2550 Intro to cybersecurity

L13: Signatures

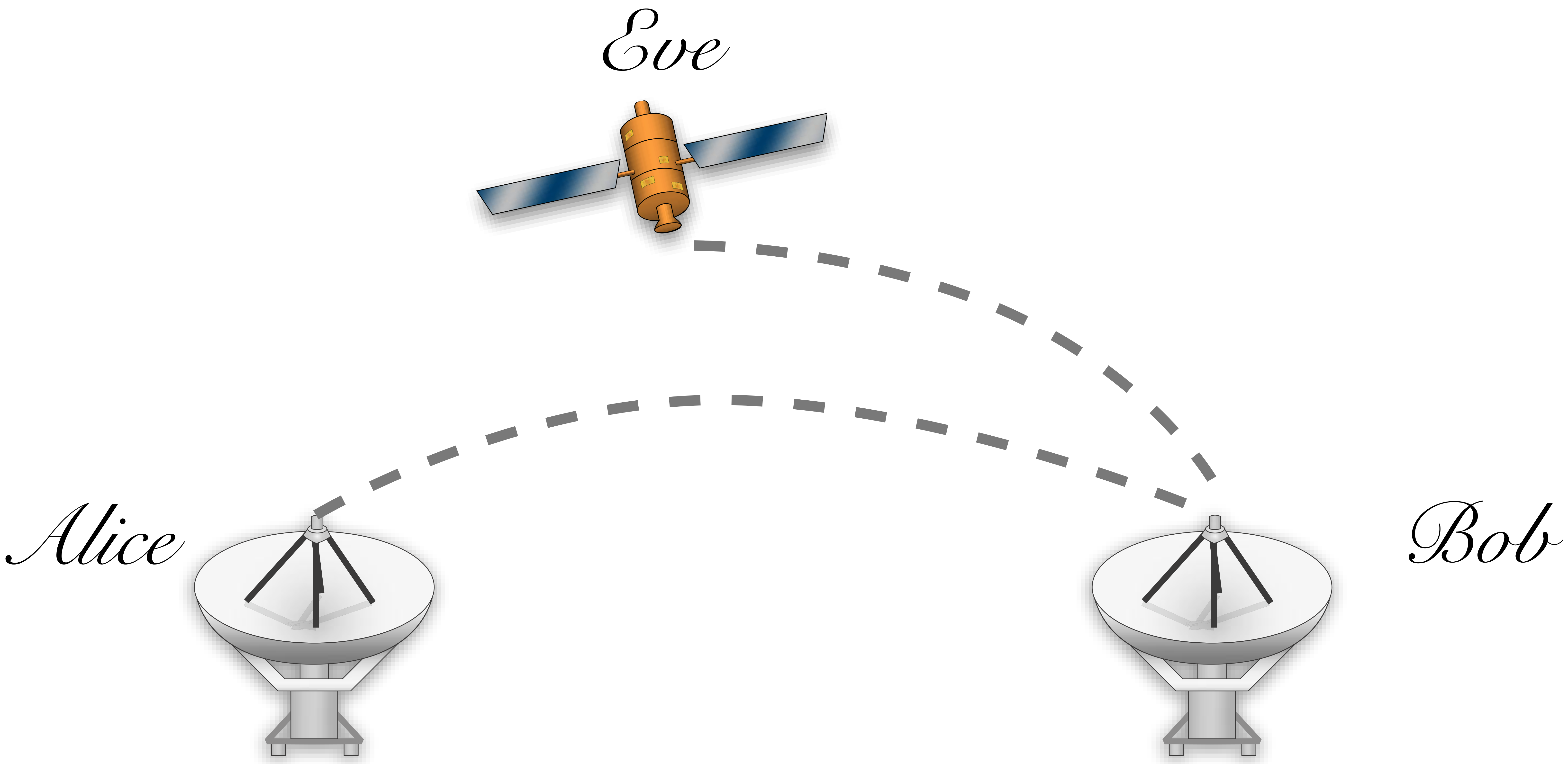
abhi shelat

Recap

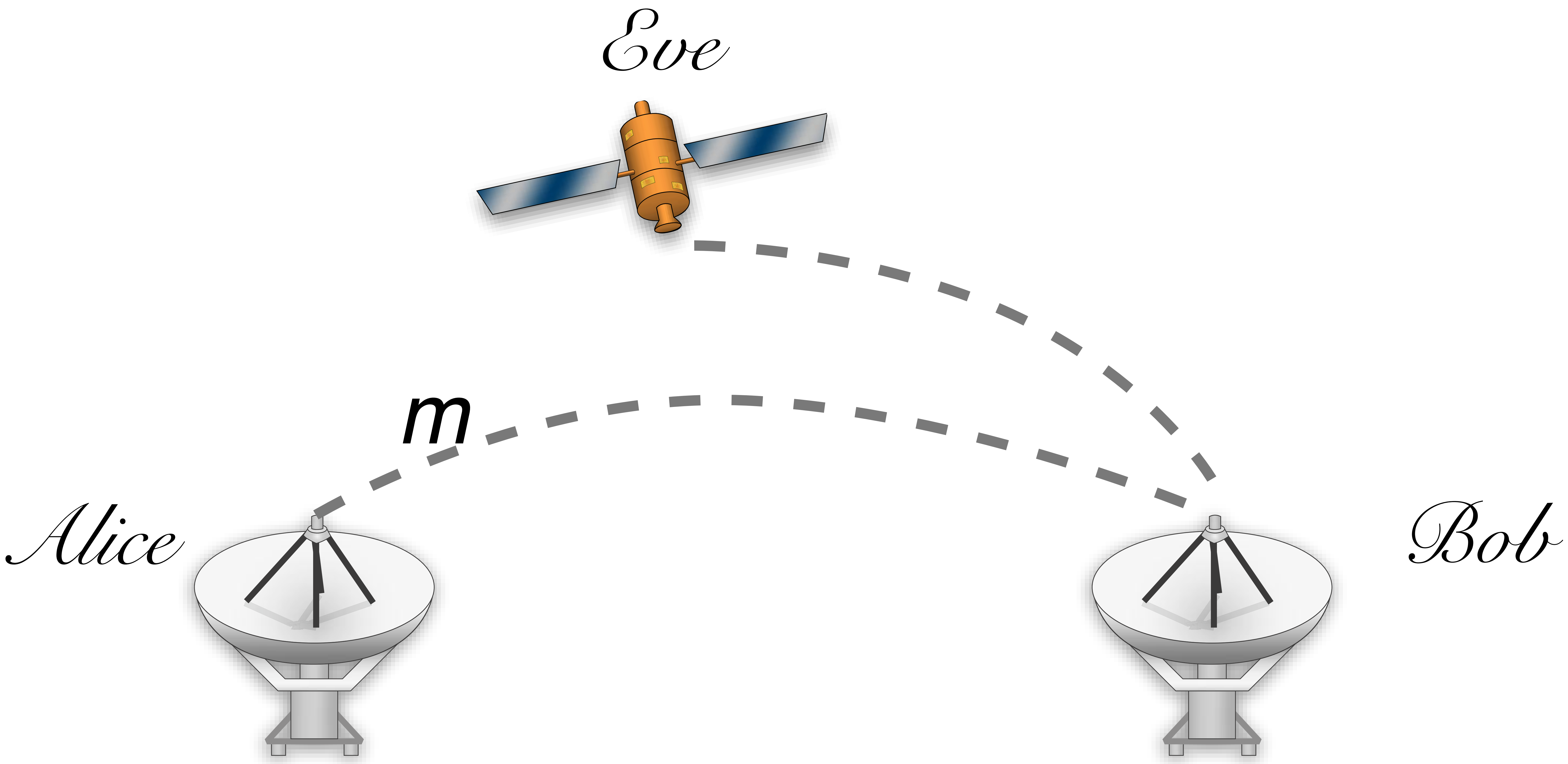
Very old problem

John Hancock

New Problem



New Problem



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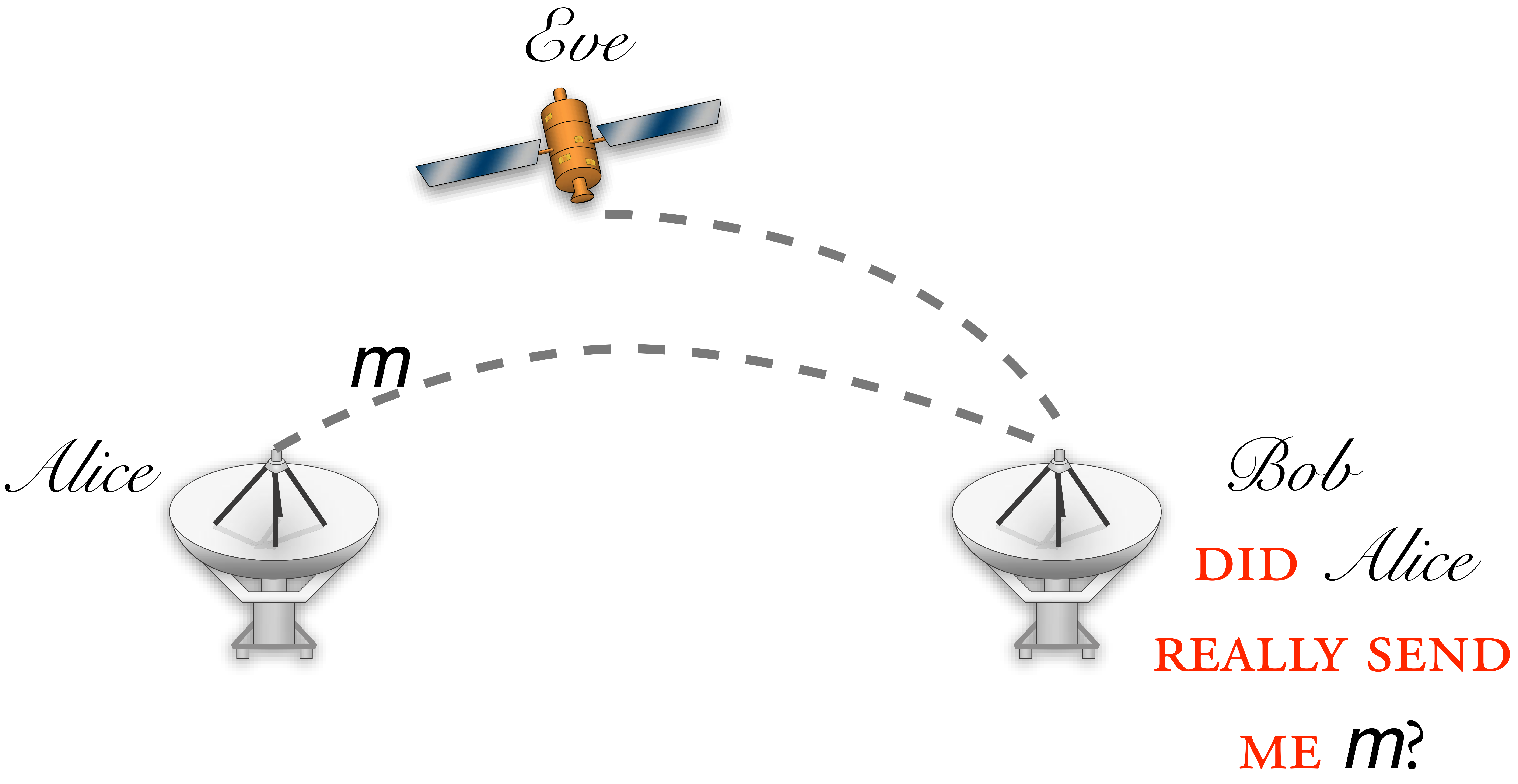
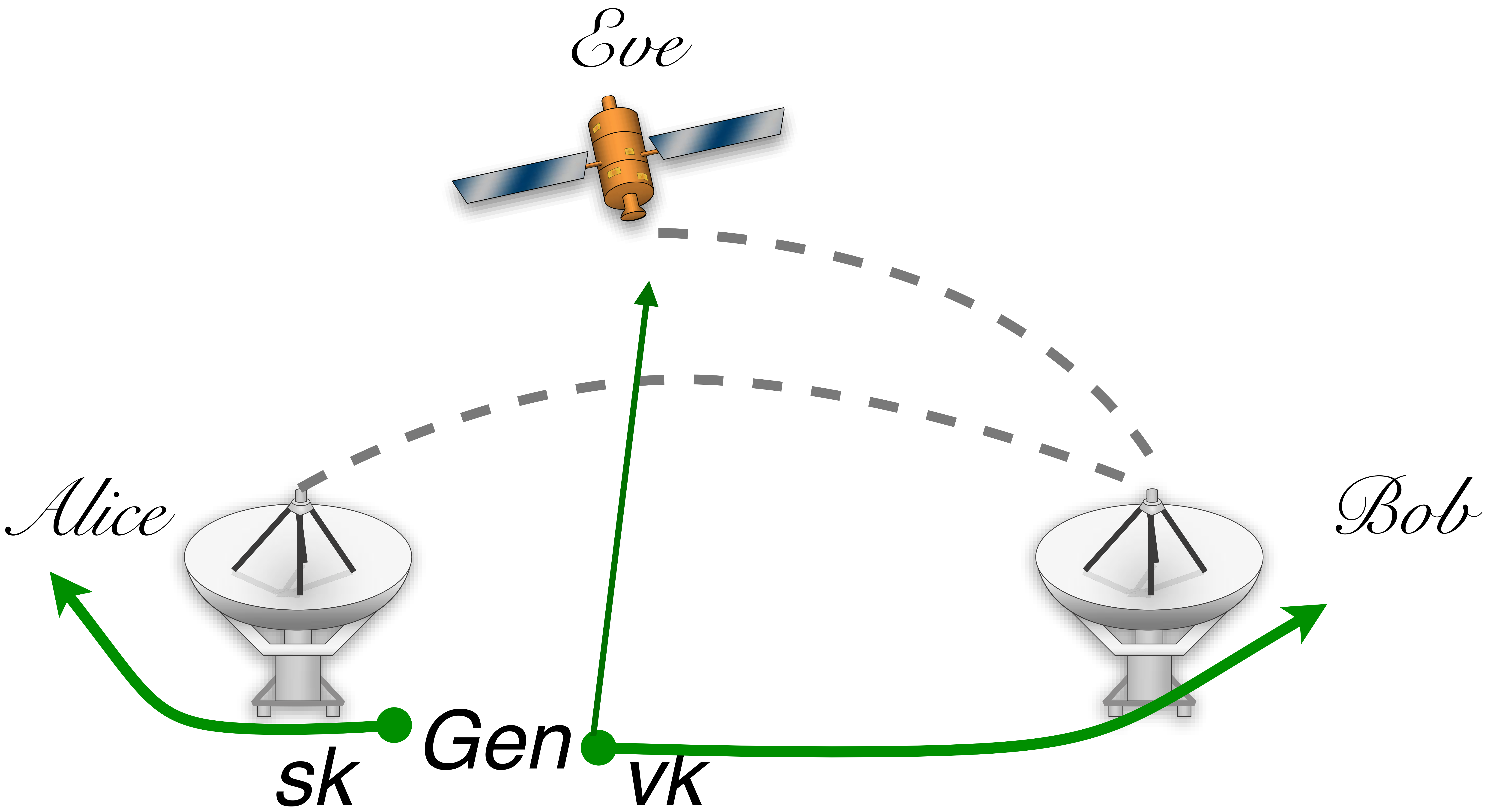


Public key digital signature



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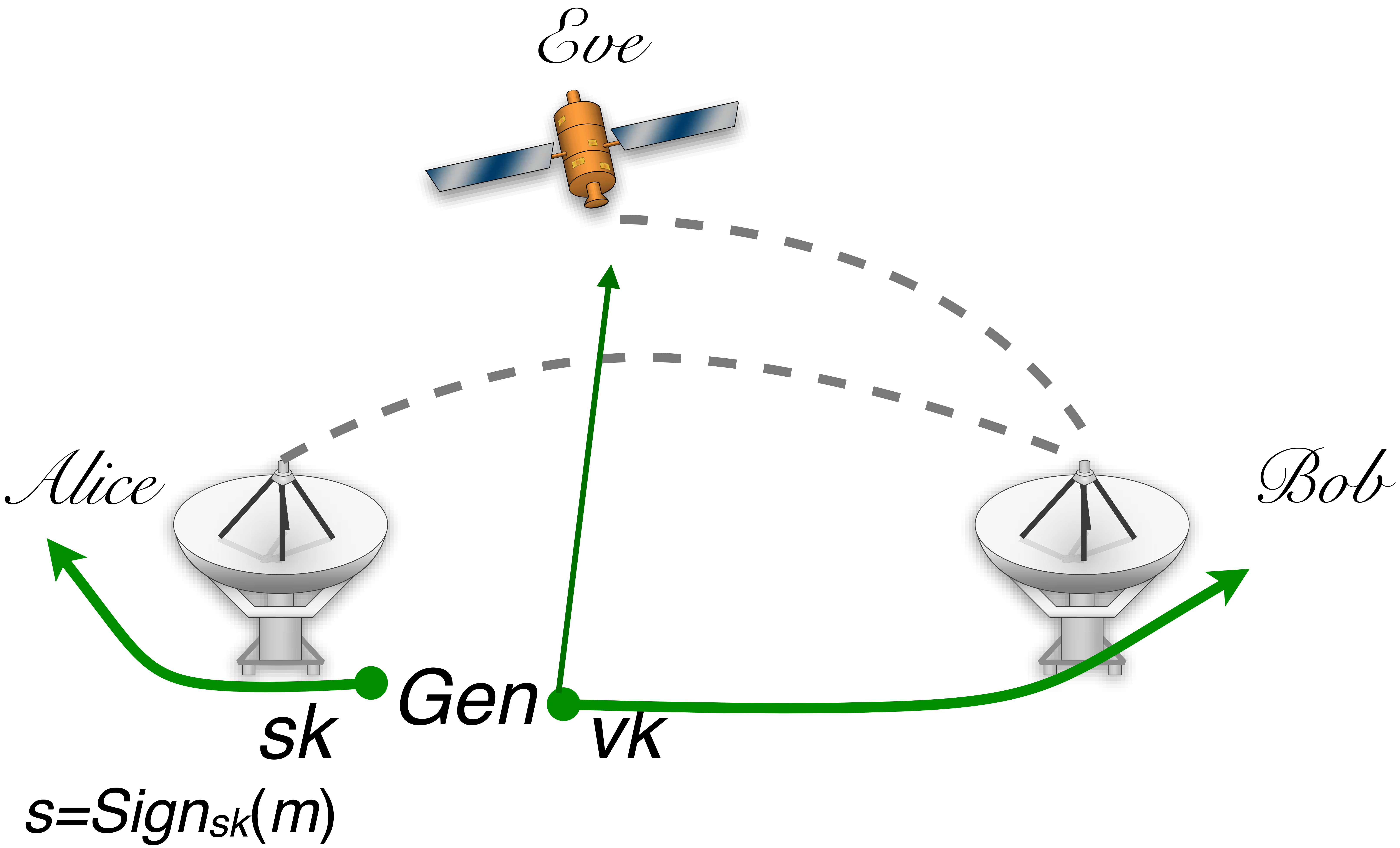

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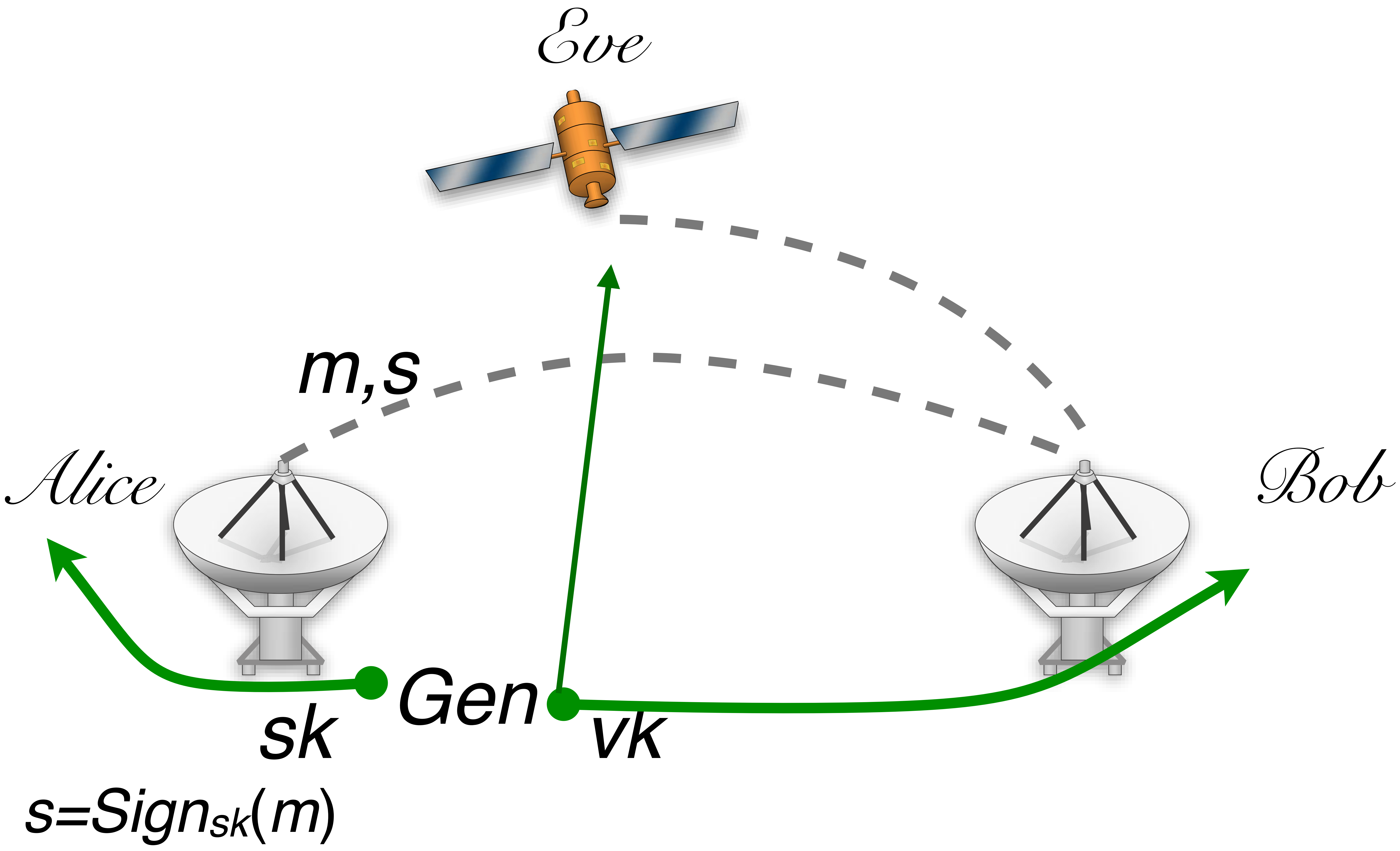

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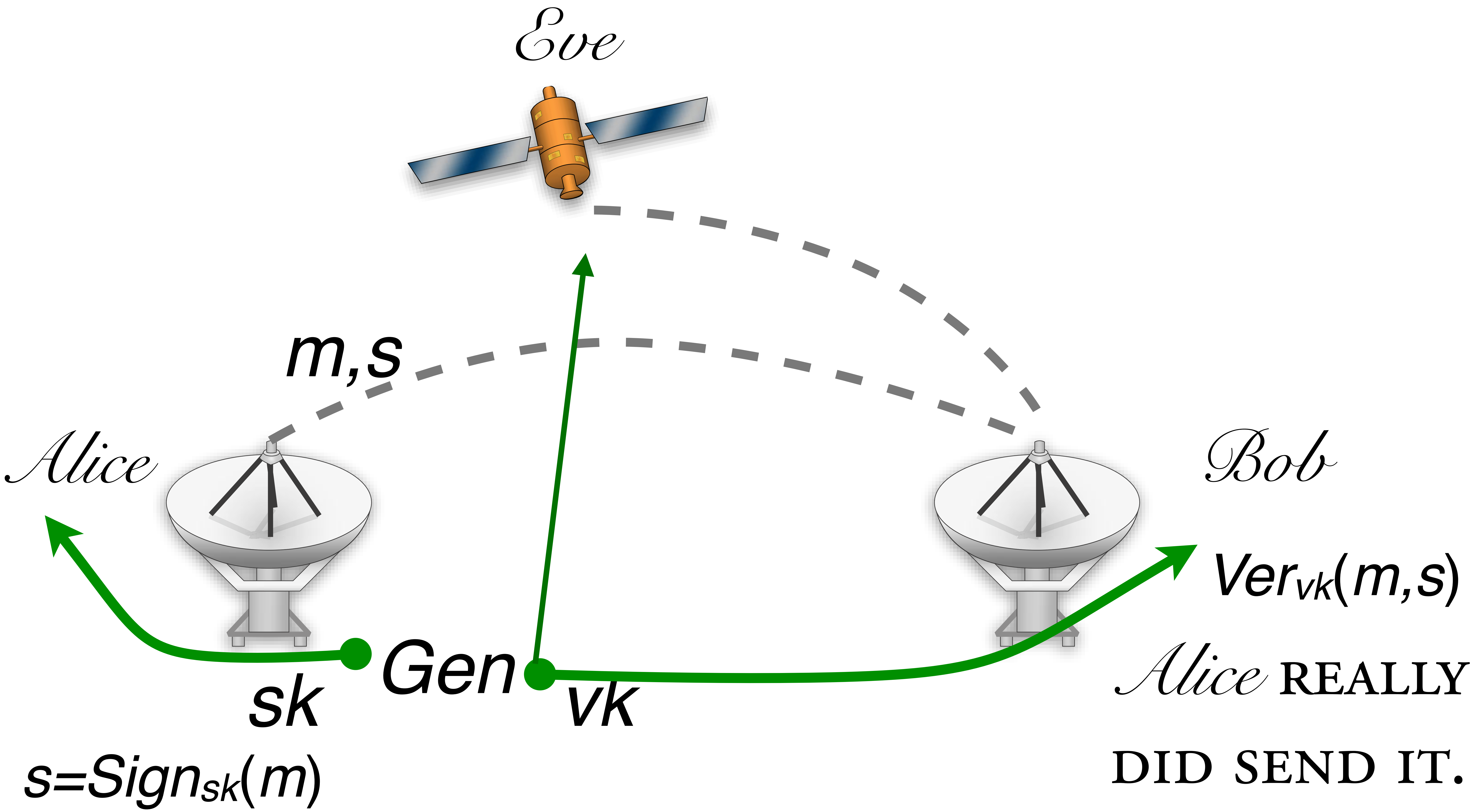


Public key digital signature

MESSAGE SPACE $\{\mathcal{M}\}_n$

$Gen(1^n)$

$Sign_{sk}(m)$

$Ver_{vk}(m,s)$

Public key digital signature

MESSAGE SPACE $\{\mathcal{M}\}_n$

$Gen(1^n)$ GENERATES A KEY PAIR sk, vk

$Sign_{sk}(m)$

$Ver_{vk}(m,s)$

Public key digital signature

MESSAGE SPACE $\{\mathcal{M}\}_n$

Gen(1^n) GENERATES A KEY PAIR *sk,vk*

Sign$_{sk}$(*m*) GENERATES A SIGNATURE *s* FOR $m \in \mathcal{M}_n$

Ver$_{vk}$(*m,s*)

Public key digital signature

MESSAGE SPACE $\{\mathcal{M}\}_n$

Gen(1^n) GENERATES A KEY PAIR *sk,vk*

Sign$_{sk}$(*m*) GENERATES A SIGNATURE *s* FOR $m \in \mathcal{M}_n$

Ver$_{vk}$(*m,s*) ACCEPTS OR REJECTS A MSG,SIG PAIR

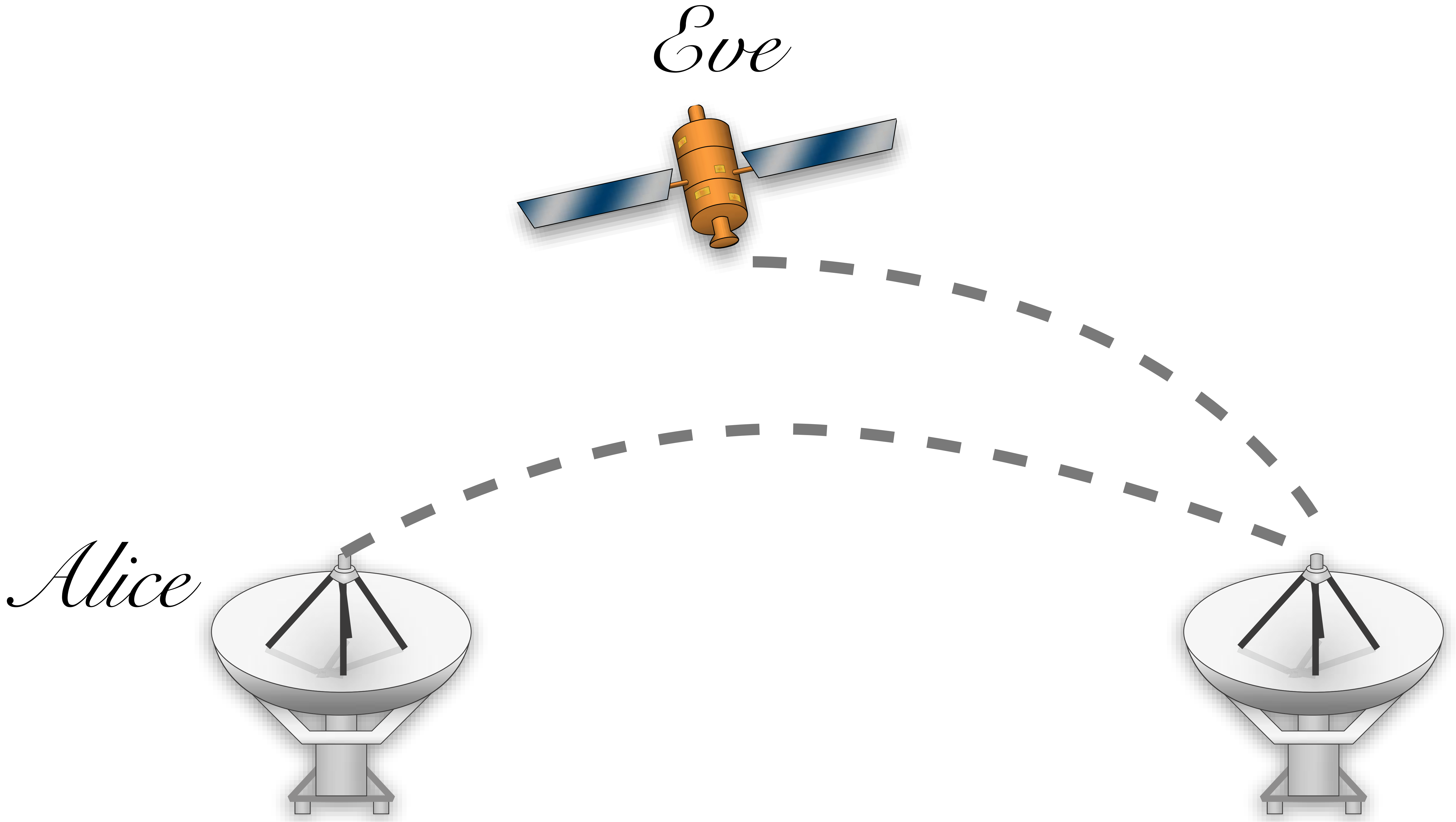
$$\Pr[k \leftarrow Gen(1^n) : Ver_{vk}(m, Sign_{sk}(m)) = 1] = 1$$

existential unforgability

"EVEN WHEN GIVEN A SIGNING ORACLE,

AN ADVERSARY CANNOT FORGE A SIGNATURE FOR

ANY MESSAGE OF ITS CHOOSING"



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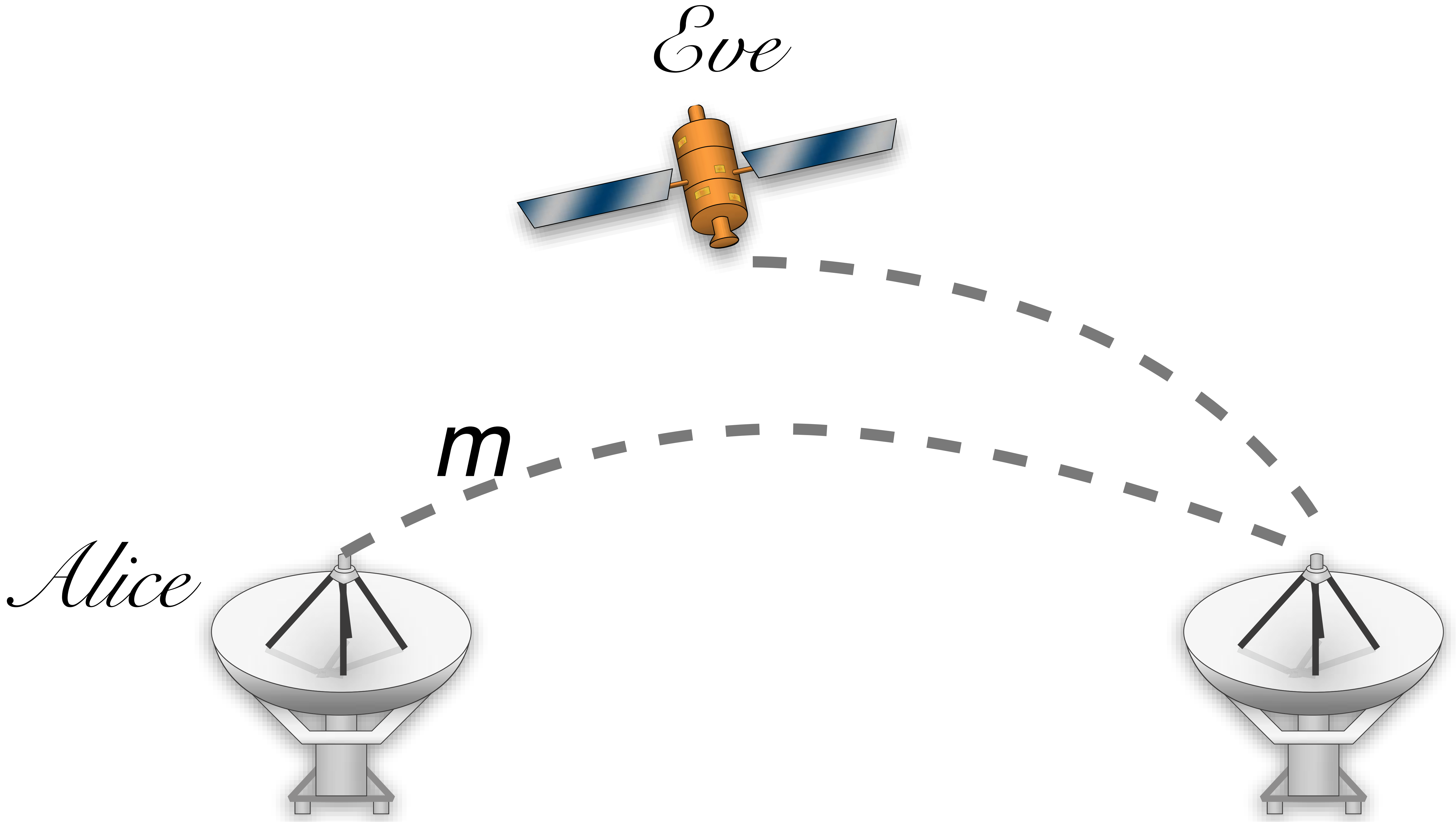
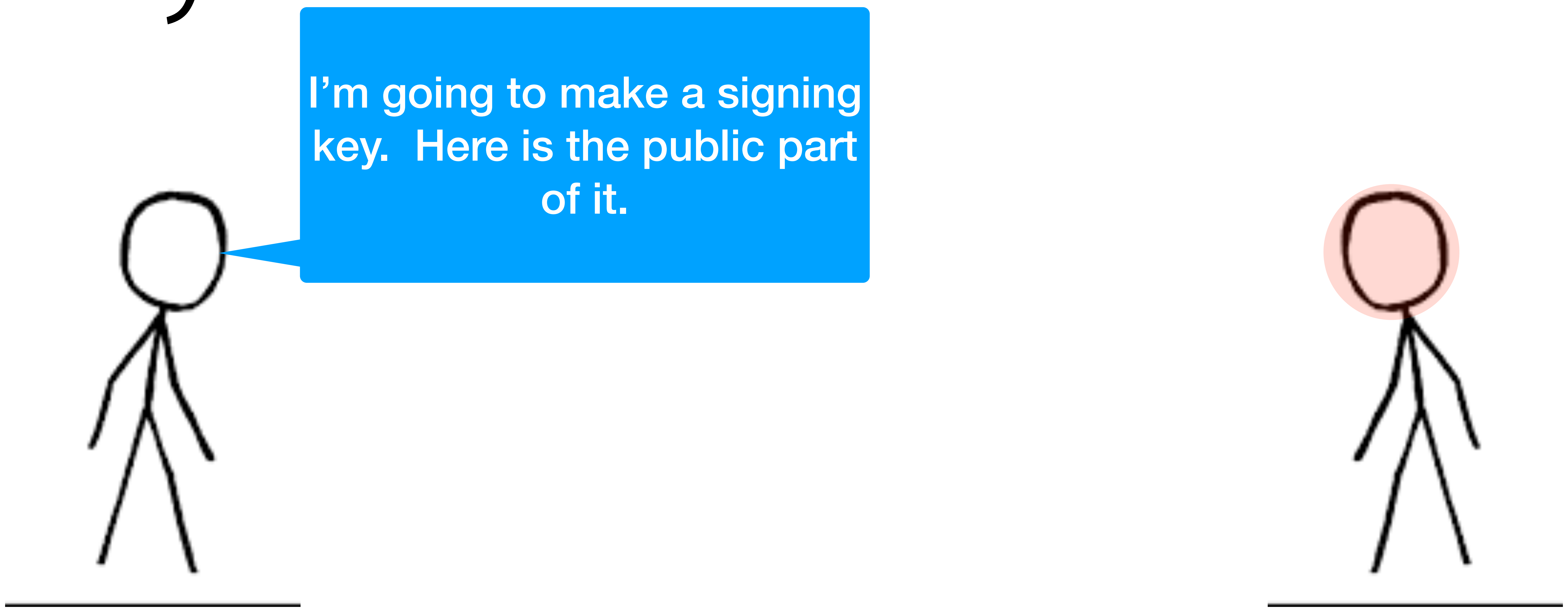


Signature security



$$(vk, sk) \leftarrow \mathrm{Gen}(1^n)$$

Signature security

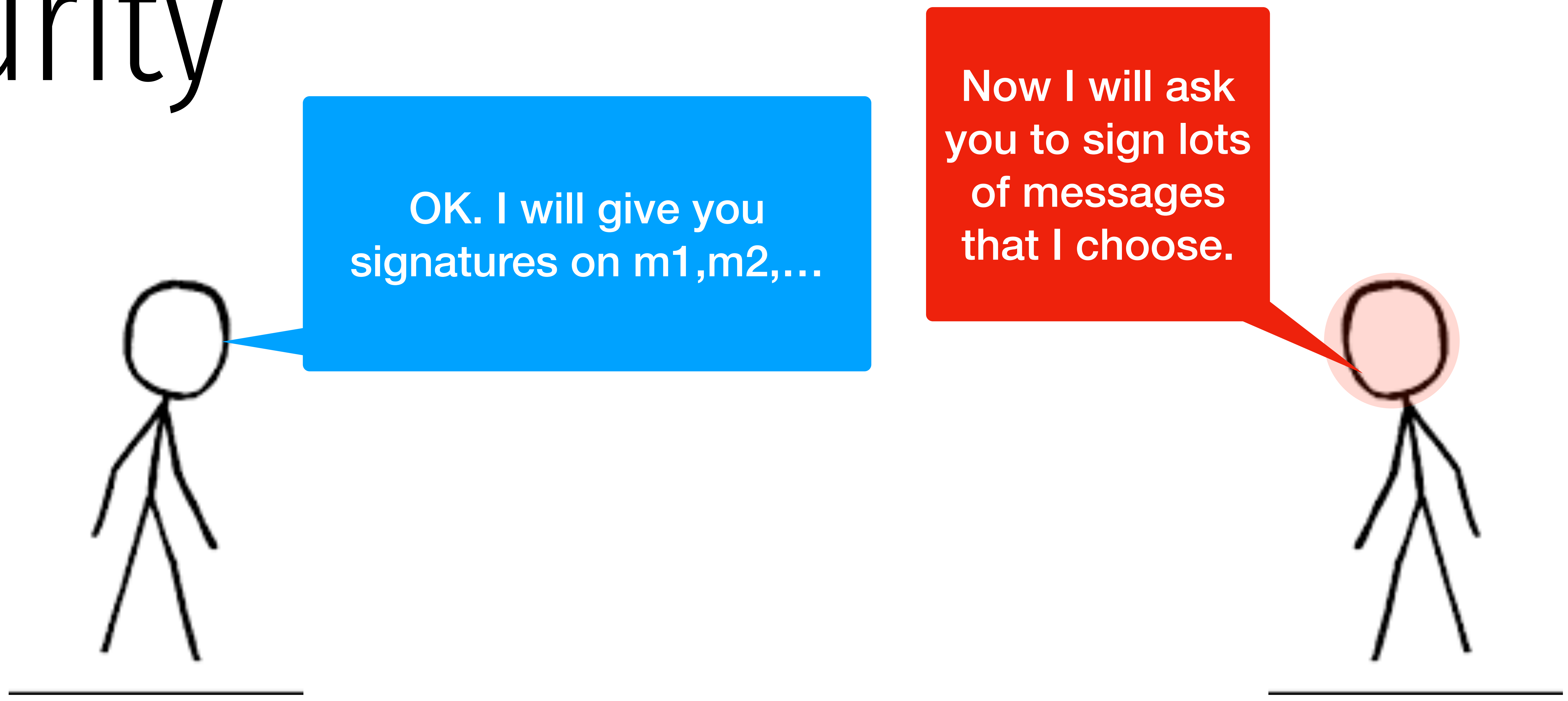
$(vk, sk) \leftarrow \text{Gen}(1^n)$

Now I will ask you to sign lots of messages that I choose.

$m_0, m_1, \ldots$

vk

Signature security



$$(vk, sk) \leftarrow \text{Gen}(1^n)$$

$$vk$$

$$s_i \leftarrow \mathsf{Sign}_{sk}(m_i)$$

Signature security

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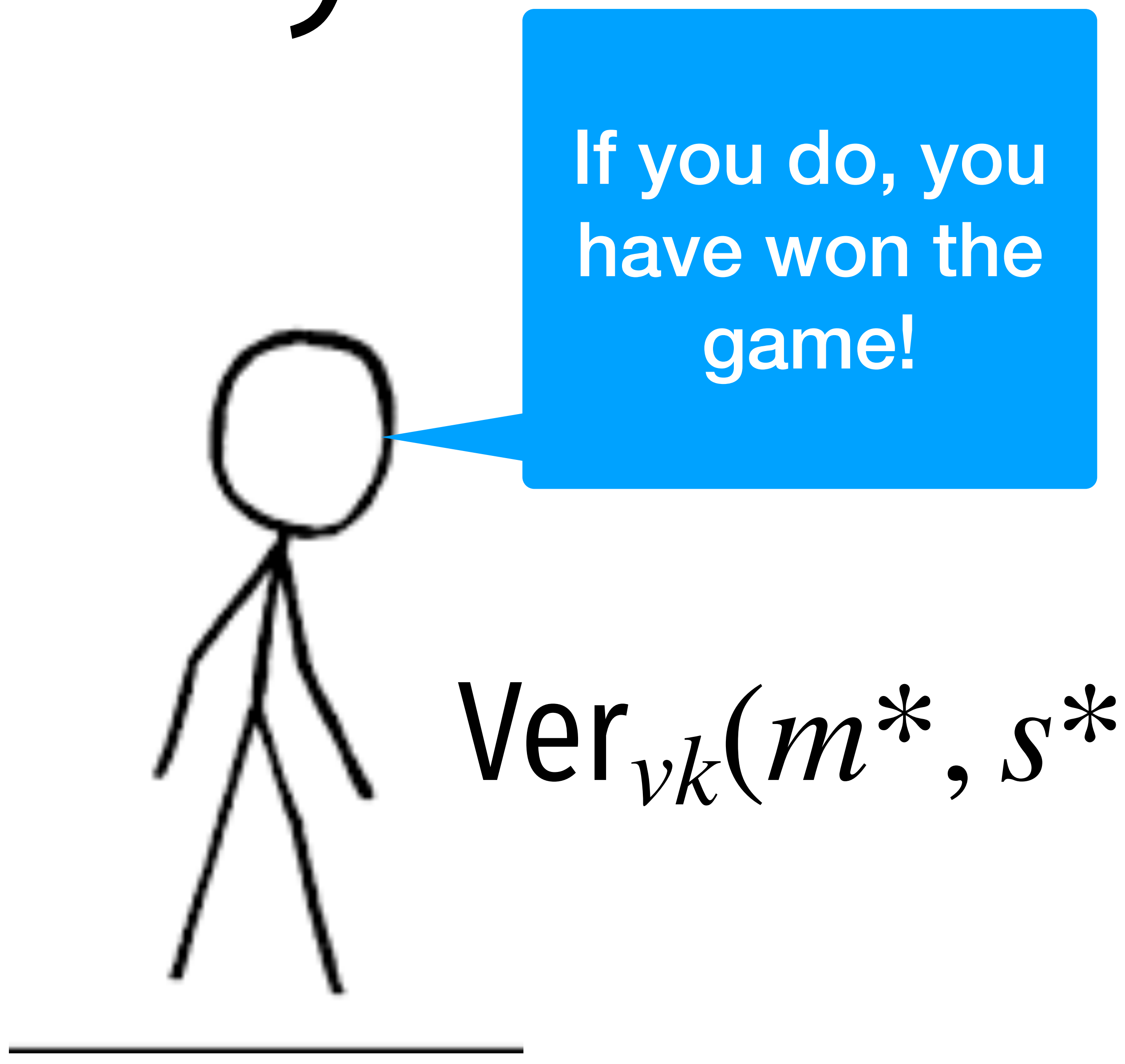
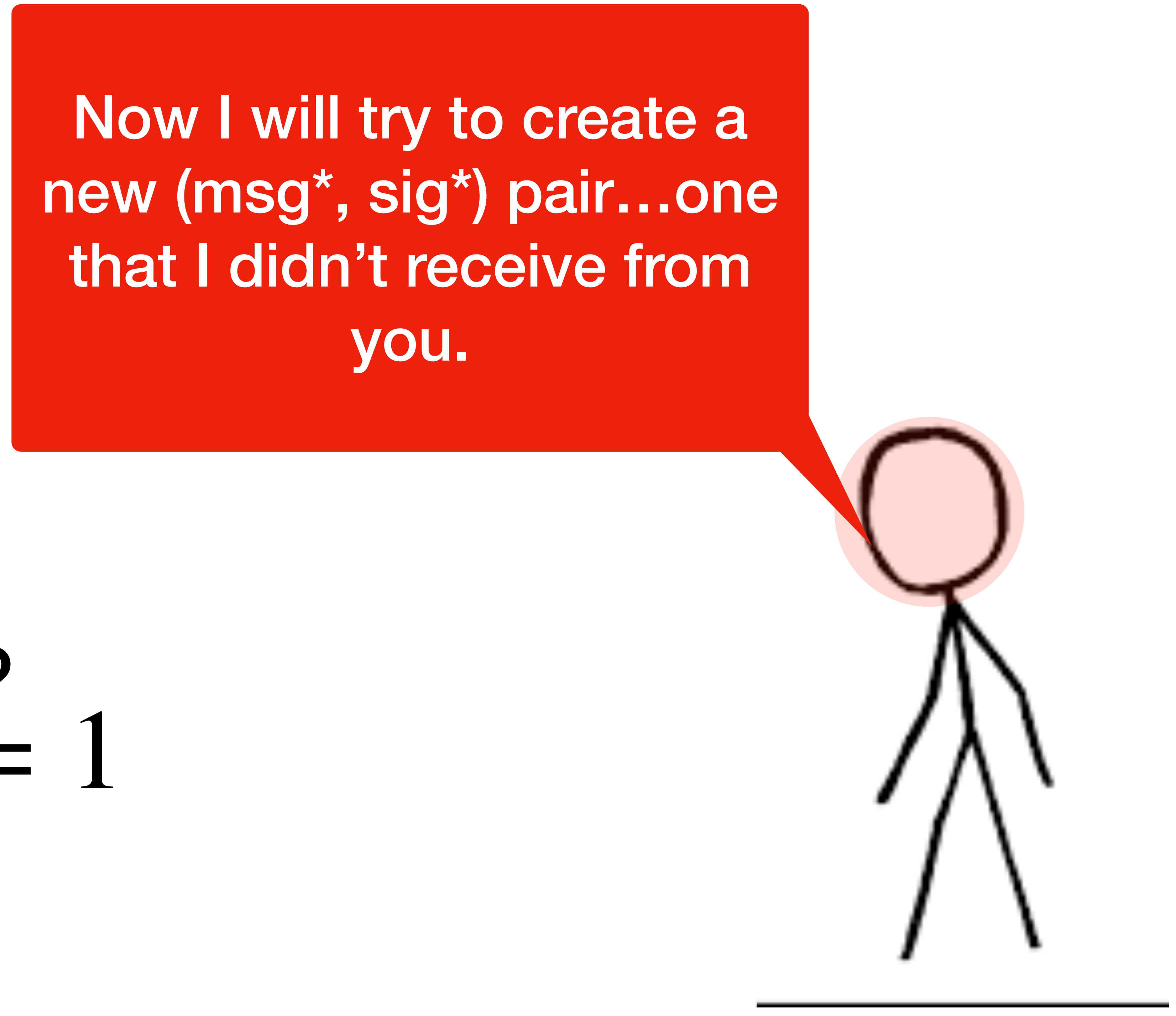
$s_1, s_2, \ldots$

Signature security



$$\mathsf{Ver}_{vk}(m^*, s^*) \stackrel{?}{=} 1$$



FOR ALL NON-UNIFORM PPT *A*

$$\Pr\left[\begin{array}{l}(vk, sk) \leftarrow Gen(1^n); (m, s) \leftarrow A^{Sign_{sk}(\cdot)} : \\ \quad Ver_{vk}(m, s) = 1 \\ \quad \text{AND } A \text{ DIDN'T QUERY } m\end{array}\right] < \mu(n)$$

Textbook RSA Signatures (insecure)

Pick N = p*q where p,q are primes.

Pick e,d such that $e \cdot d = 1 \bmod \phi(N)$

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Sign((sk=d, N) m):

Compute the signature: $\sigma \leftarrow m^d \bmod N$

Verify((pk=e, N), σ, m):

$$m \stackrel{?}{=} \sigma^e \bmod N$$

Textbook RSA (insecure) example

Lets pick a key N = 443 * 919 = 407177.

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Verify the signature (“22”, 84760) :

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Why is this scheme insecure?

Given the signature pair (“22” = 12850, 84760) ,
what is the signature on 12850 * 12850 = 165122500 ?

RSA Signatures (PKCSv1.5)

(Randomized padding to prevent basic forgery attacks. Widely used, but first full security proof was written in 2018.)

Sign((sk, N) m):

Compute the padding: $z \leftarrow 00 \cdot 01 \cdot FF \cdots FF \cdot 00 \cdot \mathsf{ID}_H \cdot H(m)$

Compute the signature: $\sigma \leftarrow z^{sk} \bmod N$

Speed

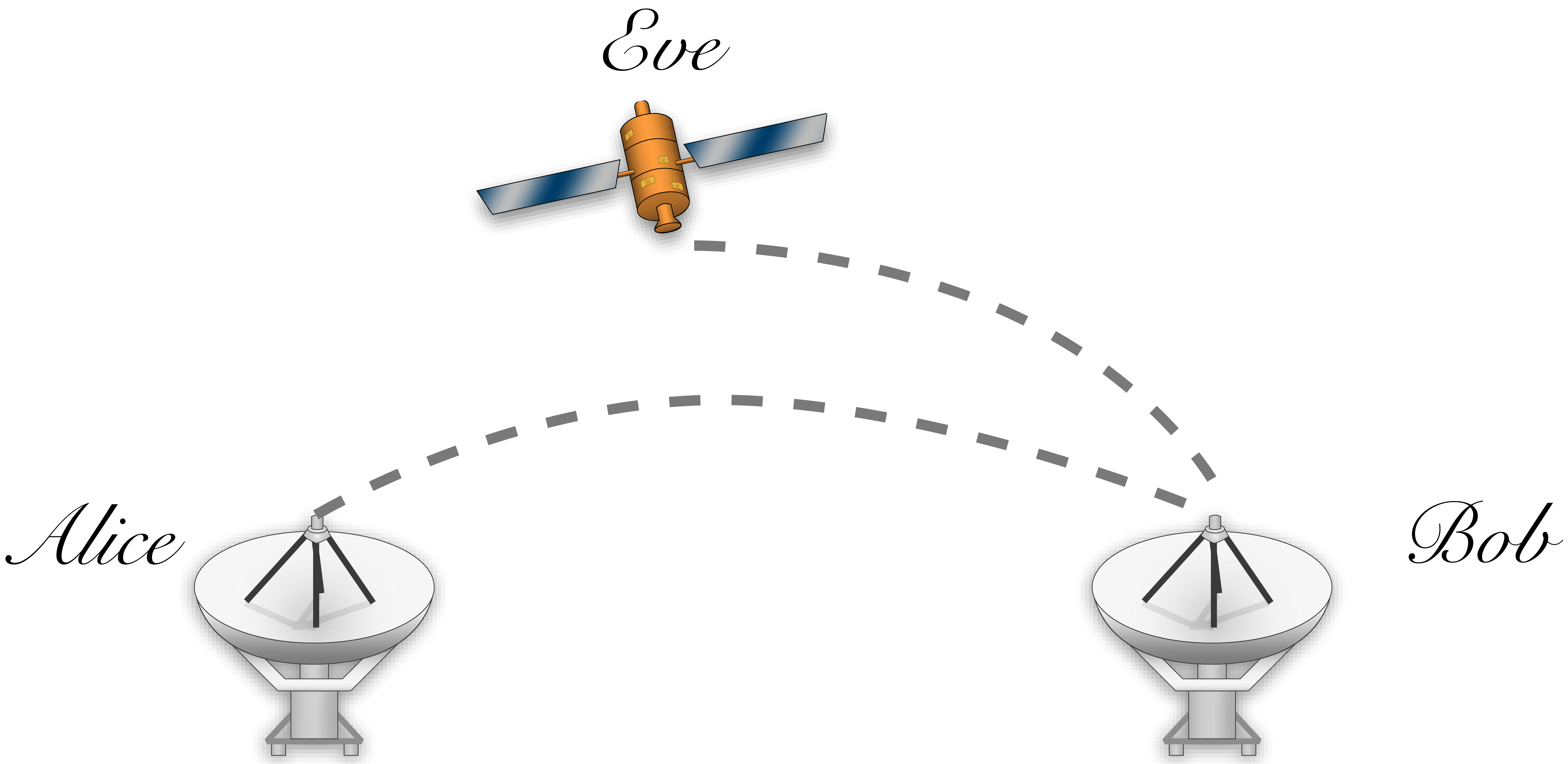
openssl speed rsa dsa ecdsa

```
Doing 1024 bits private rsa's for 10s: 86688 1024 bits private RSA's in 9.99s
Doing 1024 bits public  rsa's for 10s: 1341152 1024 bits public RSA's in 10.00s
Doing 2048 bits private rsa's for 10s: 13154 2048 bits private RSA's in 9.99s
Doing 2048 bits public  rsa's for 10s: 437080 2048 bits public RSA's in 10.00s
Doing 3072 bits private rsa's for 10s: 4243 3072 bits private RSA's in 10.00s
Doing 3072 bits public  rsa's for 10s: 211605 3072 bits public RSA's in 10.00s
Doing 4096 bits private rsa's for 10s: 1845 4096 bits private RSA's in 9.99s
Doing 4096 bits public  rsa's for 10s: 125130 4096 bits public RSA's in 9.99s

Doing 1024 bits sign   dsa's for 10s: 74467 1024 bits DSA signs in 9.95s
Doing 1024 bits verify dsa's for 10s: 95863 1024 bits DSA verify in 9.99s
Doing 2048 bits sign   dsa's for 10s: 30197 2048 bits DSA signs in 9.97s
Doing 2048 bits verify dsa's for 10s: 33802 2048 bits DSA verify in 10.00s

Doing 256 bits sign   ecdsa's for 10s: 339010 256 bits ECDSA signs in 9.89s
Doing 256 bits verify ecdsa's for 10s: 115106 256 bits ECDSA verify in 10.00s
Doing 384 bits sign   ecdsa's for 10s: 7773 384 bits ECDSA signs in 9.98s
Doing 384 bits verify ecdsa's for 10s: 10066 384 bits ECDSA verify in 10.00s
Doing 521 bits sign   ecdsa's for 10s: 25316 521 bits ECDSA signs in 9.98s
Doing 521 bits verify ecdsa's for 10s: 12896 521 bits ECDSA verify in 9.99s
Doing 283 bits sign   ecdsa's for 10s: 13860 283 bits ECDSA signs in 9.98s
Doing 283 bits verify ecdsa's for 10s: 7028 283 bits ECDSA verify in 9.99s
Doing 409 bits sign   ecdsa's for 10s: 8441 409 bits ECDSA signs in 9.99s
Doing 409 bits verify ecdsa's for 10s: 4309 409 bits ECDSA verify in 9.98s
```

Message Authentication codes



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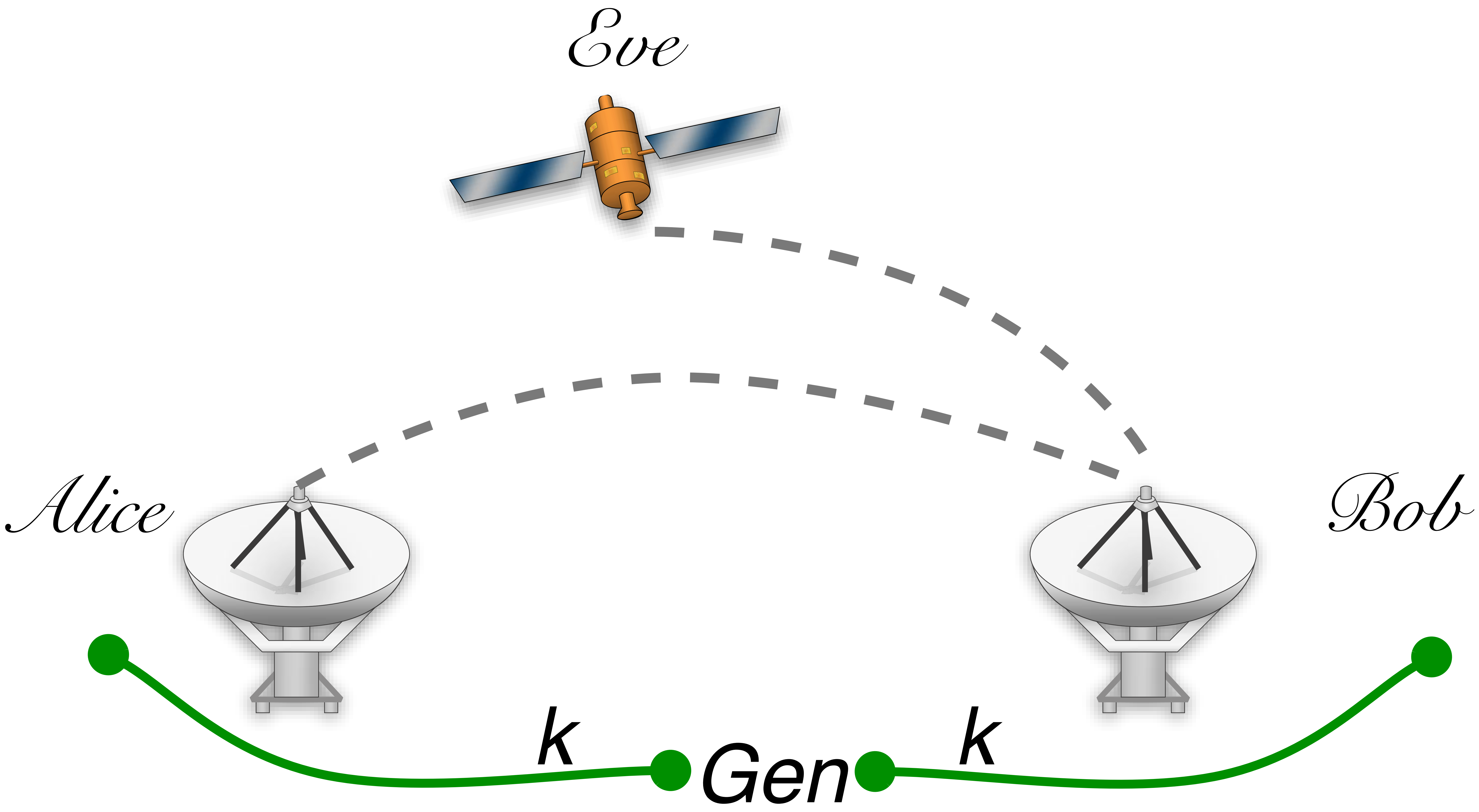

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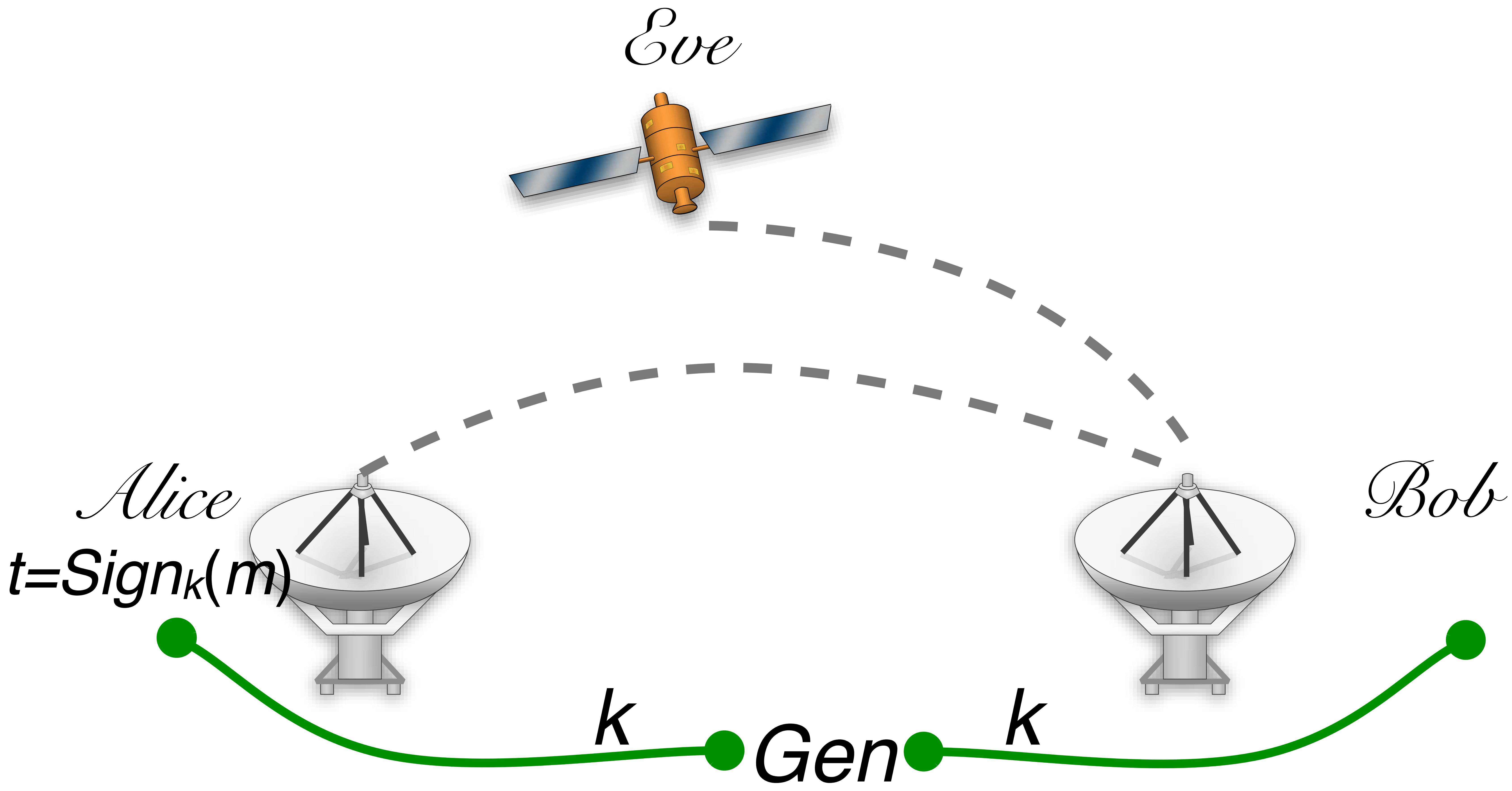

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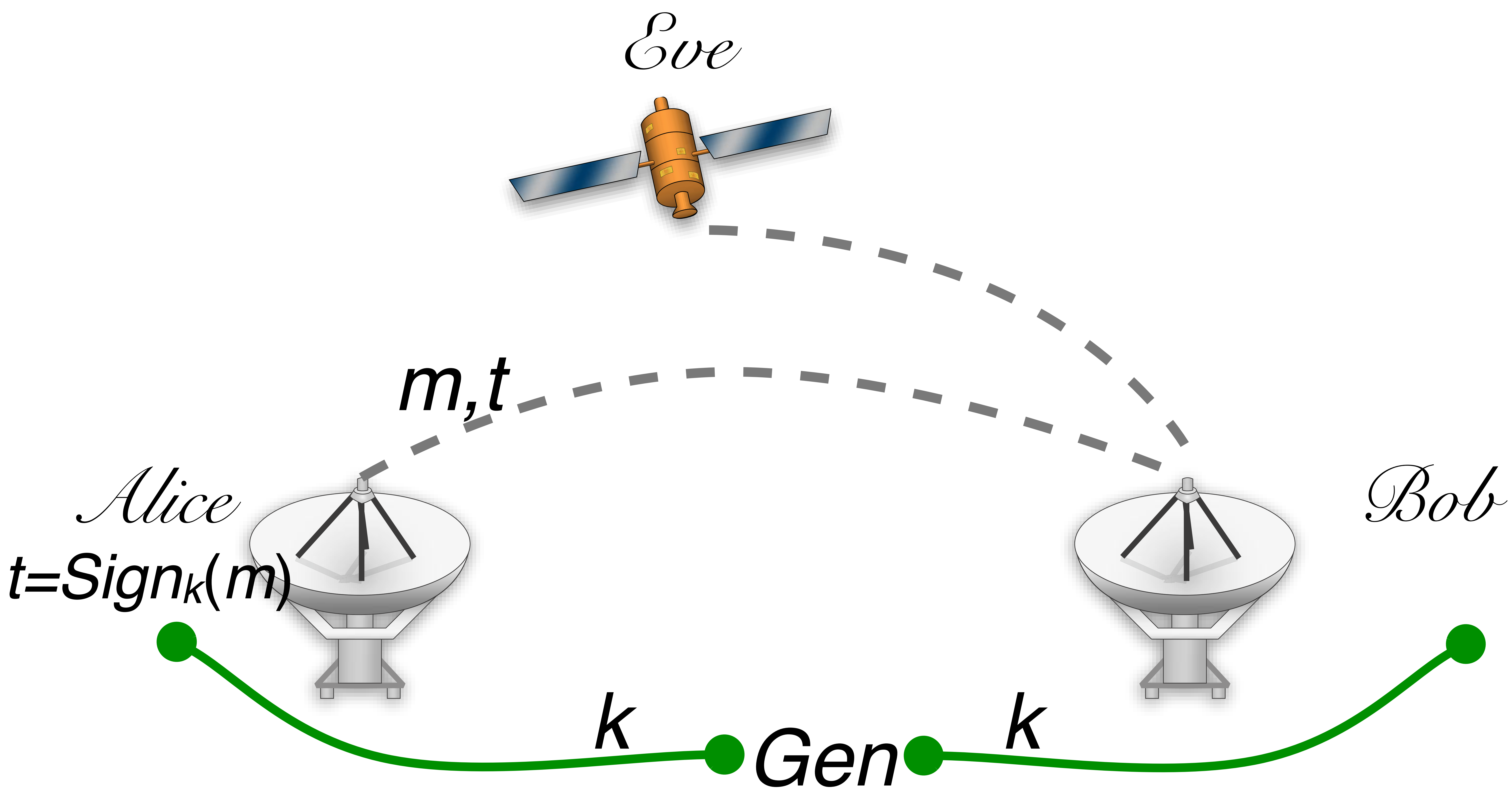

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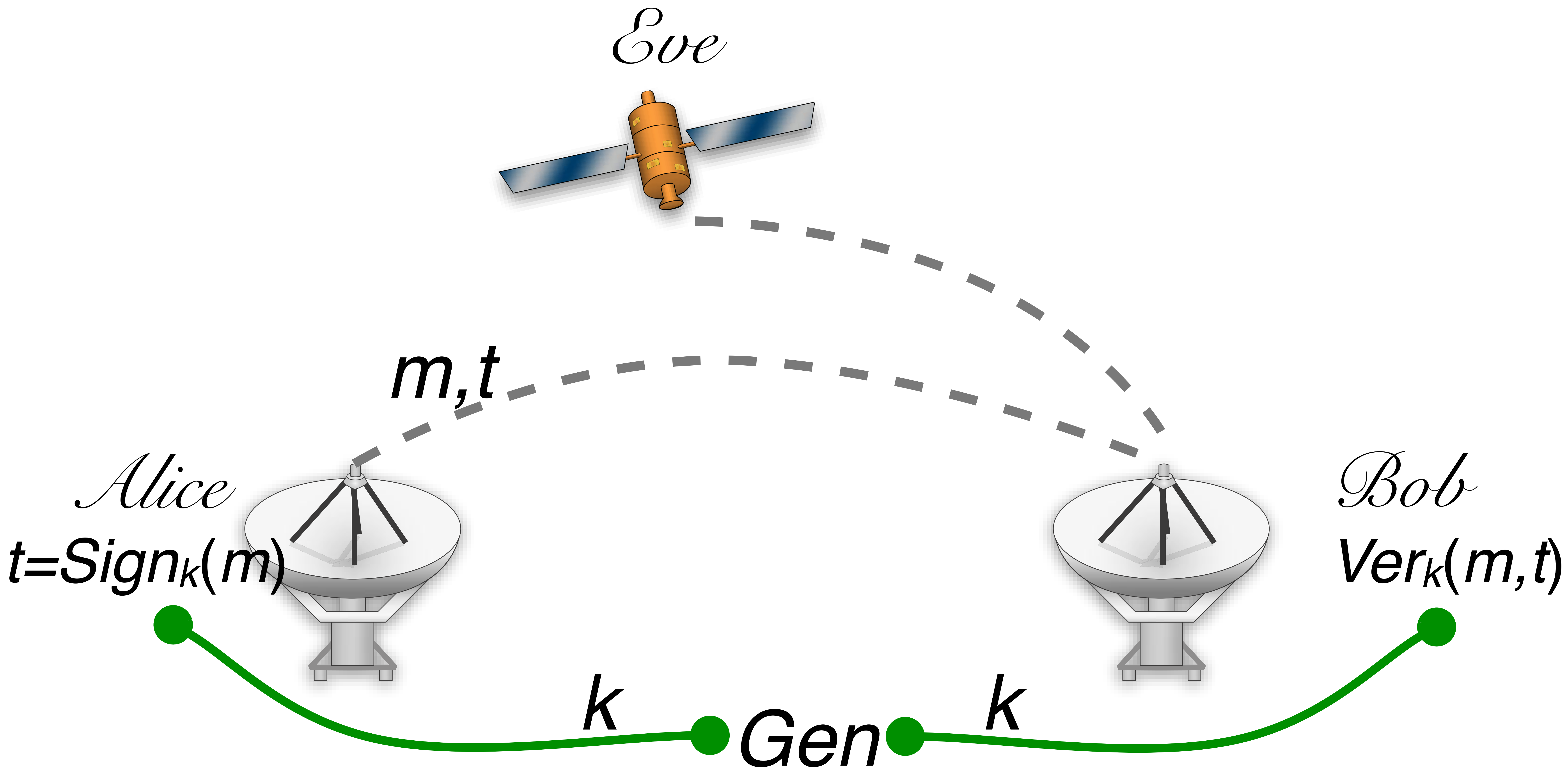

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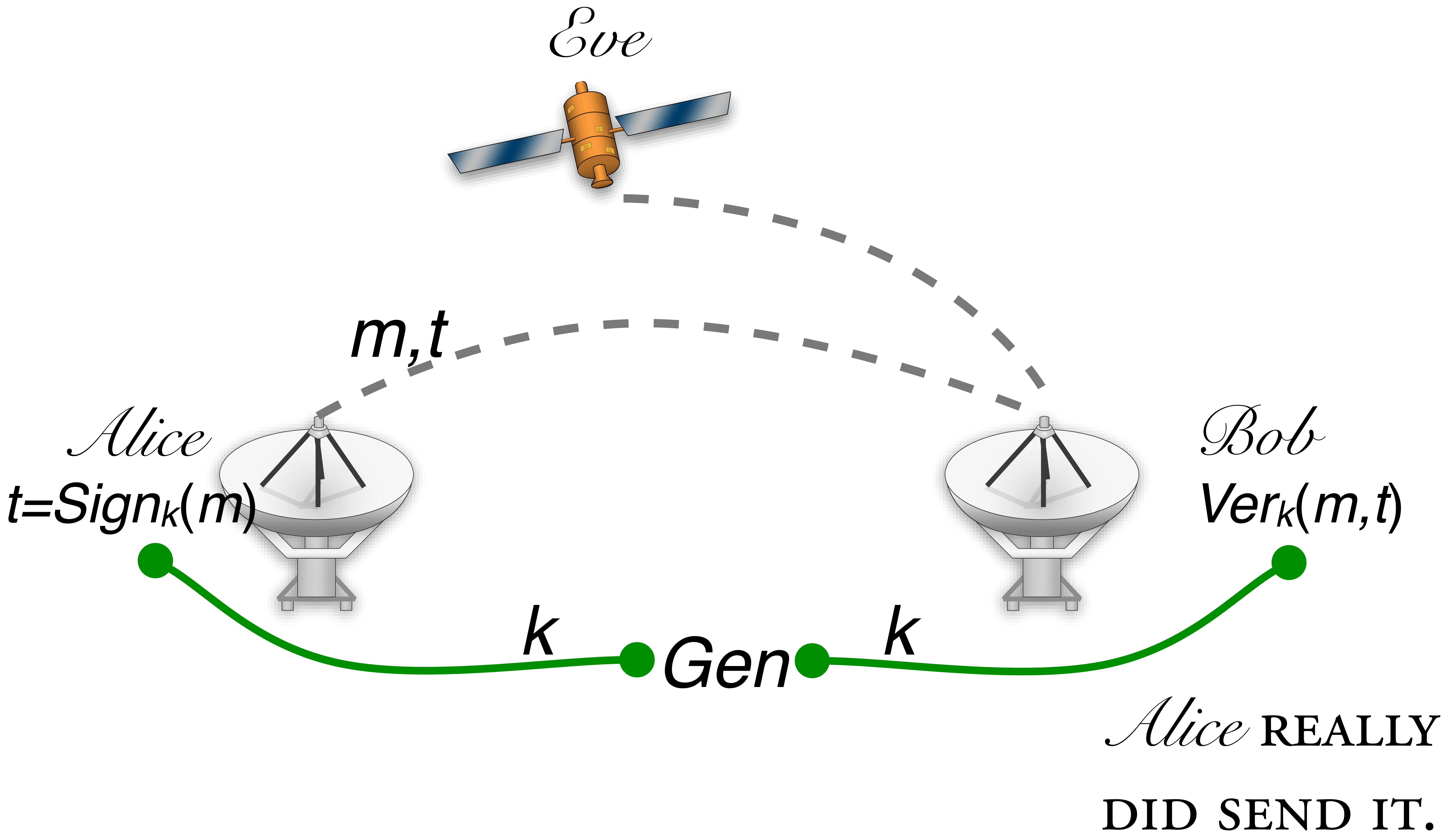


Construction of a MAC

$Gen(1^n)$:

$Sign_k(m)$:

$Ver_k(m,t)$:

Construction of a MAC

LET $\{F_k\}$ BE A PRF FAMILY LIKE AES

Gen(1^n):

Sign$_k$(*m*):

Ver$_k$(*m*,*t*):

Construction of a MAC

LET $\{F_k\}$ BE A PRF FAMILY LIKE AES

Gen(1ⁿ): $k \leftarrow U_n$

Sign$_k$(m):

Ver$_k$(m,t):

Construction of a MAC

LET $\{F_k\}$ BE A PRF FAMILY LIKE AES

Gen(1^n): $k \leftarrow U_n$

Sign_k(m): $t \leftarrow F_k(m)$

Ver_k(m,t):

Construction of a MAC

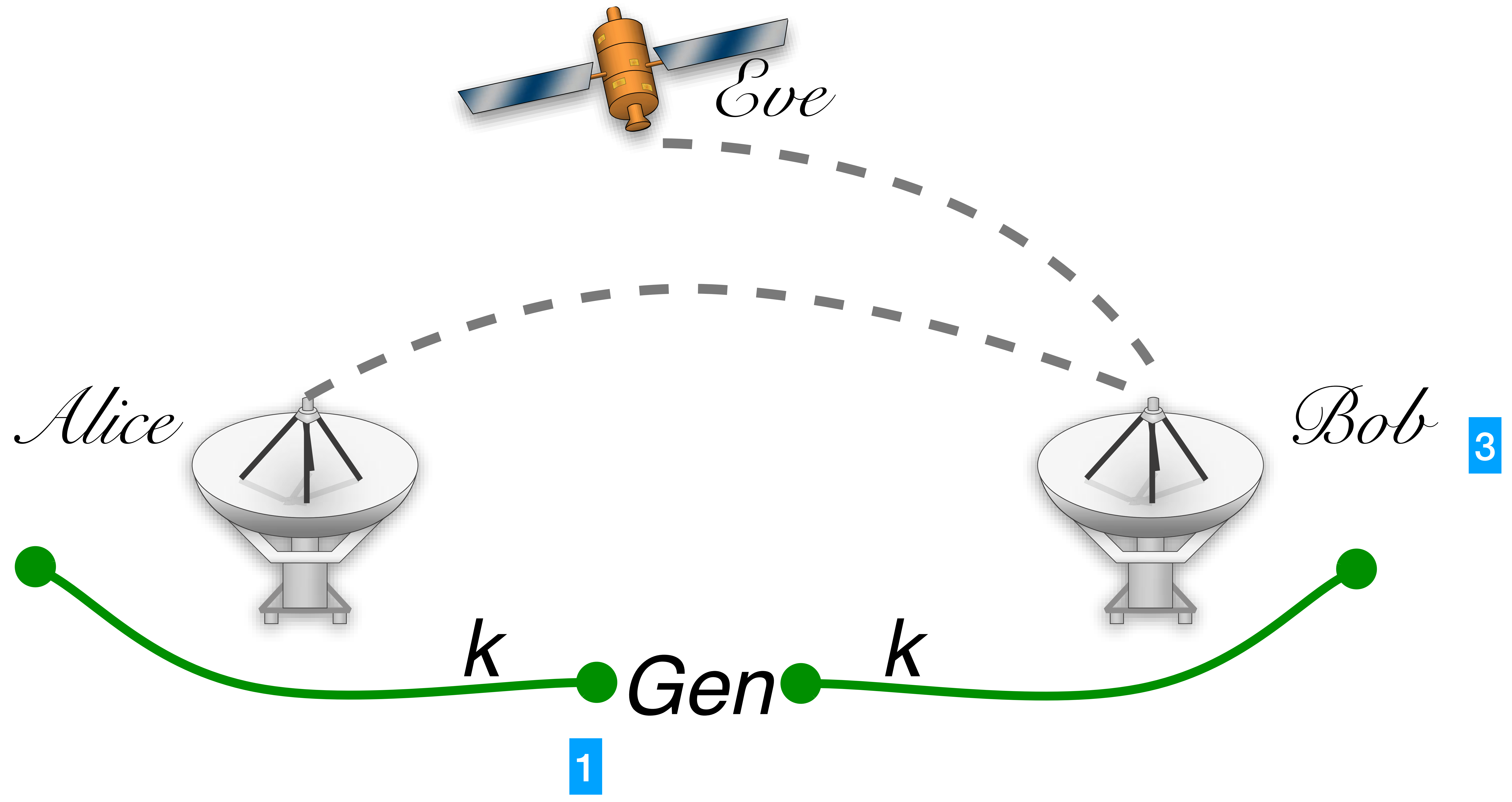
LET $\{F_k\}$ BE A PRF FAMILY LIKE AES

Gen(1^n): $k \leftarrow U_n$

Sign$_k$(m): $t \leftarrow F_k(m)$

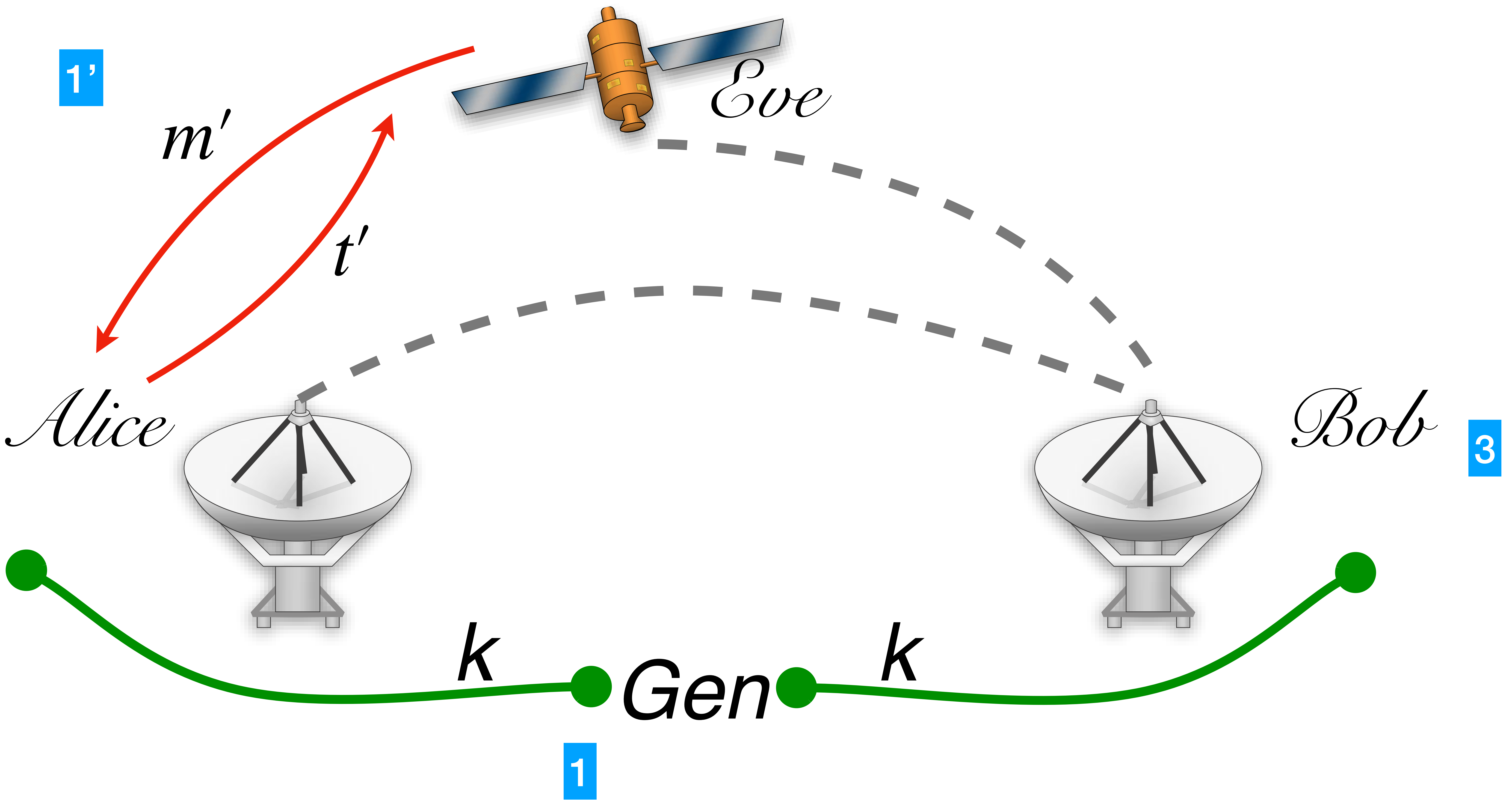
Ver$_k$(m,t): ACCEPT IF $t \stackrel{?}{=} F_k(m)$

Security for a MAC (similar to Signature)



Security for a MAC (similar to Signature)

$t' \leftarrow \mathsf{Sign}_k(m')$



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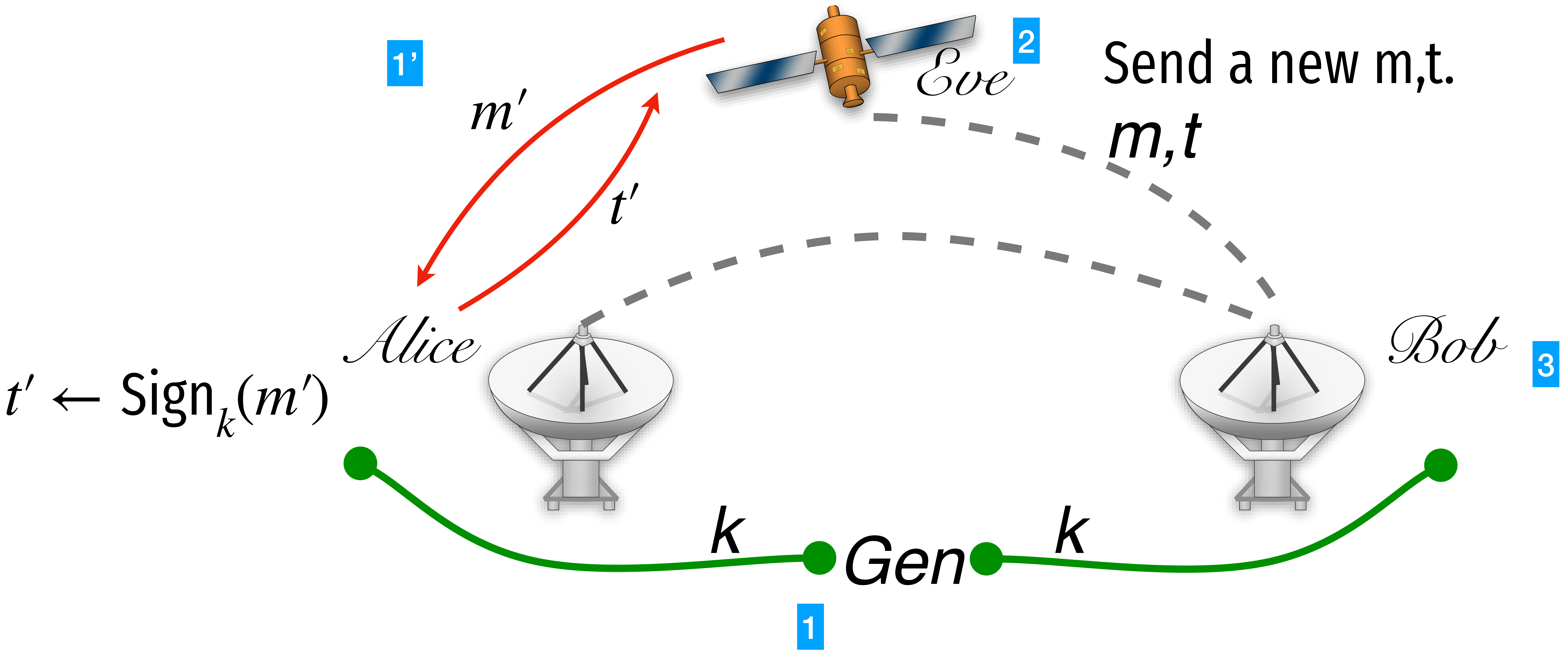

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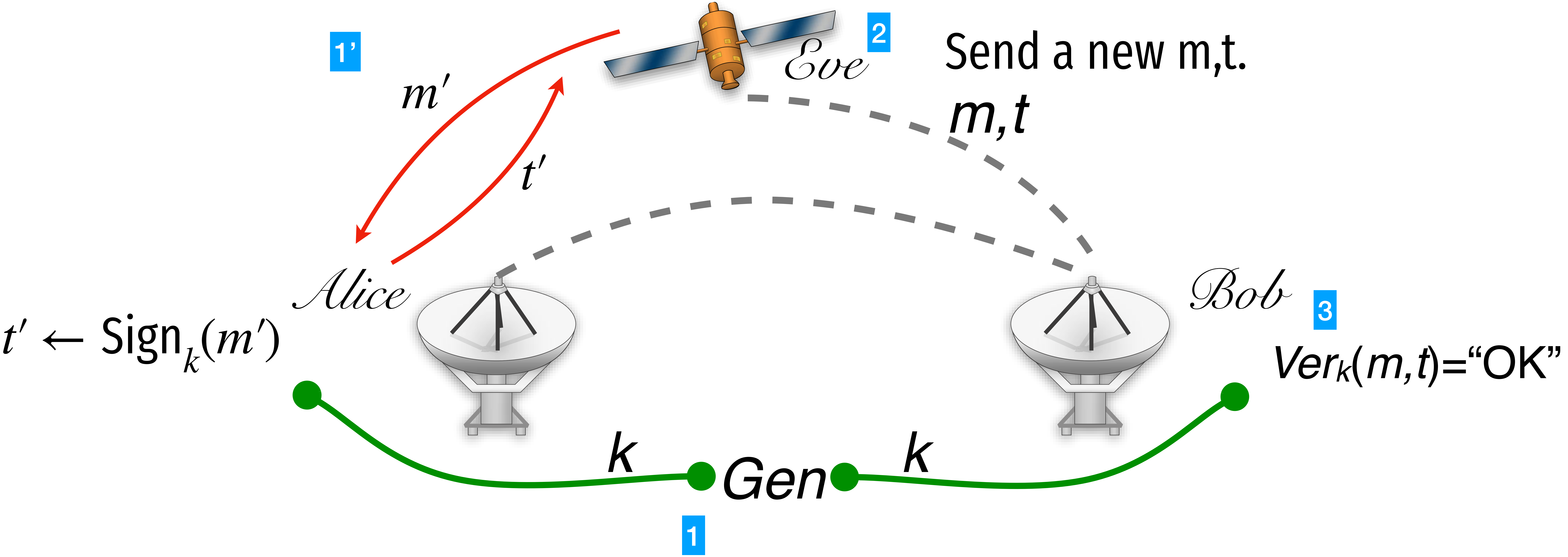
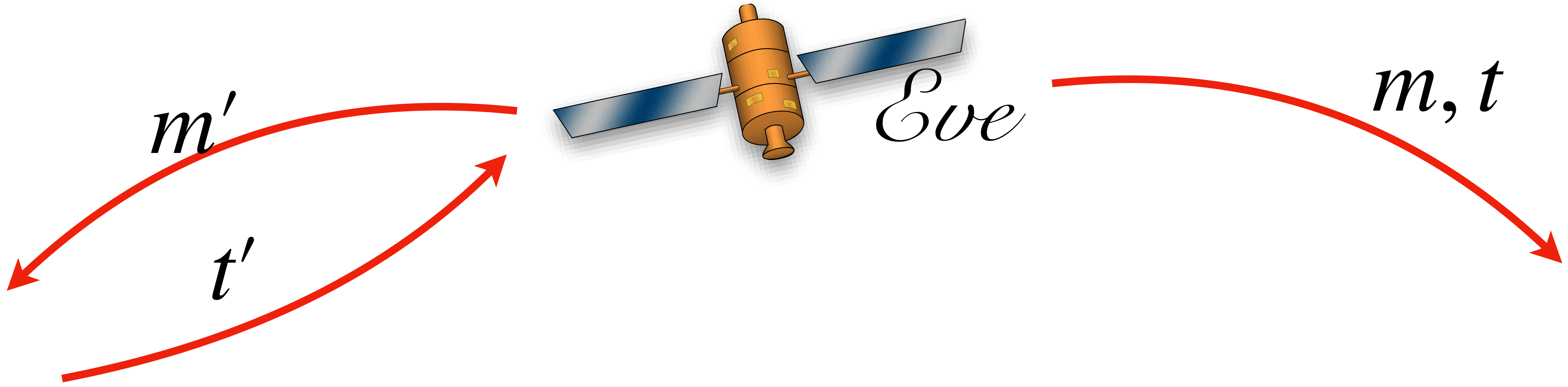


Security intuition



$$\Pr[F_k(m) = t] =$$

Lets do some class exercises with these tools.