (G, c, s, t) $G = (V, E)$ $c : E \rightarrow \mathbb{Z}_+$ $x : E \rightarrow \mathbb{Z}_+$ d



min-cost flow as lp

min-cost flow as lp

$$\min_e x_e \cdot f(e)$$

$$f(e) \leq c(e)$$

$$f(e) \geq 0$$

$$\sum_u f(u, v) = \sum_w f(v, w)$$

$$\sum_v f(s, v) - \sum_v f(v, s) = d$$