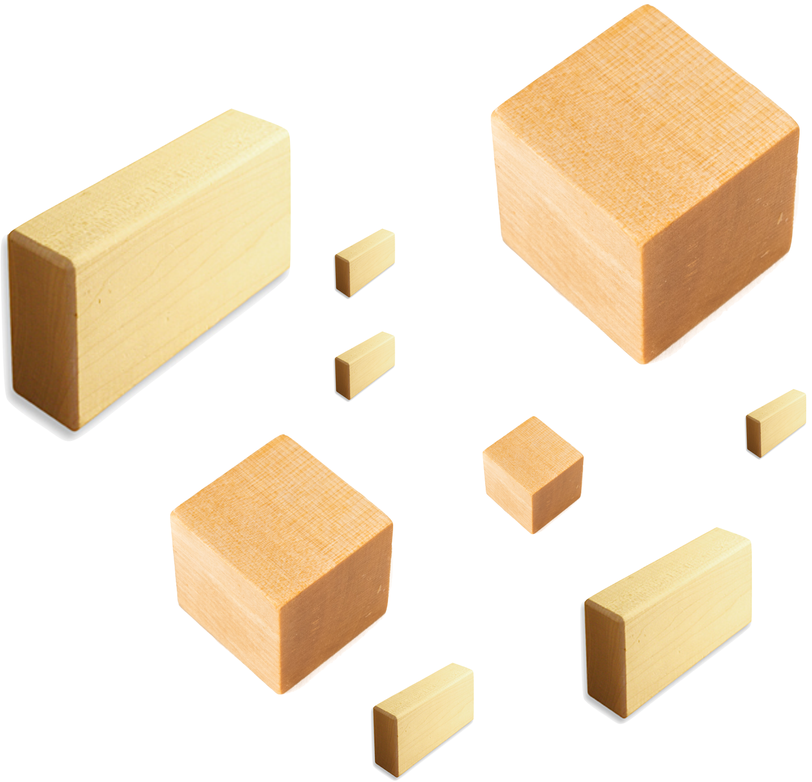
Lg 5800

feb 15/17 2022

shelat

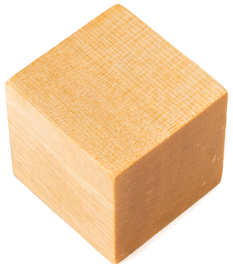
Knapsack



Sack has Capacity W

W

w_1 v_1



w_2 v_2

w_3 v_3

Each item
has a weight w_i
and a value v_i

Goal is to select
a set of items that
"**fit**" into the Knapsack
and have the **greatest** value.
Can only use each item once
(versus Logcutter)

Define a quantity that captures the optimal solution:

Best({1,…,n}, W) :

Consider the very first item. Is it part of the max solution?

w_1 v_1

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Best({1,…,n}, W) : max value obtainable from items {1…n} that fit in sack of size W

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Consider the very first item. Is it part of the max solution?

w_1 v_1

$$B(1\ldots n, W) = \max \begin{cases} B(2\ldots n, W) & \text{if not included} \\ v_1 + B(2\ldots n-1, W-w_1) & \text{if in} \end{cases}$$

Recursive structure

Either the best solution doesn't include item i

B({i…n}, W) = max

Or, it includes item i and the best solution for the remaining space, $W-w_i$

Recursive structure

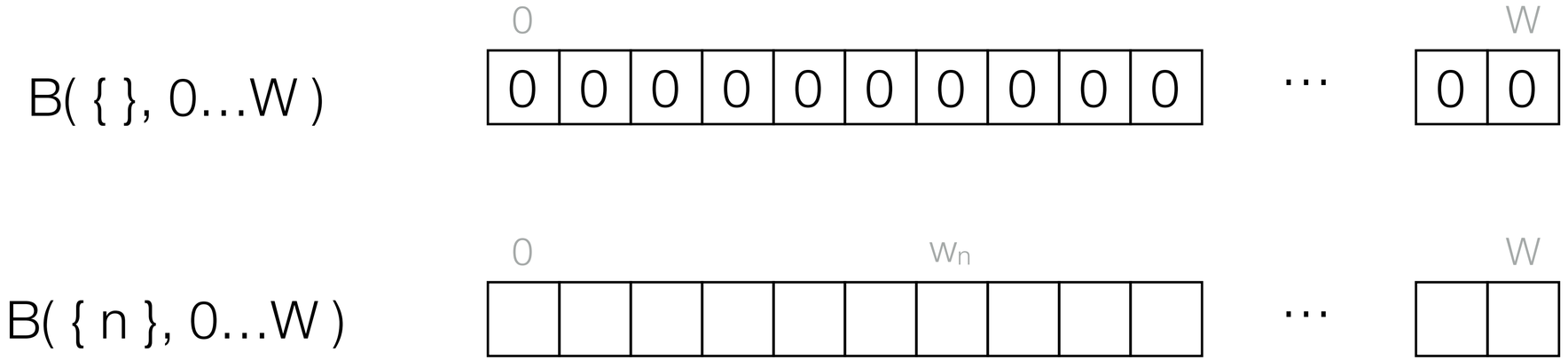
Either the best solution doesn't include item i

$$B(\{i\ldots n\}, W) = \max \begin{cases} B(\{i+1\ldots n\}, W) \\ v_i + B(\{i+1\ldots n\}, W - w_i) \end{cases} \quad \text{If } W - w_i > 0$$

Or, it includes item i and the best solution for the remaining space, $W-w_i$

Pick an order

Start from the last item

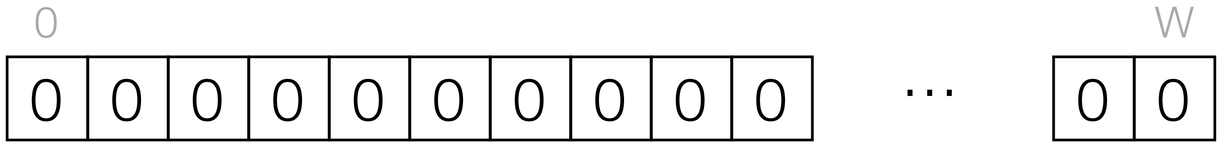


If $W - w_n > 0$

Pick an order

Start from the last item

B({ }, 0…W)



B({ n }, 0…W)

$$B(\{n\}, W) = \max \begin{cases} B(\{\}, W) \\ v_n + B(\{\}, W - w_n) \end{cases} \quad \text{If } W - w_n > 0$$

Pick an order

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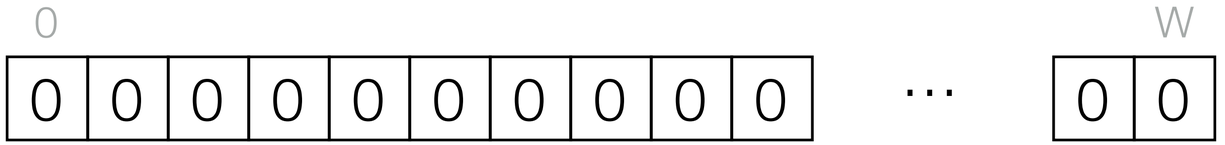
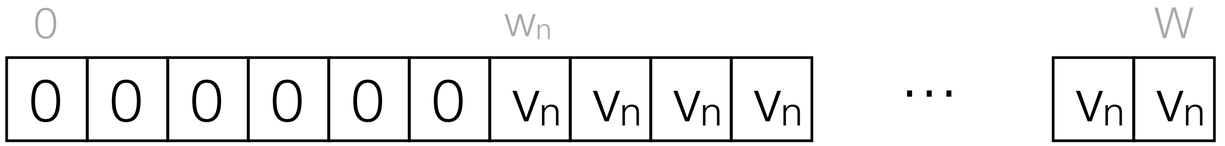
B({ }, 0…W)



B({ n }, 0…W)



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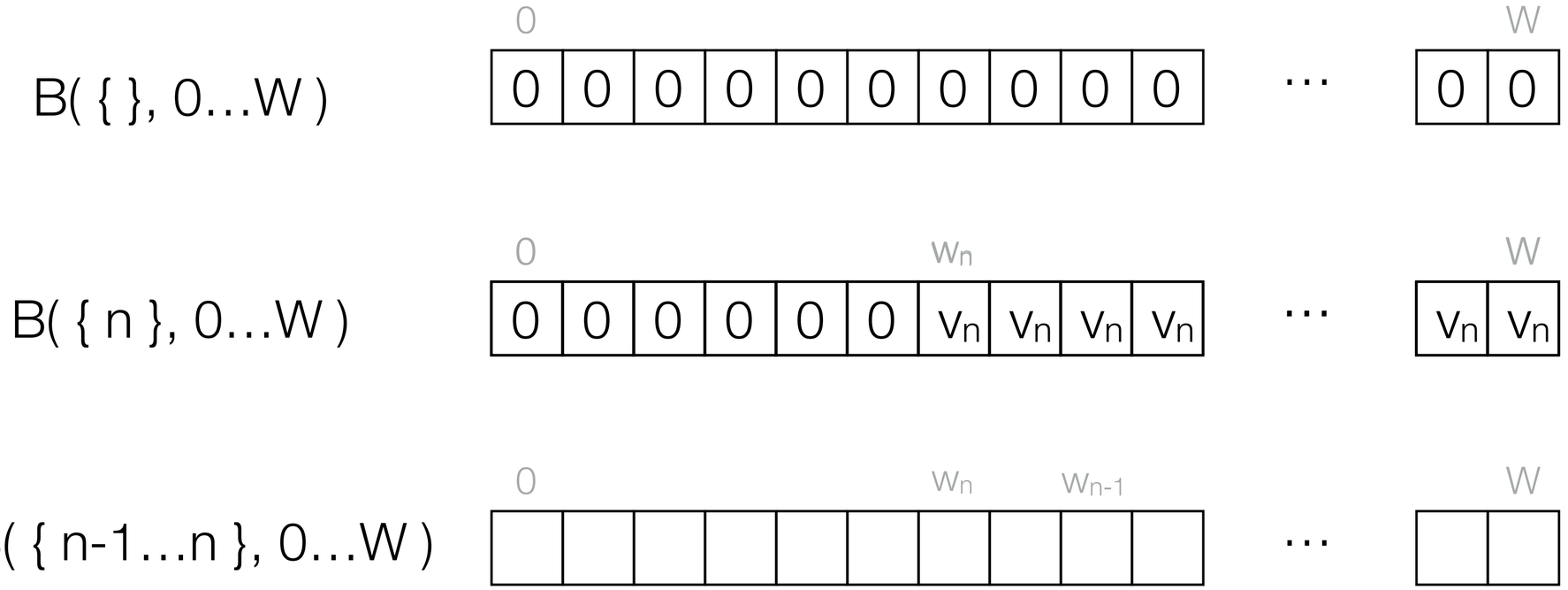
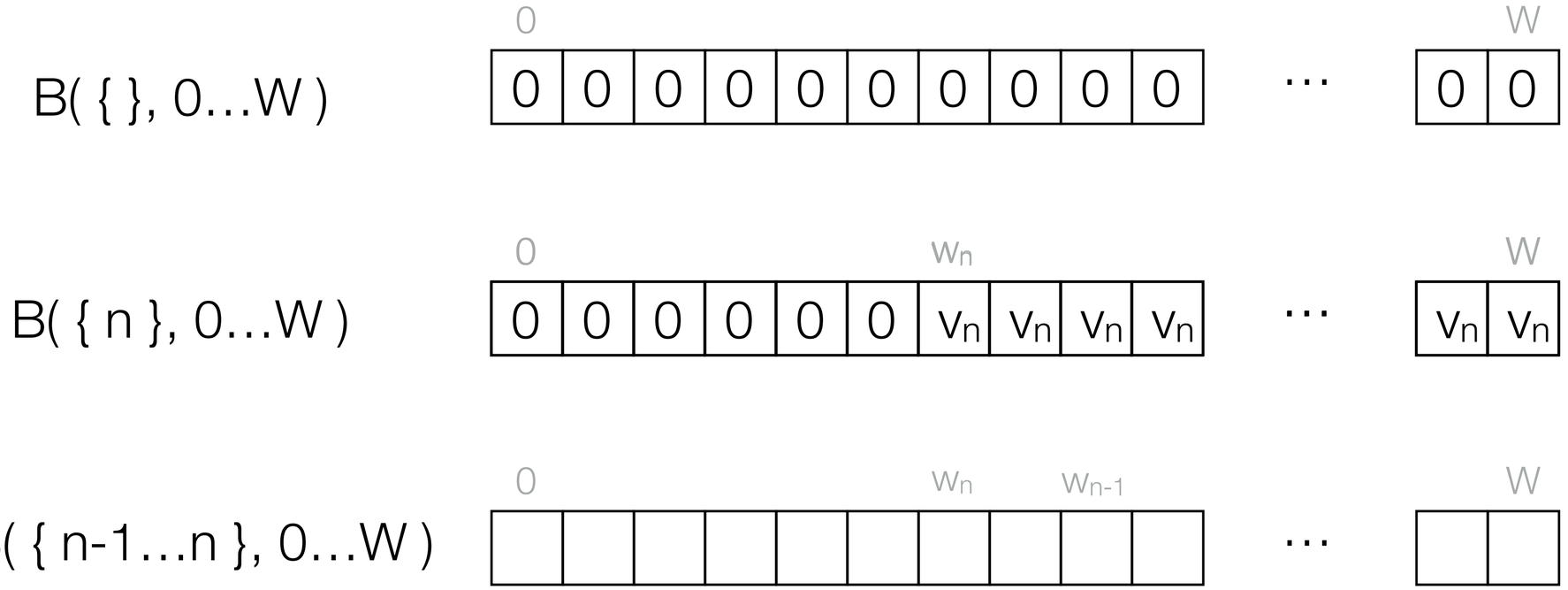


Pick an order

Start from the last item



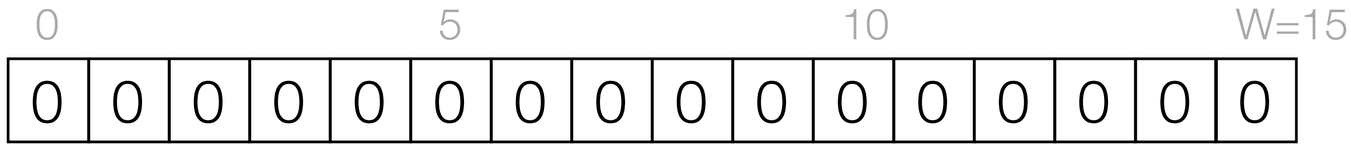
$$B(\{i \ldots n\}, W) = \max \begin{cases} B(\{i+1 \ldots n\}, W) \\ v_i + B(\{i+1 \ldots n\}, W - w_i) \end{cases}$$

Example

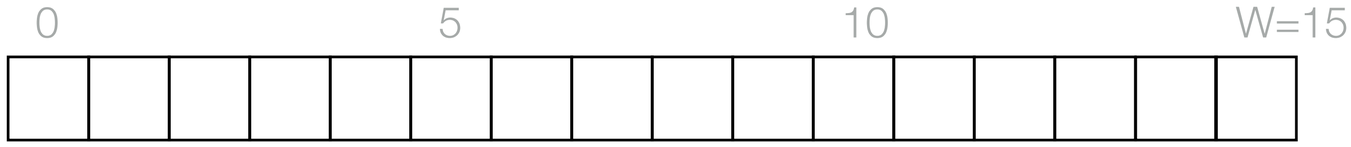
	1	2	3	4	5	6
v_i	4	5	9	1	10	3
w_i	1	3	2	4	7	2

$$C = 15$$

$B(\{\}, \ldots)$



$B(\{6\}, \ldots)$



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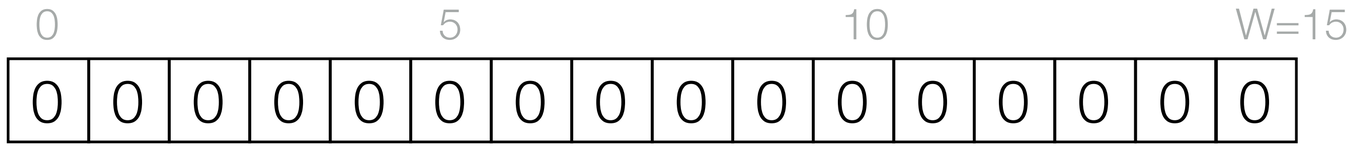

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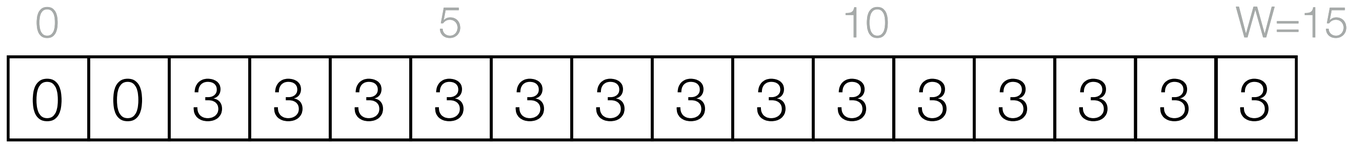

$$B(\{i \ldots n\}, W) = \max \begin{cases} B(\{i+1 \ldots n\}, W) \\ v_i + B(\{i+1 \ldots n\}, W - w_i) \end{cases} \quad \text{If } W - w_i > 0$$

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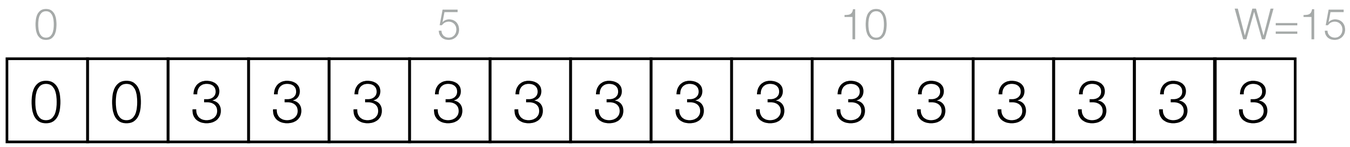
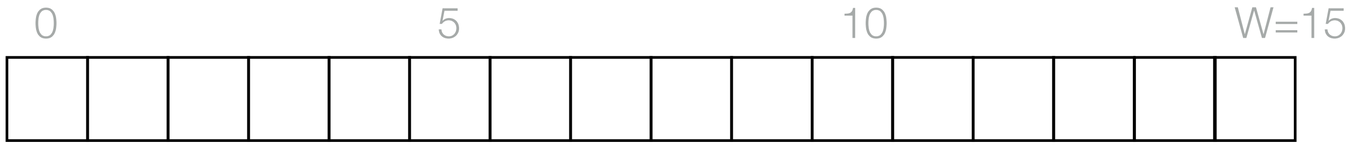


$B(\{5,6\}, \ldots)$



$$B(\{i \ldots n\}, W) = \max \begin{cases} B(\{i+1 \ldots n\}, W) \\ v_i + B(\{i+1 \ldots n\}, W - w_i) \end{cases} \quad \text{If } W - w_i > 0$$

Example

	1	2	3	4	5	6
v_i	4	5	9	1	10	3
w_i	1	3	2	4	7	2

$$C = 15$$

$B(\{6\}, \ldots)$

0	0	3	3	3	3	3	3	3	3	3	3	3	3	3	3
0					5					10					W=15

$B(\{5,6\}, \ldots)$

0	0	3	3	3	3	3	10	10	13	13	13	13	13	13	13
0					5					10					W=15

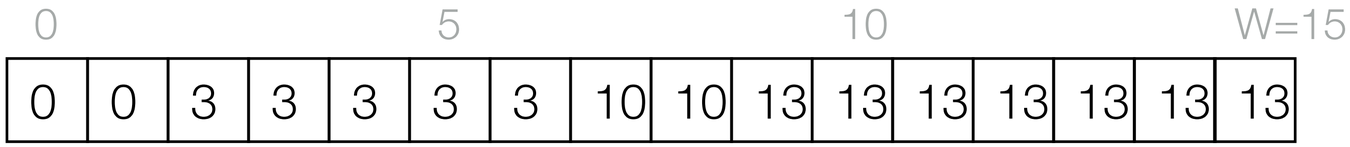
$$B(\{i \ldots n\}, W) = \max \begin{cases} B(\{i+1 \ldots n\}, W) \\ v_i + B(\{i+1 \ldots n\}, W - w_i) \end{cases} \quad \text{If } W - w_i > 0$$

Example

	1	2	3	4	5	6
v_i	4	5	9	1	10	3
w_i	1	3	2	4	7	2

$$C = 15$$

$B(\{5,6\}, \ldots)$



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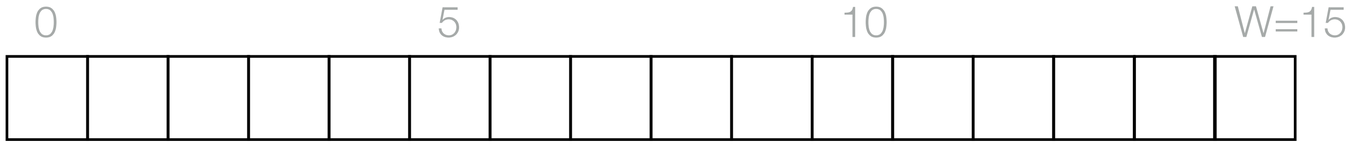


$$B(\{i \ldots n\}, W) = \max \begin{cases} B(\{i+1 \ldots n\}, W) \\ v_i + B(\{i+1 \ldots n\}, W - w_i) \end{cases} \quad \text{If } W - w_i > 0$$

Knapsack($\{w_i, v_i\}_n$, W)

Initialize B({n-1}, 0…W) = 0

for i from n to 1

for j from 0 to W

B(i, j) = max $\begin{cases} B(i+1, j) \\ v_i + B(i+1, j - w_i) \end{cases}$

as long as $j > w_i$
because otherwise,
this term is negative

Return B(1,W)

Knapsack($\{w_i, v_i\}_n$, W)

Initialize B({n-1}, 0…W) = 0

for i from n to 1

for j from 0 to W

B(i, j) = B(i+1, j)

if j > wi and B(i+1, j-wi)+ vi > S(i, j)

B(i,j) = B(i+1, j-wi)+ vi

Return B(1,W)

How can we determine WHICH items are selected?

Knapsack($\{w_i, v_i\}_n$, W)

Initialize B({n-1}, 0…W) = 0

for i from n to 1

for j from 0 to W

B(i, j) = B(i+1, j)

pick(i, j) = false

if j > wi and B(i+1, j-wi)+ vi > B(i, j)

B(i,j) = B(i+1, j-wi)+ vi

pick(i, j) = true

//Backtrack to find solution

cap = W, sol = {}

for i from 1 to n

if picked(i,cap) = true { sol = sol + {i}; cap = cap - w_i; }

Gerrymander

Congressional District 5

nationalatlas.g

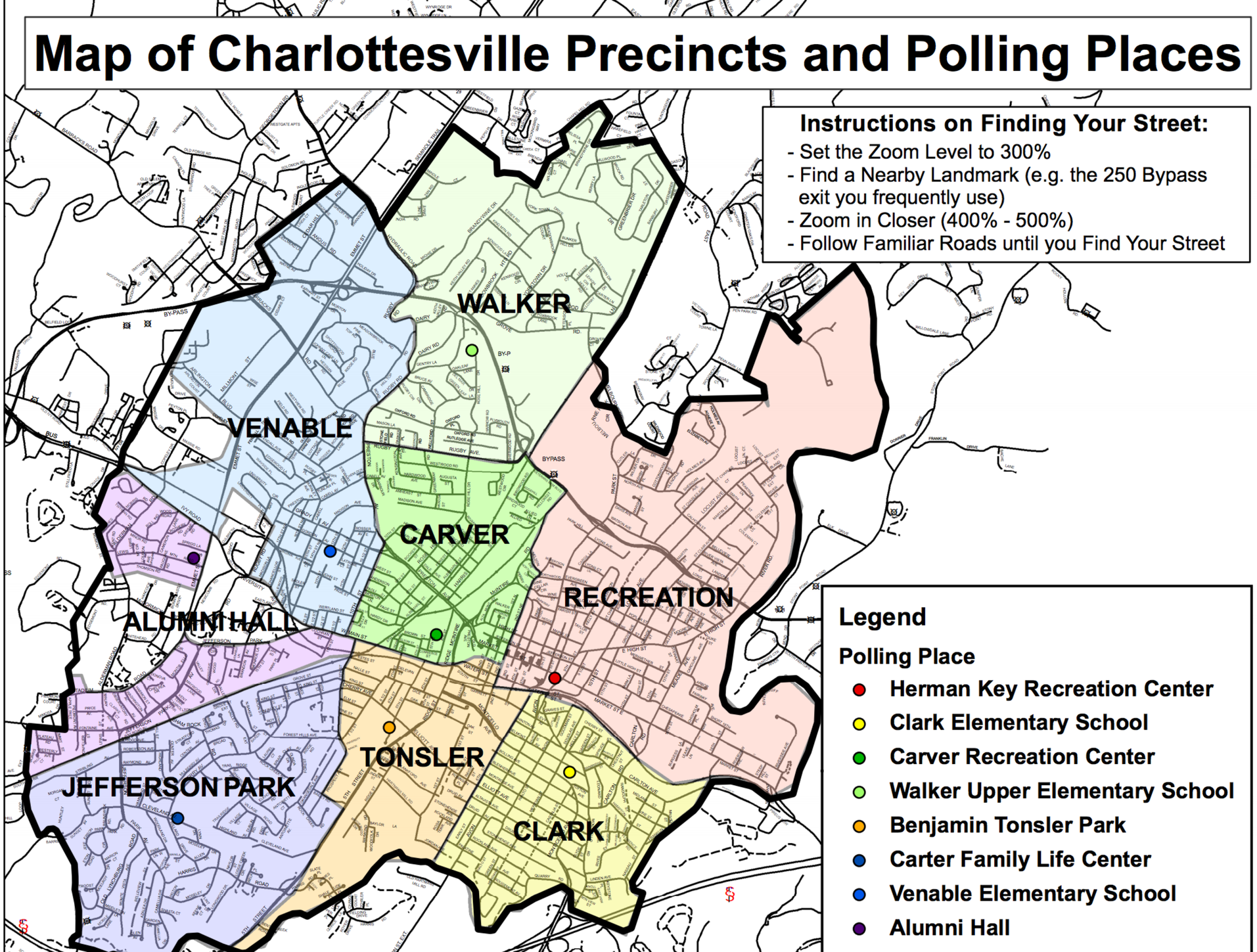
5 Congressional D

Nelson County

0 50 100 Miles

Virginia (11 Distr

Map of Charlottesville Precincts and Polling Places



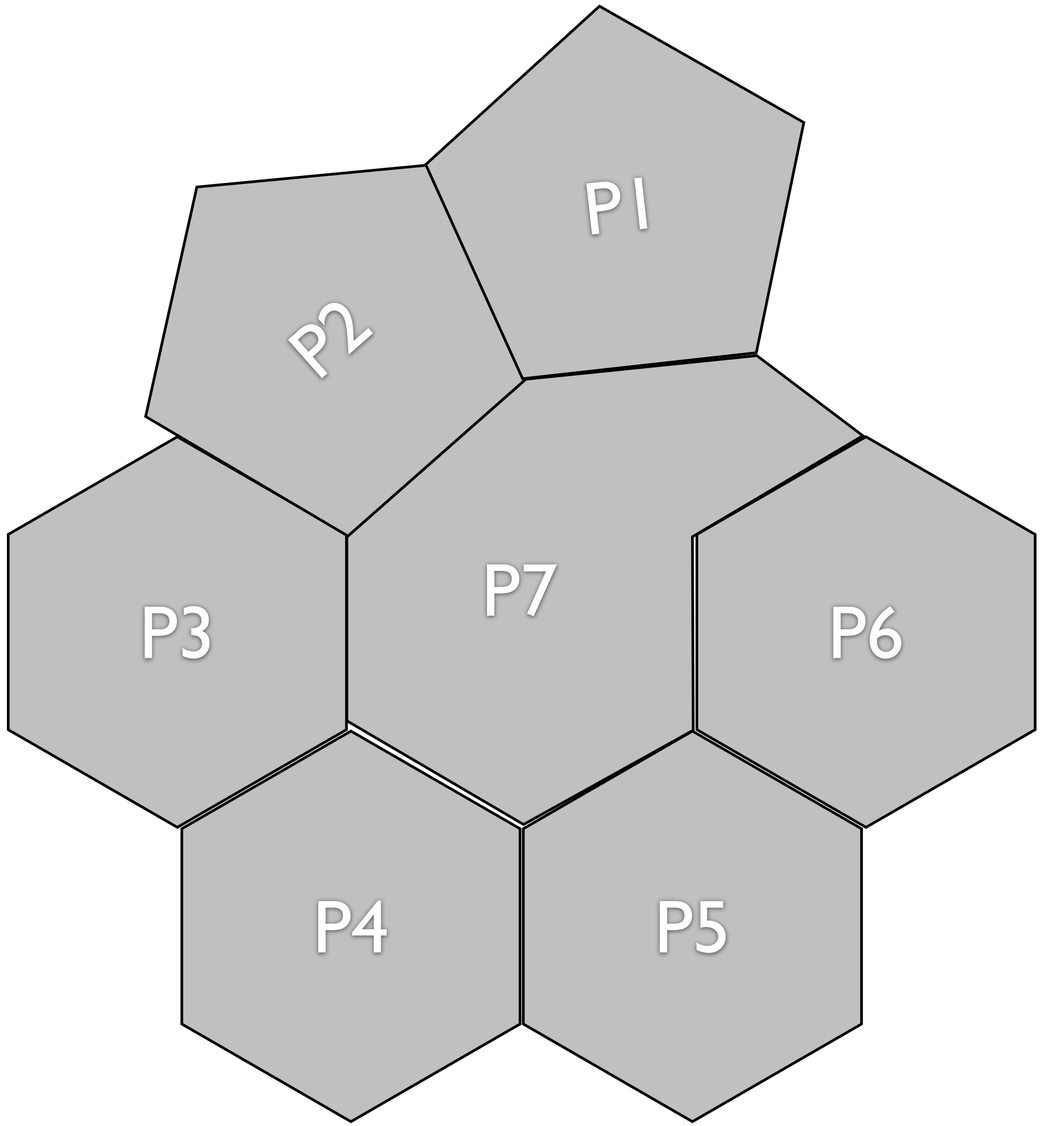
Instructions on Finding Your Street:

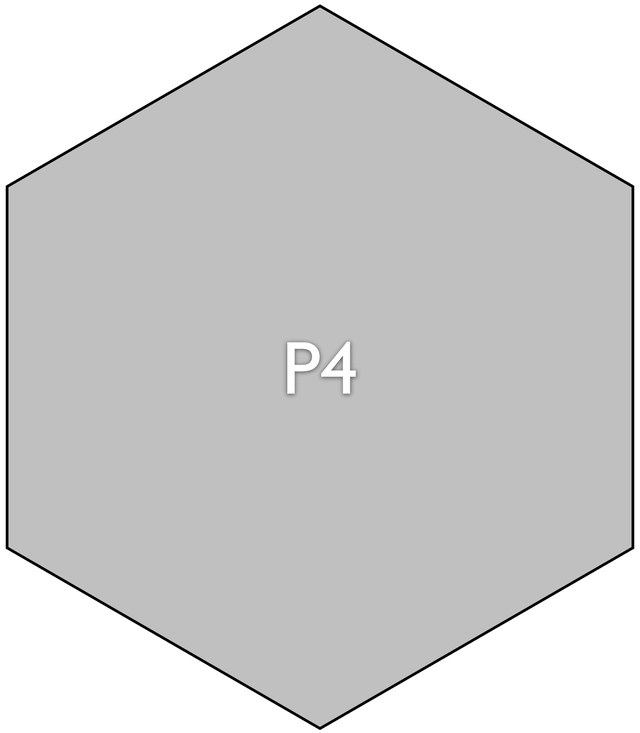
- Set the Zoom Level to 300%
- Find a Nearby Landmark (e.g. the 250 Bypass exit you frequently use)
- Zoom in Closer (400% - 500%)
- Follow Familiar Roads until you Find Your Street

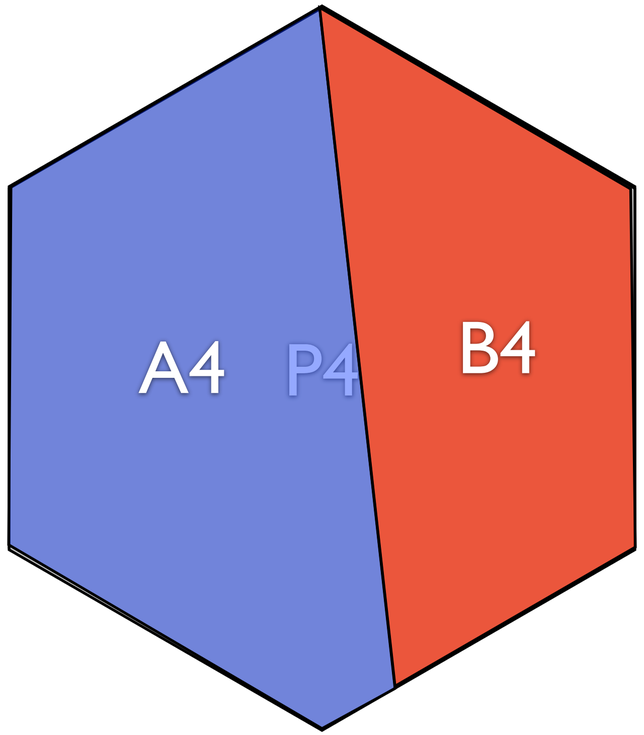
Legend

Polling Place

- Herman Key Recreation Center
- Clark Elementary School
- Carver Recreation Center
- Walker Upper Elementary School
- Benjamin Tonsler Park
- Carter Family Life Center
- Venable Elementary School
- Alumni Hall


P1
P2
P3
P7
P6
P4
P5


P4



Each precinct P_i has a A_i voters for party A and B_i voters for party B.

gerrymander problem

given:

output:

gerrymander problem

given: m $A_1, A_2, \ldots, A_n$ n is even

Total voters Voters for party A in each precinct

output: D_1, D_2

such that $|D_1| = |D_2|$

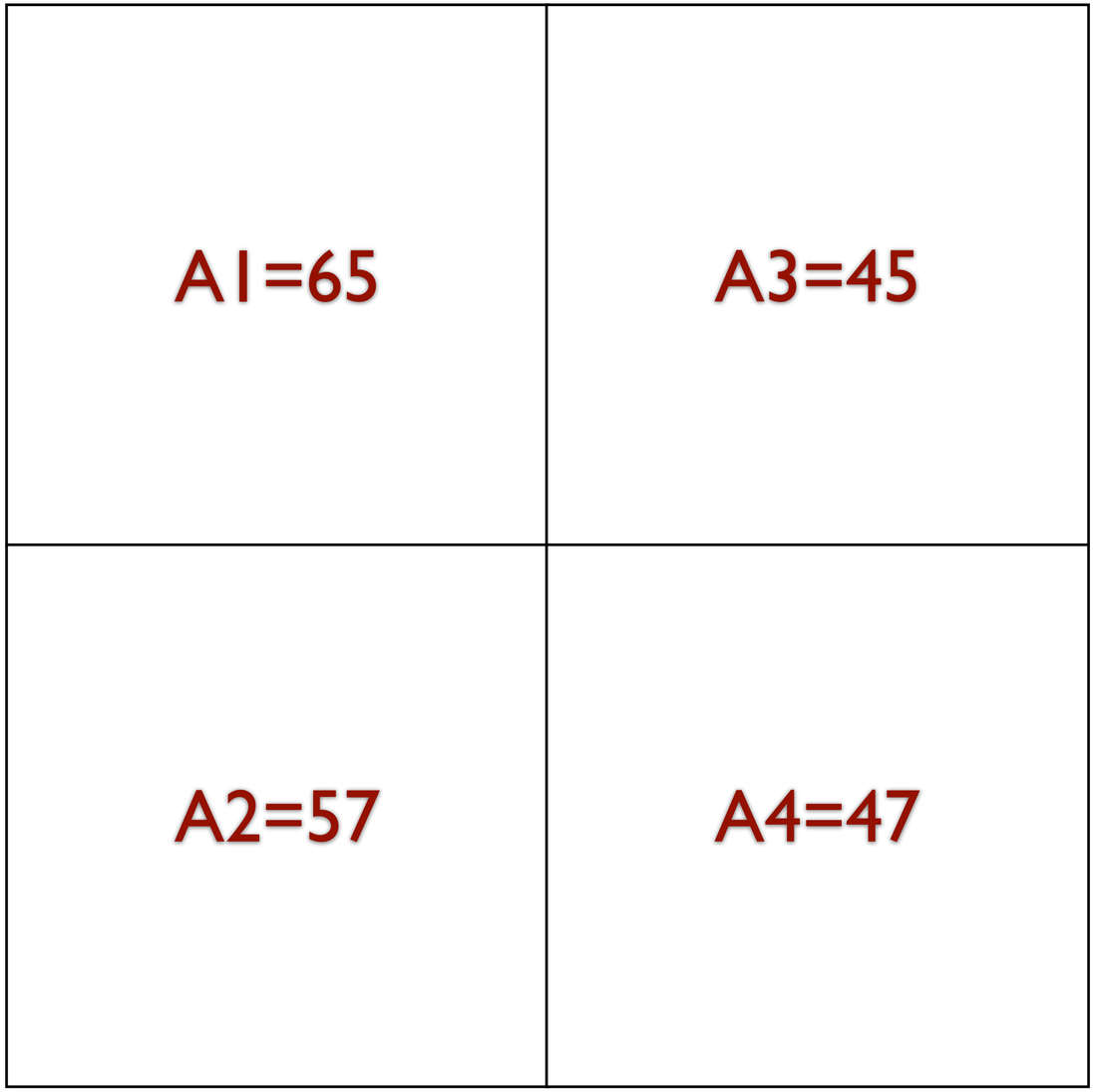
$$A(D_1) > \frac{mn}{4}$$

$$A(D_2) > \frac{mn}{4}$$

or “failure” if no such solution is possible

Example

Imagine 4 precincts divided into 2 districts.



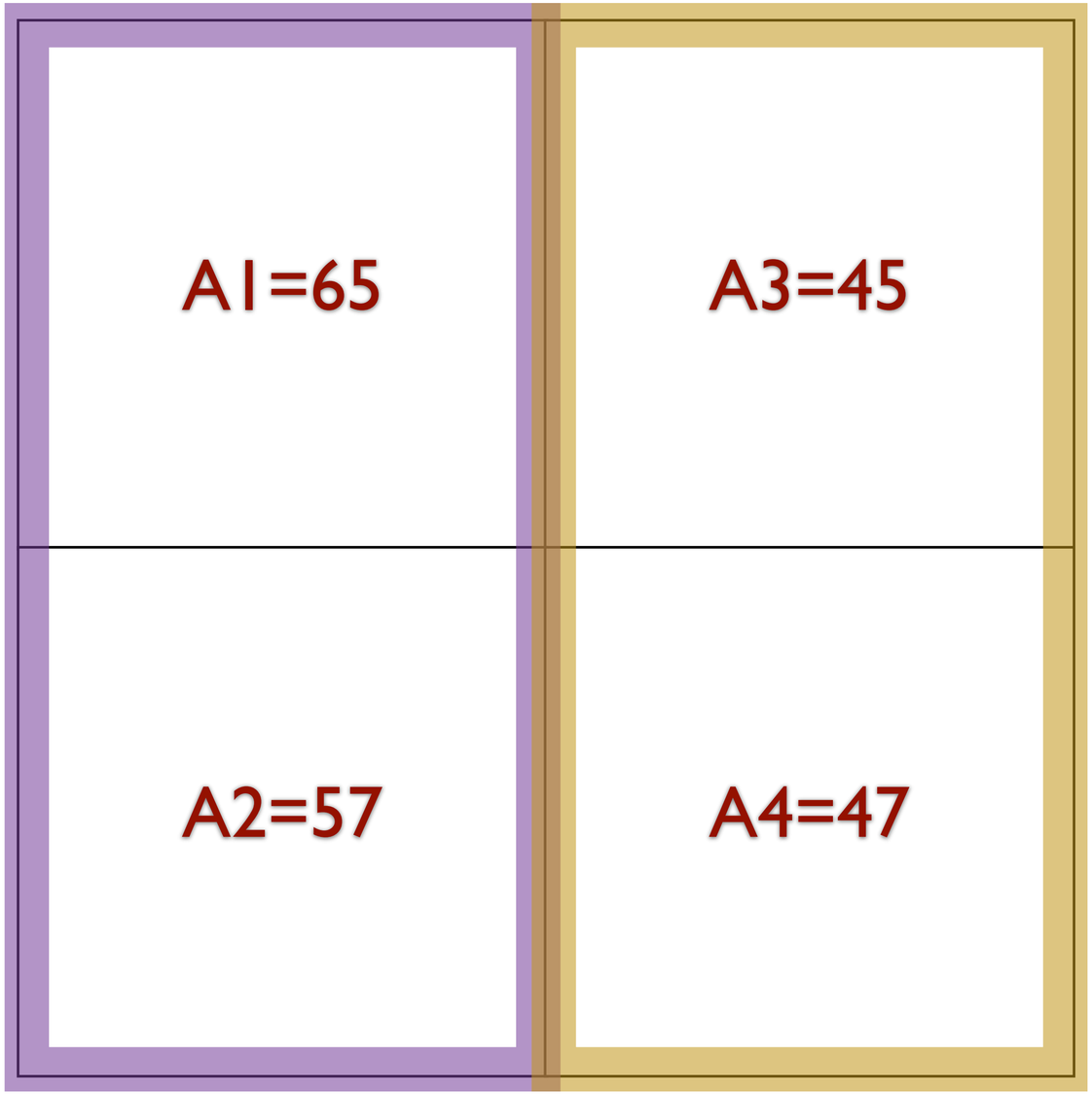
M=100

D1

D2

Example

Imagine 4 precincts divided into 2 districts.



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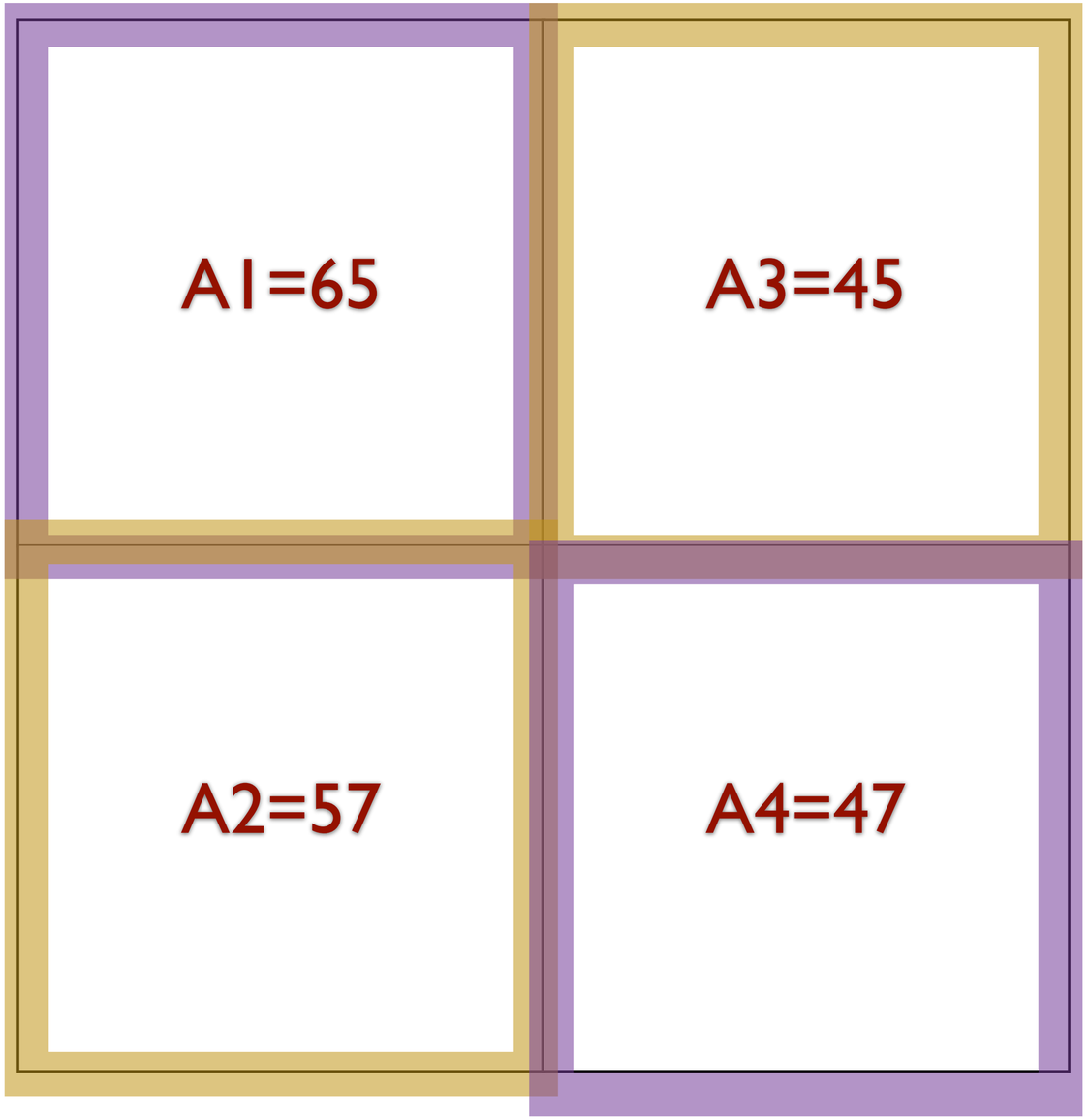


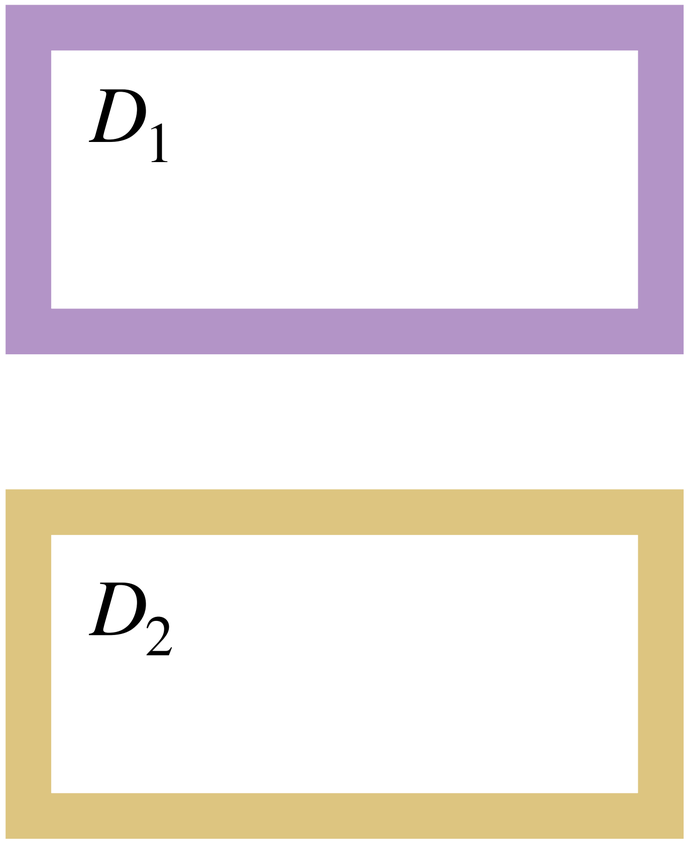
M=100

D1

D2

Gerrymander

imagine very last precinct and how it is assigned:



Gerrymander

imagine very last precinct and how it is assigned:

D_1
k precincts
x votes for A

D_2
$n - k - 1$ precincts
y votes for A

Precinct n

Which district should we assign it to?

Gerrymander

imagine very last precinct and how it is assigned:

D_1 D_2

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Gerrymander

imagine very last precinct and how it is assigned:

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$n-k-1$ precincts
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Precinct n

Which district should we assign it to?

D_1	D_2
$k+1$ precincts, $x+A_n$ votes	$n-k-1$ precincts, y votes
k precincts, x	$n-k$ precincts, $y+A_n$

Gerrymander

$$S_{j,k,x,y} =$$

Gerrymander

$S_{j,k,x,y} =$ TRUE if there exists an assignment of the first j precincts such that $|D_1| = k$, $A(D_1) = x$, $A(D_2) = y$

Gerrymander

$S_{j,k,x,y}$ = there is a split of first **j** precincts
in which |D1|=**k** and
x people in D1 vote A
y people in D2 vote A

How can we express this value in an equation:

$$S_{j,k,x,y} =$$

$$S_{j,k,x,y} = S_{j-1,k-1,x-A_j,y} \vee S_{j-1,k,x,y-A_j}$$

Gerrymander(P,A,m)

initialize array S[o,o,o,o]

$$S_{j,k,x,y} = S_{j-1,k-1,x-A_j,y} \vee S_{j-1,k,x,y-A_j}$$

Gerrymander(P,A,m)

```
initialize array S[0,0,0,0]
for j=1,...,n
  for k=1,...,j
   for x=0,...,jm
    for y=0,...,jm
        fill table according to equation

search for true entry at S[n, n/2, >(mn/4)…mn, >(mn/4)…mn]
```

PROBLEM: REDUCE IMAGE WIDTH

scaling: distortion

deleting column: distortion

delete the most invisible seam

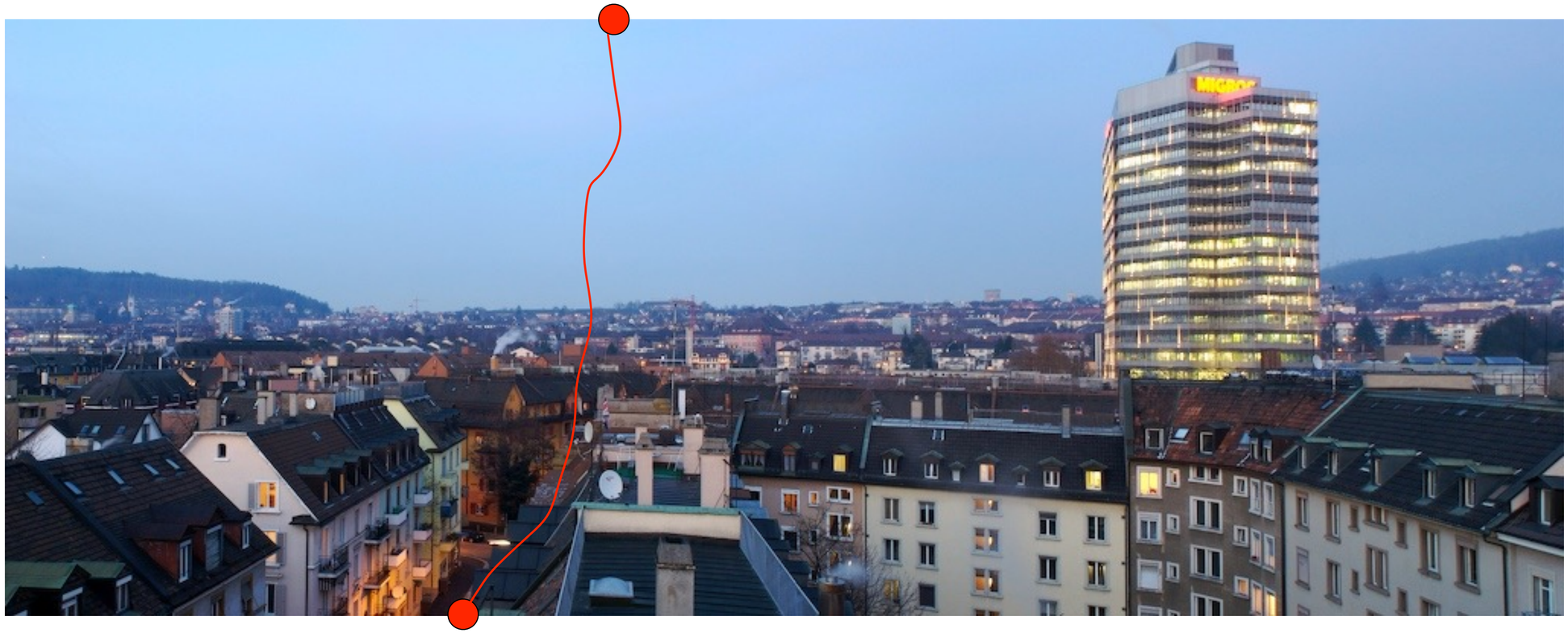
http://www.youtube.com/watch?v=qadw0BRKeMk

http://www.youtube.com/watch?v=qadw0BRKeMk

DEMO?

Demo

WHICH SEAM TO DELETE?

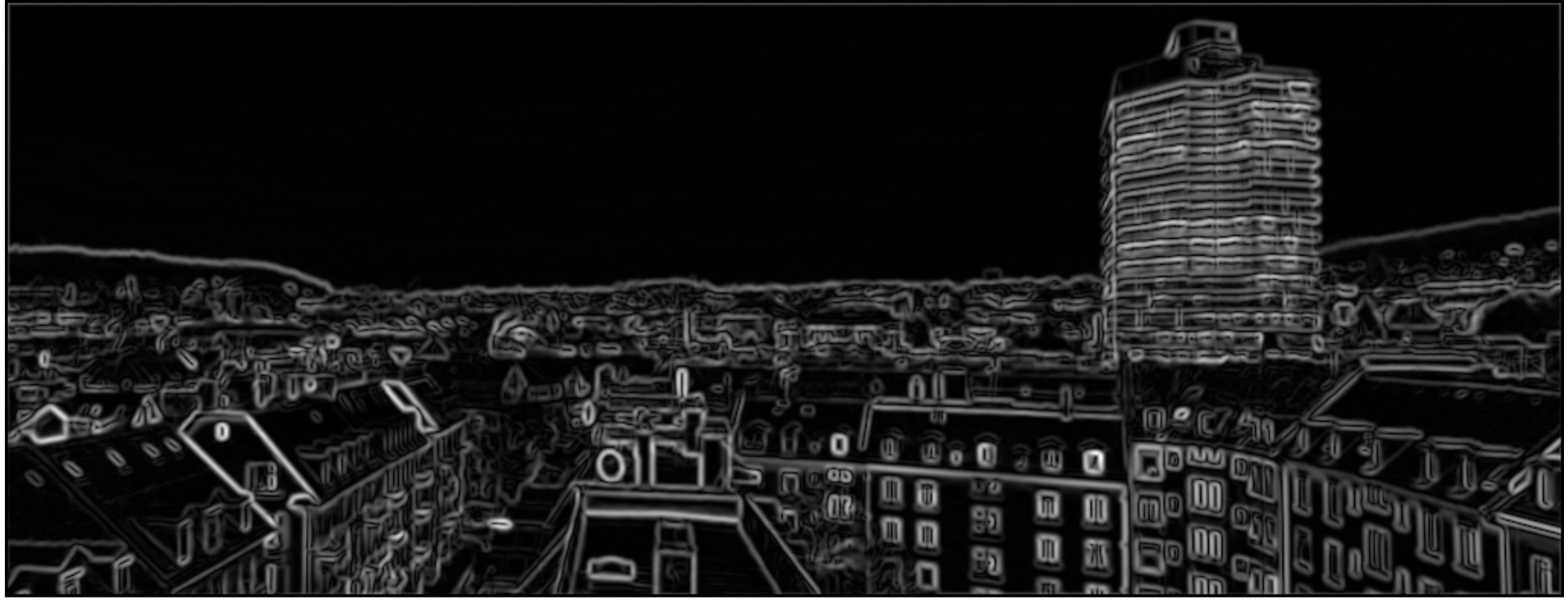


ENERGY OF AN IMAGE

$$e(\mathbf{I}) = \left|\frac{\partial}{\partial x}\mathbf{I}\right| + \left|\frac{\partial}{\partial y}\mathbf{I}\right|$$

“magnitude of gradient at a pixel”

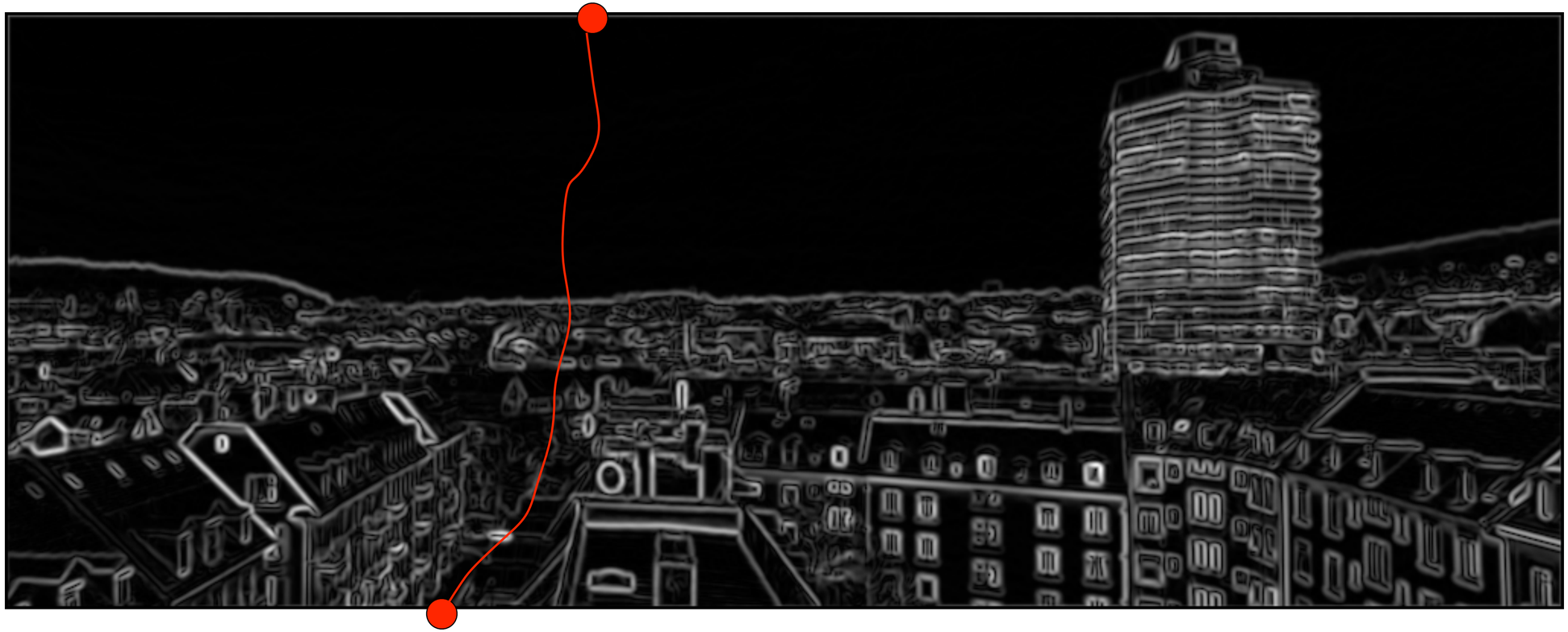
$$\frac{\partial}{\partial x} I_{x,y} = I_{x-1,y} - I_{x+1,y}$$



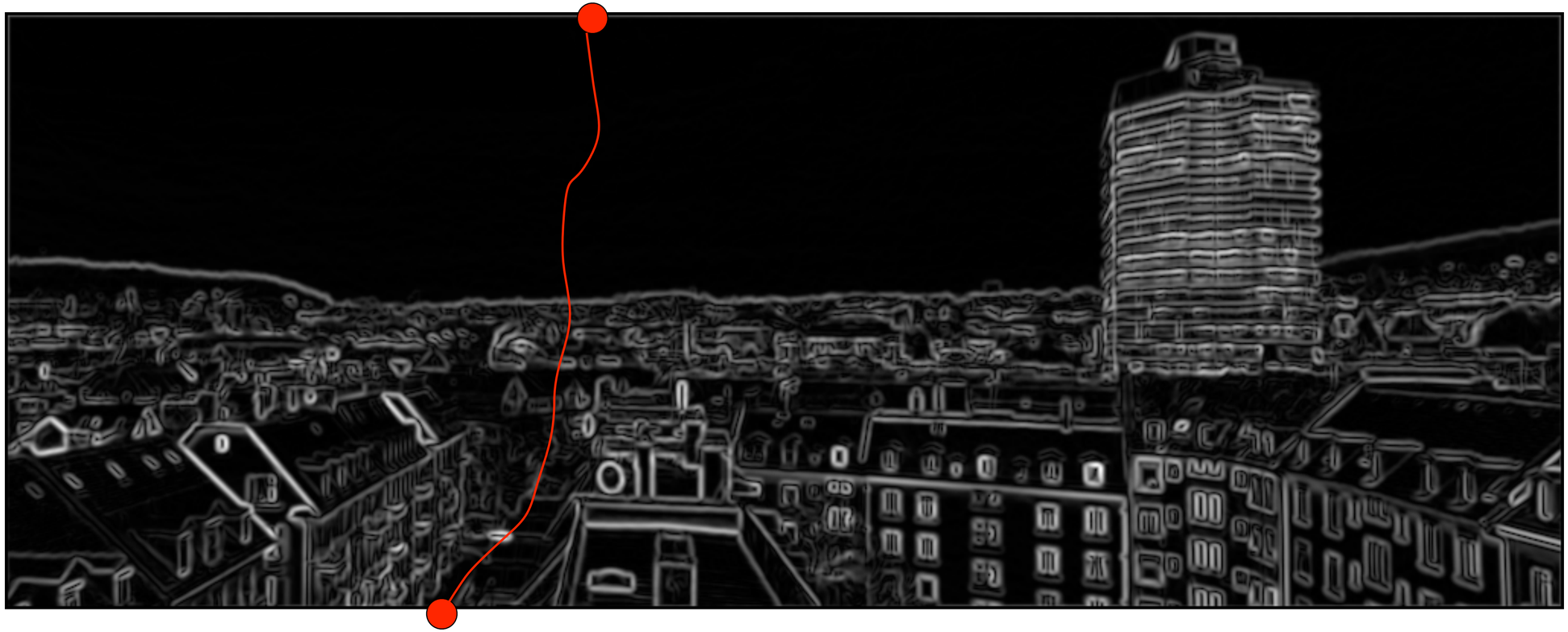
energy of sample image

thanks to Jason Lawrence for gradient software

BEST SEAM HAS LOWEST ENERGY



FINDING LOWEST ENERGY SEAM?



Define a variable:

$$S_i(j)$$

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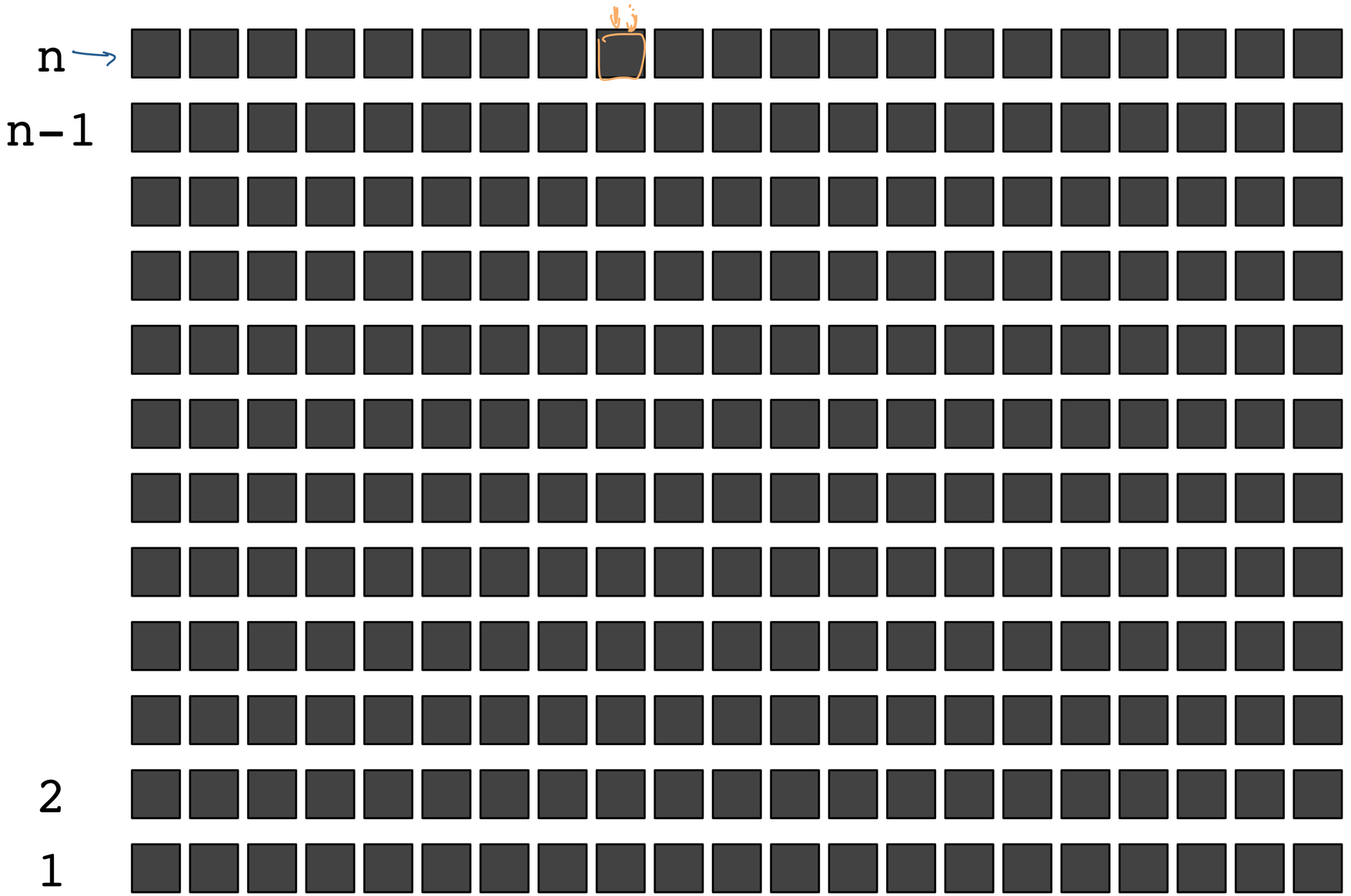
$$S_i(j)$$

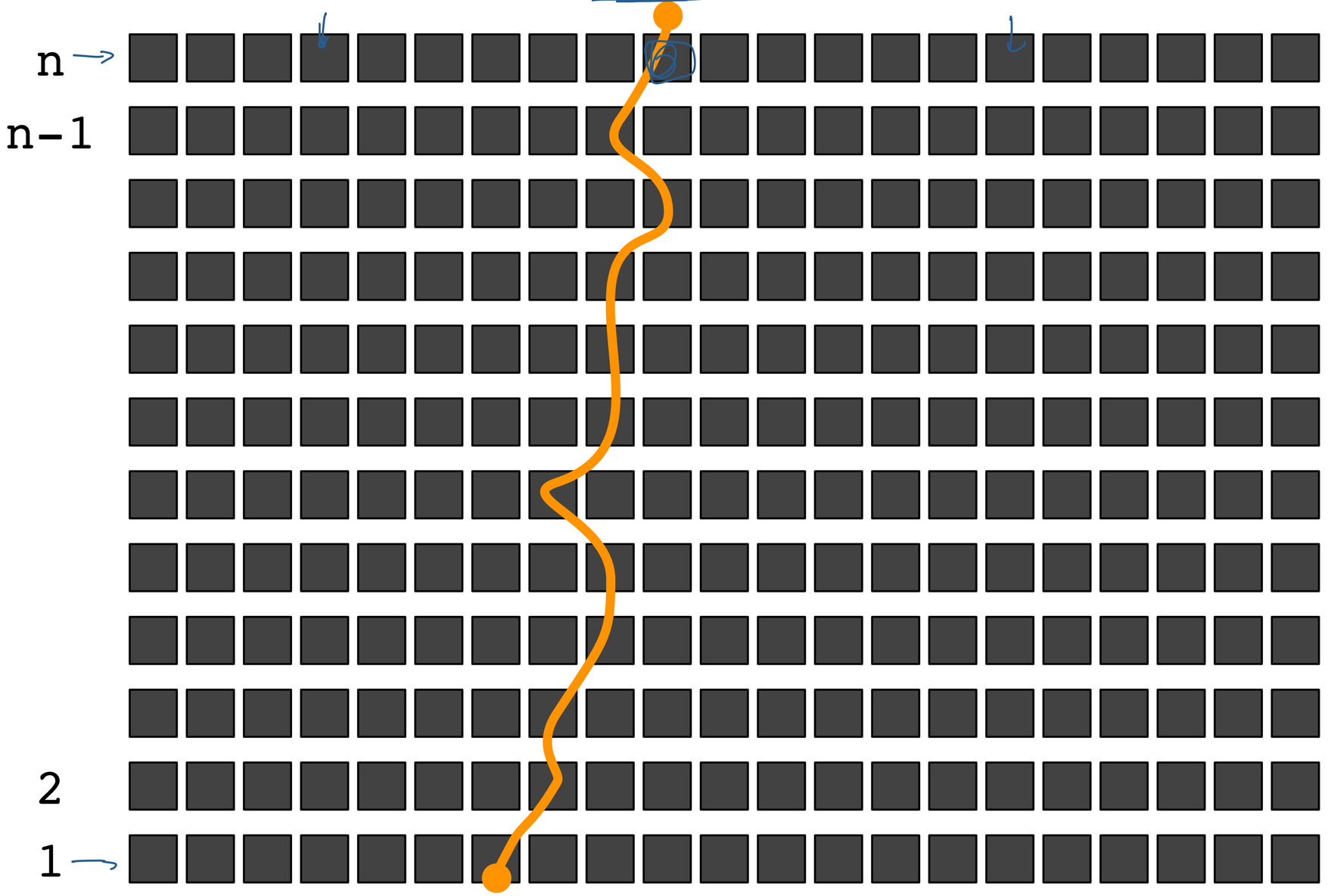
Total energy of the lowest energy seam that ends at row (I,j)

definition: $S_n(j)$ Value of lowest energy seam that ends at (n,j)



definition:

$S_n(j)$ best seam ending at (n,j)

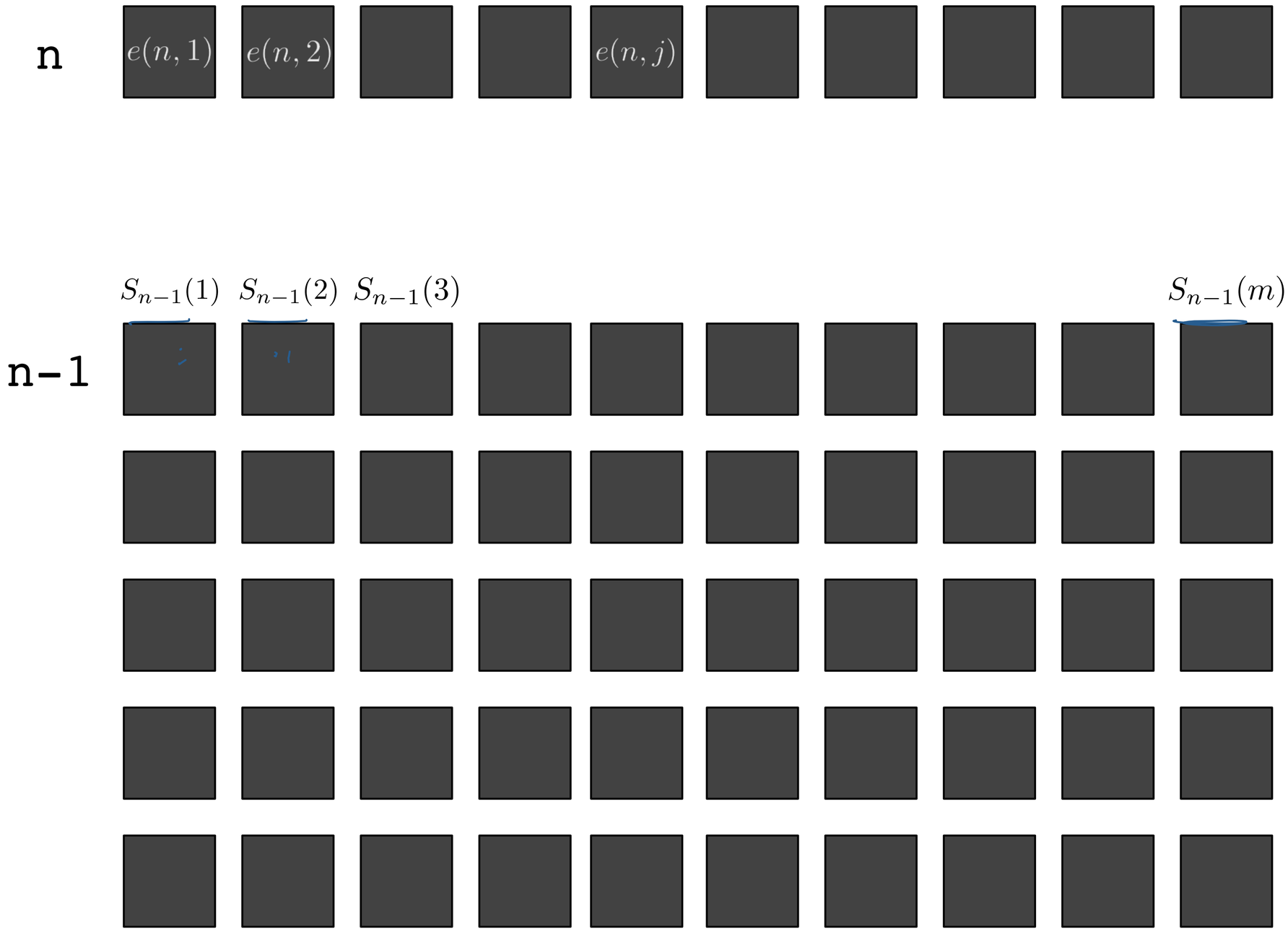


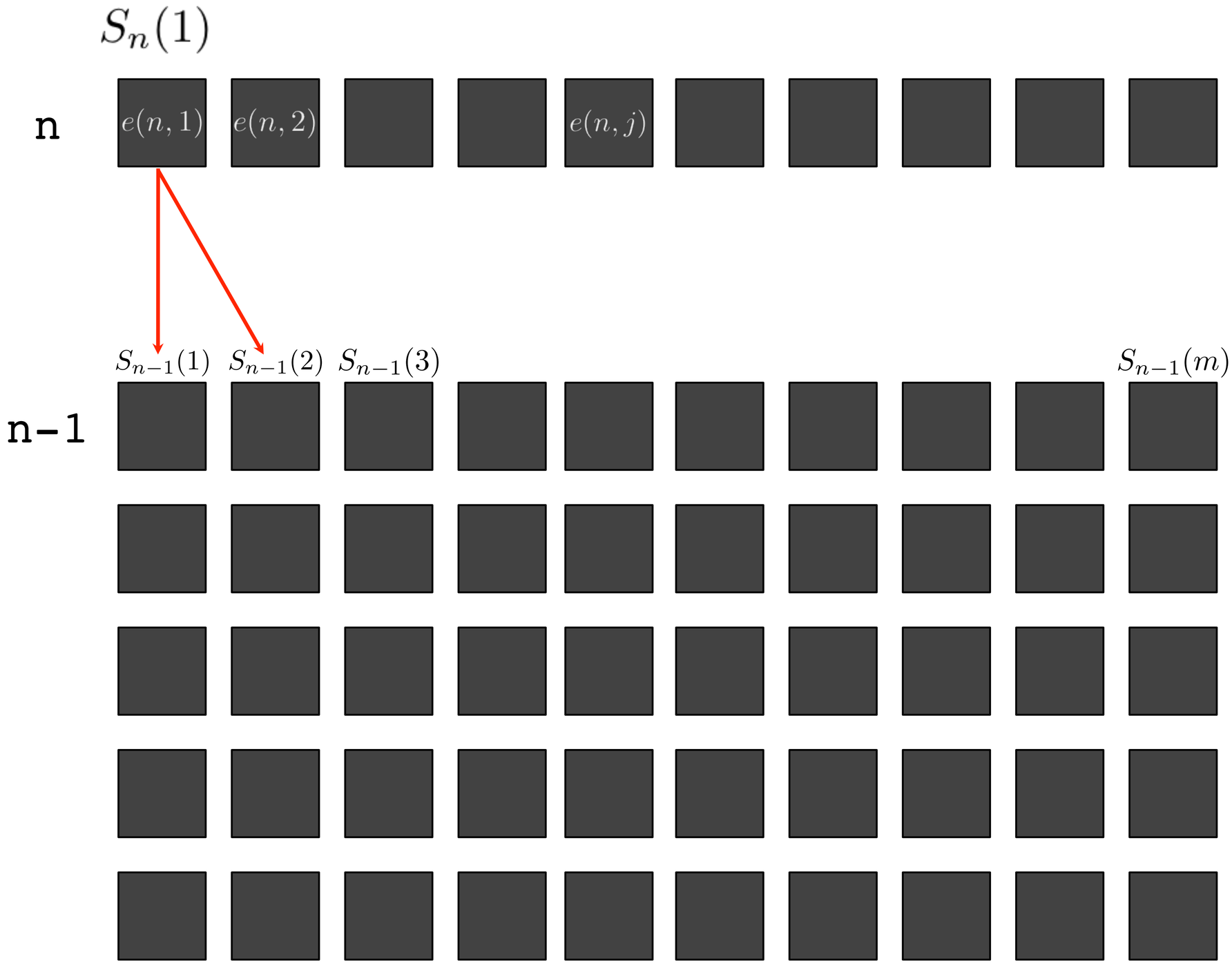
BEST SEAM TO DELETE HAS TO BE THE BEST AMONG

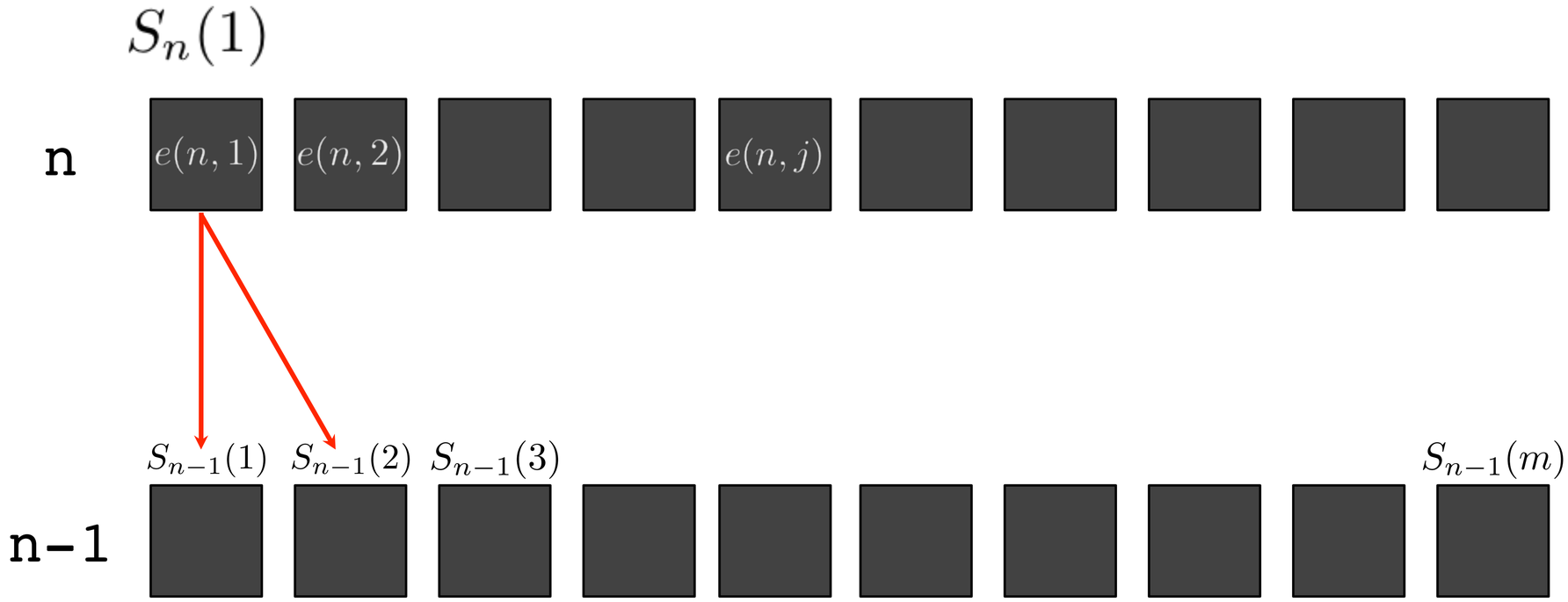
$$S_n(1), S_n(2), \ldots, S_n(m)$$

IMAGINE YOU HAVE THE SOLUTION TO THE FIRST N-1 ROWS

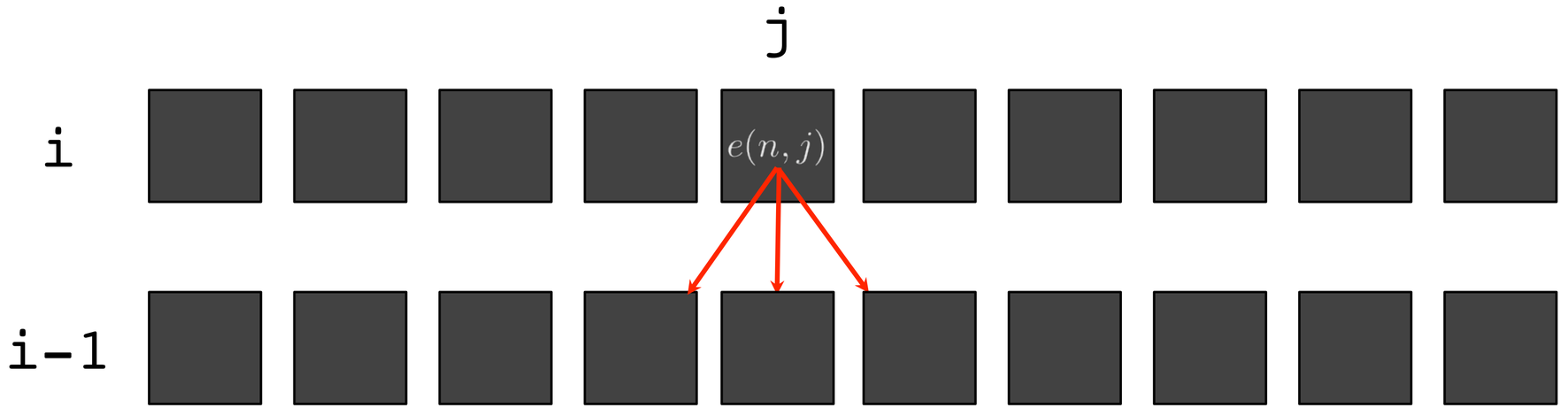
What happens on the very last row?


n
$e(n,1)$
$e(n,2)$
$e(n,j)$
n-1
$S_{n-1}(1)$
$S_{n-1}(2)$
$S_{n-1}(3)$
$S_{n-1}(m)$

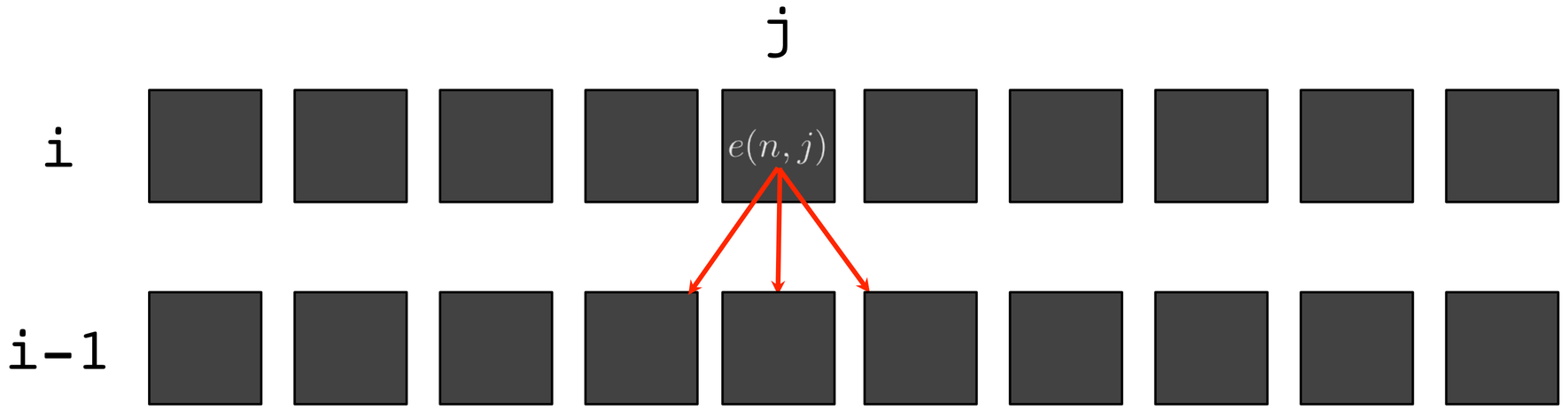

$S_n(1)$
n
$e(n,1)$
$e(n,2)$
$e(n,j)$
$S_{n-1}(1)$
$S_{n-1}(2)$
$S_{n-1}(3)$
$S_{n-1}(m)$
n-1



$$S_n(1) = e(n,1) + \min\{S_{n-1}(1), S_{n-1}(2)\}$$



$$S_i(j) =$$



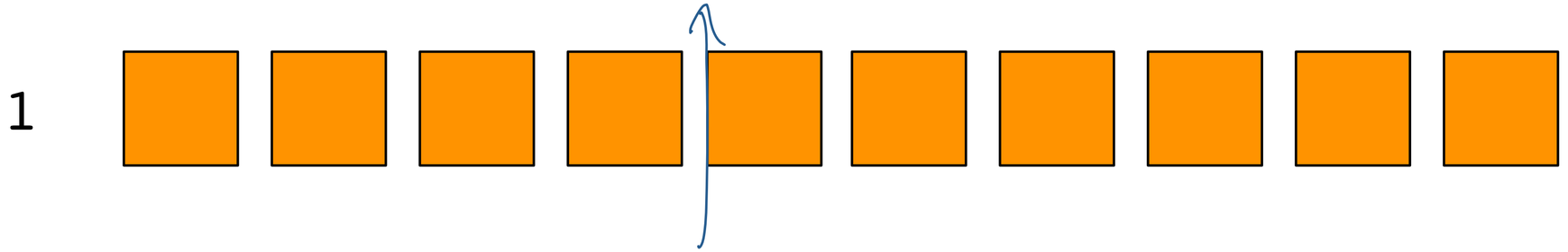
$$S_i(j) = e(i,j) + \min \begin{cases} S_{i-1}(j-1) \\ S_{i-1}(j) \\ S_{i-1}(j+1) \end{cases}$$

ALGORITHM

start at bottom of picture

ALGORITHM

start at bottom of picture. initialize $S_1(i) = e(1, i)$

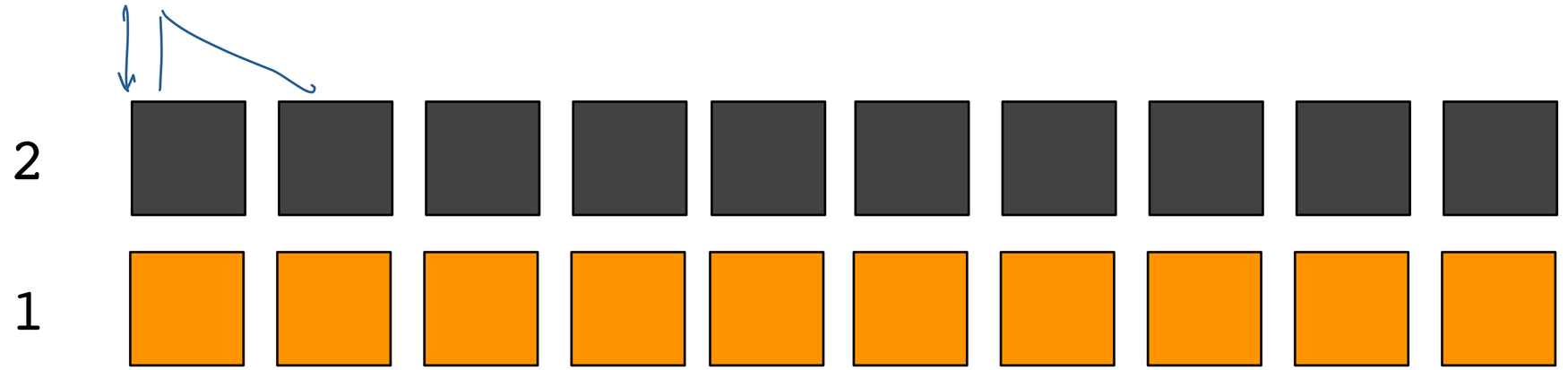


ALGORITHM

start at bottom of picture. initialize $S_1(i) = e(1, i)$

for `i=2 to n` use formula to compute $S_{i+1}(\cdot)$

$$S_i(j) = e(i, j) + \min \begin{cases} S_{i-1}(j-1) \\ S_{i-1}(j) \\ S_{i-1}(j+1) \end{cases}$$

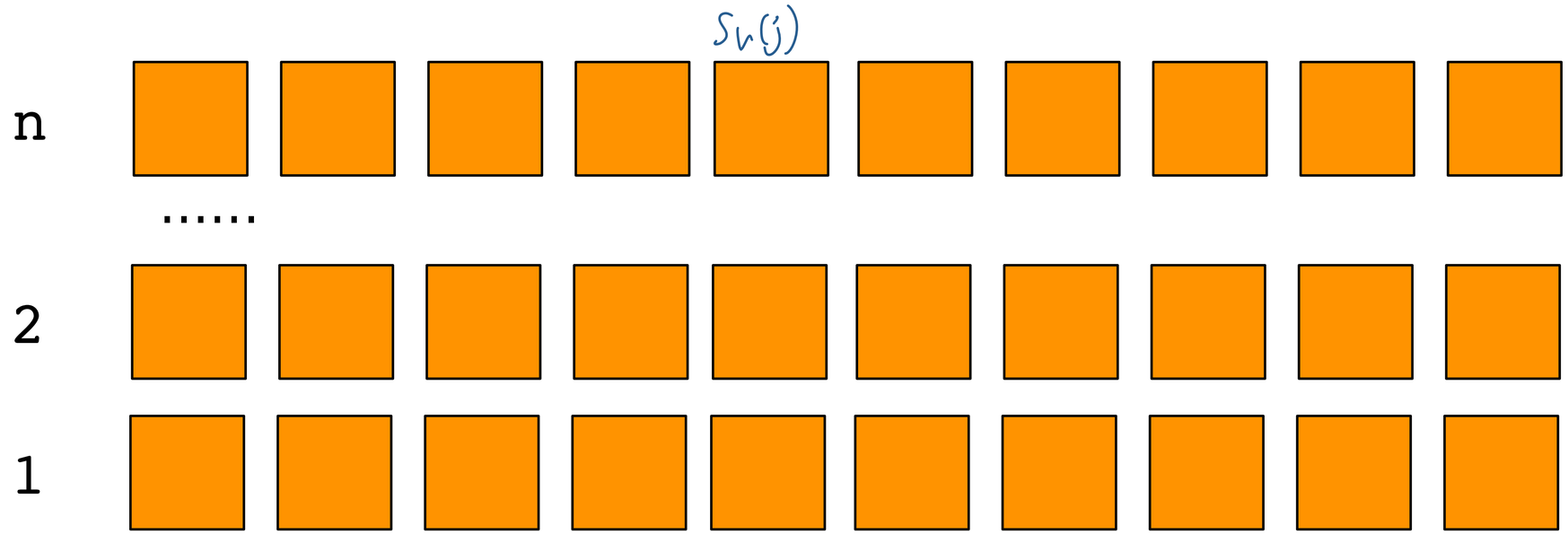


ALGORITHM

start at bottom of picture. initialize $S_1(i) = e(1,i)$

for `i=2,n` use formula to compute $S_{i+1}(\cdot)$

$$S_i(j) = e(i,j) + \min \begin{cases} S_{i-1}(j-1) \\ S_{i-1}(j) \\ S_{i-1}(j+1) \end{cases}$$



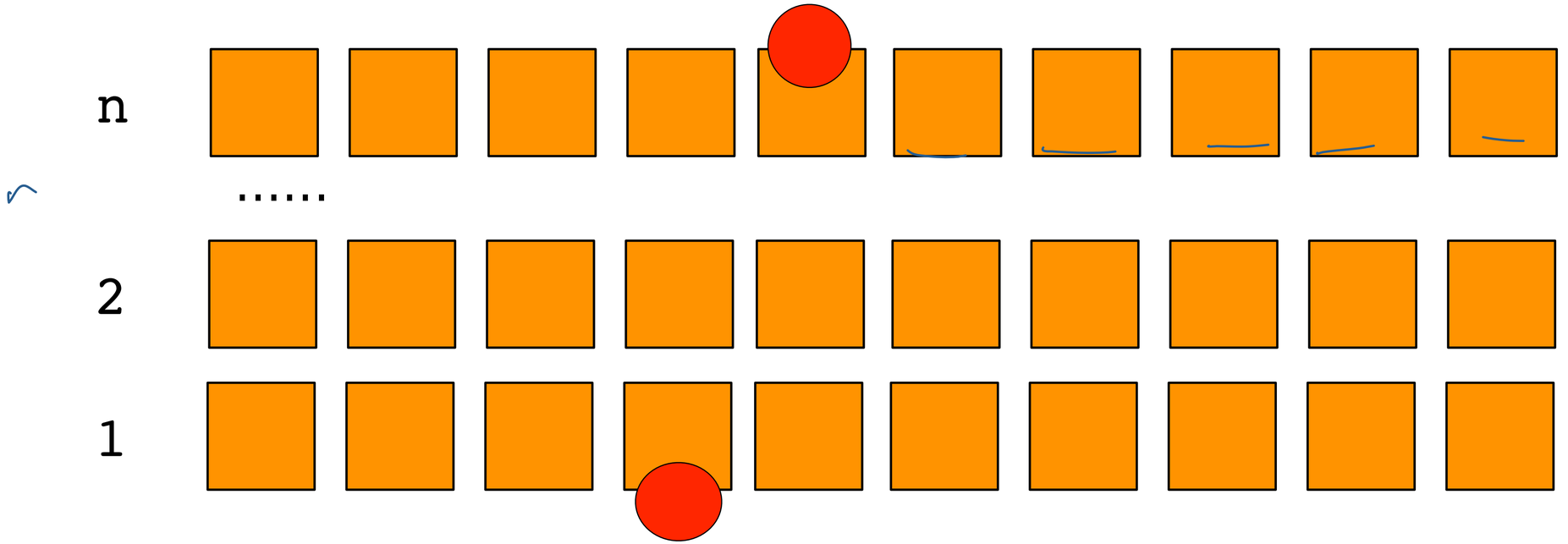
ALGORITHM

start at bottom of picture. initialize $S_1(i) = e(1,i)$

for `i=2,n` use formula to compute $S_{i+1}(\cdot)$

$$S_i(j) = e(i,j) + \min \begin{cases} S_{i-1}(j-1) \\ S_{i-1}(j) \\ S_{i-1}(j+1) \end{cases}$$

pick best among top row, backtrack.



RUNNING TIME

start at bottom of picture. initialize $S_1(i) = e(1, i)$

for `i=2,n` use formula to compute $S_{i+1}(\cdot)$

$$S_i(j) = e(i, j) + \min \begin{cases} S_{i-1}(j-1) \\ S_{i-1}(j) \\ S_{i-1}(j+1) \end{cases}$$

pick best among top row, backtrack.