

# 2550 Intro to cybersecurity

## L11: Signatures

abhi shelat

# Recap

Very old problem



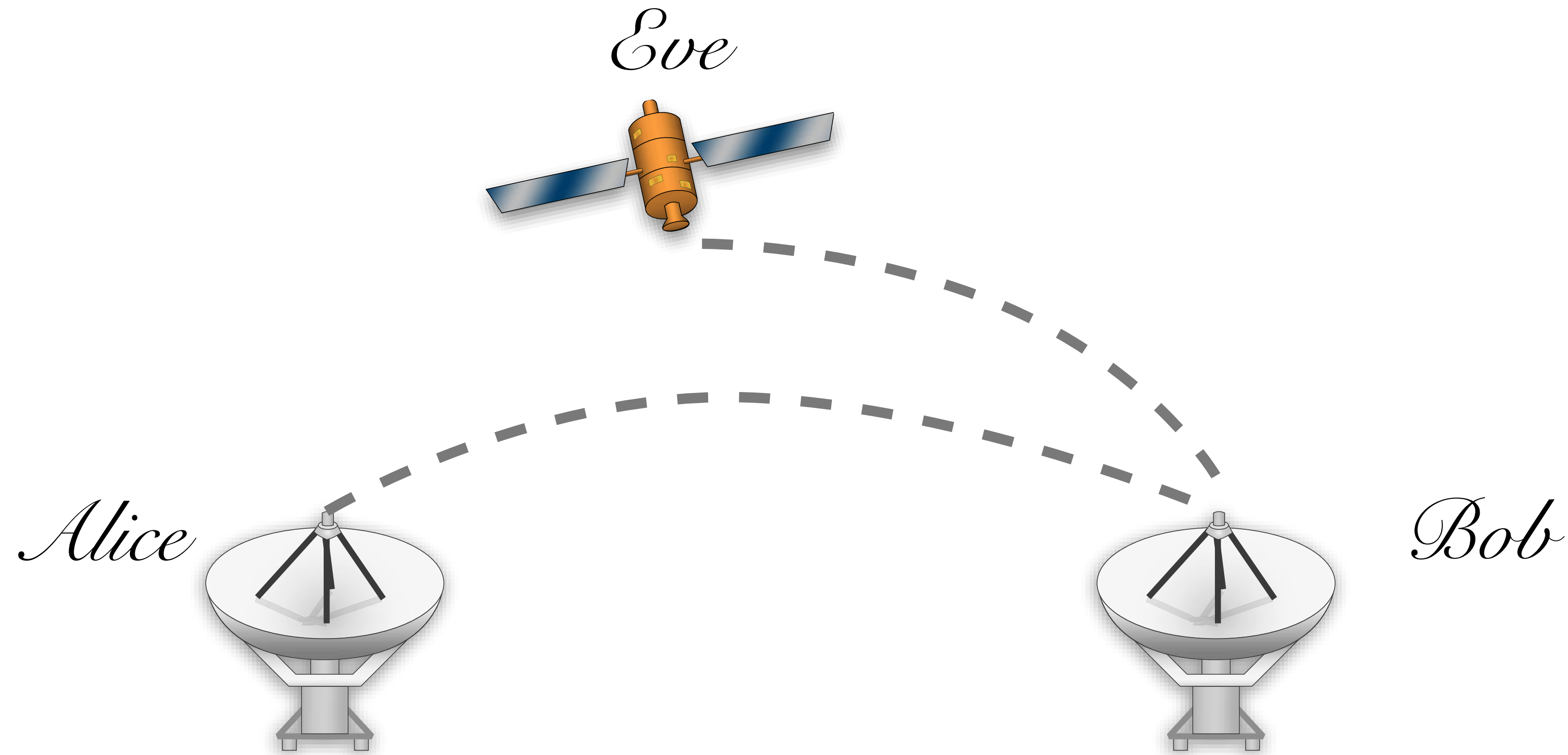
John Hancock



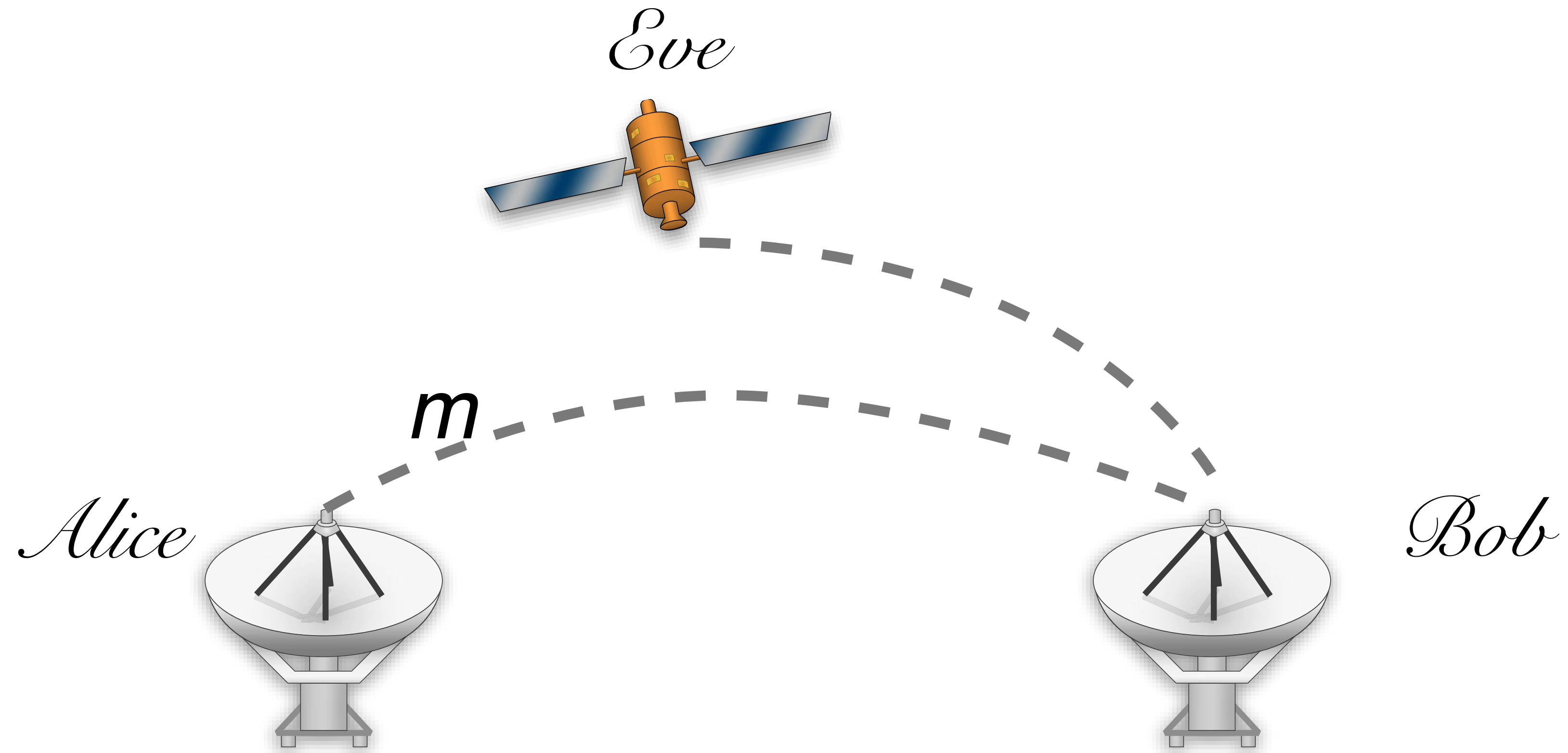




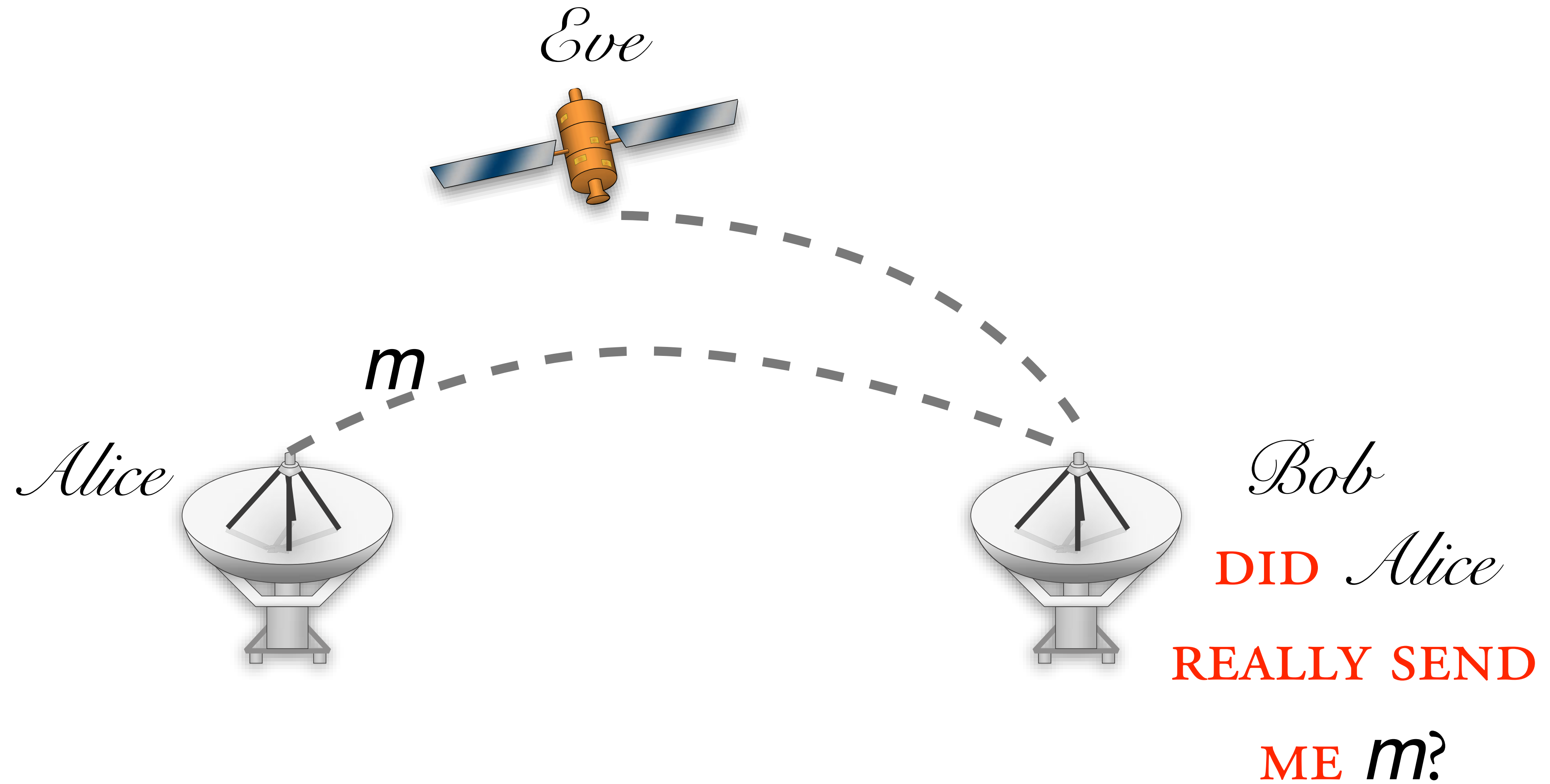
# New Problem



# New Problem

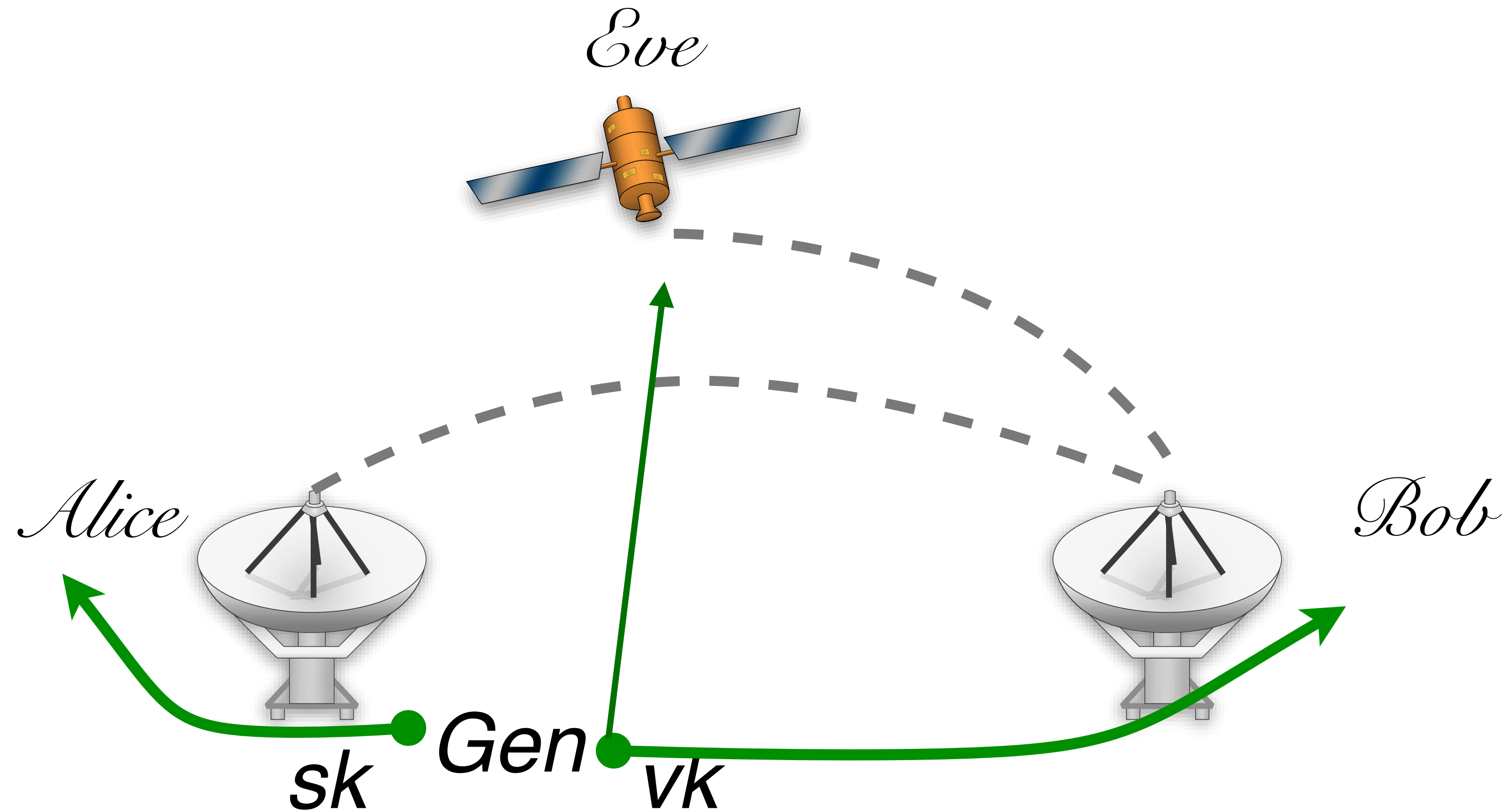


# New Problem

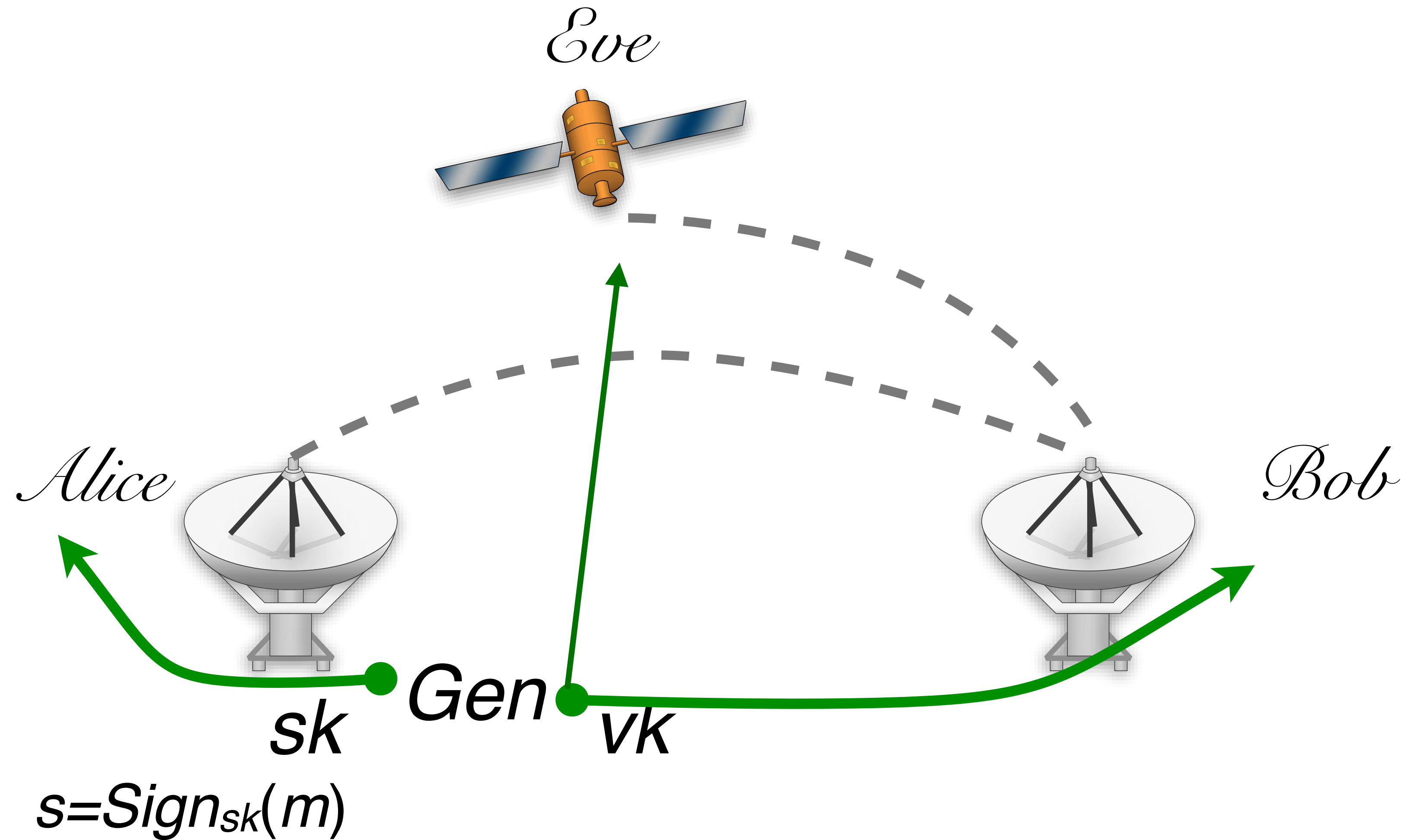




# Public key digital signature

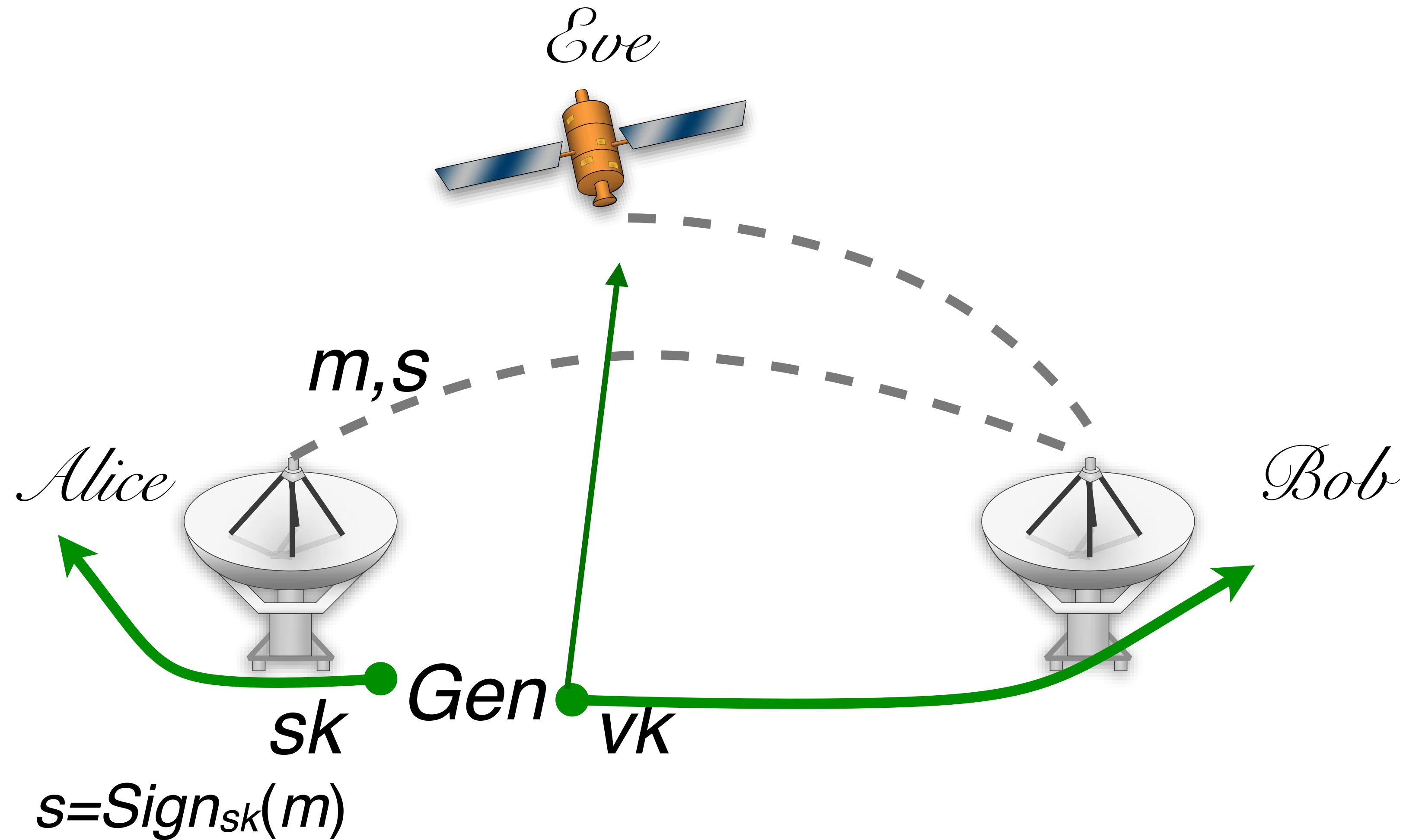


# Public key digital signature

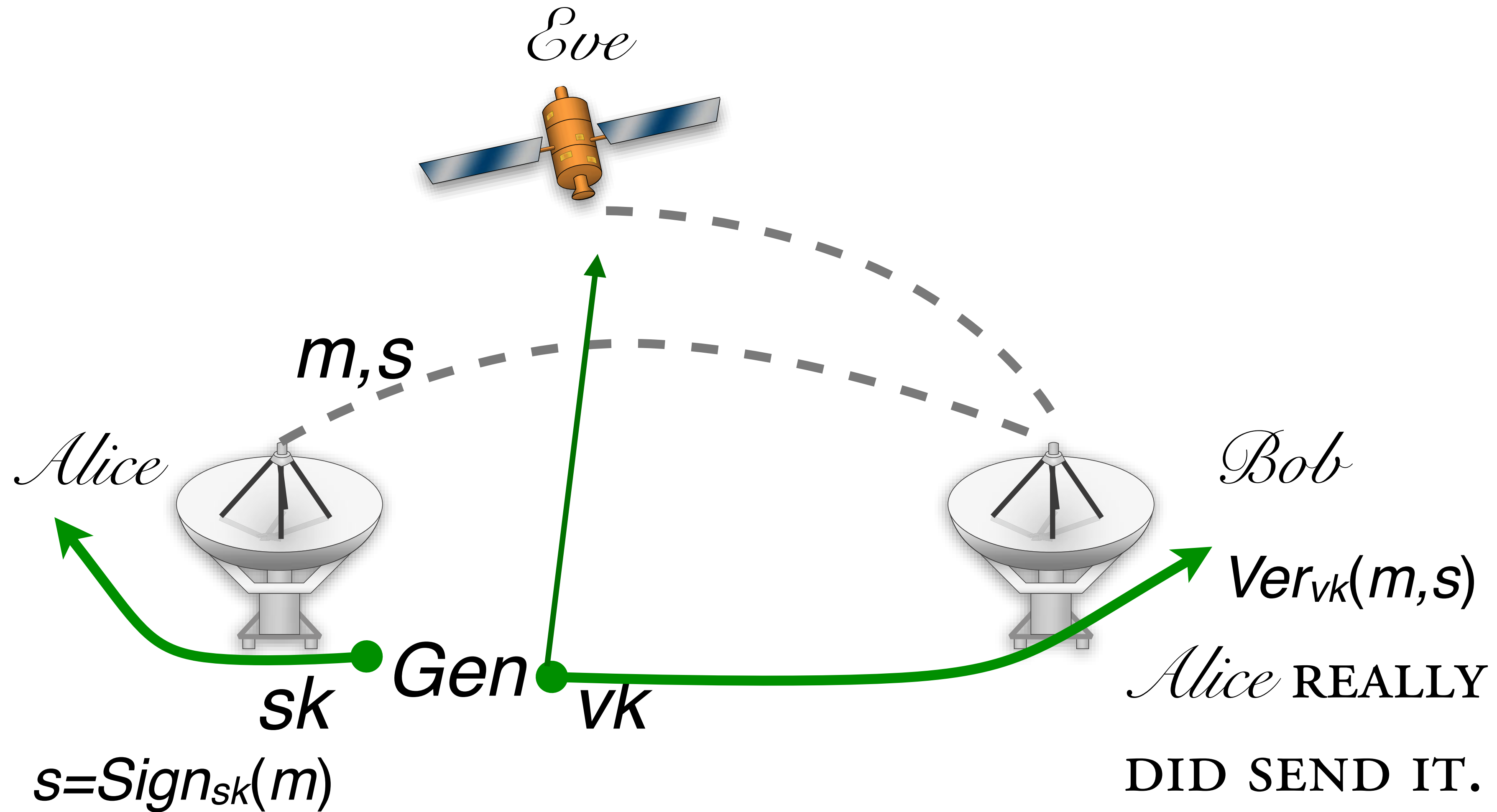




# Public key digital signature



# Public key digital signature





# Public key digital signature

MESSAGE SPACE  $\{\mathcal{M}\}_n$

*Gen*( $1^n$ )

*Sign*<sub>sk</sub>( $m$ )

*Ver*<sub>vk</sub>( $m, s$ )

# Public key digital signature

MESSAGE SPACE  $\{\mathcal{M}\}_n$

*Gen*( $1^n$ ) GENERATES A KEY PAIR  $sk, vk$

*Sign* <sub>$sk$</sub> ( $m$ )

*Ver* <sub>$vk$</sub> ( $m, s$ )



# Public key digital signature

MESSAGE SPACE  $\{\mathcal{M}\}_n$

$Gen(1^n)$  GENERATES A KEY PAIR  $sk, vk$

$Sign_{sk}(m)$  GENERATES A SIGNATURE  $S$  FOR  
 $m \in \mathcal{M}_n$

$Ver_{vk}(m, s)$

# Public key digital signature

MESSAGE SPACE  $\{\mathcal{M}\}_n$

*Gen*( $1^n$ ) GENERATES A KEY PAIR  $sk, vk$

*Sign* <sub>$sk$</sub> ( $m$ ) GENERATES A SIGNATURE  $S$  FOR  
 $m \in \mathcal{M}_n$

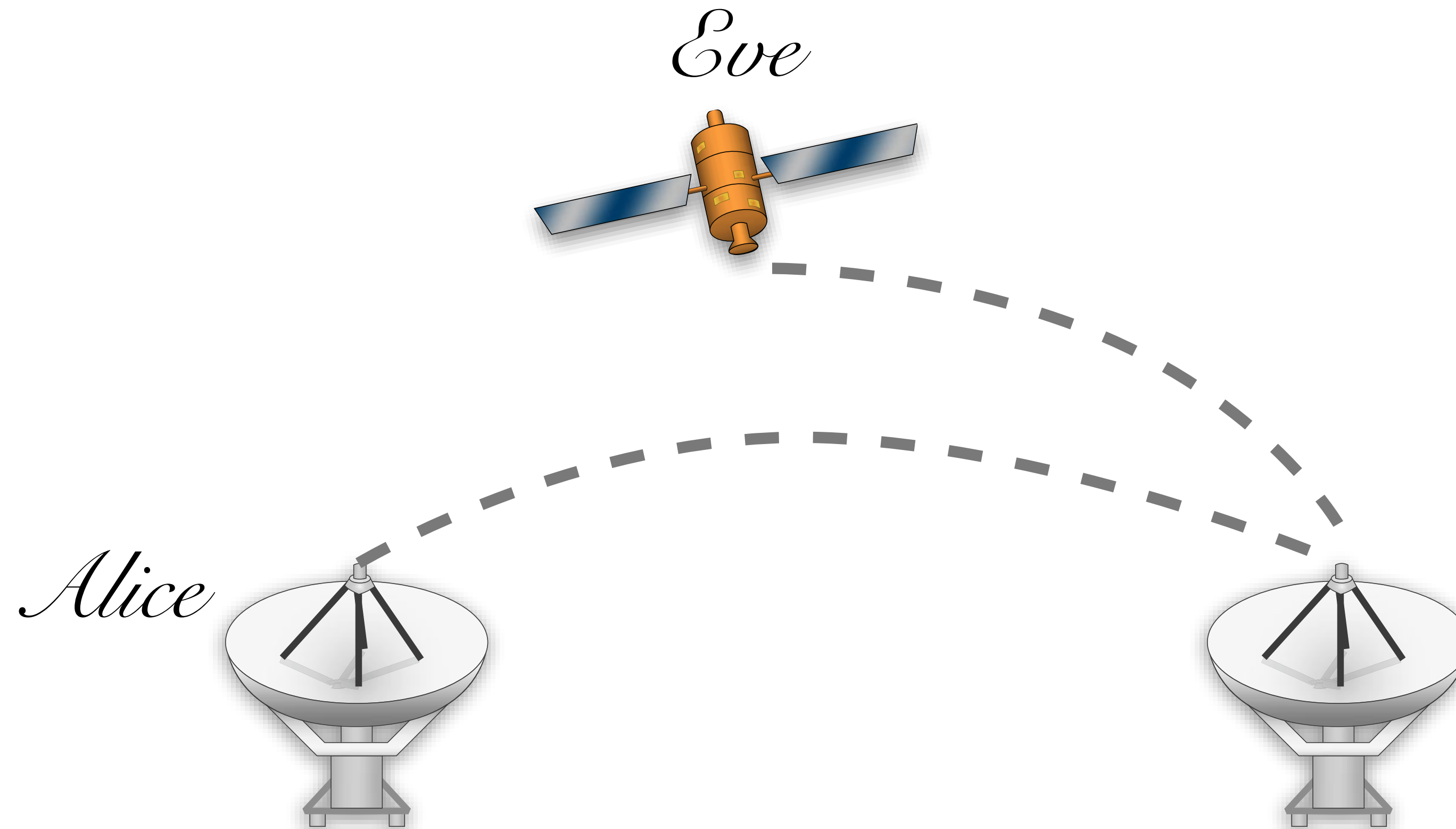
*Ver* <sub>$vk$</sub> ( $m, s$ ) ACCEPTS OR REJECTS A MSG, SIG PAIR

$$\Pr[k \leftarrow \text{Gen}(1^n) : \text{Ver}_{vk}(m, \text{Sign}_{sk}(m)) = 1] = 1$$



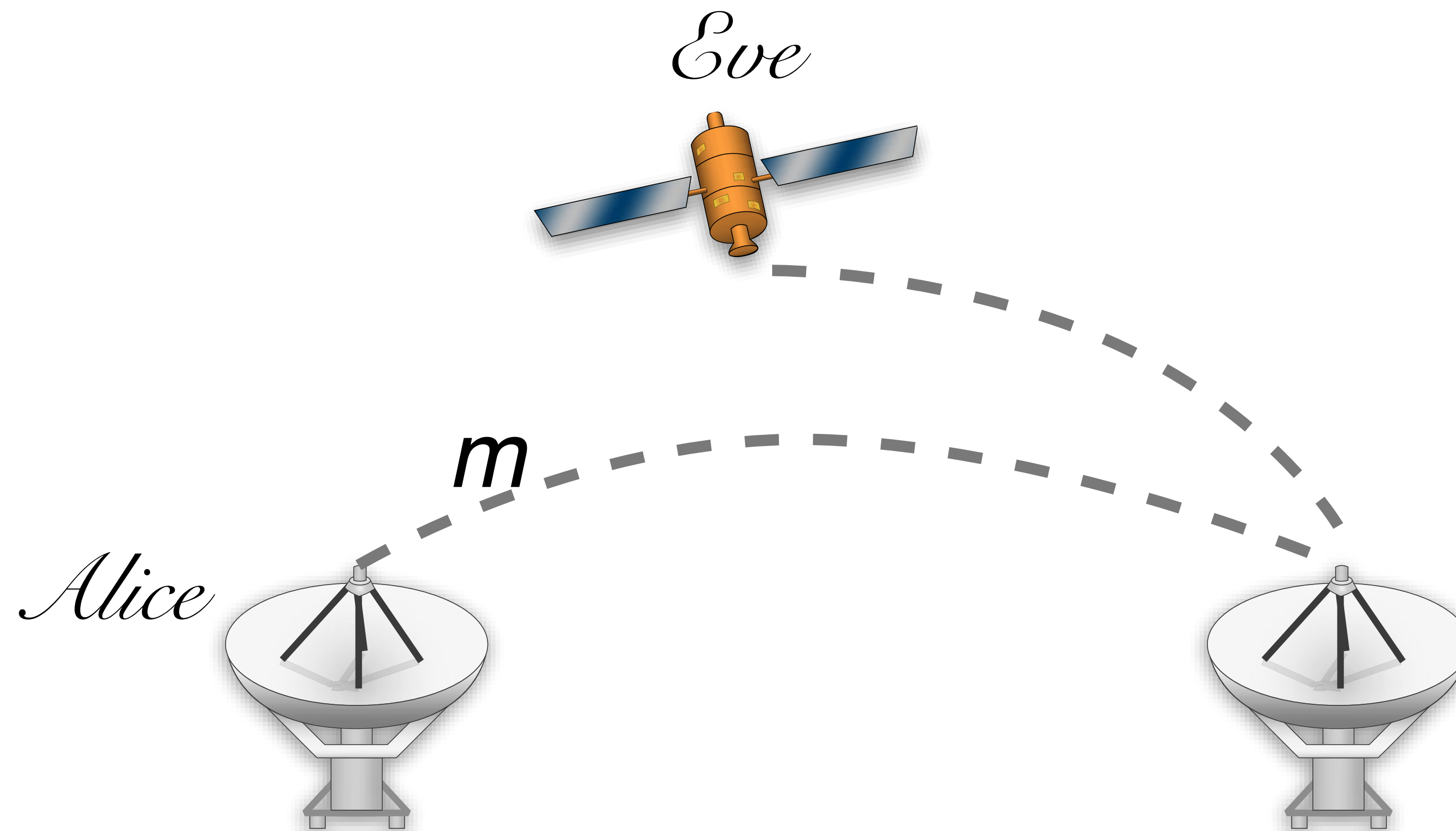
# existential unforgeability

“EVEN WHEN GIVEN A SIGNING ORACLE,  
AN ADVERSARY CANNOT FORGE A SIGNATURE FOR  
ANY MESSAGE OF ITS CHOOSING”

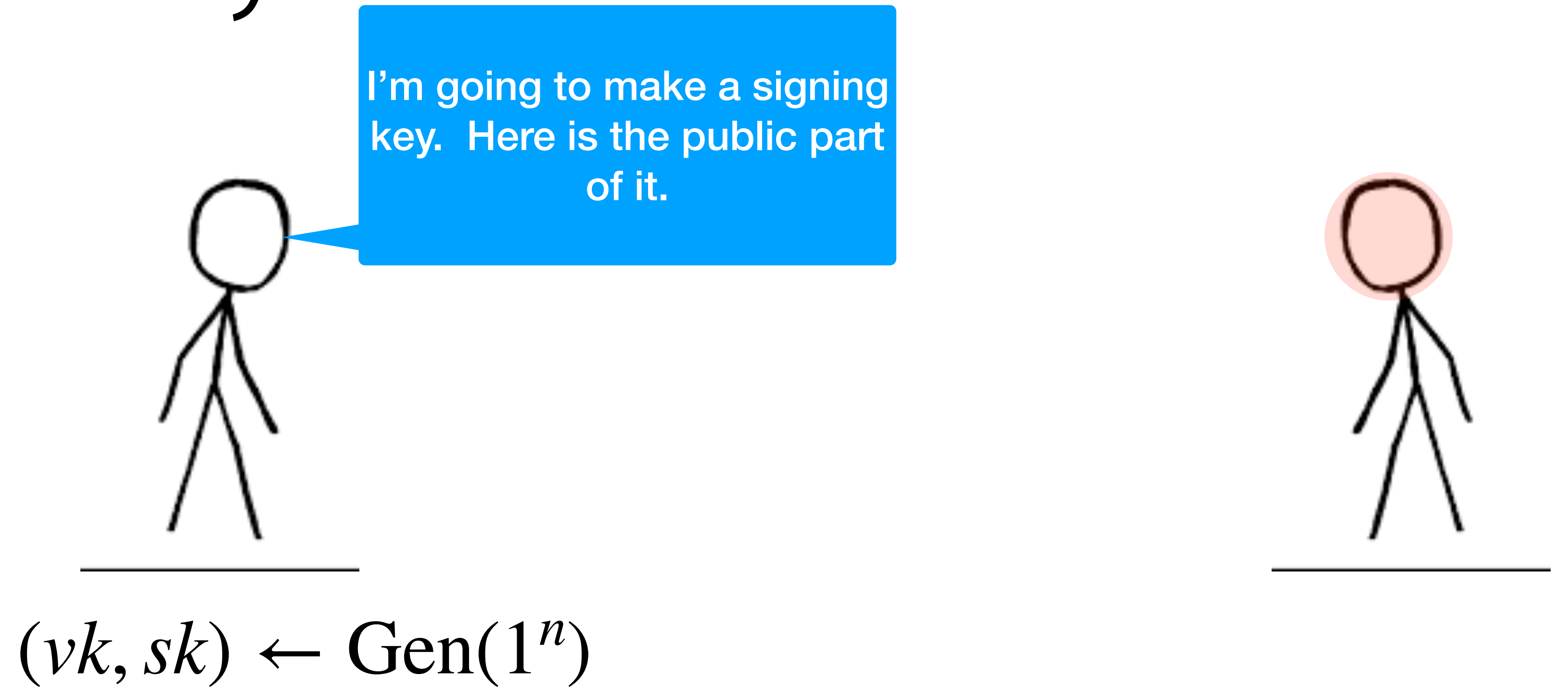


# existential unforgeability

“EVEN WHEN GIVEN A SIGNING ORACLE,  
AN ADVERSARY CANNOT FORGE A SIGNATURE FOR  
ANY MESSAGE OF ITS CHOOSING”



# Signature security





# Signature security



$(vk, sk) \leftarrow \text{Gen}(1^n)$

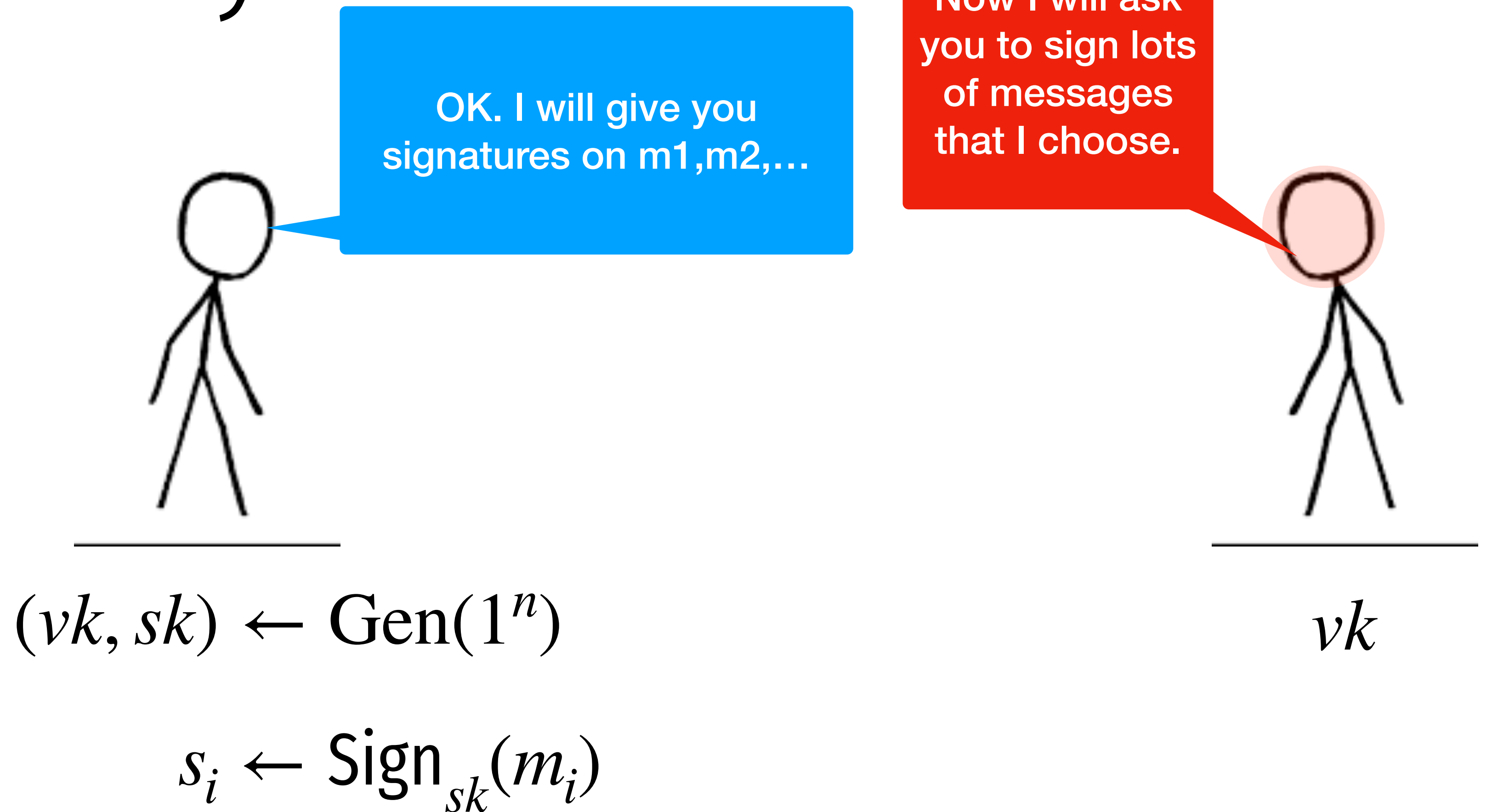
Now I will ask you to sign lots of messages that I choose.

$m_0, m_1, \dots$



$vk$

# Signature security



# Signature security



$$(vk, sk) \leftarrow \text{Gen}(1^n)$$

$$s_i \leftarrow \text{Sign}_{sk}(m_i)$$

Now I will try to create a new (signature, message) pair...one that I didn't receive from you. signature on a new message



$vk$

$s_1, s_2, \dots$



# Signature security

If you do, you  
have won the  
game!

Now I will try to create a  
new  $(msg^*, sig^*)$  pair...one  
that I didn't receive from  
you.



$$Ver_{vk}(m^*, s^*) \stackrel{?}{=} 1$$



FOR ALL NON-UNIFORM PPT  $A$

$$\Pr \left[ \begin{array}{l} (vk, sk) \leftarrow Gen(1^n); (m, s) \leftarrow A^{Sign_{sk}(\cdot)} : \\ Ver_{vk}(m, s) = 1 \\ \text{AND } A \text{ DIDN'T QUERY } m \end{array} \right] < \mu(n)$$

# Textbook RSA Signatures (insecure)

Pick  $N = p \cdot q$  where  $p, q$  are primes.

Pick  $e, d$  such that  $e \cdot d = 1 \pmod{\phi(N)}$

Sign( $(sk=d, N)$   $m$ ):

Compute the signature:  $\sigma \leftarrow m^d \pmod{N}$

Verify( $(pk=e, N)$ ,  $\sigma$ ,  $m$ ):

$$m \stackrel{?}{=} \sigma^e \pmod{N}$$

# RSA Signatures (PKCSv1.5)

Sign((sk, N) m):

Compute the padding:  $z \leftarrow 00 \cdot 01 \cdot FF \dots FF \cdot 00 \cdot ID_H \cdot H(m)$

Compute the signature:  $\sigma \leftarrow z^{sk} \bmod N$



# Speed

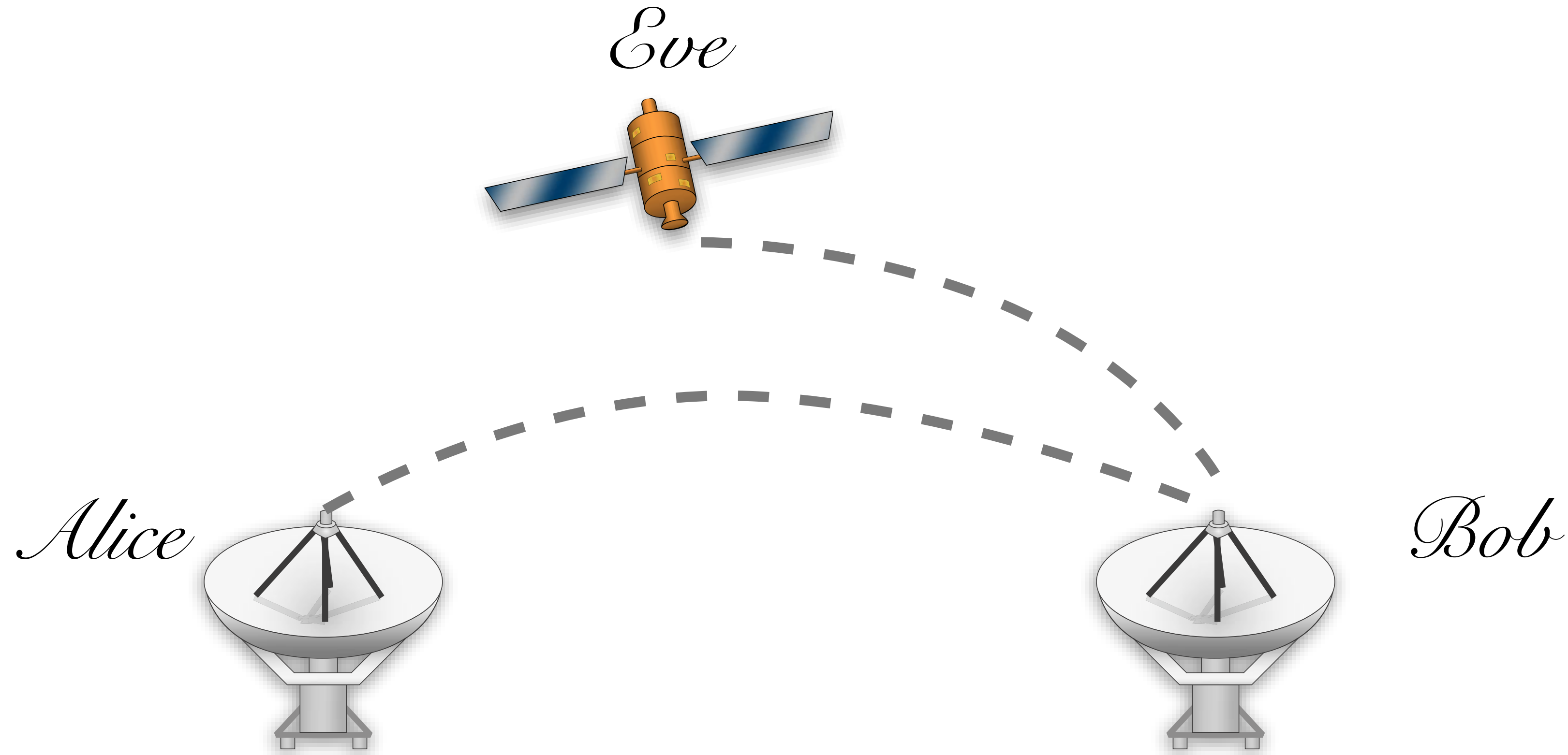
## openssl speed rsa dsa ecdsa

```
Doing 1024 bits private rsa's for 10s: 86688 1024 bits private RSA's in 9.99s
Doing 1024 bits public rsa's for 10s: 1341152 1024 bits public RSA's in 10.00s
Doing 2048 bits private rsa's for 10s: 13154 2048 bits private RSA's in 9.99s
Doing 2048 bits public rsa's for 10s: 437080 2048 bits public RSA's in 10.00s
Doing 3072 bits private rsa's for 10s: 4243 3072 bits private RSA's in 10.00s
Doing 3072 bits public rsa's for 10s: 211605 3072 bits public RSA's in 10.00s
Doing 4096 bits private rsa's for 10s: 1845 4096 bits private RSA's in 9.99s
Doing 4096 bits public rsa's for 10s: 125130 4096 bits public RSA's in 9.99s
```

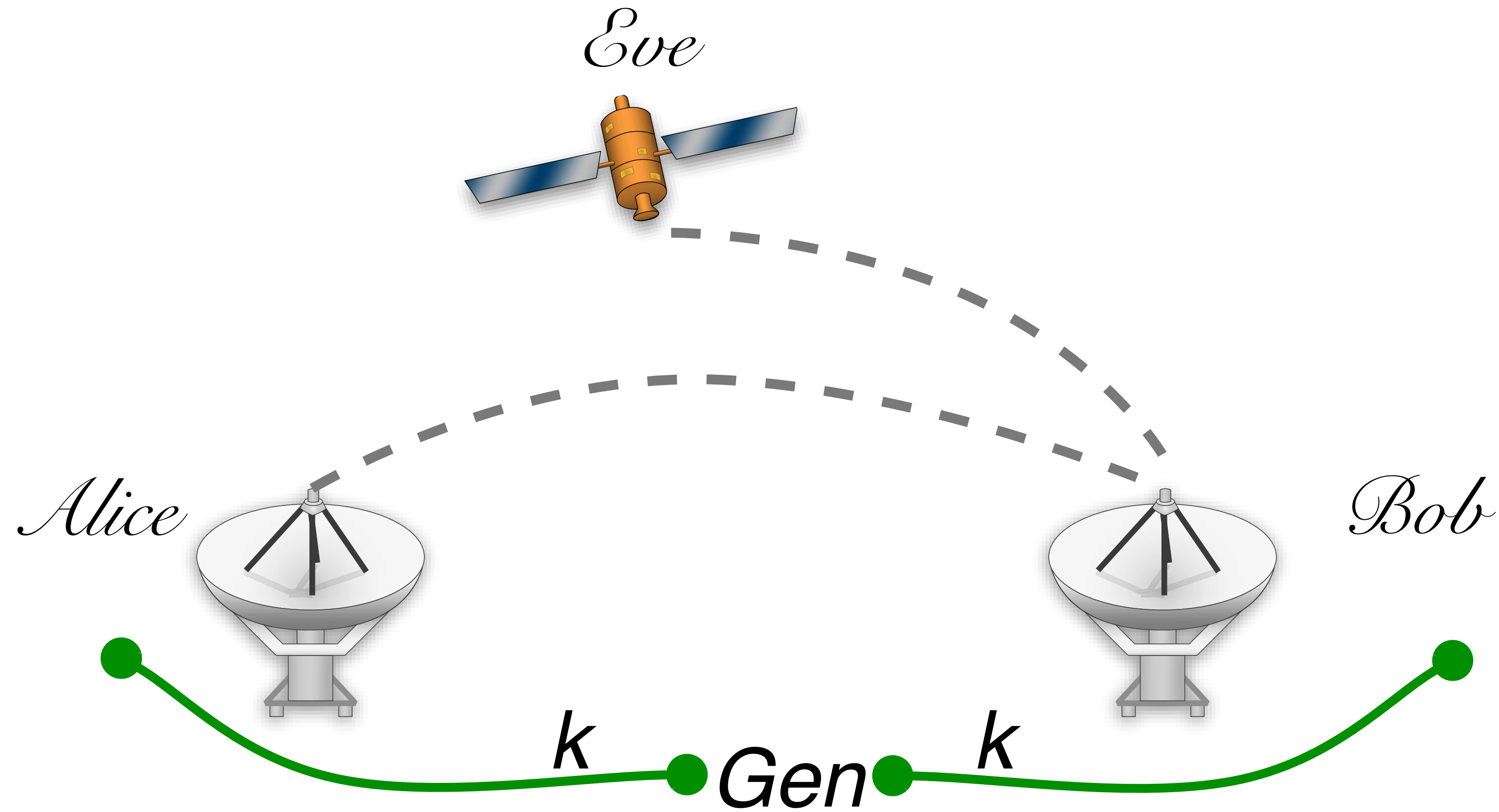
```
Doing 1024 bits sign dsa's for 10s: 74467 1024 bits DSA signs in 9.95s
Doing 1024 bits verify dsa's for 10s: 95863 1024 bits DSA verify in 9.99s
Doing 2048 bits sign dsa's for 10s: 30197 2048 bits DSA signs in 9.97s
Doing 2048 bits verify dsa's for 10s: 33802 2048 bits DSA verify in 10.00s
```

```
Doing 256 bits sign ecDSA's for 10s: 339010 256 bits ECDSA signs in 9.89s
Doing 256 bits verify ecDSA's for 10s: 115106 256 bits ECDSA verify in 10.00s
Doing 384 bits sign ecDSA's for 10s: 7773 384 bits ECDSA signs in 9.98s
Doing 384 bits verify ecDSA's for 10s: 10066 384 bits ECDSA verify in 10.00s
Doing 521 bits sign ecDSA's for 10s: 25316 521 bits ECDSA signs in 9.98s
Doing 521 bits verify ecDSA's for 10s: 12896 521 bits ECDSA verify in 9.99s
Doing 283 bits sign ecDSA's for 10s: 13860 283 bits ECDSA signs in 9.98s
Doing 283 bits verify ecDSA's for 10s: 7028 283 bits ECDSA verify in 9.99s
Doing 409 bits sign ecDSA's for 10s: 8441 409 bits ECDSA signs in 9.99s
Doing 409 bits verify ecDSA's for 10s: 4309 409 bits ECDSA verify in 9.98s
```

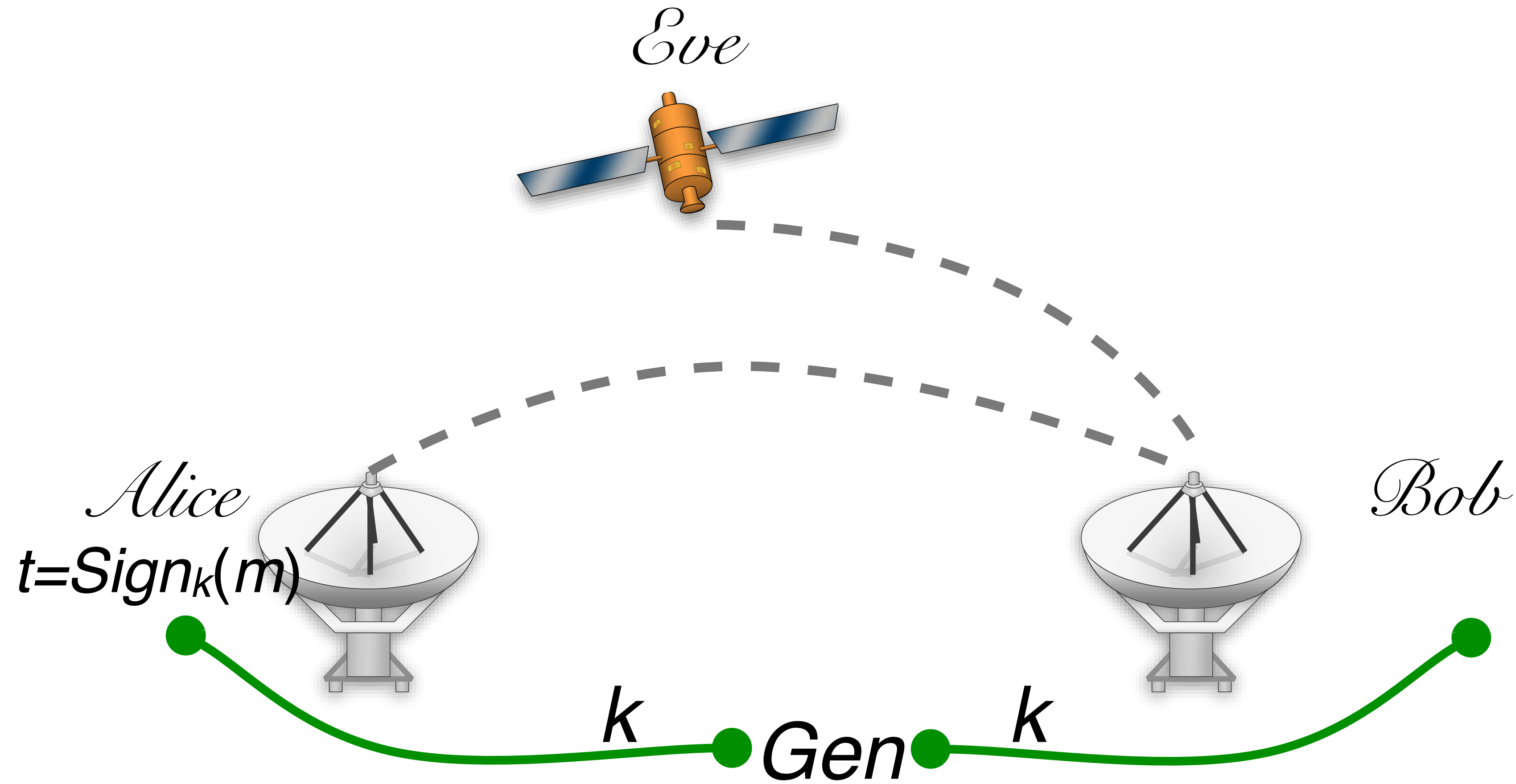
# Message Authentication codes



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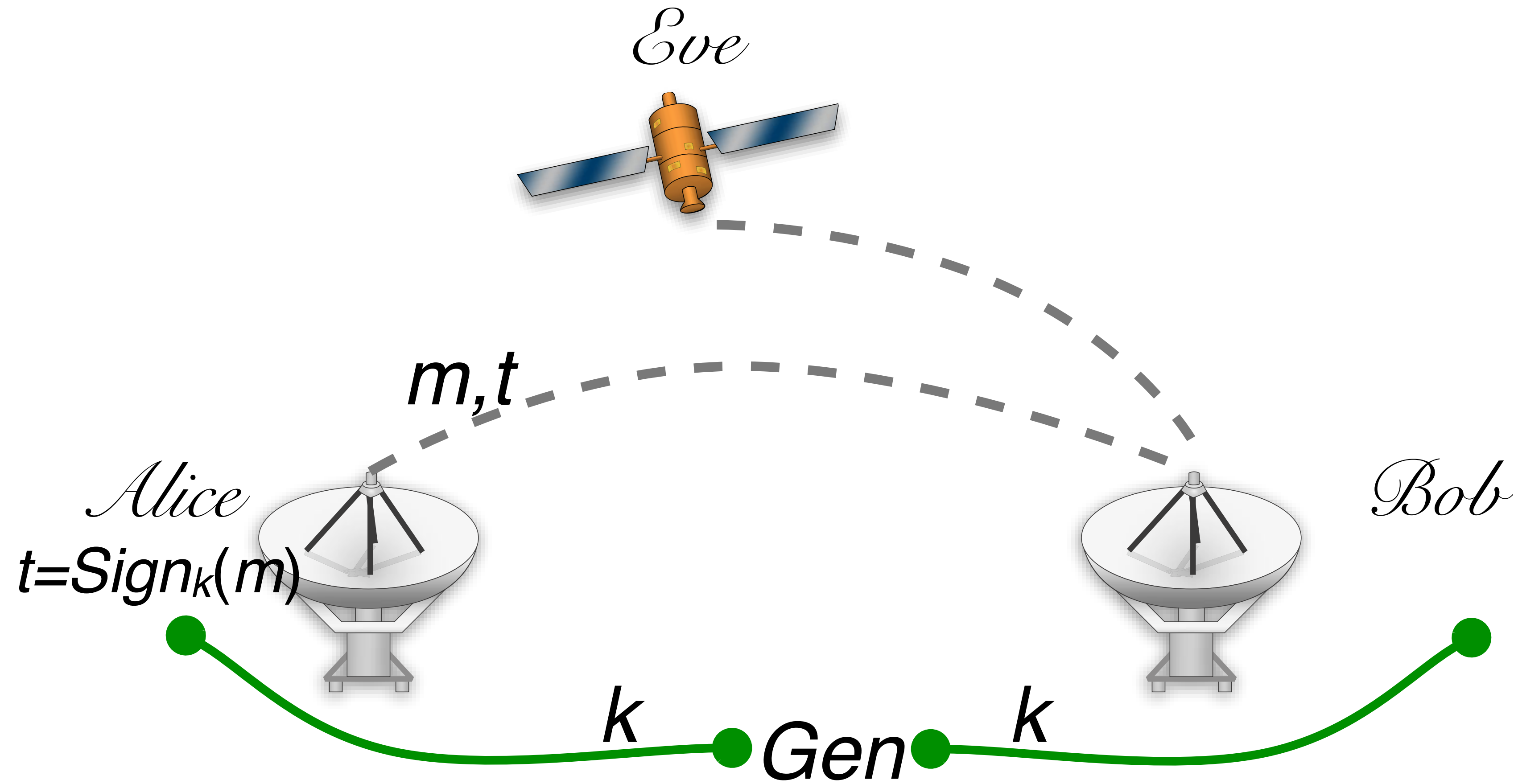


# Message Authentication codes

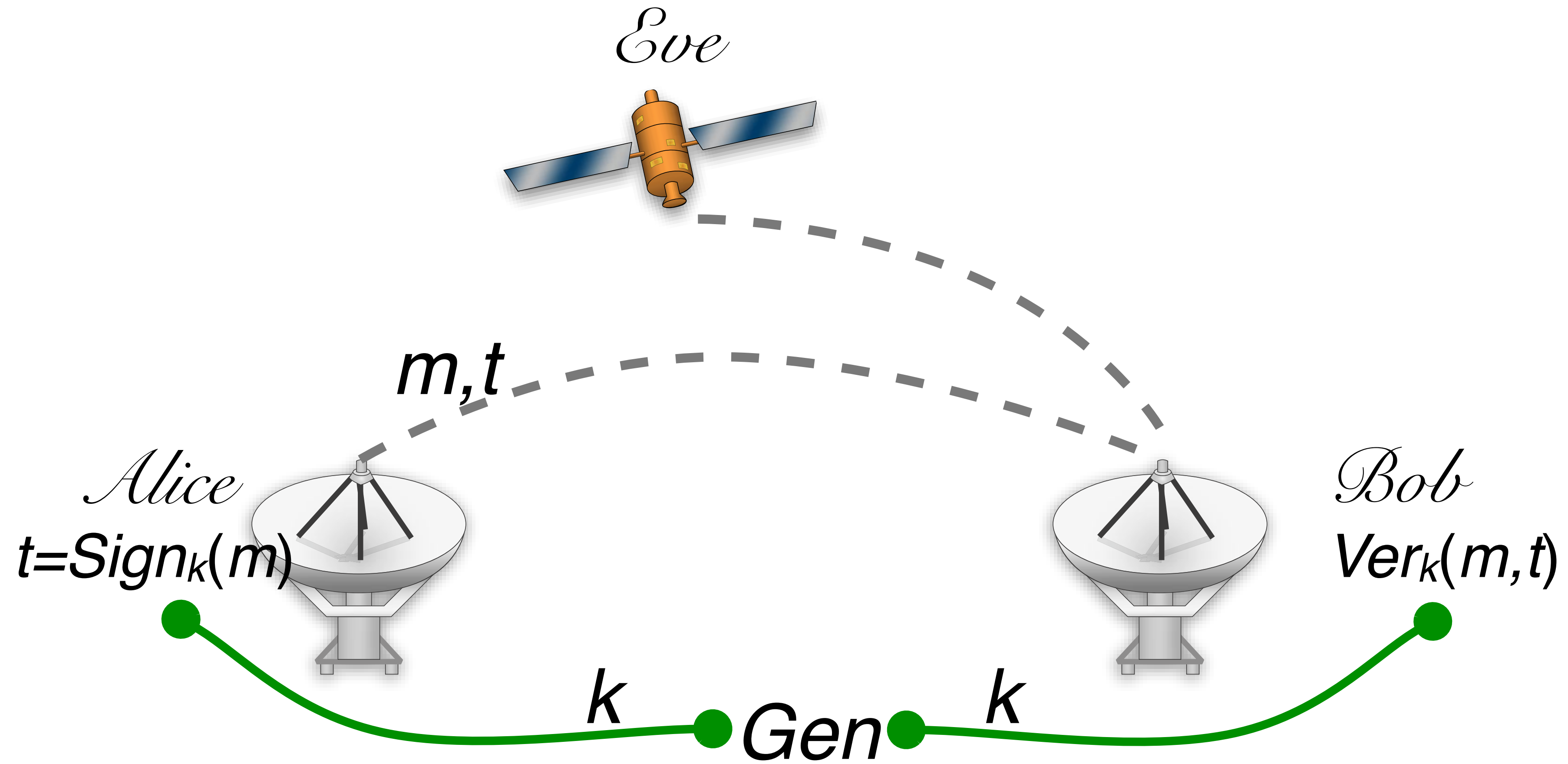




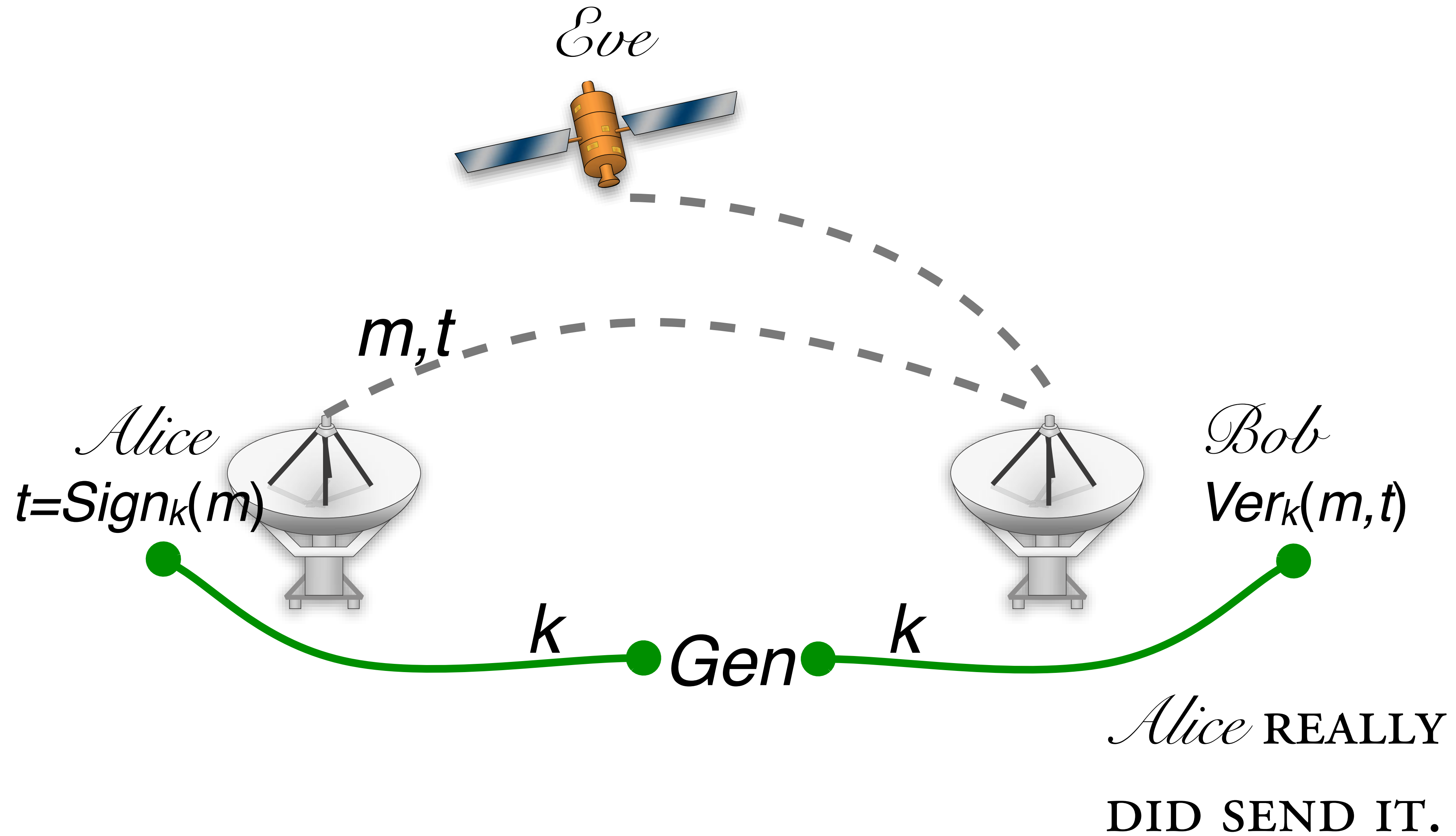
# Message Authentication codes



# Message Authentication codes



# Message Authentication codes



# Construction of a MAC

*Gen*( $1^n$ ):

*Sign* <sub>$k$</sub> ( $m$ ):

*Ver* <sub>$k$</sub> ( $m, t$ ):



# Construction of a MAC

LET  $\{F_k\}$  BE A PRF FAMILY LIKE AES

*Gen*( $1^n$ ):

*Sign* <sub>$k$</sub> ( $m$ ):

*Ver* <sub>$k$</sub> ( $m, t$ ):

# Construction of a MAC

LET  $\{F_k\}$  BE A PRF FAMILY LIKE AES

*Gen*( $1^n$ ):  $k \leftarrow U_n$

*Sign* <sub>$k$</sub> ( $m$ ):

*Ver* <sub>$k$</sub> ( $m, t$ ):

# Construction of a MAC

LET  $\{F_k\}$  BE A PRF FAMILY LIKE AES

*Gen*( $1^n$ ):  $k \leftarrow U_n$

*Sign* <sub>$k$</sub> ( $m$ ):  $t \leftarrow F_k(m)$

*Ver* <sub>$k$</sub> ( $m, t$ ):

# Construction of a MAC

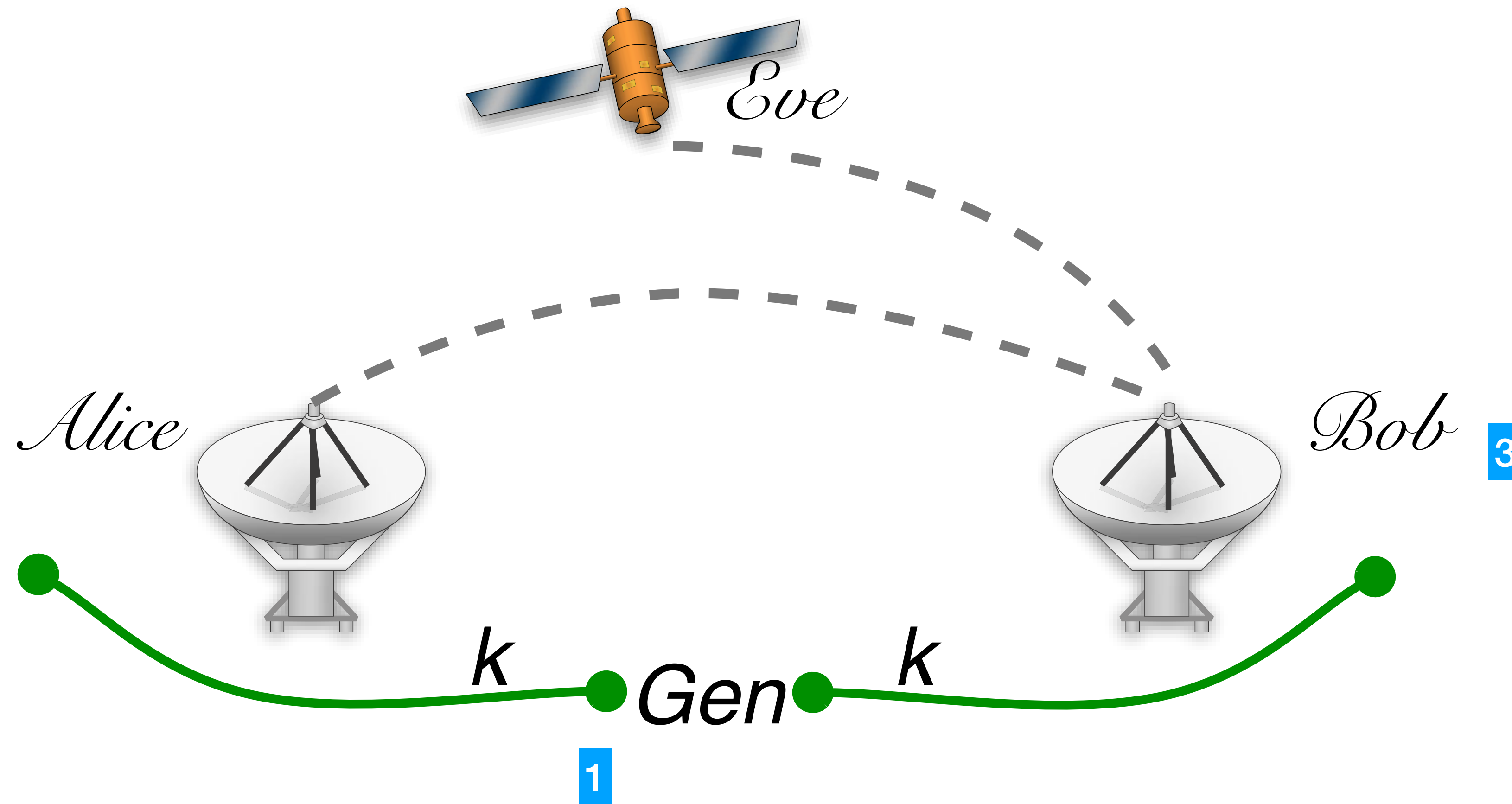
LET  $\{F_k\}$  BE A PRF FAMILY LIKE AES

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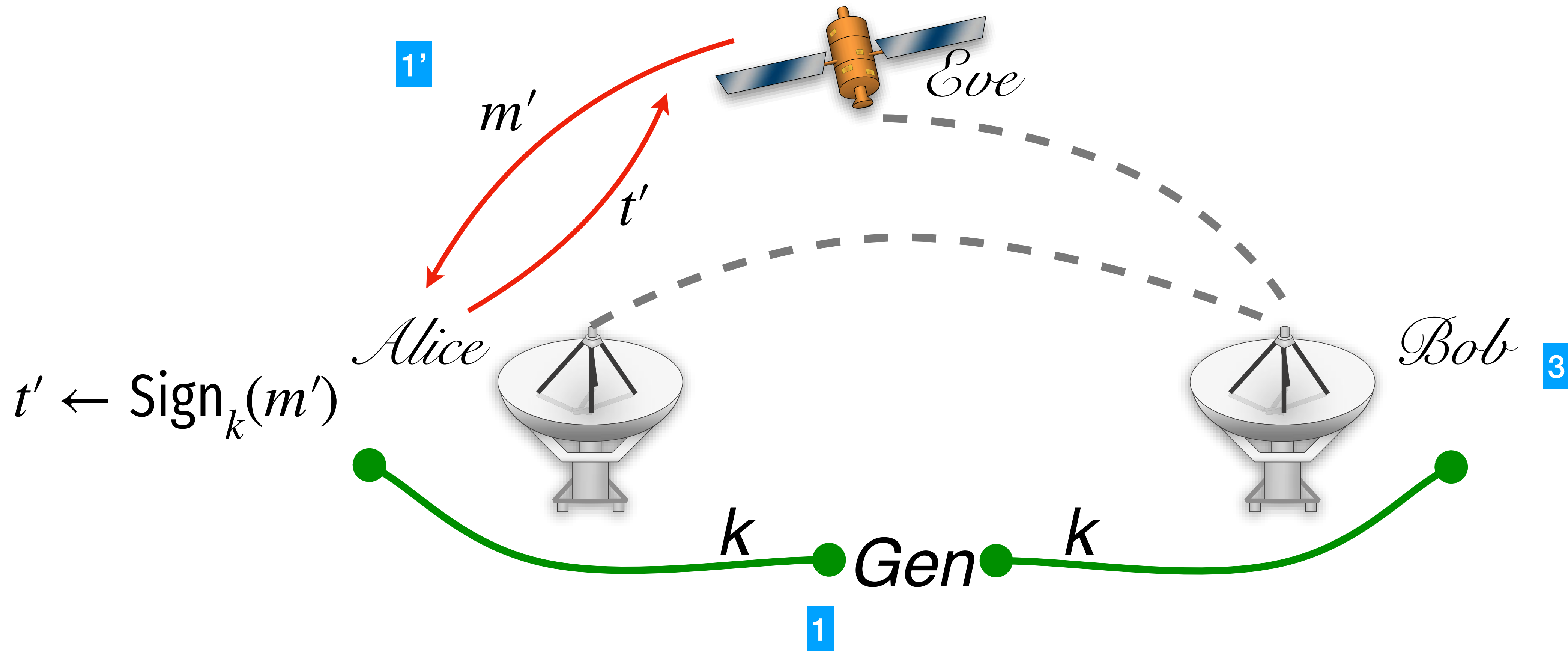
*Ver* <sub>$k$</sub> ( $m, t$ ): ACCEPT IF  $t \stackrel{?}{=} F_k(m)$

# Security for a MAC (similar to Signature)

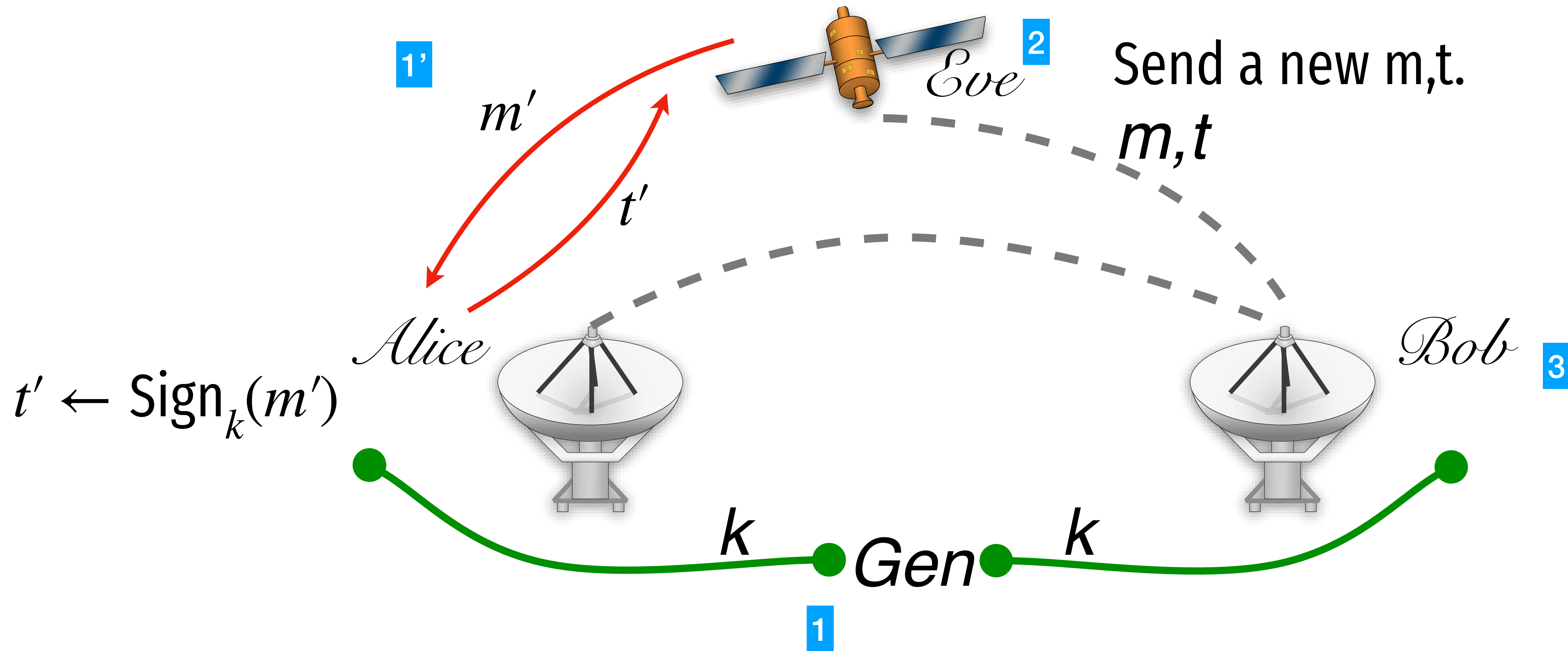




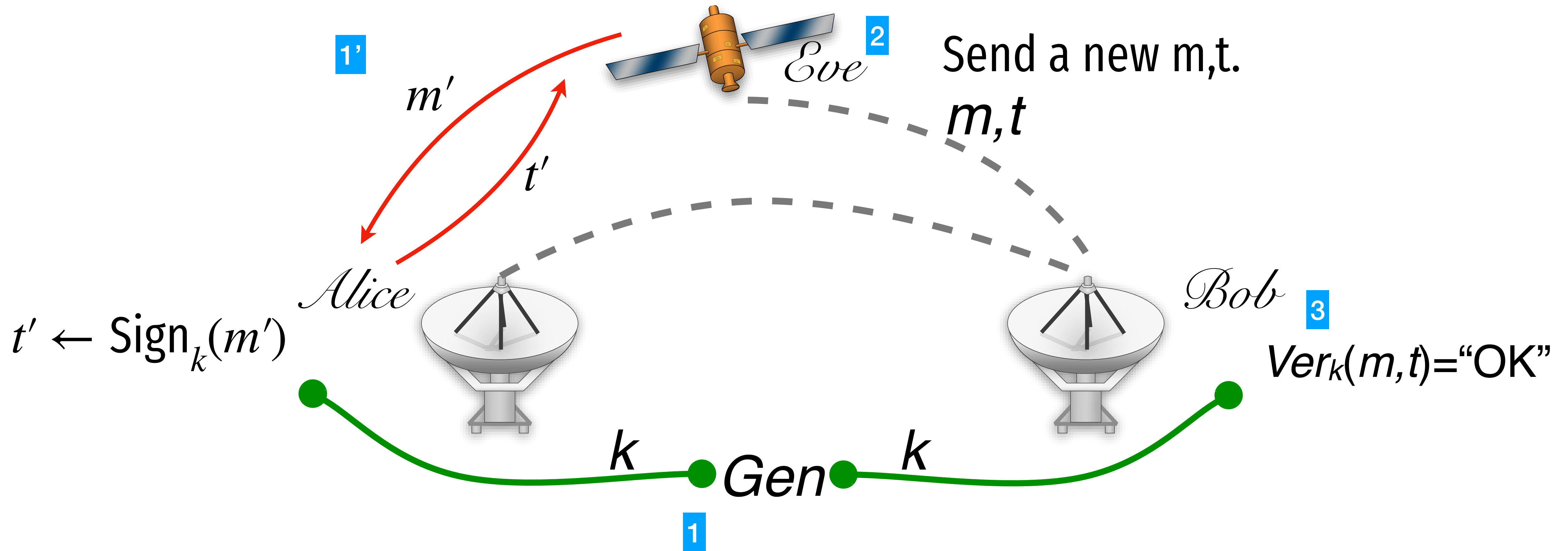
# Security for a MAC (similar to Signature)



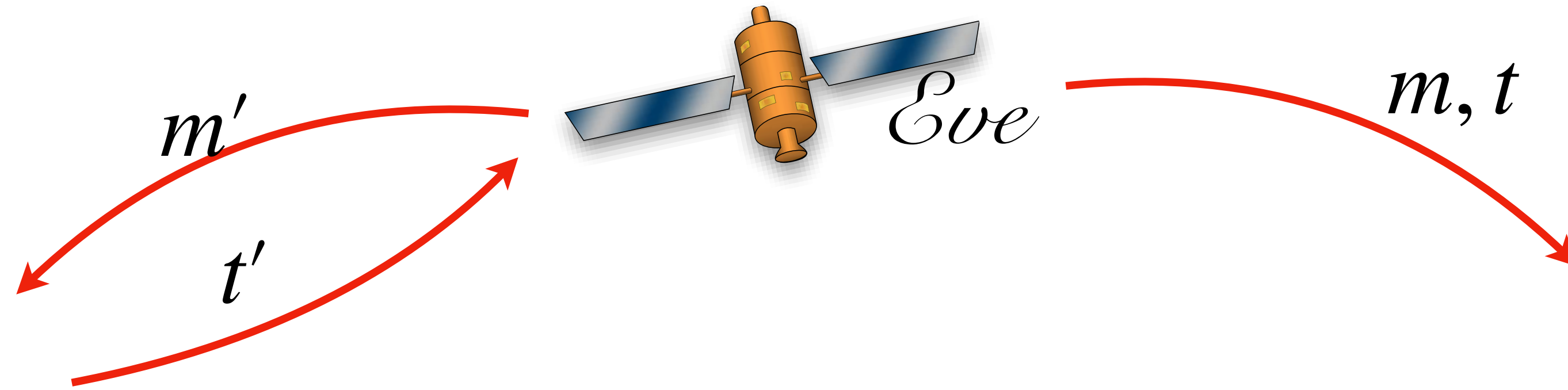
# Security for a MAC (similar to Signature)



# Security for a MAC (similar to Signature)



# Security intuition



$$\Pr[F_k(m) = t] =$$

Lets do some class exercises in Q1.